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CS 161
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Homework 5

1 Problem 1

Consider the following sentences and decide for each whether it is valid, unsatisfiable, or neither:

(a) $Smoke \Rightarrow Smoke$
(b) $Smoke \Rightarrow Fire$
(c) $Smoke \lor Fire \lor \neg Fire$
(d) $(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
(e) $(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \lor Heat) \Rightarrow Fire)$
(f) $((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$

Justify your answer using truth tables (worlds).

Solution:

(a) Valid, since the sentence is true in all models.

Table 1: Truth Table for **(a)**

Smoke	Fire	$Smoke \Rightarrow Smoke$
False	False	True
False	True	True
True	False	True
True	True	True

(b) Neither, since the sentence is satisfiable and not valid.

Table 2: Truth Table for **(b)**

Smoke	Fire	$Smoke \Rightarrow Fire$
False	False	True
False	True	True
True	False	False
True	True	True

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(c) Valid, since the sentence is true in all models.

Table 3: Truth Table for **(c)**

Smoke	Fire	$Smoke \lor Fire \lor \neg Fire$
False	False	True
False	True	True
True	False	True
True	True	True

(d) Neither, since the sentence is satisfiable and not valid.

Table 4: Truth Table for **(d)**

Smoke	Fire	$(Smoke \Rightarrow Fire) \Rightarrow (\neg Smoke \Rightarrow \neg Fire)$
False	False	True
False	True	False
True	False	True
True	True	True

Smoke	Fire	$Smoke \Rightarrow Fire$	$\neg Smoke \Rightarrow \neg Fire$
False	False	True	True
False	True	True	False
True	False	False	True
True	True	True	True

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(e) Neither, since the sentence is satisfiable and not valid.

Table 5: Truth Table for **(e)**

Smoke	Fire	Heat	$(Smoke \Rightarrow Fire) \Rightarrow ((Smoke \vee Heat) \Rightarrow Fire)$
False	False	False	True
False	False	True	False
False	True	False	True
False	True	True	True
True	False	False	True
True	False	True	True
True	True	False	True
True	True	True	True

Smoke	Fire	Heat	$Smoke \Rightarrow Fire$	$Smoke \vee Heat$	$(Smoke \vee Heat) \Rightarrow Fire$
False	False	False	True	False	True
False	False	True	True	True	False
False	True	False	True	False	True
False	True	True	True	True	True
True	False	False	False	True	False
True	False	True	False	True	False
True	True	False	True	True	True
True	True	True	True	True	True

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(f) Valid, since the sentence is true in all models.

Table 6: Truth Table for **(f)**

Smoke	Fire	Heat	$((Smoke \land Heat) \Rightarrow Fire) \Leftrightarrow ((Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire))$
False	False	False	True
False	False	True	True
False	True	False	True
False	True	True	True
True	False	False	True
True	False	True	True
True	True	False	True
True	True	True	True

Smoke	Fire	Heat	$Smoke \land Heat$	$(Smoke \land Heat) \Rightarrow Fire$
False	False	False	False	True
False	False	True	False	True
False	True	False	False	True
False	True	True	False	True
True	False	False	False	True
True	False	True	True	False
True	True	False	False	True
True	True	True	True	True

Smoke	Fire	Heat	$Smoke \Rightarrow Fire$	$Heat \Rightarrow Fire$	$(Smoke \Rightarrow Fire) \lor (Heat \Rightarrow Fire)$
False	False	False	True	True	True
False	False	True	True	False	True
False	True	False	True	True	True
False	True	True	True	True	True
True	False	False	False	True	True
True	False	True	False	False	False
True	True	False	True	True	True
True	True	True	True	True	True

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2 Problem 2

For each pair of atomic sentences, give the most general unifier if it exists:

(a) P(A, B, B), P(x, y, z).
(b) Q(y, G(A, B)), Q(G(x, x), y).
(c) Older(Father(y), y), Older(Father(x), John).
(d) Knows(Father(y), y), Knows(x, x).

