

Artificial Neural Networks

[2500WETANN]

José Oramas



Modeling Sequences with Neural Networks

José Oramas



Today's Lecture - Outline

- Intro/Recap Sequence Modeling
 - → [Algorithmically] How to approach the problem
- Sequence Modeling with Recurrent Architectures
 - \rightarrow RNNs, LSTMs, GRUs, etc.
- Predictions from Sequences & Sequence Generation
- Transformers and Attention Mechanisms



Recap: Supervised Image Recognition Task

Given: an input image x

Do: predict a label \hat{y}

(out of a set of class labels)

ILSVRC

flamingo cock ruffed grouse quail partridge ...

Egyptian cat Persian cat Siamese cat tabby lynx ...

miniature schnauzer standard schnauzer giant schnauzer

- Training data

$$\{x,y\}_i$$

- Model

$$\hat{y} \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} l(f_{\theta}(x_i), y_i)$$

- Optimization

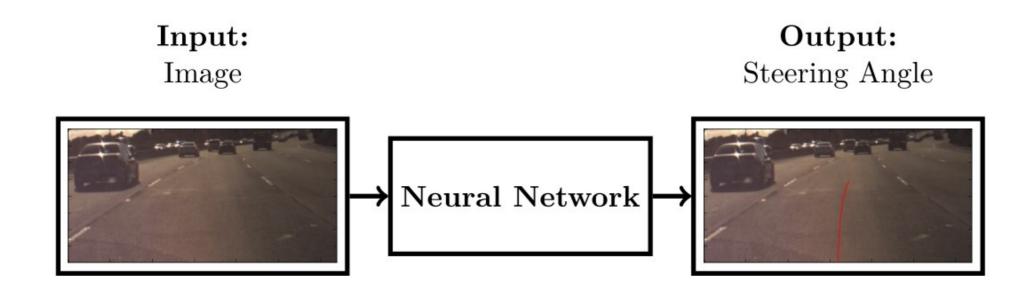
$$\theta^* = arg \ min_{\theta} \ L(\theta)$$



Now: Consider the Following Problem

Autonomous Driving

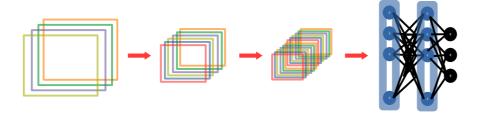
- Given: an input video x (sequence of images)
- Do: predict a label \hat{y} (out of a set of action class labels)





Ok, but...

Can't we use the previous architectures for that?







Ok, but...

Can't we use the previous

architectures for that?

and ...

Are there any other problems that we need to take into account?

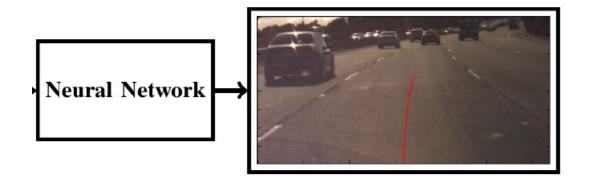


Autonomous Navigation Task

Stacking Elements from the Sequence

Output:

Steering Angle



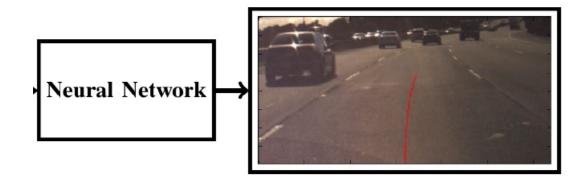


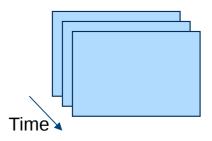
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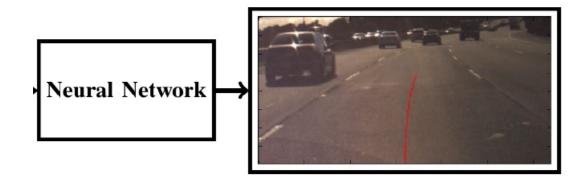


Autonomous Navigation Task

Stacking Elements from the Sequence

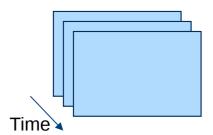
Output:

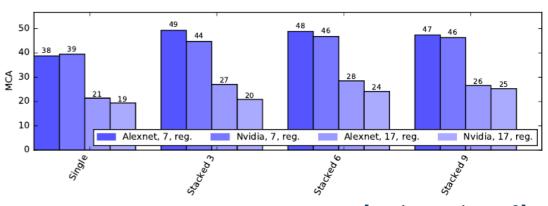
Steering Angle



Observations

- Considering sequence helps
- Reduced gains as we go further... why?





[Heylen et al., 2018]



Modeling Data Sequences

[How to do it... in theory]



Modeling Data Sequences [in theory]

Some Foundations

- Supervised Learning
- Data

$$\{x,y\}_i$$

Model

$$\hat{y} \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} l(f_{\theta}(x_i), y_i)$$

- Optimization

$$\theta^* = arg \ min_{\theta} \ L(\theta)$$



Modeling Data Sequences [in theory]

Some Foundations

- Supervised Learning
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$$\theta^* = arg \ min_{\theta} \ L(\theta)$$

- Modeling Sequences
- Data

$$\{x\}_i$$

- Model

$$p(x) \approx f_{\theta}(x)$$

- Loss

$$L(\theta) = \sum_{i=1}^{N} log \ p(f_{\theta}(x_i))$$

- Optimization

$$\theta^* = arg \ max_{\theta} \ L(\theta)$$

Modeling Data Sequences

[How to do it... in practice]



Modeling Data Sequences [in theory]

Lets consider the following sequence

• Idea: Let's focus on natural language → text

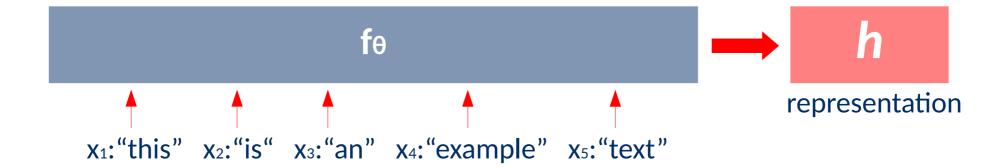
```
["this", "is", "an", "example", "text", "sequence"]
```



Modeling Data Sequences [in practice]

1. Describing/Vectorizing the Context

Idea: Learn how to represent a sub-sequence

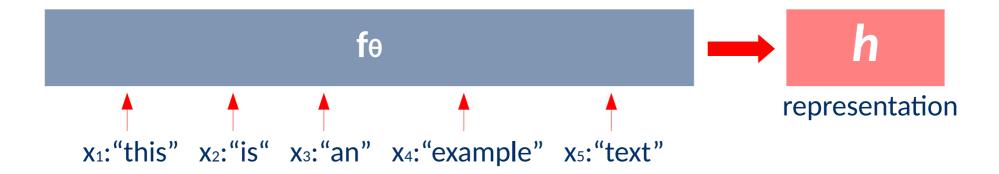




Modeling Data Sequences [in practice]

