ARPM: Advanced Risk and Portfolio Management — Advisor: Attilio Meucci

ARPM Project

Professor: Agnes Tourin — *NYU 2018*

0: Data

Stocks: Amazon, Bank of America, Best Buy, Cleveland-Cliffs, Dover Corporation, Twenty-First Century Fox, J.P. Morgan;
Index: the $S\&P500$ index;
Options: 2 Options on the $S\&P500$: a call and a put option, both with strike price = \$ 1407 and expiry date t_{end} = 26-Aug-2013;

1: Assumptions

1. The delta-moneyness implied volatility data is from ARPM database because we can not download the recent implied volatility data are from Bloomberg;
2. The yield curve is flat and constant;
3. The log implied volatility can be fitted by quadratic regression to moneyness given the time and time to maturity;
4. The joint distribution of invariant can be fitted by Gauss Copula, despite of its poor performance of capturing tail dependence;
5. The invariants calculated via linear interpolation from the empirical cumulative distribution according to given probability are accurate;
6. The utility function is risk-averse.

2: Methodology

Step 1 - Risk drivers identification

Collect the risk drivers time series for the stocks and options. For stocks, we calculate the $\ln V_{n,t}$ as the risk drivers. And for options, convert delta-moneyness to m-moneyness for each time-to-maturity and delta. Then calculate the log implied volatility surface as risk drivers by quadratic polynomial regression, which fits the moneyness to implied volatility.

Step 2 - Quest for invariance

Extract the invariants for each class respectively and forecast the evolution of risk drivers time series obtained from the former step. Use $GARCH(1,1)$ to fit invariants of stocks and index because of the great performance of this technique when dealing with stocks. Take first order difference on log implied volatility of options to be its variants because it appears to be a random walk.

Step 3 - Estimation

Estimate the distribution of invariants. For each invariant, get the empirical cumulative distribution functions with flexible probabilities, where the indicator is the smoothed and scored log return of the VIX index. Estimate joint distribution of invariants based on Sklar's Theorem since invariants are independent of time but dependent mutually. Then use Gaussian Copula to fit the model of their joint distribution and generate the distribution of invariants, whose marginal distribution function is variants' empirical distribution function.

$$Unif_i = F_{\boldsymbol{\epsilon}_i}(\boldsymbol{\epsilon}_i), i = 1, \cdots, \bar{i};$$
$$Cop^{Gauss}_{\boldsymbol{\epsilon}}(u_1, \cdots, u_{\bar{i}}) = \boldsymbol{\Phi}_{\rho^2}(\boldsymbol{\Phi}^{-1}(u_1), \cdots, \boldsymbol{\Phi}^{-1}(u_{\bar{i}})),$$
$$\text{where } \mu_{1\times\bar{i}} = (0, \cdots, 0) \text{ and, } \rho^2_{\bar{i}\times\bar{i}} = \begin{pmatrix} 1 & & \rho^{\text{Spearman}}_{i,j} \\ & \ddots & \\ \rho^{\text{Spearman}}_{2,1} & & 1 \end{pmatrix}$$

Step 4 - Projection

According to the Copula distribution function from step 3, we produce the vector of random numbers from multivariate normal distribution by the covariance matrix, which is computed via Spearman correlation coefficients for each pair of invariants. Those vectors are the inverse standard Normal of $F_{\boldsymbol{\epsilon_i}}(\boldsymbol{\epsilon_i})$, the empirical cumulative distributions of each invariant. Based on that we produce vectors of random numbers from our invariants' joint distribution.
Because our estimation is based on Copula method, we use Monte Carlo simulations to generate 5000 scenarios. Each scenario projects the path of each asset's risk driver from now along the 40-day horizon. The final output is a (5000*41*98) 3-dimension array.

Figure (1)

Figure (2)

The Figure 1 is the risk drivers time series of Stocks AMZN, which is its log value.

The Figure 2 is the path distributions of the risk drivers of Option of S&P500 (given that time to maturity = 0.082, moneyness = -0.197, which contains 125 samples and there is 40 days between now and investment horizon.

Step 5 - Pricing

Use as input the array of risk driver projection from the Projection step. A desired output is a PnL table of each of the 10 assets for 5000 simulations. The methodology is as followed. The PnL of stocks and index is computed by exponential of projected risk driver, namely log(stock price), which would just equal the future price, and then subtracting the price now.

$$V_{n,t_{hor}} = v_{n,t_{now}} \exp(\ln V_{n,t_{hor}} - \ln v_{n,t_{now}}) = v_{n,t_{now}} \exp(X_{n,t_{hor}} - x_{n,t_{now}}) \qquad n = 1, 2$$

The PnL of options makes use of Black-Scholes pricing formula in which the volatility is calculated by an interpolation of log-implied volatility, and then subtracting the price now.