Solution:

(a) MGU = {x/A, y/B, z/B}

Unify(P(A, B, B), P(x, y, z))
Unify(P(A, B, B), P(A, y, z)) ← {x/A}
Unify(P(A, B, B), P(A, B, z)) ← {x/A, y/B}
Unify(P(A, B, B), P(A, B, B)) ← {x/A, y/B, z/B}

(b) MGU does not exist.

Unify(Q(y, G(A, B)), Q(G(x, x), y))
Unify(Q(G(x, x), G(A, B)), Q(G(x, x), G(x, x))) ← {y/G(x, x)}
Unify(Q(G(A, A), G(A, B)), Q(G(A, A), G(A, A))) ← {y/G(x, x), x/A}
No substitutions left, as we cannot substitute A with B, so a most general unifier does not exist.

(c) MGU = {y/John, x/John}

Unify(Older(Father(y), y), Older(Father(x), John))
Unify(Older(Father(John), John), Older(Father(x), John)) ← {y/John}
Unify(Older(Father(John), John), Older(Father(John), John)) ← {y/John, x/John}

(d) MGU does not exist.

Unify(Knows(Father(y), y), Knows(x, x))
Unify(Knows(Father(x), x), Knows(x, x)) ← {y/x}
No substitutions left, as we cannot substitute x with Father(x), since the variable x occurs within the term Father(x), so it does not pass the occur check; a most general unifier does not exist.

3 Problem 3

Consider the following sentences:

- John likes all kinds of food.
- Apples are food.
- Chicken is food.
- Anything anyone eats and isn't killed by is food.
- If you are killed by something, you are not alive.
- Bill eats peanuts and is still alive. *
- Sue eats everything Bill eats.

For first-order syntax, feel free to use the following text file notation: | (for disjunction), & (for conjunction), - (for negation), => (for implication), <=> (for equivalence), E (for existential quantification, e.g., E x, y, Loves(x, y)), and A (for universal quantification, e.g., A x, y, Loves(x, y)).

(a) Translate these sentences into formulas in first-order logic.
(b) Convert the formulas of part (a) into CNF (also called clausal form).
(c) Prove that John likes peanuts using resolution.
(d) Use resolution to answer the question, "What food does Sue eat?"
(e) Use resolution to answer the question, "What food does Sue eat?" if, instead of the axiom marked with an asterisk above, we had:

- If you don't eat, you die.
- If you die, you are not alive.
- Bill is alive.

Solution:

(a)

1. A x, Food(x) => Likes(John, x)
2. Food(Apples)
3. Food(Chicken)
4. A x, y, (Eats(x, y) & -Kills(y, x)) => Food(y)
5. A x, y, Kills(x, y) => -Alive(y)
6. Eats(Bill, Peanuts) & Alive(Bill)
7. A x, Eats(Bill, x) => Eats(Sue, x)

(b)

1. -Food(x) | Likes(John, x)
2. Food(Apples)
3. Food(Chicken)

4. -Eats(y, z) | Kills(z, y) | Food(z)
5. -Kills(t, u) | -Alive(u)
6a. Eats(Bill, Peanuts)
6b. Alive(Bill)
7. -Eats(Bill, v) | Eats(Sue, v)

A x, Food(x) => Likes(John, x) → -Food(x) | Likes(John, x)
Food(Apples) → Food(Apples)
Food(Chicken) → Food(Chicken)
A x, y, (Eats(x, y) & -Kills(y, x)) => Food(y) → A x, y, -(Eats(x, y) & -Kills(y, x)) | Food(y) → A x, y (-Eats(x, y) | Kills(y, x)) | Food(y) → -Eats(y, z) | Kills(z, y) | Food(z)
A x, y, Kills(x, y) => -Alive(y) → -Kills(t, u) | -Alive(u)
Eats(Bill, Peanuts) & Alive(Bill) → Eats(Bill, Peanuts), Alive(Bill)
A x, Eats(Bill, x) => Eats(Sue, x) → -Eats(Bill, v) | Eats(Sue, v)

