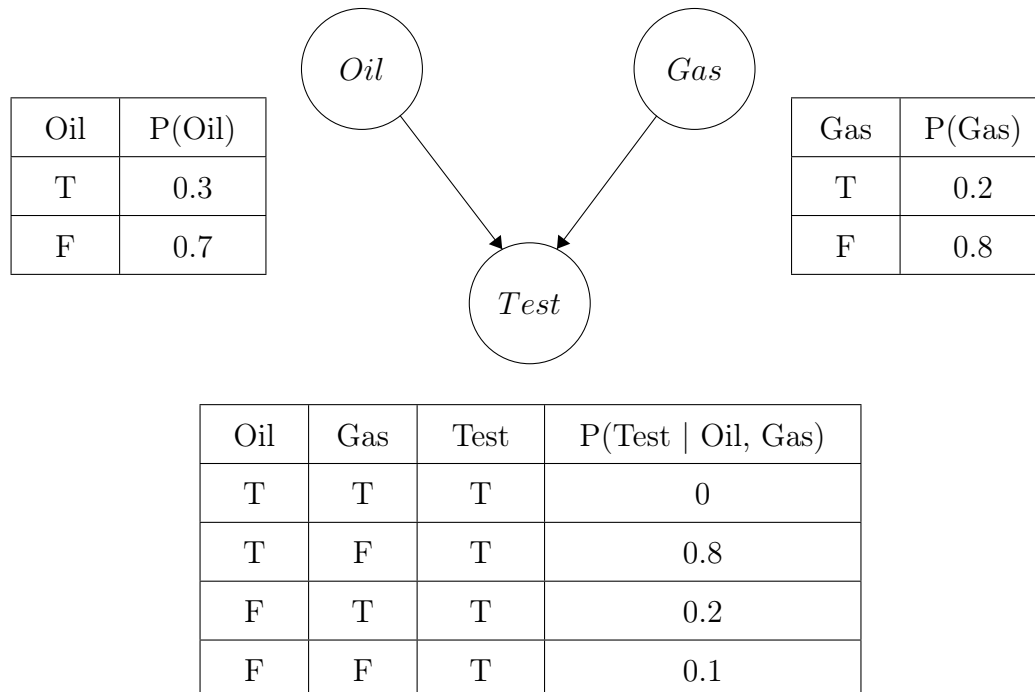# Homework 6

1 Problem 1

Consider the following:

> An oil well may be drilled on Mr. G's farm in LA. Based on what has happened to similar farms, we judge the probability of only oil being present to be .3, the probability of only natural gas being present to be .2, and the probability of neither being present to be .5. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .8; if natural gas is present, it will give a positive result with probability .2; and if neither are present, the test will be positive with probability .1.

(a) Model this problem as a Bayesian network over three variables: Oil, Gas, and Test.
(b) Suppose the test comes back positive. What's the probability that oil is present?

Solution:

(a)

Oil	P(Oil)
T	0.3
F	0.7

Gas	P(Gas)
T	0.2
F	0.8

Oil	Gas	Test	P(Test | Oil, Gas)
T	T	T	0
T	F	T	0.8
F	T	T	0.2
F	F	T	0.1

(b) $P(\text{Oil} \mid \text{Test}) = 0.7\overline{2}\overline{72}$

Work:

b)

We can use Bayes' Theorem:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

$$P(Oil \mid Test) = \frac{P(Test \mid Oil)P(Oil)}{P(Test)}$$

To get P(Test), we can construct a CPT for the event Test and use Equation 13.8 to get the marginal probability of when Test = Positive from conditioning:

Equation 13.8: $P(Y) = \sum_{z} P(Y \mid Z)P(Z)$

	Oil	Gas	Neither	Total
+	(0.3)(0.8)	(0.2)(0.2)	(0.5)(0.1)	0.33
-	(0.3)(0.2)	(0.2)(0.8)	(0.5)(0.9)	0.67
			Total	1

P(Test) = 0.33

$$P(Oil \mid Test) = \frac{P(Test \mid Oil)P(Oil)}{P(Test)}$$

$$= \frac{(0.8)(0.3)}{0.33}$$

$$= 0.72\overline{72}$$

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2 Problem 2

Consider the Bayesian network in Figure 1:

(a) Express $\Pr(A, B, C, D, E, F, G, H)$ as a multiplication of conditional and marginal probabilities, according to the factorization encoded in the network structure.

