Jason Lai
204995126
Dis 1A

CS 161
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Homework 6

1 Problem 1

Consider the following:

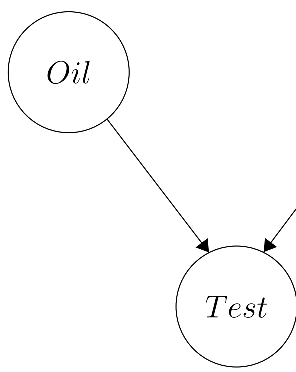
> An oil well may be drilled on Mr. G's farm in LA. Based on what has happened to similar farms, we judge the probability of only oil being present to be .3, the probability of only natural gas being present to be .2, and the probability of neither being present to be .5. Oil and gas never occur together. If oil is present, a geological test will give a positive result with probability .8; if natural gas is present, it will give a positive result with probability .2; and if neither are present, the test will be positive with probability .1.

(a) Model this problem as a Bayesian network over three variables: Oil, Gas, and Test.
(b) Suppose the test comes back positive. What's the probability that oil is present?

Solution:

(a)

Oil	P(Oil)
T	0.3
F	0.7



Gas	P(Gas)
T	0.2
F	0.8

Oil	Gas	Test	P(Test | Oil, Gas)
T	T	T	0
T	F	T	0.8
F	T	T	0.2
F	F	T	0.1

(b) P(Oil | Test) $= 0.72\overline{72}$

We can use Bayes' Theorem:

$$\text{P(A | B)} = \frac{\text{P(B | A)P(A)}}{P(B)}$$

$$\text{P(Oil | Test)} = \frac{\text{P(Test | Oil)P(Oil)}}{\text{P(Test)}}$$

To get P(Test), we can construct a CPT for the event Test and use Equation 13.8 to get the marginal probability of when Test = Positive from conditioning:

Equation 13.8: $P(Y) = \sum_{\text{z}} \text{P(Y | Z)P(Z)}$

	Oil	Gas	Neither	Total
+	(0.3)(0.8)	(0.2)(0.2)	(0.5)(0.1)	0.33
-	(0.3)(0.2)	(0.2)(0.8)	(0.5)(0.9)	0.67
			Total	1

P(Test) = 0.33

$$\text{P(Oil | Test)} = \frac{\text{P(Test | Oil)P(Oil)}}{\text{P(Test)}}$$

$$= \frac{(0.8)(0.3)}{0.33}$$

$$= 0.72\overline{72}$$

2 Problem 2

Consider the Bayesian network in Figure 1:

(a) Express Pr(A, B, C, D, E, F, G, H) as a multiplication of conditional and marginal probabilities, according to the factorization encoded in the network structure.
(b) Express Pr(E, F, G, H) in terms of factors instead of (conditional) probabilities.
(c) Express Pr(a, $\neg b$, c, d, $\neg e$, f, $\neg g$, h) in terms of the parameters in the CPTs (a denotes $A = 1$ and ¬a denotes $A = 0$). Use placeholder symbols for the parameters that are not shown in the CPTs.
(d) Compute Pr($\neg a$, b) and Pr($\neg e \mid a$). Justify your answers. Hint: leaf nodes that are not part of the probability query can be removed from the network without affecting the computed probability.
(e) List the Markovian assumptions (also known as topological semantics) encoded in the Bayesian network structure.
(f) Provide the Markov blanket for variable D.
(g) Multiply the factors (tables) corresponding to Pr($D|AB$) and Pr($E|B$).
(h) Sum out D from the factor (table) computed above.

Solution:

(a) Pr(A, B, C, D, E, F, G, H)

$$= \Pr(A) \times \Pr(B) \times \Pr(C \mid A) \times \Pr(D \mid A,\ B) \times \Pr(E \mid B) \times \Pr(F \mid C,\ D) \times \Pr(G \mid F) \times \Pr(H \mid F,\ E)$$

(b) Pr(E, F, G, H)

$$= \begin{pmatrix} Pr(E \mid b) \\ Pr(E \mid \neg b) \end{pmatrix} \times \begin{pmatrix} Pr(F \mid c, d) & Pr(F \mid \neg c, d) \\ Pr(F \mid c, \neg d) & Pr(F \mid \neg c, \neg d) \end{pmatrix} \times \begin{pmatrix} Pr(G \mid f) \\ Pr(G \mid \neg f) \end{pmatrix} \times \begin{pmatrix} Pr(H \mid f, e) & Pr(H \mid \neg f, e) \\ Pr(H \mid f, \neg e) & Pr(H \mid \neg f, \neg e) \end{pmatrix}$$

To get the factor form of each term, we fix each random variable since they are the query variables and sum out the hidden variables:

Pr(E, F, G, H)

$$= \Pr(E \mid B) \times \Pr(F \mid C,\ D) \times \Pr(G \mid F) \times \Pr(H \mid F,\ E)$$

$$= \begin{pmatrix} Pr(E \mid b) \\ Pr(E \mid \neg b) \end{pmatrix} \times \begin{pmatrix} Pr(F \mid c, d) & Pr(F \mid \neg c, d) \\ Pr(F \mid c, \neg d) & Pr(F \mid \neg c, \neg d) \end{pmatrix} \times \begin{pmatrix} Pr(G \mid f) \\ Pr(G \mid \neg f) \end{pmatrix} \times \begin{pmatrix} Pr(H \mid f, e) & Pr(H \mid \neg f, e) \\ Pr(H \mid f, \neg e) & Pr(H \mid \neg f, \neg e) \end{pmatrix}$$