1. Describing/Vectorizing the Context

Idea: Learn how to represent a sub-sequence



Desirable properties for f_{θ}

- Ranked → order matters
- Variable Length

- Learnable → differentiable
- ' Small changes, large effects → non-linear



Modeling Data Sequences [in practice]

2. Modeling Conditional Probabilities

• Idea: Predicting the next element given the context



• Objective: $p(x_t|h) \approx p(x_t|x_1, \dots, x_{t-1})$

Modeling Data Sequences [in practice]

2. Modeling Conditional Probabilities

• Idea: Predicting the next element given the context



• Objective: $p(x_t|h) \approx p(x_t|x_1, \dots, x_{t-1})$

Desirable properties for ge

- Output a distribution over vocabulary
- Small changes, large effects → non-linear



Recurrent Architectures

[RNNs, LSTMs, GRUs, etc.]



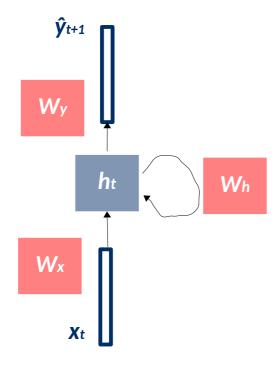
Recurrent Neural Networks

[The Most Popular Architecture]



Provide Neural Networks with Memory

Idea: Use a persistent state h that encodes past observations (context)



Defined by three equations

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

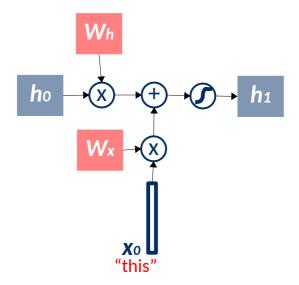
$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$



Provide Neural Networks with Memory

Idea: Use a persistent state h that encodes past observations (context)

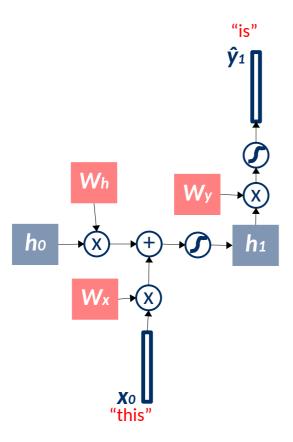


$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$



Provide Neural Networks with Memory

Idea: Use a persistent state h that encodes past observations (context)



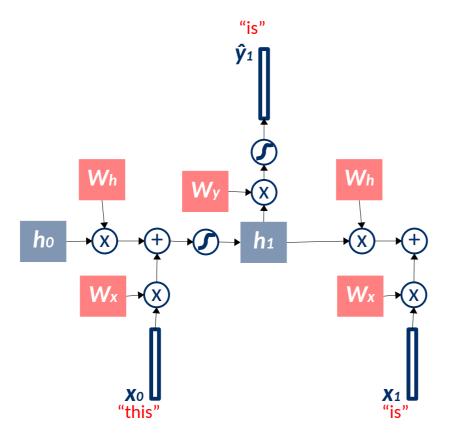
The probability of next element is obtained from the state **h**

$$p(y_{t+1}) = softmax(W_y h_t)$$



Provide Neural Networks with Memory

Idea: Use a persistent state h that encodes past observations (context)

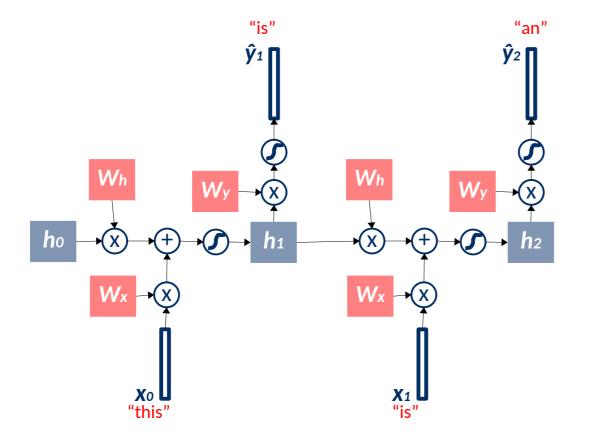


Input the next element **x**₁ from the sequence



Provide Neural Networks with Memory

Idea: Use a persistent state h that encodes past observations (context)

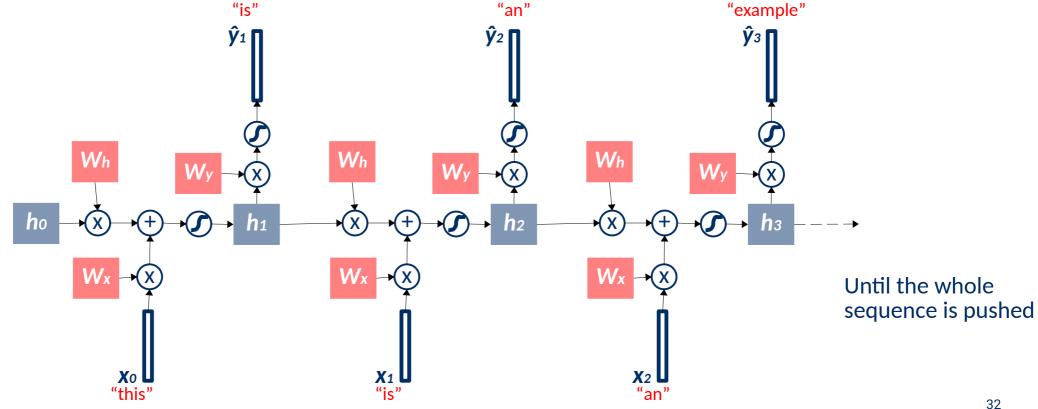


Keep going



Provide Neural Networks with Memory

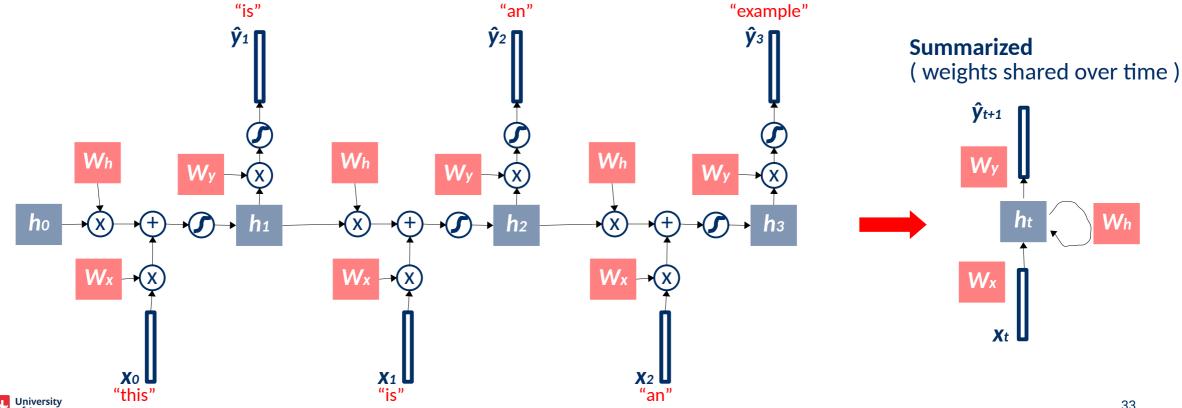
Idea: Use a persistent state **h** that encodes past observations (context)





