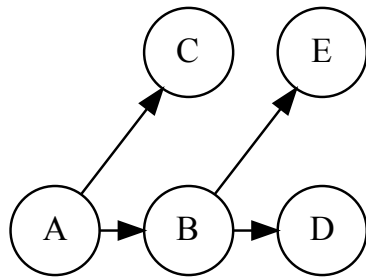


## Probability (2 points)



	P(A)	P(B   A)	+b	-b	P(C   A)	+c	-c
+a	0.25	+a	0.5	0.5	+a	0.2	0.8
-a	0.75	-a	0.25	0.75	-a	0.6	0.4

P(D   B)	+d	-d	P(E   B)	+e	-e
+b	0.6	0.4	+b	0.25	0.75
-b	0.8	0.2	-b	0.1	0.9

1. Using the Bayes net and conditional probability tables above, calculate the following quantities: (2 pts)

(a)  $P(+b | +a) = 0.5$

(b)  $P(+a, +b) = P(+a)P(+b | +a) = \frac{1}{8}$

(c)  $P(+a | +b) = \frac{P(+a | +b)}{P(+a | +b) + P(+a | -b)} = \frac{P(+b | +a)P(+a)}{P(+b | +a)P(+a) + P(+b | -a)P(-a)} = \frac{0.5 * 0.25}{0.5 * 0.25 + 0.25 * 0.75} = \frac{2}{5}$

(d)  $P(-e, +a) = 0.25 * (0.5 * 0.75 + 0.5 * 0.9) = 0.20625$

(e)  $P(D | A) =$

P(D   A)	+d	-d
+a	$\frac{7}{40}$	$\frac{3}{40}$
-a	$\frac{36}{160}$	$\frac{15}{80}$

## Independence (8 points)

2. For each of the following equations, select the minimal set of conditional independence assumptions necessary for the equation to be true. (4 pts)

(a)  $P(A, C) = P(A | B)P(C)$

- ☐  $A \perp\!\!\!\perp B$    
 ☐  $A \perp\!\!\!\perp B | C$    
 ☐  $A \perp\!\!\!\perp C$    
☒  $A \perp\!\!\!\perp C | B$   
☐  $B \perp\!\!\!\perp C$    
☐  $B \perp\!\!\!\perp C | A$    
☐ No independence assumptions needed

(b)  $P(A | B, C) = \frac{P(A)P(B|A)P(C|A)}{P(B|C)P(C)}$

- ☐  $A \perp\!\!\!\perp B$    
☒  $A \perp\!\!\!\perp B | C$    
☐  $A \perp\!\!\!\perp C$    
☐  $A \perp\!\!\!\perp C | B$   
☐  $B \perp\!\!\!\perp C$    
☒  $B \perp\!\!\!\perp C | A$    
☐ No independence assumptions needed

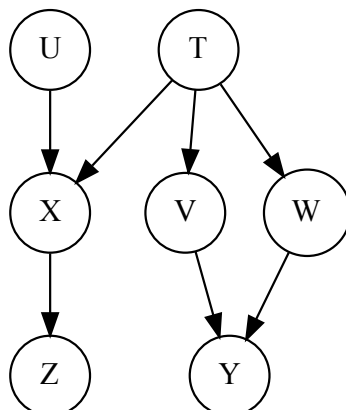
(c)  $P(A, B) = \sum_c P(A | B, c)P(B|c)P(c)$

- ☐  $A \perp\!\!\!\perp B$    
☒  $A \perp\!\!\!\perp B | C$    
☐  $A \perp\!\!\!\perp C$    
☐  $A \perp\!\!\!\perp C | B$   
☒  $B \perp\!\!\!\perp C$    
☐  $B \perp\!\!\!\perp C | A$    
☐ No independence assumptions needed

(d)  $P(A, B | C, D) = P(A | C, D)P(B | A, C, D)$

- ☐  $A \perp\!\!\!\perp B$    
☐  $A \perp\!\!\!\perp B | C$    
☐  $A \perp\!\!\!\perp C$    
☐  $A \perp\!\!\!\perp C | B$   
☐  $B \perp\!\!\!\perp C$    
☐  $B \perp\!\!\!\perp C | A$    
☒ No independence assumptions needed

3. Indicate whether each of the following conditional independence relationships is guaranteed to be true in the Bayes Net below. If the independence relationship does not hold, identify all active (d-connected) paths in the graph. (4 pts)