Step 6 - Aggregation

Evenly distribute a fixed budget to all stocks and calculate the holdings of each one. Assume 10 holdings in S&P 500 index and one holding in each option. The ex-ante performance is measured by the linear combination of PnL of each asset and respective holdings.

$$\boldsymbol{Y}_{\mathbf{h},t_{\mathrm{now}}\to t_{\mathrm{hor}}} = \boldsymbol{h}^T \times \boldsymbol{\Pi}_{t_{\mathrm{now}}\to t_{\mathrm{hor}}}$$

Step 7 - Ex-ante evaluation

Using the ex-ante performance as input, calculate the expected utility and certainty-equivalent with exponential utility function.

$$\text{utility}(y) \equiv -e^{-\lambda y}, \qquad \mathbb{E}\{\text{utility}(\Pi_h)\} = \sum_{j=1}^{\bar{j}} p^{(j)}(-\exp(-\lambda \pi_{\boldsymbol{h}}^{(j)})), \qquad \mathbb{C}ert\_eq\{\Pi_{\boldsymbol{h}}\} = -\frac{1}{\lambda}\ln(-\mathbb{E}\{\text{utility}(\Pi_{\boldsymbol{h}})\})$$

Suppose we want 95% VaR, then calculate the quantile index of satisfaction value corresponding to 95% quantile of ex-ante performance.

$$\mathbb{S}atis\{Y\} \equiv q_Y(1-c)$$

Compute the 95% cVaR by averaging the lowest 5% quantile, and then find its variance index of satisfaction.

$$\mathbb{ES}_c\{\,\boldsymbol{Y}\} \equiv \frac{1}{1-c}\int_0^{1-c} q_Y(u)du = \mathbb{E}\{\,\boldsymbol{Y}\,|\,\boldsymbol{Y} \leq q_Y(1-c)\},$$

Step 8 - Ex-ante attribution

Fit the attribution factors $\boldsymbol{Z}$ and ex-ante performance, i.e. the PnL matrix-multiplied by holdings, into a Lasso Regression, so as to select around 10 most relevant factors from around 100 factors and their respective loadings.

$$\widehat{\beta}_\lambda \equiv \operatorname{argmin}_{\boldsymbol{b}} \sum_{t=1}^{\bar{t}} p_t ||\boldsymbol{x}_t - \boldsymbol{b}\boldsymbol{z}_t||_2^2 + \lambda||\boldsymbol{b}||_1$$

The goal of selecting factors is to decompose the satisfaction into additive contributions associated with these risk factors. Marginal contribution to the variance index of satisfaction can be saved into a vector.

Step 9 - Construction

Solve for the optimal holdings h_λ that maximize the objective function under given constraints, such as only long position allowed. The objective function, mean-variance efficient frontier, is written below. For a range of λ, pick the allocation on the frontier that gives rise to the highest satisfaction (i.e. 95% cVaR under the optimized holdings h_λ).

$$\boldsymbol{h}_\lambda \equiv \operatorname{argmax}_{h\in\mathscr{C}}\{\mathbb{E}\{\boldsymbol{Y}_{\boldsymbol{h}}\} - \lambda\mathbb{V}\{\boldsymbol{Y}_{\boldsymbol{h}}\}\} \;\text{ where }\; \mathbb{E}\{\boldsymbol{Y}_{\boldsymbol{h}}\} = \boldsymbol{h}^T\mathbb{E}\{\boldsymbol{\Pi}\}, \mathbb{V}\{\boldsymbol{Y}_{\boldsymbol{h}}\} = \boldsymbol{h}^T\mathbb{C}v\{\boldsymbol{\Pi}\}\boldsymbol{h}$$

Step 10 - Execution

Derive parent orders needed for trading from the difference between initial holdings and end holdings and conduct the optimized child order scheduling in order to minimize the market impact.

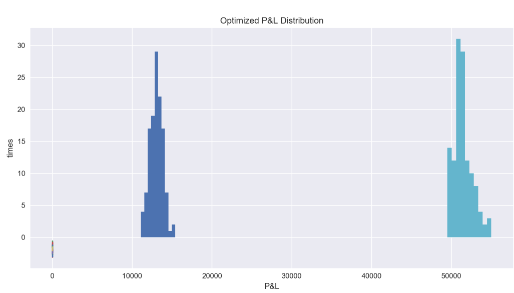

Figure (3)

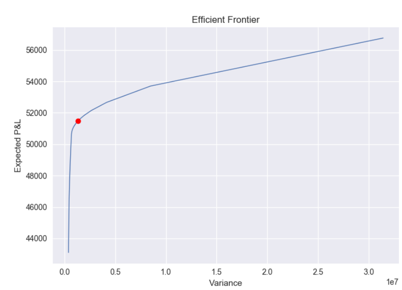

Figure (4)

The Figure 3 is optimized PnL Distribution

The Figure 4 is efficient frontier of portfolio

3: Conclusion

Based on the above steps, we generate the optimized holdings of assets which reach the highest cVaR under mean-variance optimization process. The optimized holdings are to invest 405 shares in Amazon, 42,856 shares in Fox and 56 call option contracts. The expected return of the optimized portfolio is $\$51,536$, generating a 5.15% return in 40 days, and the expected variance is 1,288,279.