(c) $\Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$

$$\begin{aligned}
&= (0.1) \times (0.4) \times \Pr(c \mid a) \times (0.6) \times (0.2) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \\
&\quad \times \Pr(h \mid f, \neg e) \\
&= (0.0048) \times \Pr(c \mid a) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e)
\end{aligned}$$

$\Pr(a, \neg b, c, d, \neg e, f, \neg g, h)$

$$\begin{aligned}
&= \Pr(a) \times \Pr(\neg b) \times \Pr(c \mid a) \times \Pr(d \mid a, \neg b) \times \Pr(\neg e \mid \neg b) \times \Pr(f \mid c, d) \\
&\quad \times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e) \\
&= (0.1) \times (0.4) \times \Pr(c \mid a) \times (0.6) \times (0.2) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \\
&\quad \times \Pr(h \mid f, \neg e) \\
&= (0.0048) \times \Pr(c \mid a) \times \Pr(f \mid c, d) \times \Pr(\neg g \mid f) \times \Pr(h \mid f, \neg e)
\end{aligned}$$

(d) $\Pr(\neg a, b) = 0.54$
$\Pr(\neg e \mid a) = 0.62$

A and B are independent:

$$\begin{aligned}
\Pr(\neg a, b) &= (0.9)(0.6) \\
&= 0.54
\end{aligned}$$

e and a are also independent:

$$\begin{aligned}
\Pr(\neg e \mid a) &= \frac{\Pr(\neg e \cap a)}{\Pr(a)} \\
&= \frac{\Pr(\neg e)\Pr(\text{a})}{\Pr(a)} \\
&= \Pr(\neg e) \\
&= \Pr(\neg e \mid b)\Pr(b) + \Pr(\neg e \mid \neg b)\Pr(\neg b) \text{ Law of Total Probability} \\
&= (0.9)(0.6) + (0.2)(0.4) \\
&= 0.62
\end{aligned}$$

(e) The Markovian assumptions for each variable are as follows:

A is independent of B and E given A's parents (none).
B is independent of A and C given B's parents (none).
C is independent of B, D, and E given A.
D is independent of C and E given A and B.
E is independent of A, C, D, F, and G given B.
F is independent of A, B, and E given C and D.
G is independent of A, B, C, D, E, and H given F.
H is independent of A, B, C, D, and G given F and E.

(f) The Markov blanket for D is {A, B, C, F}.

A Markov blanket is the set of a variable's parents, children, and children's parents.
The parents of D are {A, B}.
The child of D is {F}.
The parent of F besides D is {C}.

(g)

A	B	D	E	$\Pr(D \mid AB) \times \Pr(E \mid B)$
1	1	1	1	$(0.7)(0.1) = \mathbf{0.07}$
1	1	1	0	$(0.7)(0.9) = \mathbf{0.63}$
1	1	0	1	$(0.3)(0.1) = \mathbf{0.03}$
1	1	0	0	$(0.3)(0.9) = \mathbf{0.27}$
1	0	1	1	$(0.6)(0.8) = \mathbf{0.48}$
1	0	1	0	$(0.6)(0.2) = \mathbf{0.12}$
1	0	0	1	$(0.4)(0.8) = \mathbf{0.32}$
1	0	0	0	$(0.4)(0.2) = \mathbf{0.08}$
0	1	1	1	$(0.2)(0.1) = \mathbf{0.02}$
0	1	1	0	$(0.2)(0.9) = \mathbf{0.18}$
0	1	0	1	$(0.8)(0.1) = \mathbf{0.08}$
0	1	0	0	$(0.8)(0.9) = \mathbf{0.72}$
0	0	1	1	$(0.8)(0.8) = \mathbf{0.64}$
0	0	1	0	$(0.8)(0.2) = \mathbf{0.16}$
0	0	0	1	$(0.2)(0.8) = \mathbf{0.16}$
0	0	0	0	$(0.2)(0.2) = \mathbf{0.04}$

(h)

A	B	E	$\mathbf{f}(A,\ B,\ E)$
1	1	1	$0.07 + 0.03 = \mathbf{0.1}$
1	1	0	$0.63 + 0.27 = \mathbf{0.9}$
1	0	1	$0.48 + 0.32 = \mathbf{0.8}$
1	0	0	$0.12 + 0.08 = \mathbf{0.2}$
0	1	1	$0.02 + 0.08 = \mathbf{0.1}$
0	1	0	$0.18 + 0.72 = \mathbf{0.9}$
0	0	1	$0.64 + 0.16 = \mathbf{0.8}$
0	0	0	$0.16 + 0.04 = \mathbf{0.2}$

$\mathbf{f}(A,\ B,\ D,\ E) = \Pr(D \mid AB) \times \Pr(E \mid B)$

We sum out D from the product of $\Pr(D \mid AB)$ and $\Pr(E \mid B)$:

$\mathbf{f}(A,\ B,\ E) = \sum_d \mathbf{f}(A,\ B,\ d,\ E) + \mathbf{f}(A,\ B,\ \neg d,\ E)$