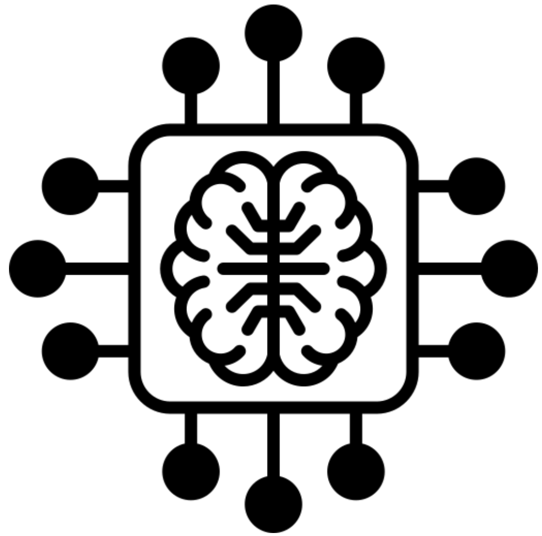
SBS4115 Fundamentals of AI & Data Analytics



AI in Action II

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Intended Learning Outcomes

- By the end of this lecture, you will be able to…
  - Describe usage of AI in engineering
  - Use measures of association
  - Describe how two variables are related to each other
  - Show correlations between each pair of variables
  - Introduce linear regression
  - Build and test a multiple regression model.

AI in Civil Engineering

- AI for monitoring large infrastructures (bridges, tunnels, buildings)
- Machine learning algorithms detect cracks, stress, and other deterioration
- Prevent catastrophic failures and prolong infrastructure lifespan

AI in Civil Engineering

- Morandi Bridge collapse in Italy (2018) led to AI-powered monitoring by University of Cambridge
- Real-time detection of early deterioration signs across Italian bridges

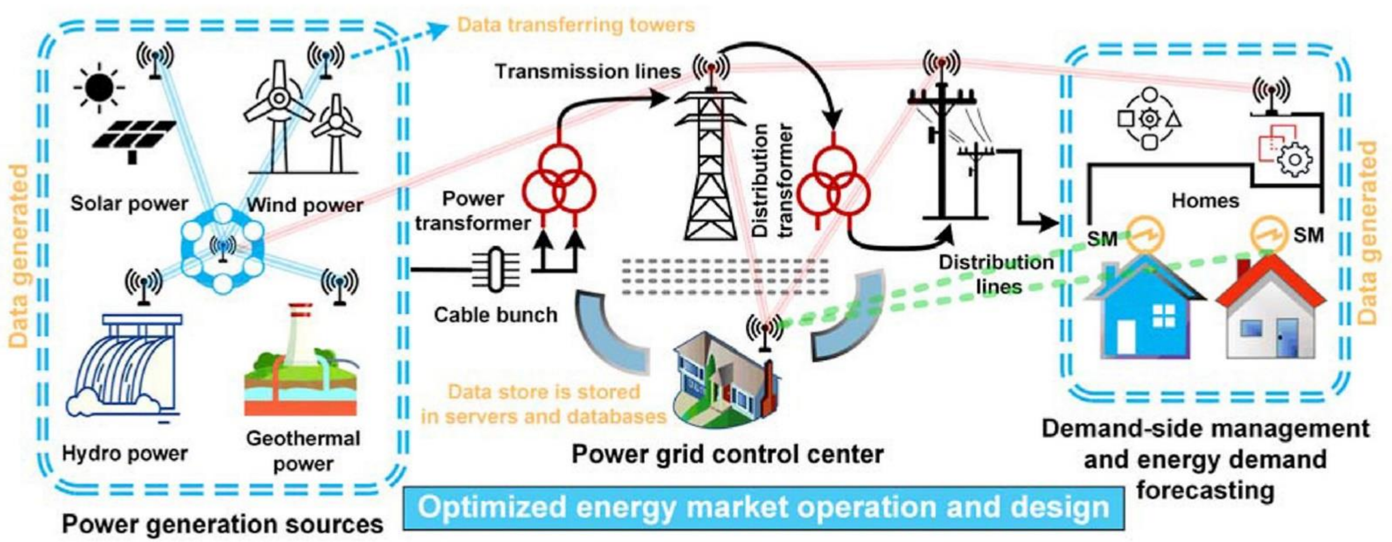
AI in Mechanical Engineering

- AI models analyze sensor data from machines
- Predict when components will fail, reducing unexpected downtime
- Improves equipment lifespan and reduces maintenance costs

AI in Electrical Engineering

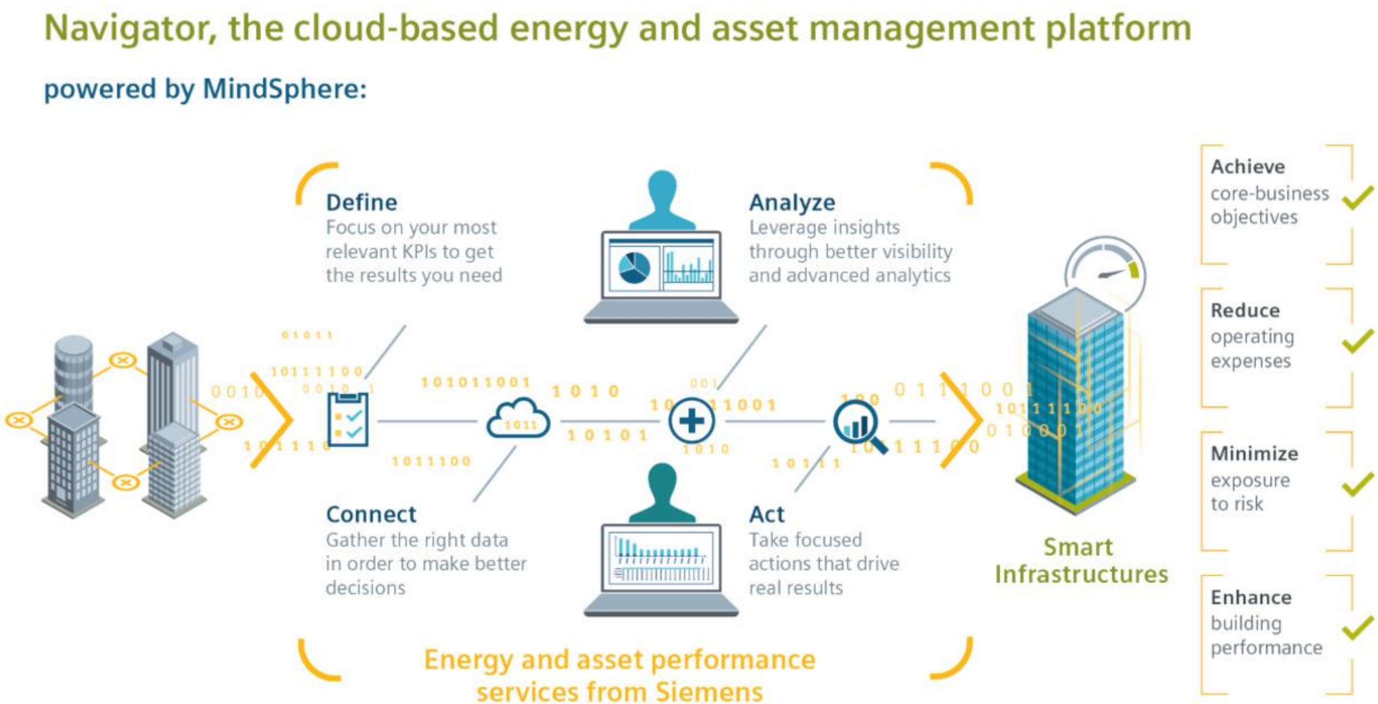
- AI optimizes energy distribution in smart grids
- Predicts demand based on consumption patterns and weather
- Minimizes power outages and enhances energy efficiency

Big data and AI applications.