(c) Proof that John likes Peanuts:

-Likes(John, Peanuts)

Resolve with **[1]** -Food(x) | Likes(John, x) and substitute {x/Peanuts}

-Food(Peanuts)

Resolve with **[4]** -Eats(y, z) | Kills(z, y) | Food(z) and substitute {z/Peanuts}

-Eats(y, Peanuts) | Kills(Peanuts, y)

Resolve with **[6a]** Eats(Bill, Peanuts) and substitute {y/Bill}

Kills(Peanuts, Bill)

Resolve with **[5]** -Kills(t, u) | -Alive(u) and substitute {t/Peanuts, u/Bill}

-Alive(Bill)

Resolve with **[6b]** Alive(Bill) and substitute {}

NIL

From the proof by resolution, we refute that John does not like peanuts; by contradiction with the facts in the knowledge base and the given substitutions, we see that this results in an empty set, therefore John must like peanuts.

(d) The food that Sue eats is Peanuts. Proof that Sue eats peanuts:

"What food does Sue eat?" is the same as: E n, Food(n) & Eats(Sue, n).
Negate the statement to get the CNF: -Food(F(n)) | -Eats(Sue, F(n)) for the proof.

-Food(F(n)) | -Eats(Sue, F(n))

Resolve with **[7]** -Eats(Bill, v) | Eats(Sue, v) and substitute {v/F(n)}

-Food(F(n)) | -Eats(Bill, F(n))

Resolve with **[6a]** Eats(Bill, Peanuts) and substitute {F(n)/Peanuts}

-Food(Peanuts)

Resolve with **[4]** -Eats(y, z) | Kills(z, y) | Food(z) and substitute {z/Peanuts}

-Eats(y, Peanuts) | Kills(Peanuts, y)

Resolve with **[6a]** Eats(Bill, Peanuts) and substitute {y/Bill}

Kills(Peanuts, Bill)

Resolve with **[5]** -Kills(t, u) | -Alive(u) and substitute {t/Peanuts, u/Bill}

-Alive(Bill)

Resolve with **[6b]** Alive(Bill) and substitute {}

NIL

From the proof by resolution, we find that F(n) = Peanuts. So, the answer to the question "What food does Sue eat?" is Peanuts. This makes sense since Sue eats everything Bill eats by [7], and Bill eats Peanuts by [6a].

(e) Sue eats every food that Bill eats. Proof that Sue eats every food that Bill eats with the new facts:

Instead of statement [6], we now have:

6a. A x, y, -Eats(x, y) => Die(x) → Eats(p, q) | Die(p)
6b. A x, Die(x) => -Alive(x) → -Die(r) | -Alive(r)
6c. Alive(Bill) → Alive(Bill)

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NOTE: Let's say v is an arbitrary food that Bill eats: G.
-Food(F(n)) | -Eats(Sue, F(n))

Resolve with **[7]** -Eats(Bill, G) | Eats(Sue, G) and substitute {F(n)/G}

-Food(G) | -Eats(Bill, G)

Resolve with **[6a]** Eats(p, q) | Die(p) and substitute {p/Bill, q/G}

-Food(G) | Die(Bill)

Resolve with **[6b]** -Die(r) | -Alive(r) and substitute {r/Bill}

-Food(G) | -Alive(Bill)

Resolve with **[6c]** Alive(Bill) and substitute {}

-Food(G)

Resolve with **[4]** -Eats(y, z) | Kills(z, y) | Food(z) and substitute {z/G}

-Eats(y, G) | Kills(G, y)

Resolve with **[6a]** Eats(p, q) | Die(p) and substitute {p/y, q/G}

Die(y) | Kills(G, y)

Resolve with **[5]** -Kills(t, u) | -Alive(u) and substitute {t/G, u/y}

Die(y) | -Alive(y)

Resolve with **[6c]** Alive(Bill) and substitute {y/Bill}

Die(Bill)

Resolve with **[6b]** -Die(r) | -Alive(r) and substitute {r/Bill}

-Alive(Bill)

Resolve with **[6c]** Alive(Bill) and substitute {}

NIL

From the proof, we find that F(n) = G, which is an arbitrary food that represents the foods that Bill eats. So, the answer to the question "What food does Sue eat?" is every food that Bill eats, which makes sense since Sue eats everything Bill eats by [7].

