Design Document for Assignment 1 part2

Jingyi Lu u5932987

Overview

In the assignment, a changeable sine wave is given. Frequency, amplitude and initial phase, which are important parameters for a sine function can be changed freely. Apart from that, many steps of calculation are reduced by using memory of the discoboard reasonably.

Introduction

Unlike sawtooth wave in the first part of the assignment, the sound that a sine wave makes is more comfortable for humans. In addition, sine wave is fundamental for Fourier series, which can decompose a complex periodic signal into many sine and cosine functions (2005). And this is attached great significance in the study of signal.

However, sine wave is more difficult to implement than a sawtooth wave. In Java, C, or other high-level programming languages, there is usually a built-in function called sin() or something similar. However, in assembly, it is required to start from beginning. In this case, Taylor series is applied to estimate the approximate value of sine function as shown below (1970).

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1} \qquad = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots \qquad \text{for all } x$$

Implementation

Firstly, to put a sine wave into practice, a counter is necessary because it takes the role of horizontal axis in the sine function. In assembly, it is hard to store the accurate value of a decimal, like $\frac{1}{2}\pi$. Hence, a discrete counter is used instead of it. For example, if the frequency is 200, then the counter will count from 0 to 240. The upper bound of the range is determined by the frequency and the characteristics of function BSP_AUDIO_OUT_Play_Sample. To be more specific, the number of cycles per second is equal to the number of frequency. What's more, the counter also indicates the times that the function BSP_AUDIO_OUT_Play_Sample has been called in a cycle. Hence, we can get a relation between the counter and x in sine function.

$$\frac{\text{Counter}}{48k/frequency} = \frac{x}{2\pi}$$

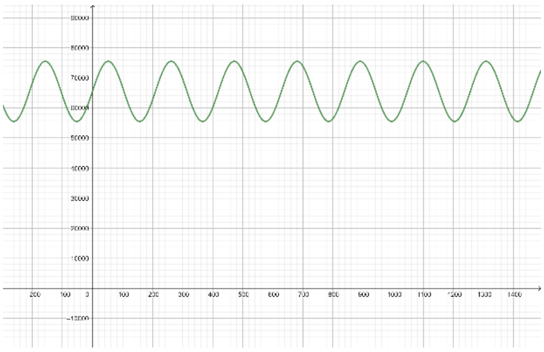
Hence, the value of x can be expressed by a formula of the counter. Then, by substituting x with the value of counter, the Taylor series can be changed as the formula below. In this case, only first three parts are chosen for convenience. Careful calculation is conducted in the assignment. And then the value of r_0 corresponding to the counter is passed to BSP_AUDIO_OUT_Play_Sample to make sound. Many points regarding to the calculation will be covered in the reflection.

$$\begin{aligned} r_0 &= amplitude \times sin(x) + 0x10000 \\ &= \mathrm{A} \times \frac{\pi \times f}{24000} C - \frac{1}{6} A \times (\frac{\pi \times f}{24000} C)^3 + \frac{1}{120} A \times (\frac{\pi \times f}{24000} C)^5 + 0x10000 \end{aligned}$$

(A denotes amplitude, f denotes frequency, C denotes Counter)

What’s more, although these things have already implemented the sine wave, the machine do many calculations in each cycle. However, in each circle, the value of r_0 for certain counter does not change. Hence, the machine waste a large amount of resources in calculating repeatedly. Instead, the code involves an initialization part for sine function. In that part, all the necessary values of r_0 are stored in the memory. And in the main part, the needed value can be simply loaded to r_0. This significantly reduce the number of redundant calculations.

Finally, the code also implements the initial phase, which is an important parameter for sine function. This can be realised by delaying until the function reaches its initial phase.



A figure for the wave when frequency is 200 and amplitude is 10000

Reflection

In the assignment, there are many tricky points that need mentioning.

Firstly, for Taylor series, we only choose the first three parts and abandon the others for convenience. Hence, with x increasing, the difference between the result calculated and the value of $\sin(x)$ will become larger. Therefore, just the values for x is from 0 to $\frac{1}{2}\pi$ are taken into consideration. It is known that $f(x) = \sin(x)$ is an odd function and a periodic function, and it is symmetric with respect to the line $x = \frac{1}{2}\pi$. Hence other values of r_0 can be expressed by the values for x from 0 to $\frac{1}{2}\pi$.

Secondly, the calculation is not accurate in many cases. Only the first three parts of Taylor series are chosen. And that will make the calculation inaccurate. Another thing is that, in assembly, all the values of registers are discrete. So, decimals cannot be perfectly dealt with perfectly in this case. For example, only an approximate value is chosen for π and this leads to inaccurate result as well.

Finally, one thing needs mentioning is that, due to the limitation of upper bound for registers, the frequency will be limited. That is because in the third part of Taylor series, $-\frac{1}{120}x^5$ involves x^5, which might not be expressed by 32 bits.

Resources

Abramowitz, Milton; Stegun, Irene A. (1970), Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables, New York: Dover Publications, Ninth printing

William E. Boyce; Richard C. DiPrima (2005). Elementary Differential Equations and Boundary Value Problems (8th ed.). New Jersey: John Wiley & Sons, Inc. ISBN 0-471-43338-1.