AI in Building Services - Energy Management

- AI-driven systems optimize HVAC, lighting, and energy usage
- Adjusts energy use based on occupancy, weather, and time of day
- Reduces energy consumption and operational costs



AI in Building Services - Smart HVAC Systems

- AI improves real-time control of HVAC systems
- Predicts occupancy and weather patterns for optimized temperature control
- Reduces energy consumption and detects system failures

AI in Building Services - Indoor Air Quality Monitoring

- AI monitors and improves indoor air quality
- Adjusts ventilation and filtration systems in real time
- Critical for maintaining healthy air in offices, hospitals, and schools

AI in Building Services - Intelligent Lighting Systems

- AI adjusts lighting based on occupancy and daylight levels
- Reduces energy consumption while enhancing comfort
- Provides insights into space usage for optimization

Measures of Association

- In statistics, **bivariate data** refers to a dataset with two variables.
- Each sample in this dataset contains values of both variables.
- A typical example is the health data – each student is a sample with two variables: height (m) and weight (kg).
- Here, we might want to study **how the two variables are related to each other using measures of association**.

```
import pandas as pd
df = pd.read_csv("health.csv")
x = df['height']
y = df['weight']
```

	sex	height	weight
0	M	1.73	65
1	F	1.61	54
2	M	1.80	72
3	F	1.56	63
4	F	1.69	58

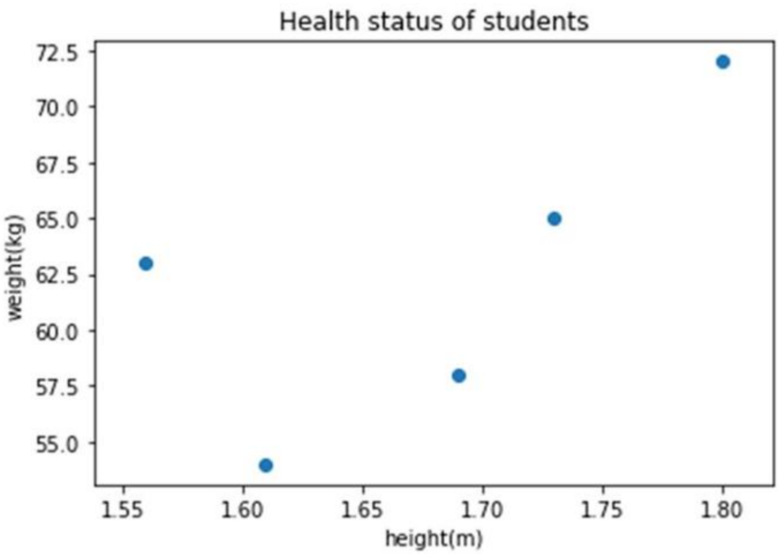
Measures of Association

```
import pandas as pd

# Read the CSV file
df = pd.read_csv(r'C:\Users\User user\Desktop\health.csv')

# Extract 'height' and 'weight' columns
x = df['height']
y = df['weight']

print(x)
print(y)
```

Measures of Association

- To visualize the relationship between two variables from a bivariate data, we can **draw a scatterplot**.
- In a scatterplot, each data point represents a sample.
- The x-coordinate and y-coordinate represent the value of the two variables.

```
plt.scatter(x,y)
plt.title("Health status of students")
plt.xlabel("height(m)")
plt.ylabel("weight(kg)")
plt.show()
```



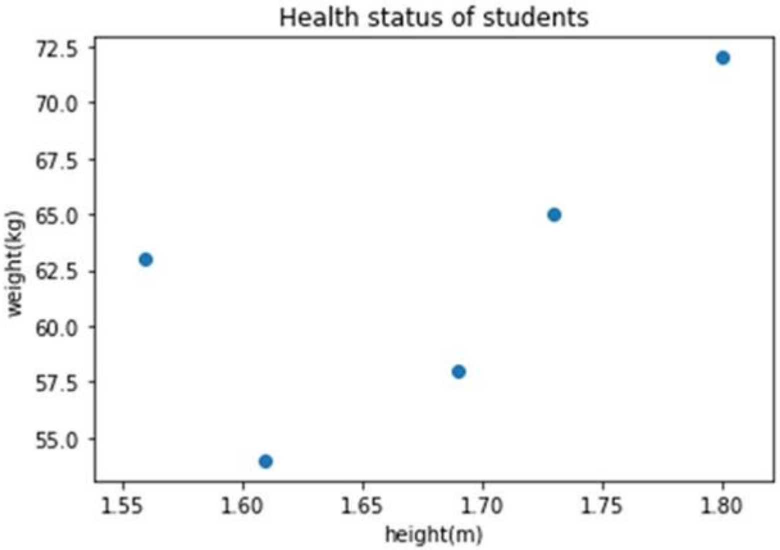
Measures of Association

```
import matplotlib.pyplot as plt

# Assuming x and y are already defined as height and weight
plt.scatter(x, y)
plt.title("Health status of students")
plt.xlabel("height(cm)")
plt.ylabel("weight(kg)")
plt.show()
```

Measures of Association

- Consider the example on the previous page.
- Do you think that weight and height are uncorrelated?
- Do you believe a tall person should be heavier (positive correlation), or do you believe a tall person should be lighter (negative correlation)?

```
plt.scatter(x,y)
plt.title("Health status of students")
plt.xlabel("height(m)")
plt.ylabel("weight(kg)")
plt.show()
```



Measures of Association

- In descriptive statistics, we have introduced variance and standard deviation as measures of dispersion, meaning how far the values apart from the mean are.
- For the variables x and y, their variance and standard deviation are σ_x^2, σ_y^2 and σ_x, σ_y respectively.
- In order measure their association, we further introduce a measure called **population covariance**:

$$\sigma_{xy} = \frac{\sum\left(x^{(i)} - \bar{x}\right)\left(y^{(i)} - \bar{y}\right)}{N}$$

where $x^{(i)}$, $y^{(i)}$ are the values of variables x, y of the i-th sample, $\bar{x}, \bar{y}$ are the arithmetic means of the two variables, N is the population size.

Measures of Association

- This is to say, for each sample we evaluate **product of difference between each variable with its mean**. (Notice that this can be positive or negative.)
- **The covariance is the average value of these products**.
  - Notice that the covariance is commutative, meaning that $\sigma_{xy} = \sigma_{yx}$.
- Also, the covariance of a variable with its own self results in the variance.
- For sample covariance, we may simply **replace N by n – 1**, where n is the sample size.
- In pandas, we can use the function `cov()` to show the sample covariance between two variables, or to give a covariance table.

