Student Name: Jingshu LI
Matriculation Number: A0262438B
(Task A3, using python)

In this task, the data sets used are numerical solutions of Burger's equation (grid method) and solutions approximated using physics-informed neural networks (non-grid method) (github.com/JasonLeeJSL/CS5346_23sp_OTOT/blob/main/OTOT_A3/burgers.mat, github.com/JasonLeeJSL/CS5346_23sp_OTOT/blob/main/OTOT_A3/burgers_pre.mat), which is generated last month for further research on physics-informed neural networks. The general form of Burger's equation is shown below. The data set contains spatial grid, time step, physical parameters and other information. Burgers equation is a partial differential equation, so the numerical solution needs to be performed on a discrete spatial grid. Within the value range of x and t, the data set collects the ground truth and the output values of the neural network on the grid nodes. Both boundary and initial conditions are recorded in this data set. The data set is stored in binary form.

$$\partial_t u + u\,\partial_x u - \left(\frac{0.01}{\pi}\right)\partial_{xx} u = 0, \qquad (t,x) \in (0,1] \times (-1,1)$$
$$u(0,x) = -\sin(\pi x), \qquad x \in [-1,1]$$
$$u(t,-1) = u(t,1) = 0, \qquad t \in (0,1]$$

Visualization of this dataset satisfies query for identifying stiff points in differential equations. At the same time, the difference between traditional method and machine learning method can be compared. In addition, it can help users have an overall understanding of Burger's equation. In more detail, first of all, visualization helps to check the validity and accuracy of the dataset. With visualization, users can see how the numerical solutions are distributed on different time steps and spatial grids, and whether the solutions satisfy the boundary conditions. This can help users determine whether the data set is suitable for a particular analysis or model training task. Meanwhile, in terms of understanding the behavior of Burgers equation, visualization can help users better understand the behavior of Burgers equation. Users can view the evolution of solutions at different time steps and observe how nonlinear terms affect the morphology and propagation of solutions. Then, visualization also benefits data set feature extraction: Visualization helps users extract useful data set features. For example, you can look at how solutions at different points in space change at different time steps to extract information about the spatial and temporal dependence of solutions. These features can be used in machine learning models. Finally, for model evaluation, visualization can help users evaluate the performance of machine learning models. By visualizing the difference between the predicted solution and the real solution, the user can determine where there are errors or inaccurate predictions.

Figure 1 shows the train data selected from the dataset. For training physics-informed neural network, only boundary and initial condition data are needed. The t-axis represents the time and x-axis represents the spatial location. Since the boundary condition is always zero, they are in the same color. And for initial condition represent by ‘x’, different colors represent different value, as shown in the legend. And the internal gray points are points using for computing force term of the equation within the domain. Since the force term is always zero, in training, we do not care about the values of the domain points. So the locations of the domain points are the only thing we need. This visualization can help user better understand the training process of physics-informed neural networks and the train data for PINNs. It visualize the location, the value of the desired points.

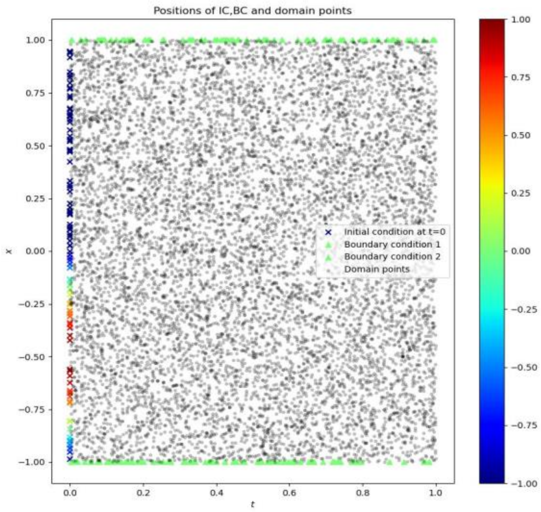

Figure 1. Train data visualization

In Figure 2, there are two parts of the visualizations. The left two heat maps show the ground truth of the Burger’s equation’s solution and the approximation from physics-informed neural networks. There are two axis and different colors are used to represent the value at each point. They can help user to compare the ground truth and the approximation value. The left four charts compare the ground truth and the approximation in different time step. The x-axis represents the location and u-axis represents the value. These charts can also show how the solution changes along the time.

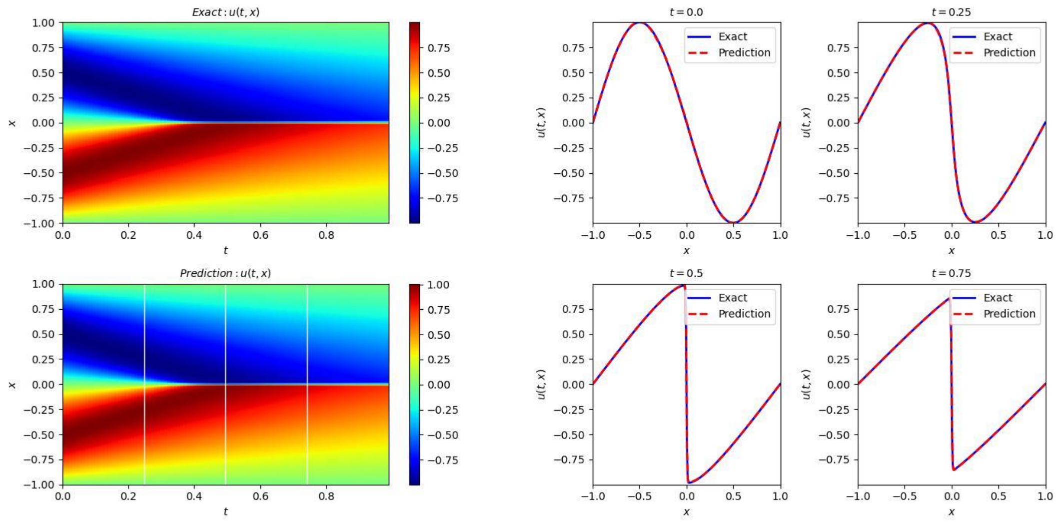

Figure 2. The heat map and line charts comparing the ground truth and approximation