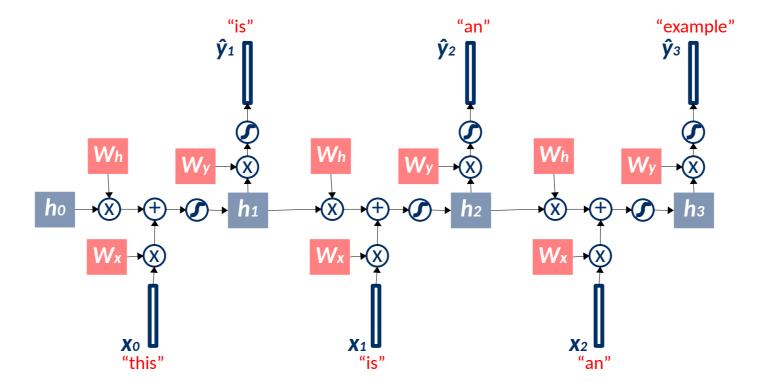
Provide Neural Networks with Memory

Idea: Use a persistent state **h** that encodes past observations (context)



Learning | Training

- Idea: Formulate the next-word prediction as a classification problem
- Number of classes = vocabulary_size → Use the cross-entropy loss.





Learning | Training

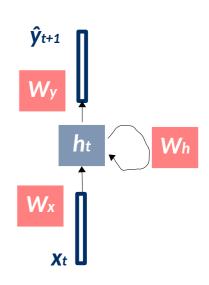
- Idea: Formulate the next-word prediction as a classification problem
- Number of classes = vocabulary_size → Use the cross-entropy loss.

Given a sequence of *T* elements:

For one element
$$t \to L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

For the sequence
$$\rightarrow L_{\theta}(y,\hat{y}) = -\sum_{t=1}^{T} y_t \log \hat{y}_t$$

Trainable parameters
$$\rightarrow \theta = \{W_x, W_h, W_y\}$$





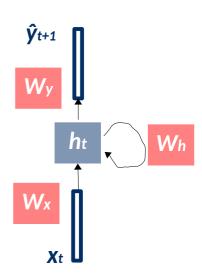
Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

$$h_t = \tanh(W_h h_{t-1} + W_x x_t)$$

$$p(y_{t+1}) = softmax(W_y h_t)$$

$$L_{\theta}(y, \hat{y})_t = -y_t \log \hat{y}_t$$

$$\frac{\partial L_{\theta,t}}{\partial W_y} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial W_y}$$
$$= (y_t - \hat{y}_t)h_t$$





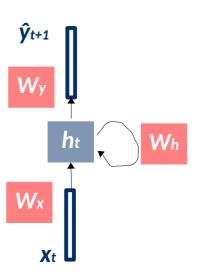
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$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$





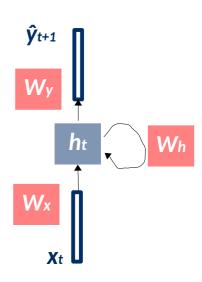
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$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$
$$\frac{\partial h_t}{\partial W_h} = \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h}$$





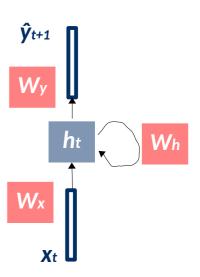
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$$\begin{split} \frac{\partial L_{\theta,t}}{\partial W_h} &= \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h} \\ \frac{\partial h_t}{\partial W_h} &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h} \\ &= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \left[\frac{\partial h_{t-1}}{\partial W_h} + \frac{\partial h_{t-1}}{\partial h_{t-2}} \frac{\partial h_{t-2}}{\partial W_h} \right] \end{split}$$



Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

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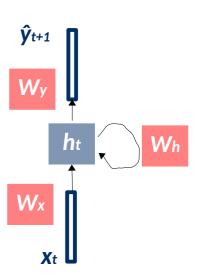
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$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$

$$\frac{\partial h_t}{\partial W_h} = \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \frac{\partial h_{t-1}}{\partial W_h}$$

$$= \frac{\partial h_t}{\partial W_h} + \frac{\partial h_t}{\partial h_{t-1}} \left[\frac{\partial h_{t-1}}{\partial W_h} + \frac{\partial h_{t-1}}{\partial h_{t-2}} \frac{\partial h_{t-2}}{\partial W_h} \right]$$

$$= \sum_{k=1}^{t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$





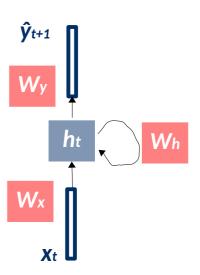
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Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

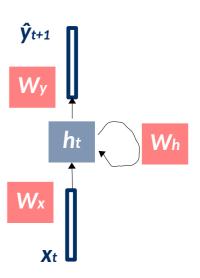
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Differentiating wrt. Wh

$$\frac{\partial L_{\theta,t}}{\partial W_h} = \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial W_h}$$





Back-Prop. - Differentiation wrt. the parameters $\theta = \{W_x, W_h, W_y\}$

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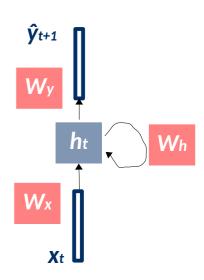
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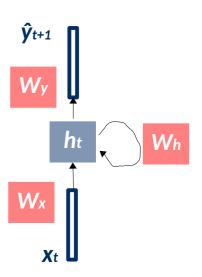
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$$= \sum_{k=1}^t \frac{\partial L_{\theta,t}}{\partial \hat{y}_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial \hat{y}_t}{\partial h_t} \frac{\partial h_t}{\partial h_k} \frac{\partial h_k}{\partial W_h}$$





Summarizing

- + Good at modeling sequences of variable length
- + Trainable via Back-Prop. → differentiable