Measures of Association

```
x.cov(y)
```

```
0.43849999999999995
```

- However, the value of covariance also **depends on the scale of the variables**.
- For example, if we use centimeter and pounds as the units for height and weight, the covariance will be different.

Measures of Association

Try:

df.cov()

and see what you get

Non-numerical variables (e.g. sex) is ignored!

Measures of Association

You need to extract 'height' and 'weight' columns to forma new DataFrame

```
new_df = df[['height', 'weight’]]
print(new_df)
```

Try again:

```
new_df.cov()
```

Measures of Association

- In order to study the association between two variables without considering scale and unit, we can **standardize the covariance** by the standard deviation of the two variables.

- This gives the **correlation coefficient**: $$r_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

- The complete formula is written as:

$$r_{xy} = \frac{\sum\left(x^{(i)} - \bar{x}\right)\left(y^{(i)} - \bar{y}\right)}{\sqrt{\sum(x^{(i)} - \bar{x})^2}\ \sqrt{\sum(y^{(i)} - \bar{y})^2}}$$

Measures of Association

- The result of correlation coefficient r_{xy} is a value **between –1 and 1** inclusively.
- We can interpret the relationship between the two variables by the value of r_{xy} :
  - $r_{xy} = 0$ indicates no linear relationship, or in other words uncorrelated.
  - $r_{xy} = 1$ indicates a perfect positive linear relationship: as one variable increases in its values, the other variable also increases in its values through an exact linear rule.
  - $r_{xy} = -1$ indicates a perfect negative linear relationship: as one variable increases in its values, the other variable decreases in its values through an exact linear rule.

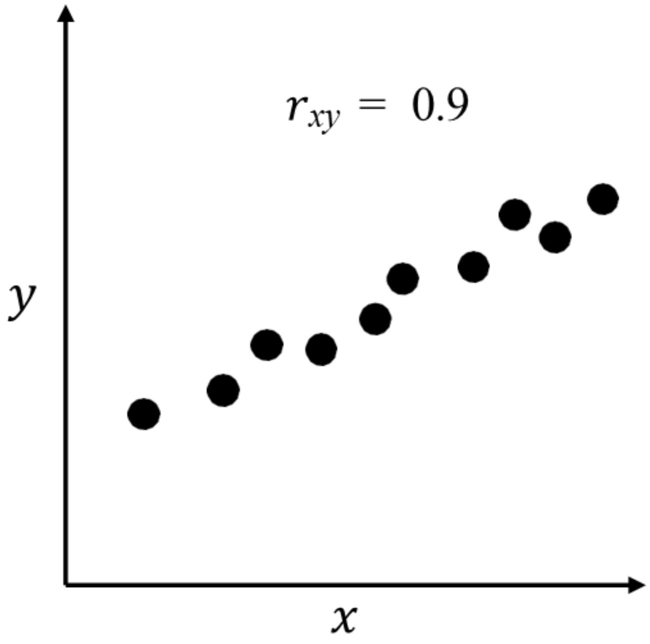
Measures of Association

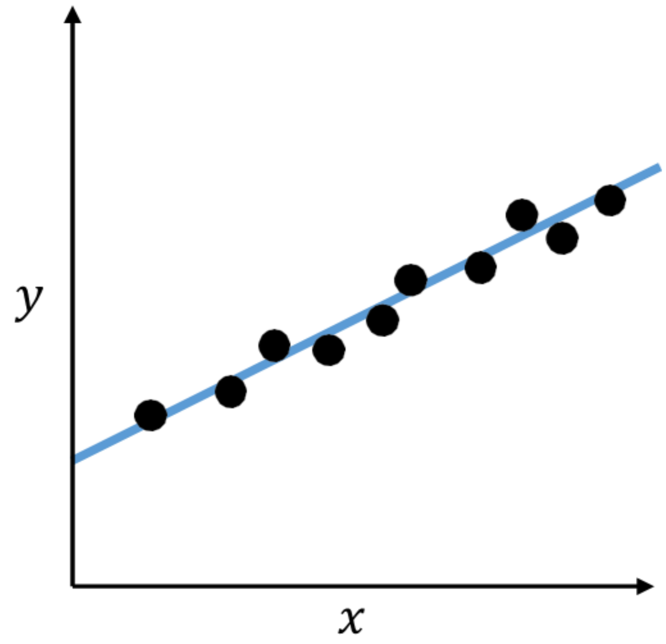
- Other than these special cases, we can also distinguish the linear relationship between the variables as follows:

	weak	**moderate**	**strong**
positive	$0 < r_{xy} < 0.3$	$0.3 < r_{xy} < 0.7$	$0.7 < r_{xy} < 1$
negative	$-0.3 < r_{xy} < 0$	$-0.7 < r_{xy} < -0.3$	$-1 < r_{xy} < -0.7$

Measures of Association

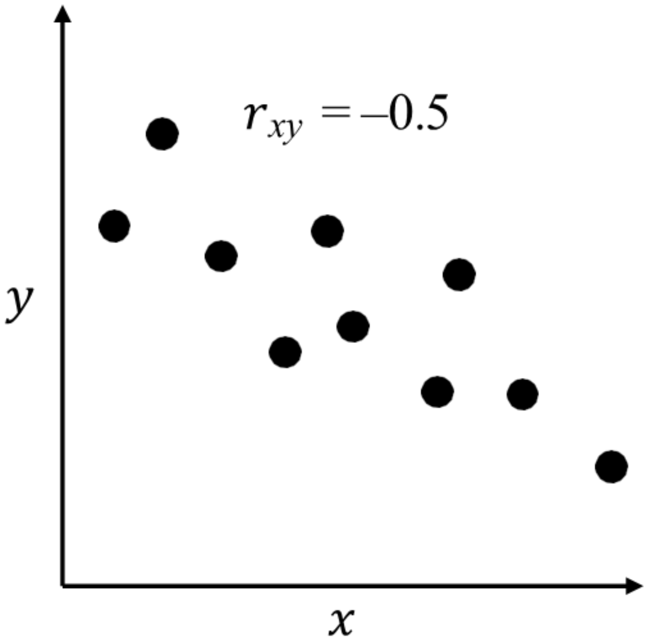
- To illustrate this idea, compare the three figures below:
  1. Strong positive correlation
     - We can easily draw a straight line with positive slope with the points lying close to it.