4 Problem 4

Consider the following:

> *If the unicorn is mythical, then it is immortal, but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned.*

(a) Represent the above information using a propositional logic knowledge base (set of sentences in propositional logic).
(b) Convert the knowledge base into CNF.
(c) Can you use the knowledge base to prove that the unicorn is mythical? How about magical? Horned?

Justify your answers using resolution by providing corresponding resolution derivations. Make sure to clearly define all propositional symbols (variables) first, then define your knowledge base, and finally give your derivations.

Solution:

(a)

1. Mythical $\Rightarrow$ ¬Mortal
2. ¬Mythical $\Rightarrow$ (Mortal $\wedge$ Mammal)
3. (¬Mortal $\vee$ Mammal) $\Rightarrow$ Horned
4. Horned $\Rightarrow$ Magical

(b)

1. ¬Mythical $\vee$ ¬Mortal
2. (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal)
3. (Mortal $\vee$ Horned) $\wedge$ (¬Mammal $\vee$ Horned)
4. ¬Horned $\vee$ Magical

Mythical $\Rightarrow$ ¬Mortal $\rightarrow$ ¬Mythical $\vee$ ¬Mortal
¬Mythical $\Rightarrow$ (Mortal $\wedge$ Mammal) $\rightarrow$ ¬¬Mythical $\vee$ (Mortal $\wedge$ Mammal) $\rightarrow$ (Mythical $\vee$ Mortal) $\wedge$ (Mythical $\vee$ Mammal)
(¬Mortal $\vee$ Mammal) $\Rightarrow$ Horned $\rightarrow$ ¬(¬Mortal $\vee$ Mammal) $\vee$ Horned $\rightarrow$ (¬¬Mortal $\wedge$ ¬Mammal) $\vee$ Horned $\rightarrow$ (Mortal $\vee$ Horned) $\wedge$ (¬Mammal $\vee$ Horned)
Horned $\Rightarrow$ Magical $\rightarrow$ ¬Horned $\vee$ Magical

(c)

We have the knowledge base:

1. ¬Mythical ∨ ¬Mortal
2a. Mythical ∨ Mortal
2b. Mythical ∨ Mammal
3a. Mortal ∨ Horned
3b. ¬Mammal ∨ Horned
4. ¬Horned ∨ Magical

Mythical? No, we cannot prove that the unicorn is Mythical using the given knowledge base.

¬Mythical

Resolve with **[2a]** and **[2b]**

Mortal, Mammal

Resolve Mortal with **[1]** and Mammal with **[3b]**

¬Mythical, Horned

¬Mythical is our starting assumption, resolve Horned with **[4]**

¬Mythical, Magical

We cannot apply anymore resolutions

Given that the resolution proof of Mythical does not result in a contradiction and we reach a point where the statements can no longer be resolved, we cannot conclude that the unicorn is mythical.

Magical? Yes, we can prove that the unicorn is Magical using resolution and the given knowledge base.

¬Magical

Resolve with **[4]**

¬Horned

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Resolve with **[3a]** and **[3b]**

Mortal, ¬Mammal

Resolve Mortal with **[1]** and ¬Mammal with **[2b]**

Mythical, ¬Mythical

We have reached a contradiction.

Under the assumption that the unicorn is not Magical, we find that there is a contradiction in the knowledge base with the statements that the unicorn is both Mythical and not Mythical, therefore, the unicorn must be Magical.

Horned? Yes, we can prove that the unicorn is Horned using resolution and the given knowledge base.

¬Horned

Resolve with **[3a]** and **[3b]**

Mortal, ¬Mammal

Resolve Mortal with **[1]** and ¬Mammal with **[2b]**

Mythical, ¬Mythical

We have reached a contradiction

Under the assumption that the unicorn is not Horned, we find that there is a contradiction in the knowledge base with the statements that the unicorn is both Mythical and not Mythical, therefore, the unicorn must be Horned.