Suffer from vanishing gradients for long sequences



Long Short-Term Memory Networks

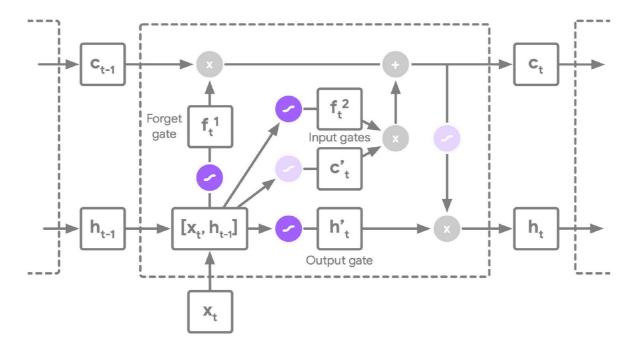
[RNNs with Extra Memory]



[Hochreiter & Jurgen , 1997]

Provide the capability of choosing what to remember/forget

• Idea: Provide special gates to control the flow of "memories"

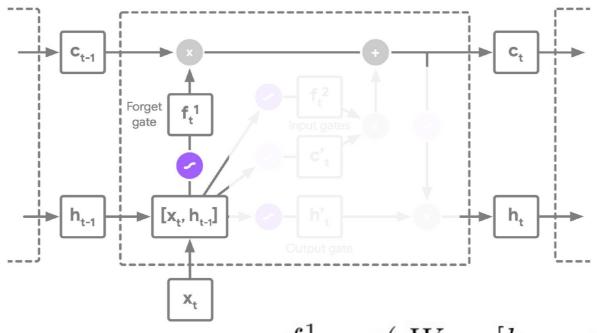




[Hochreiter & Jurgen , 1997]

Provide the capability of choosing what to remember/forget

• f1: Forget Gate



regulate what information to keep/ignore

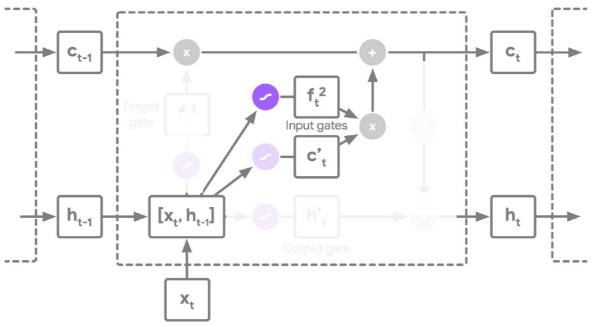
$$f_t^1 = \sigma(W_{f^1} \cdot [h_{t-1}, x_t] + b_{f^1})$$



[Hochreiter & Jurgen , 1997]

Provide the capability of choosing what to remember/forget

• f₂: Input Gate



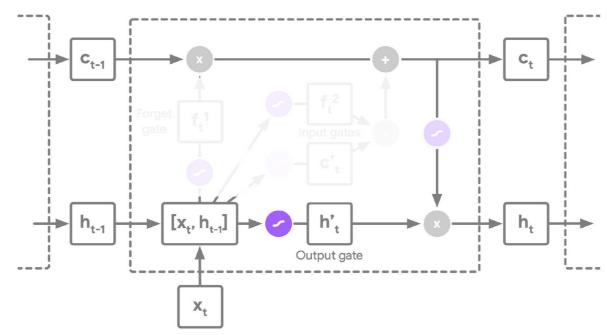
Decides what information to update

$$= \sigma(W_{f^2} \cdot [h_{t-1}, x_t] + b_{f^2}) \odot \tanh(W_{c'}[h_{t-1}, x_t] + b_{c'})$$

[Hochreiter & Jurgen , 1997]

Provide the capability of choosing what to remember/forget

h't: Output Gate



$$= \sigma(W_{h'_t} \cdot [h_{t-1}, x_t] + b_{h'_t}) \odot \tanh(c_t)$$



Gated Recurrent Units

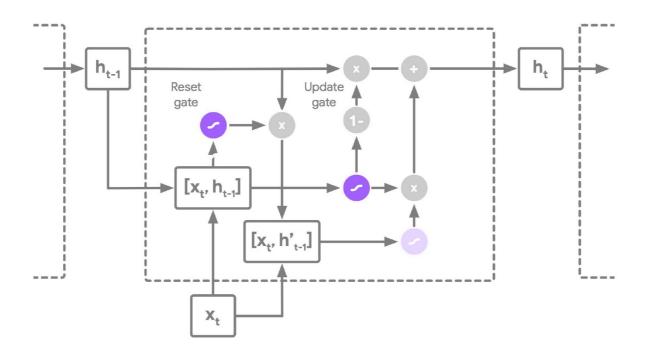
[A Simplified LSTM]



Gated Recurrent Units (GRUs) [Cho et al., 2014]

A Simplifed LSTM Network

• Idea: Provide special gates to control the flow of "memories"





LSTMs and GRUs

Summarizing

- + Good at modeling sequences of variable length
- + Trainable via Back-Prop. → differentiable

+ Capable of handling long sequences

(robust to vanishing/exploding gradients)



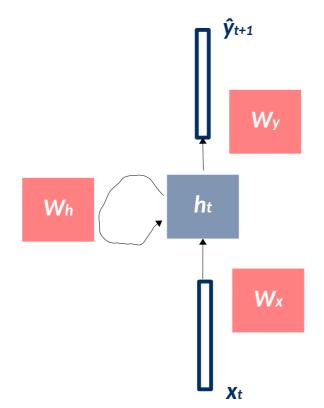
Predictions from Sequences



Predictions from Sequences

Training Classifiers/Regressors from Sequences

- 1) Attach a related head (classification, regression, etc.) to the persistent state
- 2) Measure the loss wrt. the prediction task