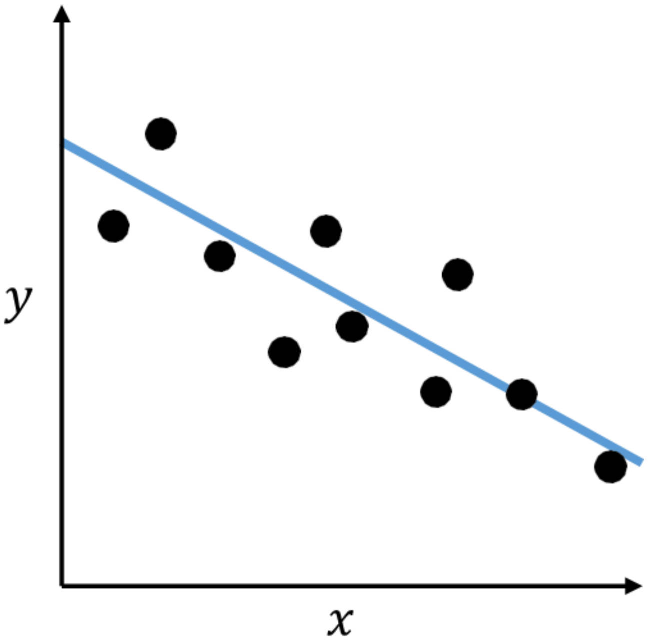




Measures of Association

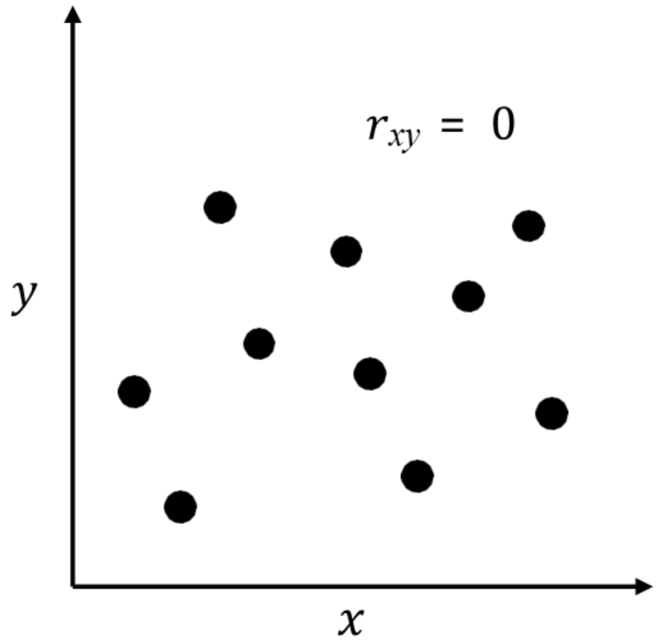
2. Moderate negative correlation
   - We can also draw a line with negative slope, but the points are farther apart from it.





Measures of Association

3. Zero correlation
   - We can hardly draw a linear relationship between the points.



Measures of Association

- We can evaluate the coefficient of correlation using pandas easily.
- Let’s go back to the example of health data of five students.
- Read the csv file as a DataFrame first.
- We can then regard height and weight as the two variables.
- For convenience, we may assign the two columns as two Series x and y.

- Then we can evaluate the correlation coefficient by `corr()`.
- Notice that this operation is commutative meaning that `x.corr(y)` and `y.corr(x)` give the same result.

Measures of Association

```
x.corr(y)
```

```
0.669476572167
6758
```

- The coefficient 0.669 indicates that height and weight are (moderate) positively correlated.
- In case there are more than two variables in the dataset, it would be inconvenient to compare every pair of them.
- Instead, we might apply `corr()` to the whole DataFrame.
- This will give a table of correlation coefficient between each column pairwisely.

Measures of Association

x.corr(y)

new_df.corr()

Measures of Association

- Notice that non-numerical variables (e.g. sex) is ignored.
- **Only variables with numerical values are accounted**.

- The table is symmetry with the diagonal values equal to 1 since the correlation coefficient of a variable with itself must be 1.

	height	weight
height	1.000000	0.669477
weight	0.669477	1.000000

Measures of Association

- To further illustrate the idea of correlation between different variables, we will study a famous example.
- In 1978, *David Harrison Jr.* and *Daniel L. Rubinfeld* published a paper called "Hedonic housing prices and the demand for clean air".
- To support their findings, they referred to the data for census tracts in the Boston Standard Metropolitan Statistical Area (SMSA) in 1970.
- This dataset is clean with lots of variables including the following:

Measures of Association

feature variables (factors)

CRIM – per capita crime rate by town
ZN – proportion of residential land zoned for lots over 25,000 sq.ft. INDUS – proportion of non-retail business acres per town.
CHAS – Charles River dummy variable (1 if tract bounds river; 0 otherwise)
NOX – nitric oxides concentration (parts per 10 million)
RM – average number of rooms per dwelling
AGE – proportion of owner-occupied units built prior to 1940 DIS – weighted distances to five Boston employment centres RAD – index of accessibility to radial highways
TAX – full-value property-tax rate per 10,000
PTRATIO – pupil-teacher ratio by town
B – 1000(Bk – 0.63)^2, where Bk is the proportion of blacks by town LSTAT – % lower status of the population

MEDV – Median value of owner-occupied homes in $1000’s

target variable

Measures of Association

- This dataset has been store in "boston.csv".
- It contains 506 rows and 14 columns.
- We can first read it as a DataFrame.

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
0	0.00632	18.0	2.31	0	0.538	6.575	65.2	4.0900	1	296	15.3	396.90	4.98	24.0
1	0.02731	0.0	7.07	0	0.469	6.421	78.9	4.9671	2	242	17.8	396.90	9.14	21.6
2	0.02729	0.0	7.07	0	0.469	7.185	61.1	4.9671	2	242	17.8	392.83	4.03	34.7
3	0.03237	0.0	2.18	0	0.458	6.998	45.8	6.0622	3	222	18.7	394.63	2.94	33.4
4	0.06905	0.0	2.18	0	0.458	7.147	54.2	6.0622	3	222	18.7	396.90	5.33	36.2

Measures of Association

```
import pandas as pd

df = pd.read_csv(r'C:\Users\User user\Desktop\Boston.csv’)

print(df)
```

Measures of Association

- As this dataframe contains quite a number of variables, we can show **correlations between each pair of variables** in table form.

```
df.corr()
```

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	DIS	RAD	TAX	PTRATIO	B	LSTAT	MEDV
CRIM	1.000000	-0.200469	0.406583	-0.055892	0.420972	-0.219247	0.352734	-0.379670	0.625505	0.582764	0.289946	-0.385064	0.455621	-0.388305
ZN	-0.200469	1.000000	-0.533828	-0.042697	-0.516604	0.311991	-0.569537	0.664408	-0.311948	-0.314563	-0.391679	0.175520	-0.412995	0.360445
INDUS	0.406583	-0.533828	1.000000	0.062938	0.763651	-0.391676	0.644779	-0.708027	0.595129	0.720760	0.383248	-0.356977	0.603800	-0.483725
CHAS	-0.055892	-0.042697	0.062938	1.000000	0.091203	0.091251	0.086518	-0.099176	-0.007368	-0.035587	-0.121515	0.048788	-0.053929	0.175260
NOX	0.420972	-0.516604	0.763651	0.091203	1.000000	-0.302188	0.731470	-0.769230	0.611441	0.668023	0.188933	-0.380051	0.590879	-0.427321
RM	-0.219247	0.311991	-0.391676	0.091251	-0.302188	1.000000	-0.240265	0.205246	-0.209847	-0.292048	-0.355501	0.128069	-0.613808	0.695360
AGE	0.352734	-0.569537	0.644779	0.086518	0.731470	-0.240265	1.000000	-0.747881	0.456022	0.506456	0.261515	-0.273534	0.602339	-0.376955
DIS	-0.379670	0.664408	-0.708027	-0.099176	-0.769230	0.205246	-0.747881	1.000000	-0.494588	-0.534432	-0.232471	0.291512	-0.496996	0.249929
RAD	0.625505	-0.311948	0.595129	-0.007368	0.611441	-0.209847	0.456022	-0.494588	1.000000	0.910228	0.464741	-0.444413	0.488676	-0.381626
TAX	0.582764	-0.314563	0.720760	-0.035587	0.668023	-0.292048	0.506456	-0.534432	0.910228	1.000000	0.460853	-0.441808	0.543993	-0.468536
PTRATIO	0.289946	-0.391679	0.383248	-0.121515	0.188933	-0.355501	0.261515	-0.232471	0.464741	0.460853	1.000000	-0.177383	0.374044	-0.507787
B	-0.385064	0.175520	-0.356977	0.048788	-0.380051	0.128069	-0.273534	0.291512	-0.444413	-0.441808	-0.177383	1.000000	-0.366087	0.333461
LSTAT	0.455621	-0.412995	0.603800	-0.053929	0.590879	-0.613808	0.602339	-0.496996	0.488676	0.543993	0.374044	-0.366087	1.000000	-0.737663
MEDV	-0.388305	0.360445	-0.483725	0.175260	-0.427321	0.695360	-0.376955	0.249929	-0.381626	-0.468536	-0.507787	0.333461	-0.737663	1.000000