In Figure 3, the difference (i.e., the error) between the ground truth and the approximation are shown. Also, the two axis are time and location. And the color varies from blue to red as the error becomes larger. This figure can directly show the error's magnitude and the distribution of the error and help users to understand the machine learning model better.

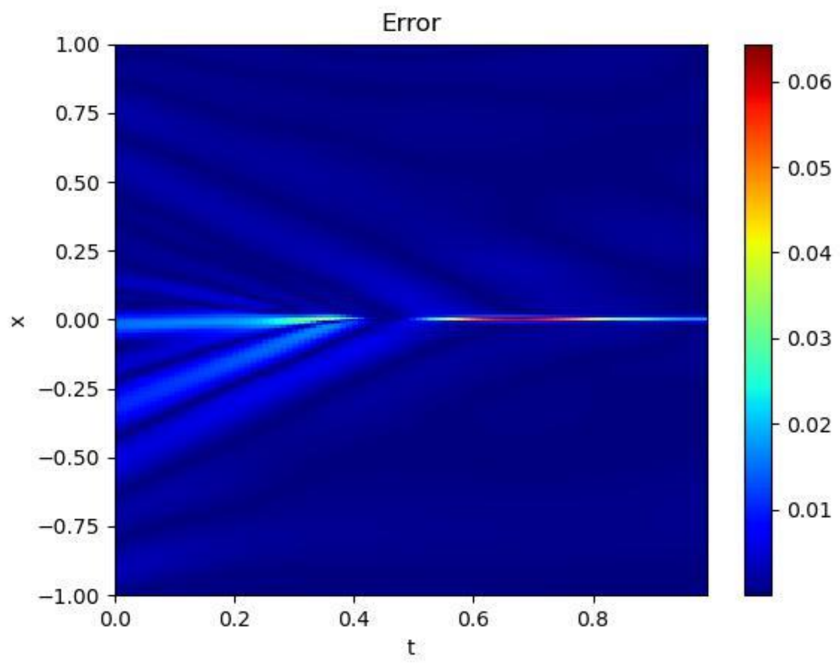

Figure 3. The heat map of the error

Figure 4 and Figure 5 show the shape of the solution and the error more directly in 3D. The x and t axis represent the location and the time, the vertical axis represents the value in Figure 4 and error in Figure 5. With these two visualizations, users can see the solution and how good the machine learning method is more directly and easily.

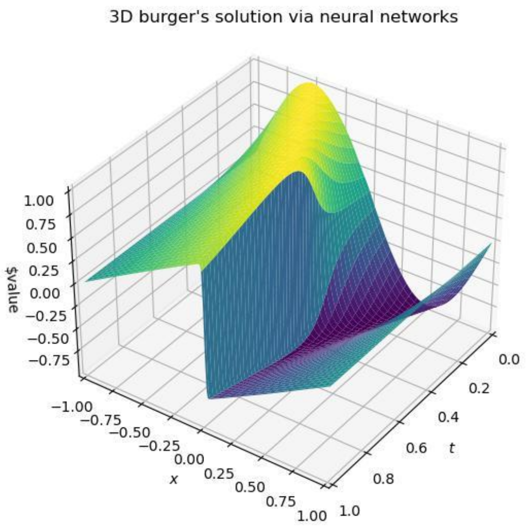

Figure 4. 3D visualization of the solution of Burger's equation

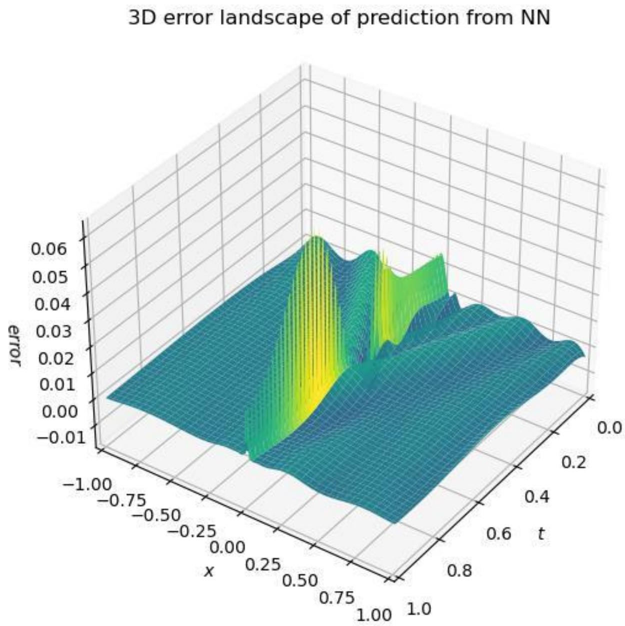

Figure 5. 3D visualization of the error of the PINNs approximation

The link to the visualizations (my github repo): https://github.com/JasonLeeJSL/CS5346_23sp_OTOT/tree/main/OTOT_A3.