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5 Problem 5

Prove each of the following assertions:

(a) α is valid if and only if *True* $\models \alpha$.
(b) For any α, *False* $\models \alpha$.
(c) $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid.
(d) $\alpha \models \beta$ if and only if the sentence $\alpha \wedge \neg\beta$ is unsatisfiable.

Solution:

(a)

Proof by contradiction:

[1] Suppose that α is valid and *True* $\not\models \alpha$ (*True* does not entail α).

[2] Then, there exists some model of α for which α is False, since the constant *True* does not entail α.

[3] However, this contradicts the definition of validity, as for α to be valid, α must be True in all models.

[4] Therefore, α is valid if and only if *True* $\models \alpha$, by contradiction.

Proof by truth tables:

True $\models \alpha$

True	α
True	True

α is valid

α
True

From the truth tables, we see that for α to be valid, then *True* must entail α. That is, α cannot be False (not valid); if α were False, then *True* would not entail α.

(b)

Proof by contradiction:

[1] Suppose there exists a statement α, such that *False* $\not\models \alpha$ (*False* does not entail α).

[2] Then, this would mean that α could be neither True nor False, since *False* $\models \alpha$ means that α could either be True or False.

[3] However, this is not possible, as a statement must be either True or False.

[4] Therefore, there cannot exist an α for which *False* does not entail α; thus, for any α, *False* $\models \alpha$, by contradiction.

Proof by truth tables:

False $\models \alpha$

False	α
False	False
False	True

Any α

α
False
True

From the truth tables, we see that for any α, α can either be True or False, and because of this, then it follows that for any α, *False* must entail α.

(c)

Proof by contradiction:

[1] Suppose that $\alpha \models \beta$ and $(\alpha \Rightarrow \beta)$ is not valid.

[2] Then, there exists some model where α = True and β = False, so that the implication $(\alpha \Rightarrow \beta)$ is False, by definition of implication and validity.

[3] However, this contradicts the statement that $\alpha \models \beta$, as β should be True for every model that α is True.

[4] Therefore, $\alpha \models \beta$ if and only if the sentence $(\alpha \Rightarrow \beta)$ is valid, by contradiction.

Proof by truth tables:

$(\alpha \Rightarrow \beta)$

α	β	$(\alpha \Rightarrow \beta)$
False	False	True
False	True	True
True	True	True
True	False	False

$\alpha \models \beta$

α	β
False	False
False	True
True	True
True	False

From the truth tables, we see that for α to entail β, then the sentence that $(\alpha \Rightarrow \beta)$ must be valid. That is, α cannot be True and β cannot be False simultaneously; if α were True and β were False, then $(\alpha \Rightarrow \beta)$ would be False, and therefore not valid i.e. the row in red.

(d)

Proof by contradiction:

[1] Suppose that $\alpha \models \beta$ and $\alpha \land \neg\beta$ is satisfiable.

[2] Then, there exists some model where α = True and β = False, so that the conjunction $\alpha \land \neg\beta$ is True, by definition of satisfiability.

[3] However, this contradicts the statement that $\alpha \models \beta$, as β should be True for every model that α is True.

[4] Therefore, $\alpha \models \beta$ if and only if the sentence $\alpha \land \neg\beta$ is unsatisfiable, by contradiction.

Proof by truth tables:

$\alpha \land \neg\beta$

α	β	$\alpha \land \neg\beta$
False	False	False
False	True	False
True	True	False
True	False	True

$\alpha \models \beta$

α	β
False	False
False	True
True	True
True	False

From the truth tables, we see that for α to entail β, then the sentence that $\alpha \land \neg\beta$ must be unsatisfiable. That is, α cannot be True and β cannot be False simultaneously; if α were True and β were False, then $\alpha \land \neg\beta$ would be True, and therefore satisfiable i.e. the row in red.