Measures of Association

- In particular, we would like to study how the target variable median house price (MEDV) is being correlated to two factors average room number (RM) and proportion of old buildings (AGE).
- First we can extract these columns from the DataFrame as three Series.

```
x1 = df['RM']
x2 = df['AGE']
x3 = df['MEDV']
```

- To visualize the data, **make a scatter plot for each pair of variables**.

Measures of Association

```
x1 = df['rm']
x2 = df['age']
x3 = df['medv']

print(x1)
print(x2)
print(x3)
```

Measures of Association

- We can see that the house price is positively correlated to the average room number, meaning that in general more rooms result in higher price.

```
plt.scatter(x1,x3)
plt.title("house price vs room number")
plt.xlabel("average room number")
plt.ylabel("median house price")
plt.show()
```

Measures of Association

```
import matplotlib.pyplot as plt

plt.scatter(x1, x3)
plt.title("house price vs room number")
plt.xlabel("average room number")
plt.ylabel("median house price")
plt.show()
```

Measures of Association

- However, it is negatively correlated to the proportional of old buildings, meaning that for more old buildings in the district the house price is more likely to be lower.

```
plt.scatter(x2,x3)
plt.title("house price vs age")
plt.xlabel("proportion of old buildings")
plt.ylabel("median house price")
plt.show()
```

Measures of Association

```
plt.scatter(x2, x3)
plt.title("house price vs age")
plt.xlabel("proportion of old buildings")
plt.ylabel("median house price")
plt.show()
```

Classwork

- The file "hr.csv" contains the human resource record of a company with 30 staff, showing their experience (years) working in the company and annual salary (USD).

  a. Read the file as a DataFrame.
  b. Evaluate the correlation between experience and salary.
  c. Make a scatter plot of these two variables.
  d. What can you conclude the relationship between experience and salary?

Introduction to Linear Regression

- In the previous section, we have introduced the correlation coefficient for measuring the association between two variables.
- The value of this coefficient suggests whether it is a linear relationship between the two variables.
- To further analyze the association and make reasonable prediction, we can use a statistical technique called **linear regression**.

- For simplicity, we are looking for a straight line that best fit the data set of two variables.
- Recall the figure in the previous section.
- We can draw a straight line graphically to fit the points.
- In general, we need to know how to find the equation of such line, and how to evaluate the performance of the line fitting the points.

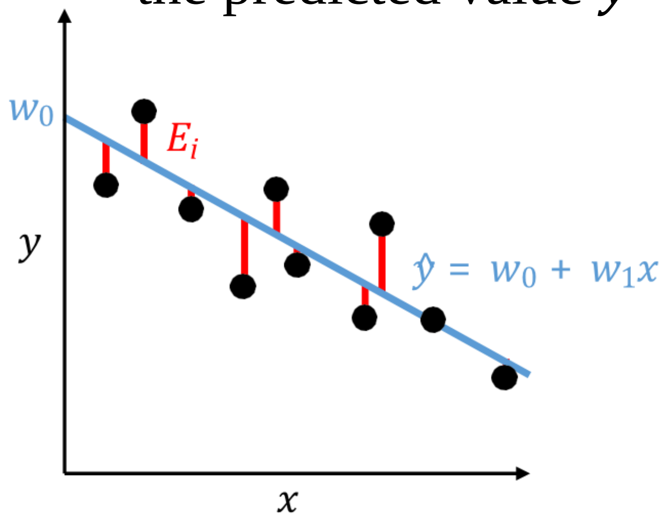
Introduction to Linear Regression

- We define $\hat{y}$ as the predicted value, which is a linear function x given by:

$$\hat{y} = w_0 + w_1 x$$

where the weights w_0, w_1 represents the y-intercept and slope of the line.

- For the i-th sample, we define the error E_i as the difference between the predicted value $\hat{y}^{(i)}$ and the actual value $y^{(i)}$.

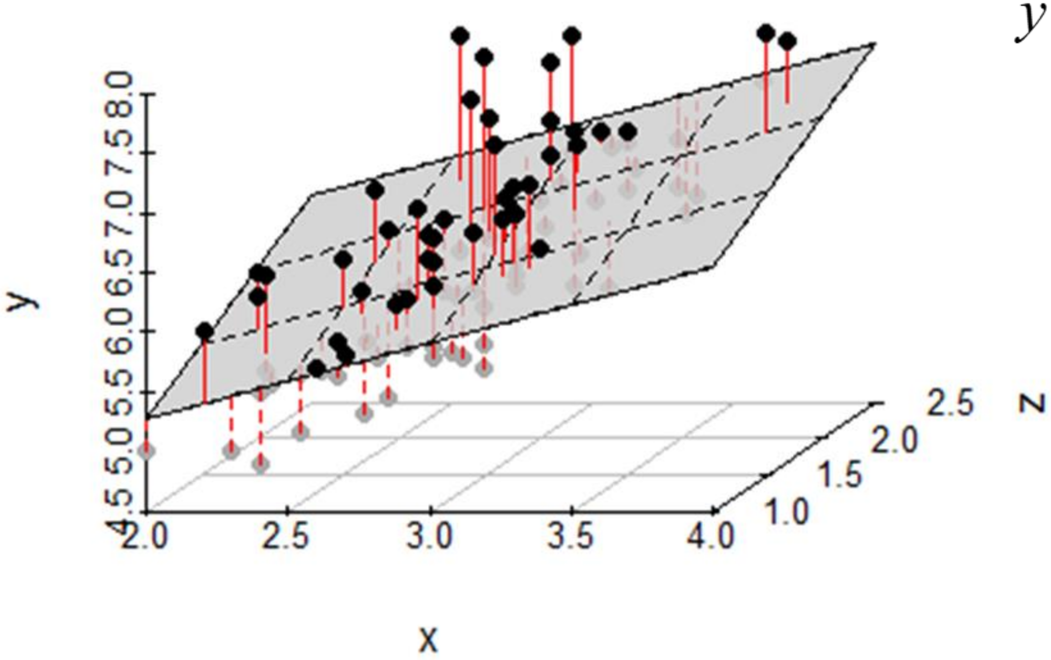


- Our target is to find the weights with the **least sum of square error (SSE)**.
- In other words, try to minimize:

$$\text{SSE} = \sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} \left(y^{(i)} - \hat{y}^{(i)}\right)^2$$

Introduction to Linear Regression

- In case the target variable y is correlated to several variable, we can express the predicted value as the following expression giving a plane or hyperplane rather than a line.



$$\hat{y} = w_0 + w_1 x_1 + \cdots + w_m x_m$$

- In such situation, we need to use a **multiple regression model**.