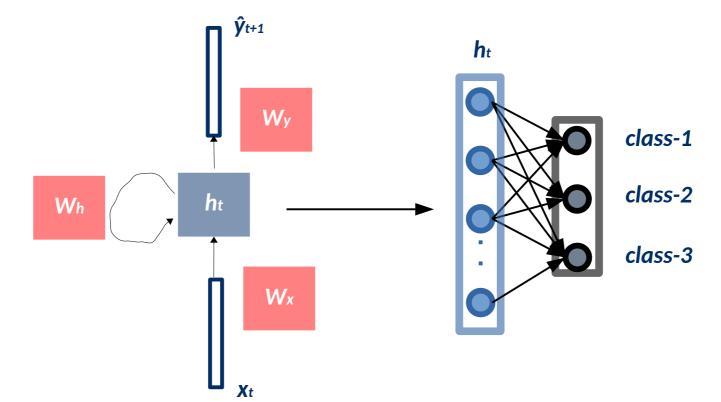




Predictions from Sequences

Training Classifiers/Regressors from Sequences

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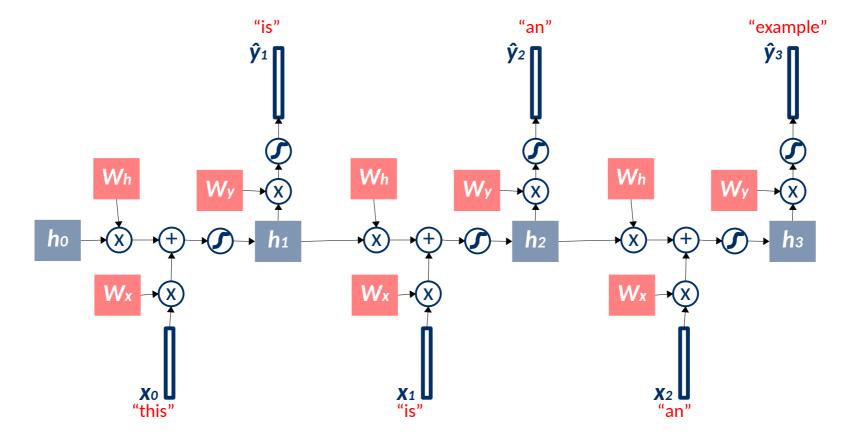
Generating Sequences



Generating Sequences

Sample the next best element from the predicted distribution

• Idea: Use the predicted element \hat{y}_t as input in the next iteration

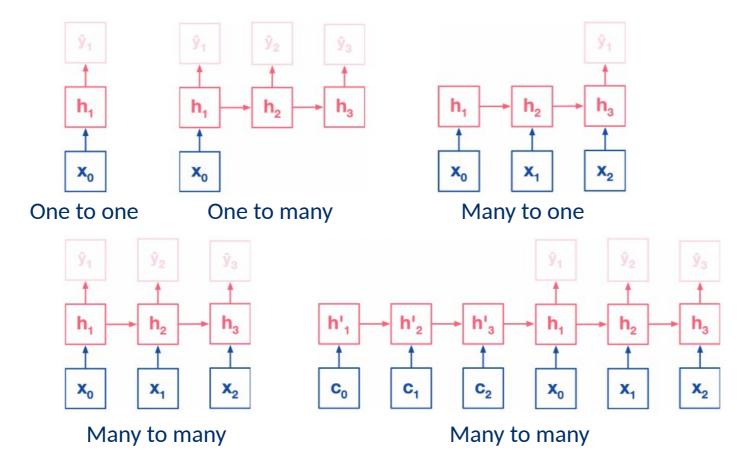




Generating Sequences

Several options are possible - beyond text sequences

Idea: Different ways to define inputs, context and outputs



Some Applications

- Language Translation
- Speech-to-Text
- Contextual Search
- Image Captioning

Break

[Let's meet again in 15 mins.]



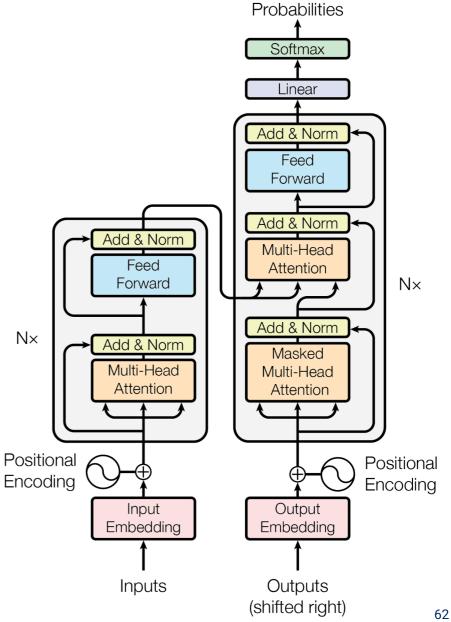
Transformers



Transformers [Vaswani et al., 2017]

Some specs

- Removed Recurrence components
- Based solely on the attention mechanism
- Originally addressed translation tasks [English-German | English-French]



Output



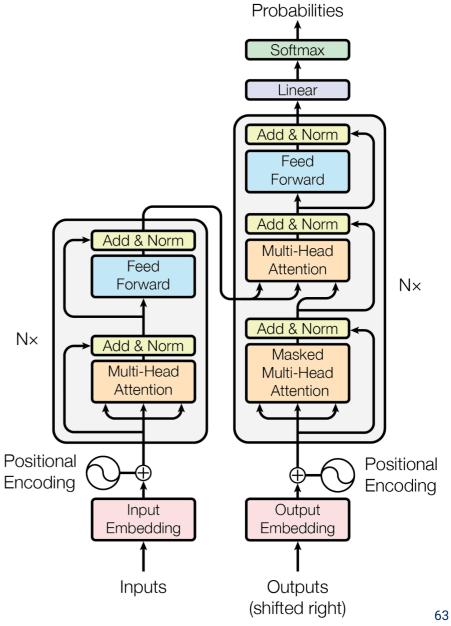
Transformers [Vaswani et al., 2017]

Some specs

- Removed Recurrence components
- Based solely on the attention mechanism
- Originally addressed translation tasks [English-German | English-French]



Scary-looking yes, difficult not Let's follow Thomas' presentation



Output



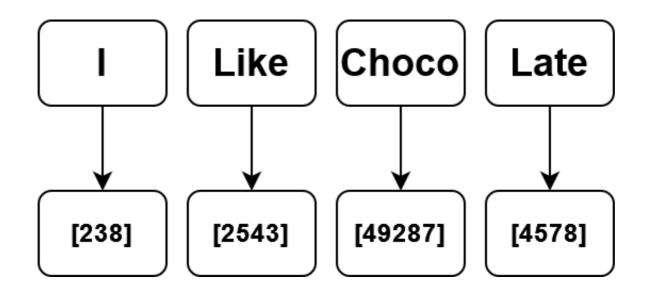
Lets consider the following sequence

"I like chocolate"



Tokenization: defining granular unit of processing

- Break the input into smaller units (tokens)
- Different levels of codification possible (character, word, etc.)