(b) Express $\Pr(E, F, G, H)$ in terms of factors instead of (conditional) probabilities.

(c) Express $\Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$ in terms of the parameters in the CPTs (a denotes $A = 1$ and $\neg a$ denotes $A = 0$). Use placeholder symbols for the parameters that are not shown in the CPTs.

(d) Compute $\Pr(\neg a, b)$ and $\Pr(\neg e \mid a)$. Justify your answers. Hint: leaf nodes that are not part of the probability query can be removed from the network without affecting the computed probability.

(e) List the Markovian assumptions (also known as topological semantics) encoded in the Bayesian network structure.

(f) Provide the Markov blanket for variable D.

(g) Multiply the factors (tables) corresponding to $\Pr(D|AB)$ and $\Pr(E|B)$.

(h) Sum out D from the factor (table) computed above.

Solution:

(a) $\Pr(A, B, C, D, E, F, G, H)$
$$= \Pr(A) \times \Pr(B) \times \Pr(C \mid A) \times \Pr(D \mid A, B) \times \Pr(E \mid B) \times \Pr(F \mid C, D)$$
$$\times \Pr(G \mid F) \times \Pr(H \mid F, E)$$

(b) $\Pr(E, F, G, H)$
$$= \mathbf{f}_6(G, F)\mathbf{f}_7(H, F, E)\mathbf{f}_{11}(E, F)$$

(My TA said leaving it in this form should be ok, but note that $\mathbf{f}_6$ is a 2x2 matrix, $\mathbf{f}_7$ is a 2x2x2 matrix, and $\mathbf{f}_{11}$ is a 2x2 matrix.)

(c) $\Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$
$$= (0.1) \times (0.4) \times \Pr(c \mid a) \times (0.6) \times (0.2) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f)$$
$$\times \Pr(h \mid f, \neg e)$$
$$= (0.0048) \times \Pr(c \mid a) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e)$$

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(d) $\Pr(\neg a,\ b) = 0.54$
$\Pr(\neg e \mid a) = 0.62$

A and B are independent:
$$\Pr(\neg a,\ b) = (0.9)(0.6)$$
$$= 0.54$$

e and a are also independent:
$$\Pr(\neg e \mid a) = \frac{\Pr(\neg e \cap a)}{\Pr(a)}$$
$$= \frac{\Pr(\neg e)\Pr(a)}{\Pr(a)}$$
$$= \Pr(\neg e)$$
$$= \Pr(\neg e \mid b)\Pr(b) + \Pr(\neg e \mid \neg b)\Pr(\neg b) \text{ Law of Total Probability}$$
$$= (0.9)(0.6) + (0.2)(0.4)$$
$$= 0.62$$

(e) The Markovian assumptions for each variable are as follows:

A is independent of B and E given A's parents (none).
B is independent of A and C given B's parents (none).
C is independent of B, D, and E given A.
D is independent of C and E given A and B.
E is independent of A, C, D, F, and G given B.
F is independent of A, B, and E given C and D.
G is independent of A, B, C, D, E, and H given F.
H is independent of A, B, C, D, and G given F and E.

(f) The Markov blanket for D is {A, B, C, F}.

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Dis 1A

(g)

A	B	D	E	$\Pr(D \mid AB) \times \Pr(E \mid B)$
1	1	1	1	$(0.7)(0.1) = \mathbf{0.07}$
1	1	1	0	$(0.7)(0.9) = \mathbf{0.63}$
1	1	0	1	$(0.3)(0.1) = \mathbf{0.03}$
1	1	0	0	$(0.3)(0.9) = \mathbf{0.27}$
1	0	1	1	$(0.6)(0.8) = \mathbf{0.48}$
1	0	1	0	$(0.6)(0.2) = \mathbf{0.12}$
1	0	0	1	$(0.4)(0.8) = \mathbf{0.32}$
1	0	0	0	$(0.4)(0.2) = \mathbf{0.08}$
0	1	1	1	$(0.2)(0.1) = \mathbf{0.02}$
0	1	1	0	$(0.2)(0.9) = \mathbf{0.18}$
0	1	0	1	$(0.8)(0.1) = \mathbf{0.08}$
0	1	0	0	$(0.8)(0.9) = \mathbf{0.72}$
0	0	1	1	$(0.8)(0.8) = \mathbf{0.64}$
0	0	1	0	$(0.8)(0.2) = \mathbf{0.16}$
0	0	0	1	$(0.2)(0.8) = \mathbf{0.16}$
0	0	0	0	$(0.2)(0.2) = \mathbf{0.04}$