Building a Linear Regression Model

- In scikit-learn, there is a **linear regression model using the least square error method**.
- We will revisit the Boston house price example to illustrate the techniques of such model.
- As we have found from the previous session, the average number of rooms (rm) has the highest positive correlation with the house price (medv).

Building a Linear Regression Model

- To create a simple regression model, we regard rm as one column in X and medv as y, extract the related columns from the DataFrame.
- Notice that X is a DataFrame which might contain one or more than one columns.
- For a simple linear regression model, we are only using one variable.
- On the other hand, y is a Series which is the only target variable to be predicted.

```
import pandas as pd
df = pd.read_csv('boston.csv')
X = df[['RM']]
y = df['MEDV']
```

Measures of Association

```
import pandas as pd

# Load the dataset
df = pd.read_csv(r'C:\Users\User user\Desktop\Boston.csv')

# Extract the 'rm' column as the feature
X = df[['rm']]

# Extract the 'medv' column as the target
y = df['medv']
```

Building a Linear Regression Model

- Create a linear regression model by `LinearRegression` in scikit-learn, then **use the fit method** to train the model using the variable X and target y.

```
from sklearn.linear_model import LinearRegression
slr = LinearRegression()
slr.fit(X, y)
```

- After evaluation, the slope w_1 and the y-intercept w_0 can be found by `coef_` and `intercept_` under the object.
- Notice that `coef_` is an array of numbers as there can be more than one variables w_1, w_2, w_3, … in a multiple regression model.

```
slr.intercept_
```

```
-34.67062077643854
```

```
slr.coef_
```

```
array([9.10210898])
```

Measures of Association

```
from sklearn.linear_model import LinearRegression

# Create an instance of the LinearRegression model
slr = LinearRegression()

# Fit the model to the data
slr.fit(X, y)


slr.intercept_


slr.coef_
```

Building a Linear Regression Model

- Therefore we have $w_0 \approx -34.67$, $w_1 \approx 9.1$.
- The regression line is given by: $\hat{y} = -34.67 + 9.1x$

  where $\hat{y}$ is the predicted median house price and x is the average number of rooms.
- With this regression model, we can **evaluate all the predict values** $\hat{y}^{(i)}$ directly using `predict()`:

```
y_pred = slr.predict(X)
```

- One may also make prediction of a single value.
- For example, the expected price (in $1000) of a house with 5 rooms is given by:

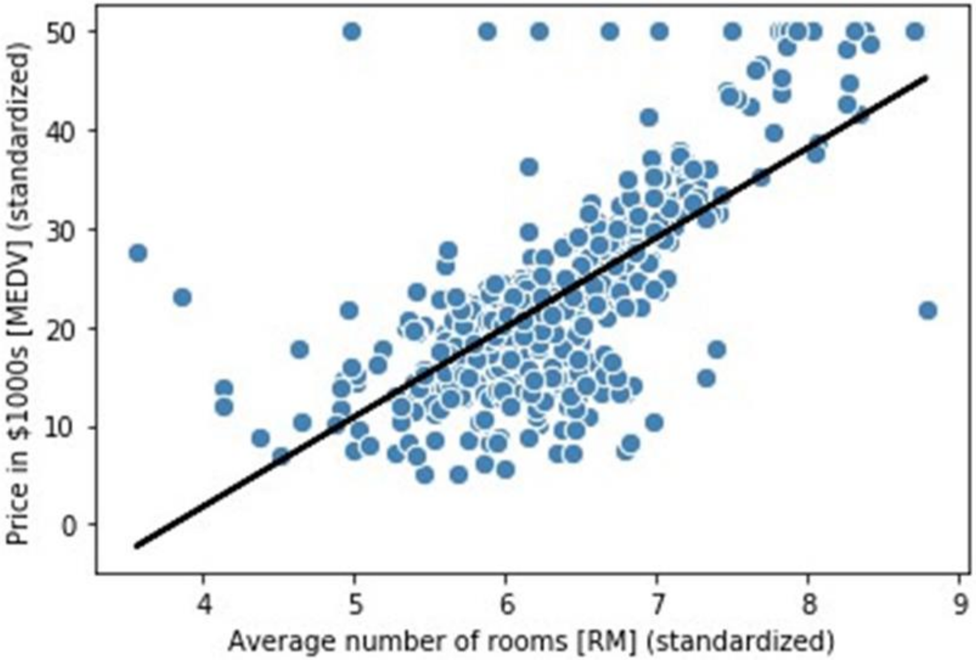
```
slr.predict([[5]])
```

```
array([10.83992413])
```

Building a Linear Regression Model

```
import matplotlib.pyplot as plt
plt.scatter(X, y, c='steelblue', edgecolor='white', s=70)
plt.plot(X, slr.predict(X), color='black', lw=2)
plt.xlabel('Average number of rooms [RM] (standardized)')
plt.ylabel('Price in $1000s [MEDV] (standardized)')
plt.show()
```

- To visualize the data and also the regression, show the line graph and scatter plot:

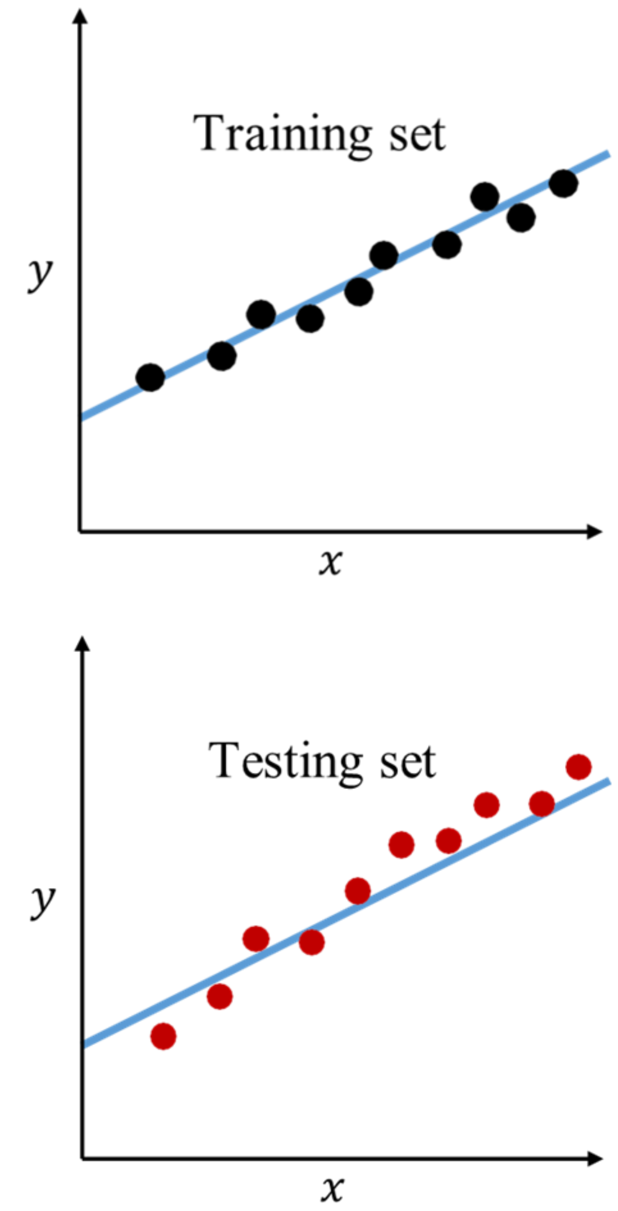


Measures of Association

```
import matplotlib.pyplot as plt

# Create a scatter plot of the data points
plt.scatter(X, y, c='steelblue', edgecolor='white', s=70)

# Plot the regression line
plt.plot(X, slr.predict(X), color='black', lw=2)

# Set the labels for the axes
plt.xlabel('Average number of rooms [rm] (standardized)')
plt.ylabel('Price in $1000s [medvv] (standardized)')

# Display the plot
plt.show()
```

Performance of Linear Regression Model

- In the previous section, we have introduced how to fit a regression model on training data.
- However, just as training a classification model, it is crucial to **test the model on data** that it has not seen during training to obtain a more unbiased estimate of its generalization performance.
- We will use scikit-learn to **split the dataset into training data and test data, and compare their performance**.



Performance of Linear Regression Model

- With `train_test_split` we can split both X and y into training data and test data:

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(
    X, y, test_size=0.3, random_state=0)
```

- The linear regression model is then trained with only the training data:

```
from sklearn.linear_model import LinearRegression
slr2 = LinearRegression()
slr2.fit(X_train, y_train)
```

- After the model is trained, apply it to X_train and X_test separately to make prediction:

```
y_train_pred = slr2.predict(X_train)
y_test_pred = slr2.predict(X_test)
```

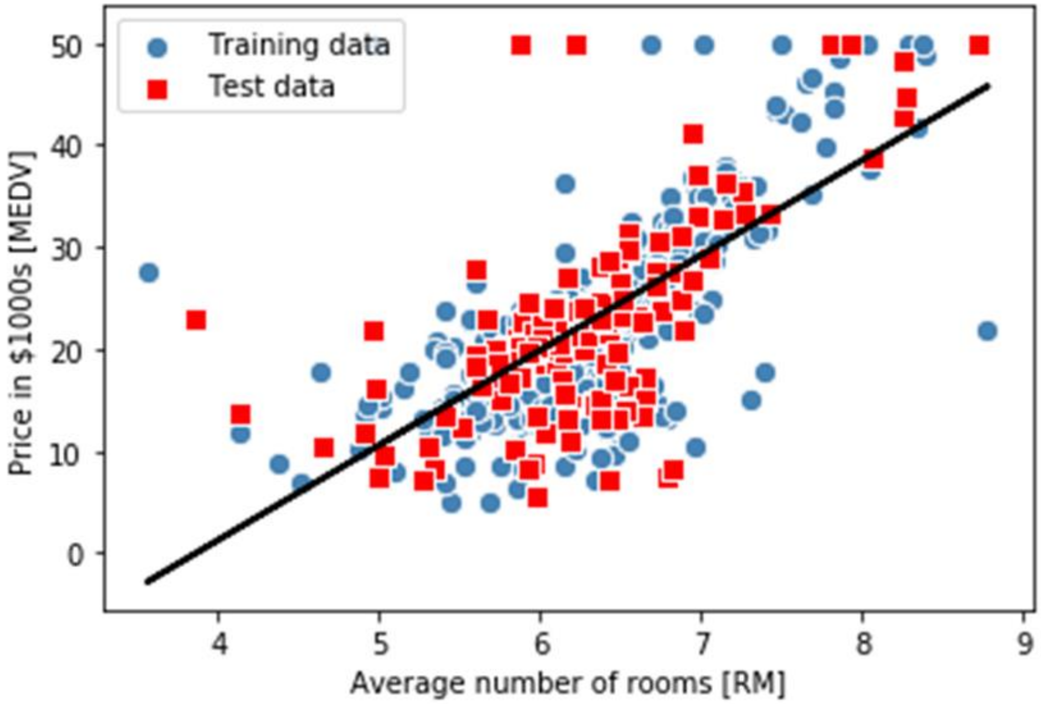
Measures of Association

```
from sklearn.model_selection import train_test_split

# Split the data into training and testing sets
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.3, random_state=0)
```

Measures of Association

```
from sklearn.linear_model import LinearRegression

# Create an instance of the LinearRegression model
lr2 = LinearRegression()

# Fit the model to the training data
lr2.fit(X_train, y_train)

# Make predictions on the training and testing data
y_train_pred = lr2.predict(X_train)
y_test_pred = lr2.predict(X_test)
```

```
plt.scatter(X_train, y_train, c='steelblue', marker='o',
            edgecolor='white', s=70, label='Training data')
plt.scatter(X_test, y_test, c='red', marker='s',
            edgecolor='white', s=70, label='Test data')
plt.plot(X_train, y_train_pred, color='black', lw=2)
plt.legend(loc='upper left')
plt.xlabel('Average number of rooms [RM]')
plt.ylabel('Price in $1000s [MEDV]')
plt.show()
```



- To compare, create scatter plot of the training data and test data with different styles, and also the line graph of the regression model on the same figure.

Measures of Association

```
import matplotlib.pyplot as plt

# Scatter plot for training data
plt.scatter(X_train, y_train, c='steelblue', marker='o', edgecolor='white', s=70,
label='Training data')

# Scatter plot for test data
plt.scatter(X_test, y_test, c='red', marker='s', edgecolor='white', s=70, label='Test data')

# Plot the regression line
plt.plot(X_train, y_train_pred, color='black', lw=2)
```

Measures of Association

```
# Add legend and labels
plt.legend(loc='upper left')
plt.xlabel('Average number of rooms [RM]')
plt.ylabel('Price in $1000s [MEDV]')