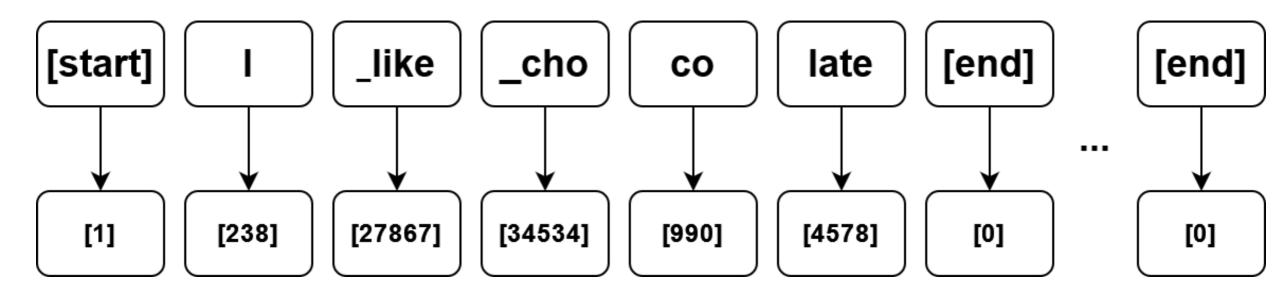




Tokenization: defining granular unit of processing

- Break the input into smaller units (tokens)
- Different levels of codification possible (character, word, etc.)

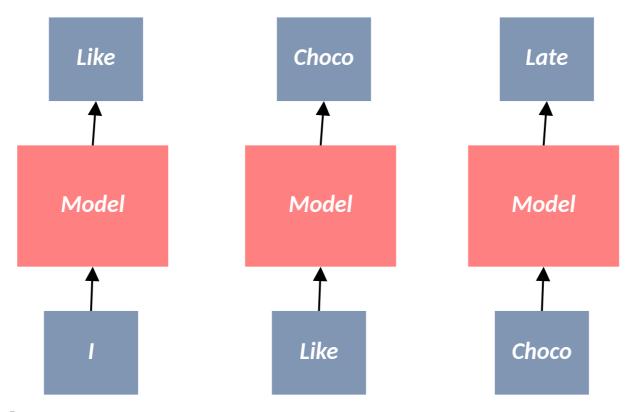
*In reality it is not that straight-forward





[Dooms, 2024]

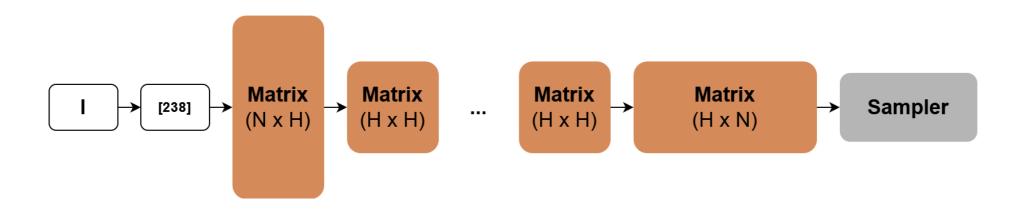
Predicting the next element (token)





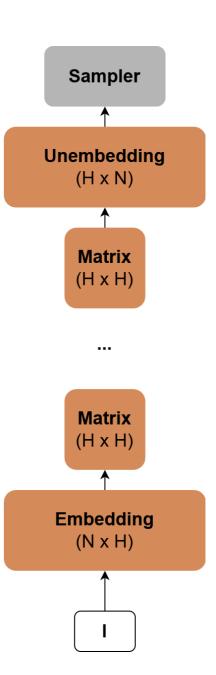
A Very Popular Recipe

Moving to a lower dimensional space (determined by H)



A Very Popular Recipe

Moving to a lower dimensional space (determined by H)

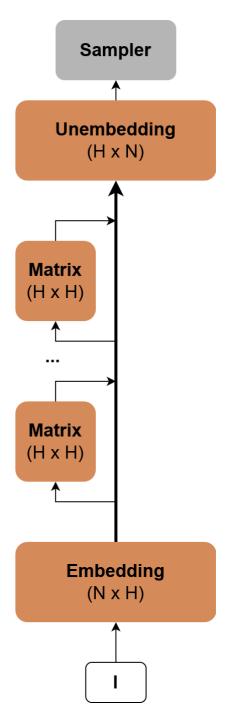




[Dooms, 2024]

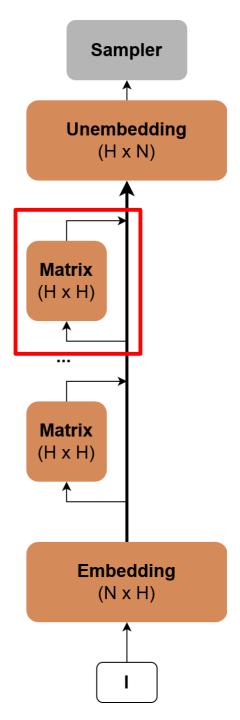
A Very Popular Recipe

- Moving to a lower dimensional space (determined by H)
- Using residual layers (why?)



A Very Popular Recipe

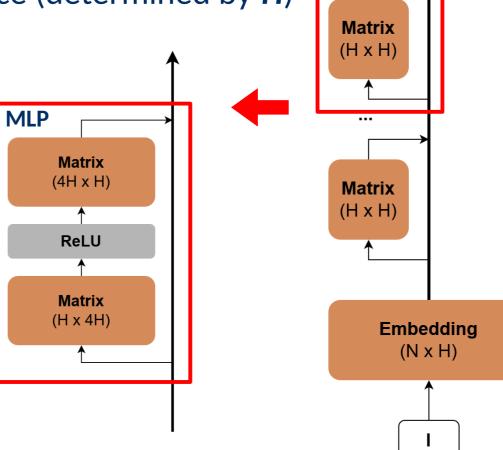
- Moving to a lower dimensional space (determined by H)
- Using residual layers





A Very Popular Recipe

- Moving to a lower dimensional space (determined by H)
- Using residual layers MLP



Sampler

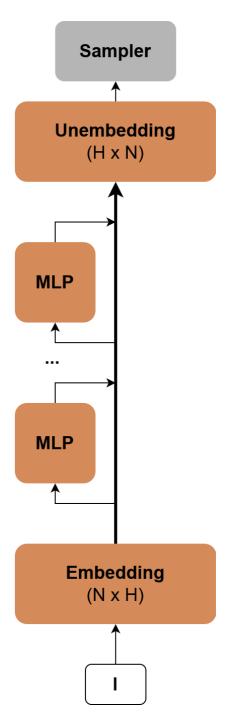
Unembedding (H x N)



[Dooms, 2024]

A Very Popular Recipe

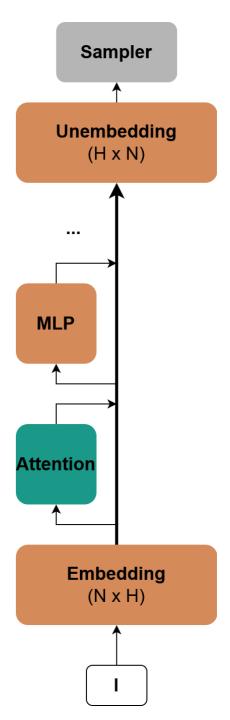
- Moving to a lower dimensional space (determined by H)
- Using residual layers MLP