(h)

A	B	E	$\mathbf{f}(A,\ B,\ E)$
1	1	1	$0.07 + 0.03 = \mathbf{0.1}$
1	1	0	$0.63 + 0.27 = \mathbf{0.9}$
1	0	1	$0.48 + 0.32 = \mathbf{0.8}$
1	0	0	$0.12 + 0.08 = \mathbf{0.2}$
0	1	1	$0.02 + 0.08 = \mathbf{0.1}$
0	1	0	$0.18 + 0.72 = \mathbf{0.9}$
0	0	1	$0.64 + 0.16 = \mathbf{0.8}$
0	0	0	$0.16 + 0.04 = \mathbf{0.2}$

Work:

b)

To get the factor form of each term, we fix each random variable since they are the query variables and sum out the hidden variables:

$\Pr(E,\ F,\ G,\ H)$
$$= \sum_{A,B,C,D} Pr(A)Pr(B)Pr(C|A)Pr(D|A,B)Pr(E|B)Pr(F|C,D)Pr(G|F)Pr(H|F,E)$$
$$= \sum_{A,B,C,D} \mathbf{f}_0(A)\mathbf{f}_1(B)\mathbf{f}_2(C,A)\mathbf{f}_3(D,A,B)\mathbf{f}_4(E,B)\mathbf{f}_5(F,C,D)\mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)$$
$$= \mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)\sum_B \mathbf{f}_1(B)\mathbf{f}_4(E,B)\sum_D\sum_C \mathbf{f}_5(F,C,D)\sum_A \mathbf{f}_0(A)\mathbf{f}_2(C,A)\mathbf{f}_3(D,A,B)$$
$$= \mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)\sum_B \mathbf{f}_1(B)\mathbf{f}_4(E,B)\sum_D\sum_C \mathbf{f}_5(F,C,D)\mathbf{f}_8(C,D,B)$$
$$= \mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)\sum_B \mathbf{f}_1(B)\mathbf{f}_4(E,B)\sum_D \mathbf{f}_9(F,D,B)$$
$$= \mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)\sum_B \mathbf{f}_1(B)\mathbf{f}_4(E,B)\mathbf{f}_{10}(F,B)$$
$$= \mathbf{f}_6(G,F)\mathbf{f}_7(H,F,E)\mathbf{f}_{11}(E,F)$$

c)

$\Pr(a,\ \neg b,\ c,\ d,\ \neg e,\ f,\ \neg g,\ h)$
$$= \Pr(a) \times \Pr(\neg b) \times \Pr(c \mid a) \times \Pr(d \mid a, \neg b) \times \Pr(\neg e \mid \neg b) \times \Pr(f \mid c, d)$$
$$\times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e)$$
$$= (0.1) \times (0.4) \times \Pr(c \mid a) \times (0.6) \times (0.2) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f)$$
$$\times \Pr(h \mid f, \neg e)$$
$$= (0.0048) \times \Pr(c \mid a) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e)$$

f)

A Markov blanket is the set of a variable's parents, children, and children's parents.
The parents of D are {A, B}.
The child of D is {F}.
The parent of F besides D is {C}.

h)

$$\mathbf{f}(A,\ B,\ D,\ E) = \Pr(D \mid AB) \times \Pr(E \mid B)$$

We sum out D from the product of $\Pr(D \mid AB)$ and $\Pr(E \mid B)$:

$$\mathbf{f}(A,\ B,\ E) = \sum_d \mathbf{f}(A,\ B,\ d,\ E) + \mathbf{f}(A,\ B,\ \neg d,\ E)$$