# Display the plot
plt.show()
```

Performance of Linear Regression Model

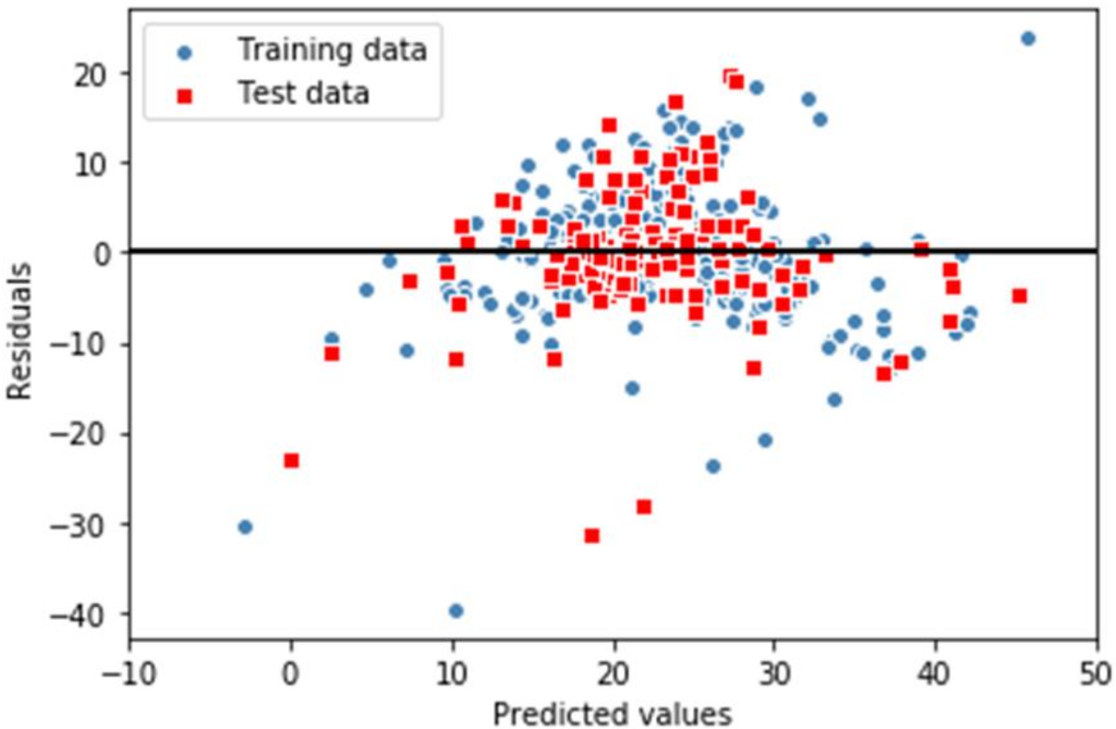
- The figure, however, is not very effective in visualizing the performance of the model.
- Moreover, if it is a multiple regression model with variables x_1, x_2, x_3, … it would be impossible to visualize it on a 2D figure.

- To visualize the performance of a regression model, we can use **residual plot**.
- The **residual** of a point refers to the difference between its actual and predicted values, i.e. $\hat{y}^{(i)} - y^{(i)}$.
- It is also referred as **error** of the prediction at a point.

```python
plt.scatter(y_train_pred,  y_train_pred - y_train,
            c='steelblue', marker='o', edgecolor='white',
            label='Training data')
plt.scatter(y_test_pred,  y_test_pred - y_test,
            c='red', marker='s', edgecolor='white',
            label='Test data')
plt.xlabel('Predicted values')
plt.ylabel('Residuals')
plt.legend(loc='upper left')
plt.hlines(y=0, xmin=-10, xmax=50, color='black', lw=2)
plt.xlim([-10, 50])
plt.show()
```

- Notice that the residual can be **positive or negative**, mean that the model **over-estimate or under-estimate** the actual value.

- The case of **zero** residual means **the prediction about this point is exact**.
- Residual plot refers to the scatter plot of the residuals against predicted values.
- Despite the number of variables in X, the residual plot is always a 2D visualization.



Measures of Association

```
import matplotlib.pyplot as plt

# Scatter plot for training data residuals
plt.scatter(y_train_pred, y_train - y_train_pred, c='steelblue', marker='o',
edgecolor='white', label='Training data')

# Scatter plot for test data residuals
plt.scatter(y_test_pred, y_test - y_test_pred, c='red', marker='s', edgecolor='white',
label='Test data')
```

Measures of Association

```
# Add labels and legend
plt.xlabel('Predicted values')
plt.ylabel('Residuals')
plt.legend(loc='upper left')

# Plot a horizontal line at y=0
plt.hlines(y=0, xmin=-10, xmax=50, color='black', lw=2)

# Set x-axis limits
plt.xlim([-10, 50])

# Display the plot
plt.show()
```

Performance of Linear Regression Model

- In the case of a perfect prediction, the residuals would be exactly zero, which we will probably never encounter in realistic and practical applications.

- However, for a good regression model, we would **expect the errors to be randomly distributed** and **the residuals to be randomly scattered around the centreline**.

- If we see patterns in a residual plot, it means that our model is unable to capture some explanatory information, which has leaked into the residuals.

Performance of Linear Regression Model

- Another useful quantitative measure of a model's performance is the **mean squared error (MSE)** which is simply the averaged value of the SSE.
- The MSE is useful for **comparing different regression models** or for tuning their parameters via grid search and cross-validation, as it normalizes the SSE by the sample size:

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(\hat{y}^{(i)} - y^{(i)}\right)^2$$

Performance of Linear Regression Model

- Compute MSE of the training and test predictions in our example:

```
from sklearn.metrics import mean_squared_error
```

```
mean_squared_error(y_train, y_train_pred)
```

```
42.1576508631225
```

```
mean_squared_error(y_test, y_test_pred)
```

```
47.0330474797551 8
```

Performance of Linear Regression Model

- However, please be aware that the MSE is unbounded in contrast to the classification accuracy, for example.
- In other words, **the interpretation of the MSE depends on the dataset and feature scaling**.

- For example, if the house prices were presented as multiples of 1,000, the same model would yield a lower MSE compared to a model that worked with unscaled features.

Performance of Linear Regression Model

- Thus, it may sometimes be more useful to report the **coefficient of determination (R^2)**, which can be understood as a standardized version of the MSE, for better interpretability of the model's performance.
- In other words, R^2 is the fraction of response variance that is captured by the model. The mathematical expression is as follows:

$$R^2 = 1 - \frac{\text{SSE}}{\text{SST}} = 1 - \frac{\frac{1}{n}\sum_{i=1}^{n}\left(y^{(i)} - \hat{y}^{(i)}\right)^2}{\frac{1}{n}\sum_{i=1}^{n}(y^{(i)} - \bar{y})^2} = 1 - \frac{\text{MSE}}{Var(y)}$$

  - SST means total sum of squares
  - **Notice that R^2 is a value in between 0 to 1. If the error is smaller, value of R^2 become bigger. In case of perfect fit, all the errors are 0 resulting $R^2 = 1$.**

Performance of Linear Regression Model

- To computer R^2 using scikit-learn:

```
from sklearn.metrics import r2_score
```

```
r2_score(y_train, y_train_pred)
```

```
0.502649763004827
```

```
r2_score(y_test, y_test_pred)
```

```
0.435143648321519З
```

- We can see that the value of R^2 of the training data is **higher** than that of the test data. This suggests our regression model **gives less error** for the training data.

Classwork

- The file "hr.csv" contains the human resource record of a company with 30 staff, showing their experience (years) working in the company and annual salary (USD).
  a. Read the file as a DataFrame.
  b. Create a linear regression model with experience as the feature variable and salary as the target variable. Find the unknown weights in this regression line $\hat{y} = w_0 + w_1 x$.
  c. Suppose an employee has been working for 10 years in this company, predict his/her salary.
  d. Show the scatter plot and regression line on the same graph.

Checklist

- Can you:
  1. Describe the application of AI in engineering?
  2. Use measures of association?
  3. Describe how two variables are related to each other?
  4. Show correlations between each pair of variables?
  5. Introduce linear regression?
  6. Build and test a multiple regression model?