A Very Popular Recipe

- Moving to a lower dimensional space (determined by H)
- Using residual layers MLP
- Add an [self] attention layer

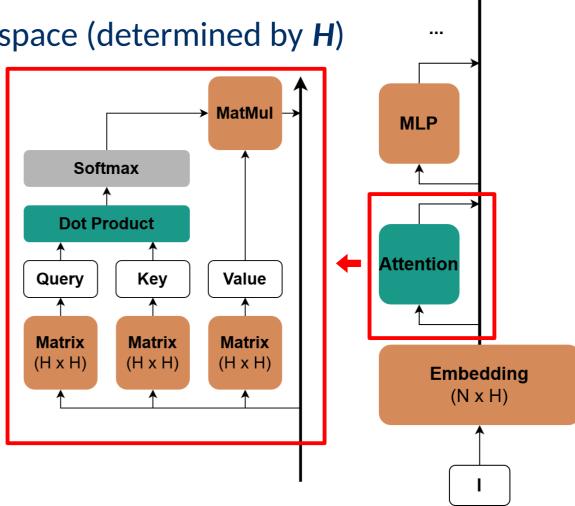




A Very Popular Recipe

Moving to a lower dimensional space (determined by H)

- Using residual layers MLP
- Add an [self] attention layer



Sampler

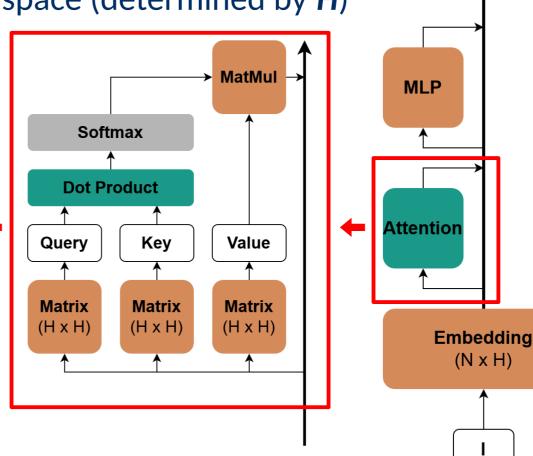
Unembedding (H x N)



A Very Popular Recipe

- Moving to a lower dimensional space (determined by **H**)
- Using residual layers MLP
- Add an [self] attention layer

$$\operatorname{Attention}(Q,K,V) = \operatorname{softmax}(\frac{QK^T}{\sqrt{d_k}})V$$





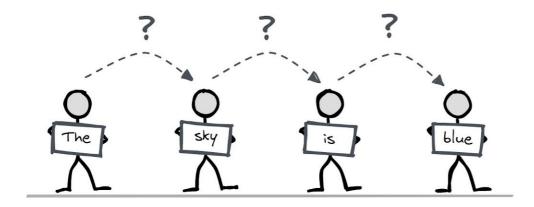
Sampler

Unembedding $(H \times N)$

 $(N \times H)$

Attention Mechanism - An Intuition

Each person in the line tries to guess what word the person in front of them is holding.

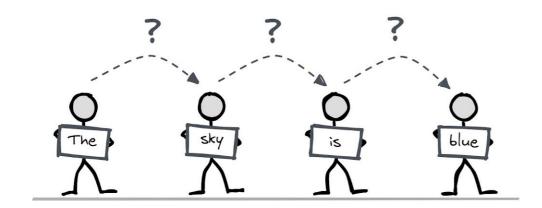




Attention Mechanism - An Intuition

Query
what question
is asked
The sky is

Each person in the line tries to guess what word the person in front of them is holding.







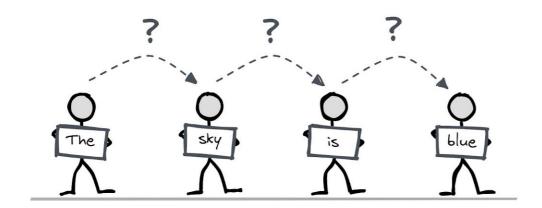


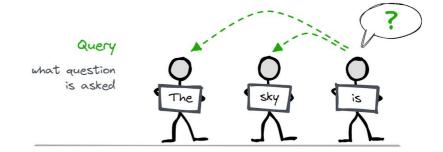
[Dooms, 2024]

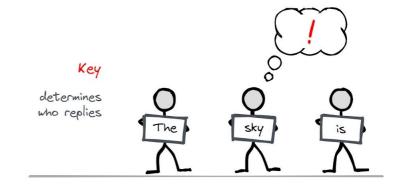
Attention Mechanism - An Intuition

- $Q \rightarrow the question$
- K \rightarrow critical element to focus on
- V → the information to be forwarded

Each person in the line tries to guess what word the person in front of them is holding.











Attention Mechanism - An Intuition

- $Q \rightarrow \text{the question}$
- $K \rightarrow critical$ element to focus on
- V → the information to be forwarded

A database Analogy

	Query = 5	Values	Output
Key = 1	0	1	0
Key = 5	1	2	2
Key = 3	0	3	0
Key = 4	0	4	0
Key = 5	1	5	5

Attention Mechanism - An Intuition

- $Q \rightarrow \text{the question}$
- K \rightarrow critical element to focus on
- V → the information to be forwarded

Like convolution but with dynamic weight (query-key similarity)