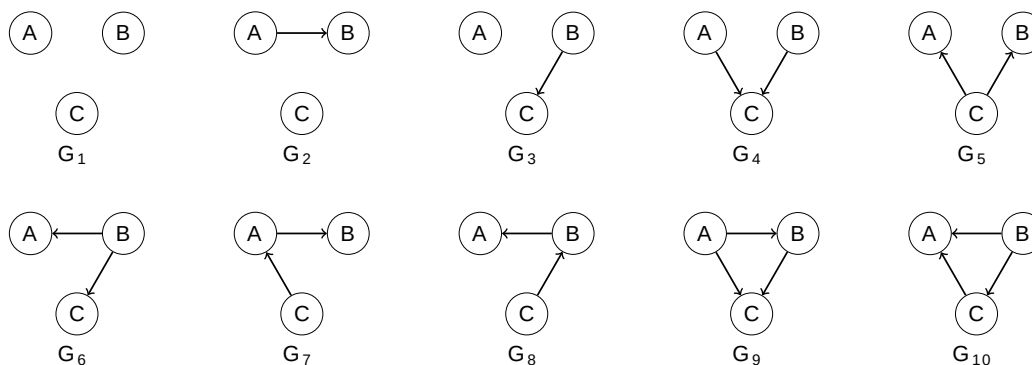


- (a)  $T \perp\!\!\!\perp Y$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$T \rightarrow V \rightarrow Y$*   
 *$T \rightarrow W \rightarrow Y$*
- (b)  $T \perp\!\!\!\perp Y \mid W$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$T \rightarrow V \rightarrow Y$*
- (c)  $U \perp\!\!\!\perp T$   
 Independence is ☒ *Guaranteed* ☐ Not Guaranteed
- (d)  $U \perp\!\!\!\perp T \mid Z$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$((U \rightarrow X) \text{ and } (T \rightarrow X)) \rightarrow Z$  because  $Z$  is a descendant of  $X$  and  $X$  forms a v-structure. While  $Z$  is observed, the v-structure is active.*
- (e)  $Z \perp\!\!\!\perp U$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$U \rightarrow X$*
- (f)  $Z \perp\!\!\!\perp Y \mid V$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$T \rightarrow X \rightarrow Z$*   
 *$T \rightarrow W \rightarrow Y$*   
 *$X \leftarrow T \rightarrow W$*
- (g)  $Z \perp\!\!\!\perp Y \mid T, W$   
 Independence is ☒ *Guaranteed* ☐ Not Guaranteed
- (h)  $Z \perp\!\!\!\perp W$   
 Independence is ☐ Guaranteed ☒ *Not Guaranteed*  
 *$T \rightarrow X \rightarrow Z$*   
 *$X \leftarrow T \rightarrow W$*

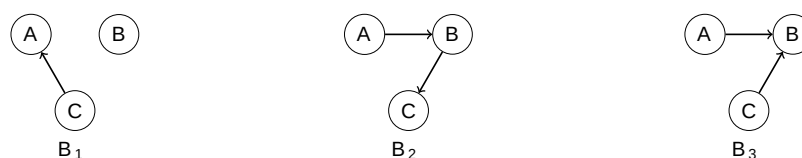
## Representation (4 points)

4. We are given the following ten Bayes nets, labeled  $G_1$  to  $G_{10}$ :

(4 pts)



and the following three Bayes nets, labeled  $B_1$  to  $B_3$ :



(a) Assume we know that a joint distribution  $d_1$  (over  $A, B, C$ ) can be represented by Bayes net  $B_1$ . Mark all of the following Bayes nets that are guaranteed to be able to represent  $d_1$ .

- ☐  $G_1$    ☐  $G_2$    ☐  $G_3$    ☒  $G_4$    ☒  $G_5$   
☐  $G_6$    ☒  $G_7$    ☐  $G_8$    ☒  $G_9$    ☒  $G_{10}$   
☒ None of the above

(b) Assume we know that a joint distribution  $d_2$  (over  $A, B, C$ ) can be represented by Bayes net  $B_2$ . Mark all of the following Bayes nets that are guaranteed to be able to represent  $d_2$ .

- ☐  $G_1$    ☐  $G_2$    ☐  $G_3$    ☐  $G_4$    ☐  $G_5$   
☐  $G_6$    ☐  $G_7$    ☒  $G_8$    ☒  $G_9$    ☐  $G_{10}$   
☐ None of the above

(c) Assume we know that a joint distribution  $d_3$  (over  $A, B, C$ ) **cannot** be represented by Bayes net  $B_3$ . Mark all of the following Bayes nets that are guaranteed to be able to represent  $d_3$ .

- ☐  $G_1$    ☐  $G_2$    ☐  $G_3$    ☐  $G_4$    ☐  $G_5$   
☒  $G_6$    ☐  $G_7$    ☐  $G_8$    ☐  $G_9$    ☒  $G_{10}$   
☐ None of the above

(d) Assume we know that a joint distribution  $d_4$  (over  $A, B, C$ ) can be represented by Bayes nets  $B_1$ ,  $B_2$  and  $B_3$ . Mark all of the following Bayes nets that are guaranteed to be able to represent  $d_4$ .

- ☐  $G_1$    ☐  $G_2$    ☐  $G_3$    ☐  $G_4$    ☐  $G_5$   
☐  $G_6$    ☐  $G_7$    ☐  $G_8$    ☐  $G_9$    ☐  $G_{10}$   
☒ None of the above

## Inference (4 points)

5. Using the same Bayes Net from question 3, we want to compute  $P(Y \mid +z)$ . All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering:  $U, T, X, V, W$ . (4 pts)

Complete the following description of the factors generated in this process:

- (a) After inserting evidence, we have the following factors to start out with:

$$P(U), P(T), P(X \mid U, T), P(V \mid T), P(W \mid T), P(+z \mid X), P(Y \mid V, W)$$

- (b) When eliminating  $U$  we generate a new factor  $f_1$  as follows, which leaves us with the factors:  $f_1(X, T) = \sum_u P(u)P(X \mid u, T)$

$$\text{Factors: } P(T), P(V \mid T), P(W \mid T), P(+z \mid X), P(Y \mid V, W), f_1(X, T)$$

- (c) When eliminating  $T$  we generate a new factor  $f_2$  as follows, which leaves us with the factors:

$$f_2(\dots) =$$

Factors:

- (d) When eliminating  $X$  we generate a new factor  $f_3$  as follows, which leaves us with the factors:

$$f_3(\dots) =$$

Factors:

- (e) When eliminating  $V$  we generate a new factor  $f_4$  as follows, which leaves us with the factors:

$$f_4(\dots) =$$

Factors:

- (f) When eliminating  $W$  we generate a new factor  $f_5$  as follows, which leaves us with the factors:

$$f_5(\dots) =$$

Factors:

- (g) How would you obtain  $P(Y \mid +z)$  from the factors left above?

- (h) What is the size of the largest factor that gets generated during the above process?

- (i) Does there exist a better elimination ordering (one which generates smaller largest factors)? Argue why not or give an example.