A database Analogy

Query = 5

0.125

1

Values

0.125

Output

Key = 5

1

2

2

Key = 3

Key = 1

0.25

3

0.75

Key = 4

0.5

2

Key = 5

1

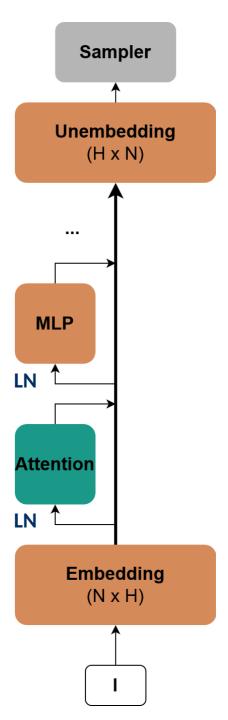
5

4

5

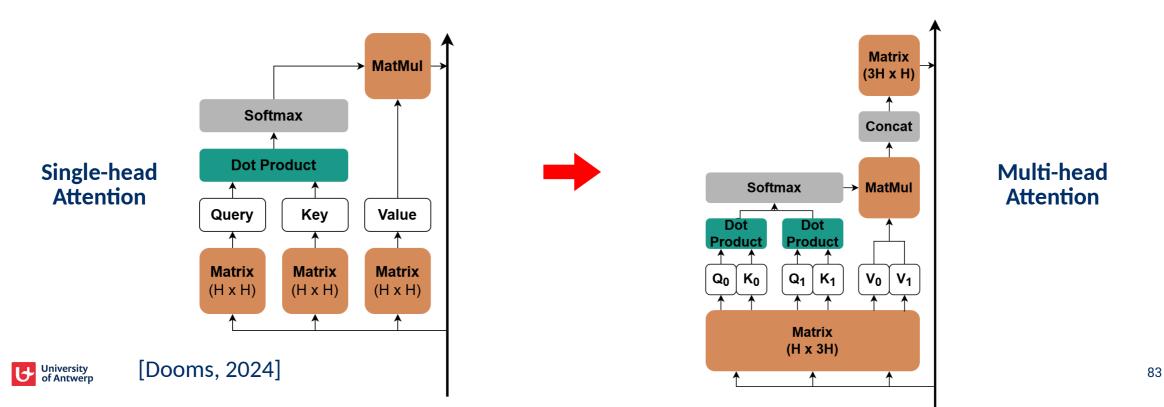
A Very Popular Recipe

- Moving to a lower dimensional space (determined by H)
- Using residual layers MLP
- Add an [self] attention layer
- Add some intermediate normalization



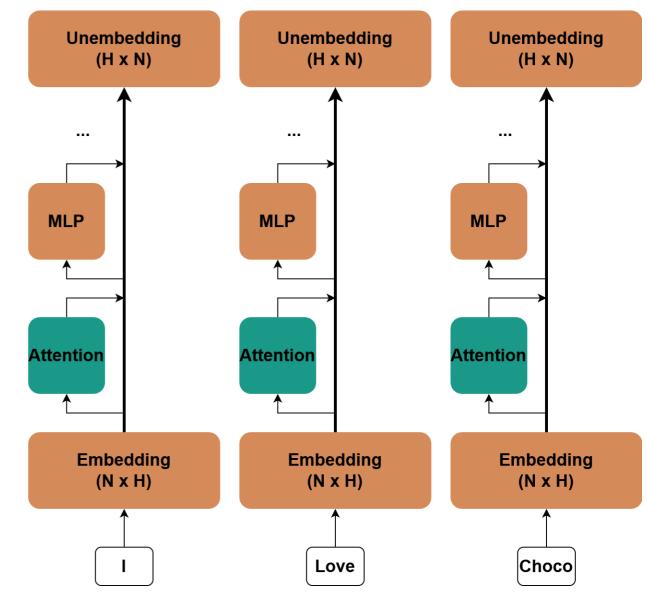
Single vs. Multi-head Attention

- Single: Each layer ask one question via the single QKV matrices.
- Multi: Split the QKV matrices into smaller ones → ask more/simpler questions



Strengths: Communication

- Depth: more specialized and structured
- Through attention: across tokens

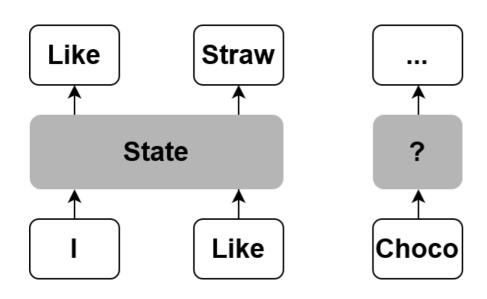


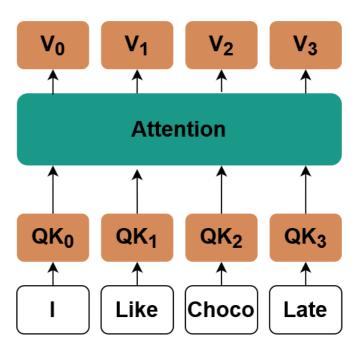


RNNs VS Transfomers

Context Sources

- Information from memory vs. tokens (error propagation)
 - → Sentence vs. Word -based



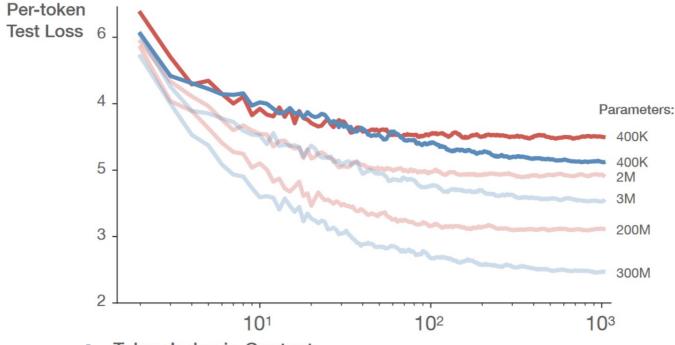




RNNs VS Transfomers

Performance / size trade-off

- Higher performance as # of considered tokens increases
- Better use of parameters



[Kaplan, 2020]



[Finally:D]



- Two Step-Approach
 - 1. Model Context
 - 2. Predict Next Element given Context

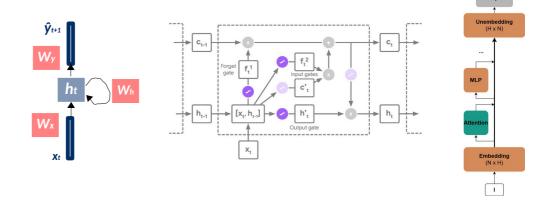




- Two Step-Approach
 - 1. Model Context

- ["this", "is", "an", "example", "text"] \longrightarrow f_{θ} \longrightarrow h \longrightarrow "sequence"
- 2. Predict Next Element given Context
- Serveral architectures with different capabilities

RNNs | LSTMs | GRUs | Transformers





- Two Step-Approach
 - 1. Model Context
 - 2. Predict Next Element given Context
- ["this", "is", "an", "example", "text"] \longrightarrow for h \longrightarrow g_{θ} "sequence"

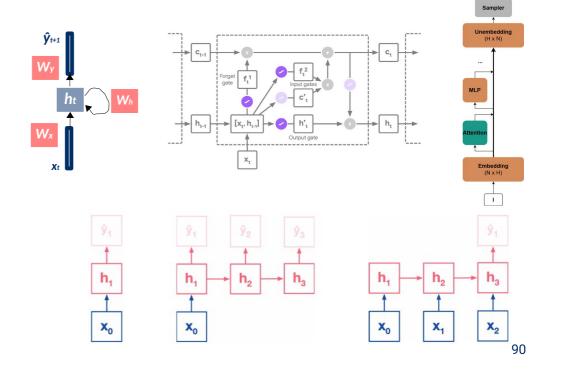
Serveral architectures with different capabilities

RNNs | LSTMs | GRUs | Transformers

High Flexibility towards different problems

one-to-one | many-to-one | many-to-many ...





References

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- Understanding LSTM Networks COlah's Blog
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Modeling Sequences with Neural Networks

José Oramas

