Homework 1

Problem #1

Decision variables

$x_1, x_2, \dots, x_{13}$

Where,

x_1 is the number of servings of peas

x_2 is the number of servings of green beans

…

x_{13} is the number of servings of Jello

Objective function

Minimize total cost

$$0.1\,x_1 + 0.12\,x_2 + 0.13\,x_3 + 0.09\,x_4 + 0.10\,x_5 + 0.07\,x_6 + 0.70\,x_7 + 1.20\,x_8 + 0.63\,x_9 + 0.28\,x_{10} + 0.42\,x_{11} + 0.15\,x_{12} + 0.12\,x_{13}$$

Subject to Constraints

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \geq 1$ (At least one equivalent serving of vegetables is desired)

$x_7 + x_8 + x_9 \geq 1$ (At least one equivalent serving of meat is desired)

$x_{10} + x_{11} + x_{12} + x_{13} \geq 1$ (At least one equivalent serving of dessert is desired)

$x_1 + x_2 + x_3 + 2\,x_4 + 4\,x_5 + 5\,x_6 + 2\,x_7 + 3\,x_8 + 3\,x_9 + x_{10} + x_{11} + x_{12} + x_{13} \geq 5$ (Minimal requirement of carbohydrate)

$3\,x_1 + 5\,x_2 + 5\,x_3 + 6\,x_4 + 2\,x_5 + x_6 + x_7 + 8\,x_8 + 6\,x_9 + 3\,x_{10} + 2\,x_{11} \geq 10$ (Minimal requirement of vitamins)

$x_1 + 2\,x_2 + x_3 + x_4 + x_5 + x_6 + 3\,x_7 + 5\,x_8 + 6\,x_9 + x_{10} \geq 10$ (Minimal requirement of proteins)

$2\,x_4 + x_5 + x_6 + x_7 + 2\,x_8 + x_9 \geq 2$ (Minimal requirement of fat)

$x_i \geq 0$ For i = 1, 2… 13 (non-negativity constraint)

Problem #2

Decision variables

X_{ij} (Which is the amount or quantity of *i* type of fuel (in tons) to be used at plant *j*)

Where, $i = 1,2, \dots, m$

$j = 1,2, \dots, n$

So, there are m*n decision variables

Objective function

There are m types of fuel and each fuel type *i* costs c_i dollars per ton.

The total cost of fuel type *i* is

$c_i * (X_{i1} + X_{i2} + \cdots + X_{in})$

So, the objective function is:

Minimize total cost of fuel which is,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_i * X_{ij}$$

Subject to Constraints

$X_{ij} \geq 0$ (Non-negativity constraints)

$\sum_{i=1}^{m} X_{ij} * a_{ij} \geq b_j$ For j = 1, 2…n

(Explanation: The total energy needed at each plant j is b_j. $X_{ij} * a_{ij}$ represents the energy generated by fuel type *i* at plant *j* which when summed over all fuel type *i* gives the total energy generated at plant *j*. Since plant *j* needs b_j British thermal units per day, so at least b_j British thermal units of energy should be generated at plant *j*)

$\sum_{i=1}^{m} X_{ij} * e_{ij} \leq \gamma_j$ For j = 1, 2…n

(Explanation: $X_{ij} * e_{ij}$ represents effluent emission per ton by fuel type *i* at plant *j* which when summed over all fuel type *i* gives the total effluent emission at plant *j.* Since plant *j* has meteorological parameter as γ_j, so at most there can be effluent emission of γ_j at plant *j*.)

$\sum_{i=1}^{m}\sum_{j=1}^{n} e_{ij} * X_{ij} \leq b$ (The level of air pollution in the region is to not exceed b micrograms per cubic meter)

<u>Problem #3</u>

Decision variables

x_1, x_2, x_3, x_4

Where,

x_1 is the number of product A to be produced next week

x_2 is the number of product B to be produced next week

x_3 is the number of product C to be produced next week

x_4 is the number of product D to be produced next week

Objective function

Maximize total profit (in $)

$18\,x_1 + 15\,x_2 + 13\,x_3 + 14\,x_4$

Subject to Constraints

$3\,x_1 + \,x_2 + 2\,x_3 + \,x_4 \leq 2400$ (Time available for pouting next week is 40 hours = 2400 minutes)

$8\,x_1 + 12\,x_2 + 6\,x_3 + 7\,x_4 \leq 4800$ (Time available for cleaning next week is 80 hours = 4800 minutes)

$10\,x_1 + 6\,x_2 + 9\,x_3 + 7\,x_4 \leq 4800$ (Time available for grinding next week is 80 hours = 4800 minutes)

$x_1 + \,x_2 + \,x_3 + \,x_4 \leq 1200$ (Time available for inspection next week is 20 hours = 1200 minutes)

$3\,x_1 + \,5\,x_2 + 3\,x_3 + 2\,x_4 \leq 2400$ (Time available for packing next week is 40 hours = 2400 minutes)

$x_1 \geq 200$ (At least 200 units of product A must be produced)

$x_2 \geq 0$ (Non-negativity constraint)

$x_3 \geq 0$ (N-negativity constraint)

$x_4 \geq 300$ (At least 300 units of product D must be produced)

<u>Problem #4</u>

(a) Let $y_0 = \frac{1}{2x_1+x_3+4x_4+3}$

Also, let

$x_1 y_0 = y_1 \quad x_2 y_0 = y_2 \quad x_3 y_0 = y_3 \quad x_4 y_0 = y_4$

Then the reformulation would be as follows:

MAXIMIZE $Z = 4y_1 + y_2 - 3y_4 + y_0$

SUBJECT TO

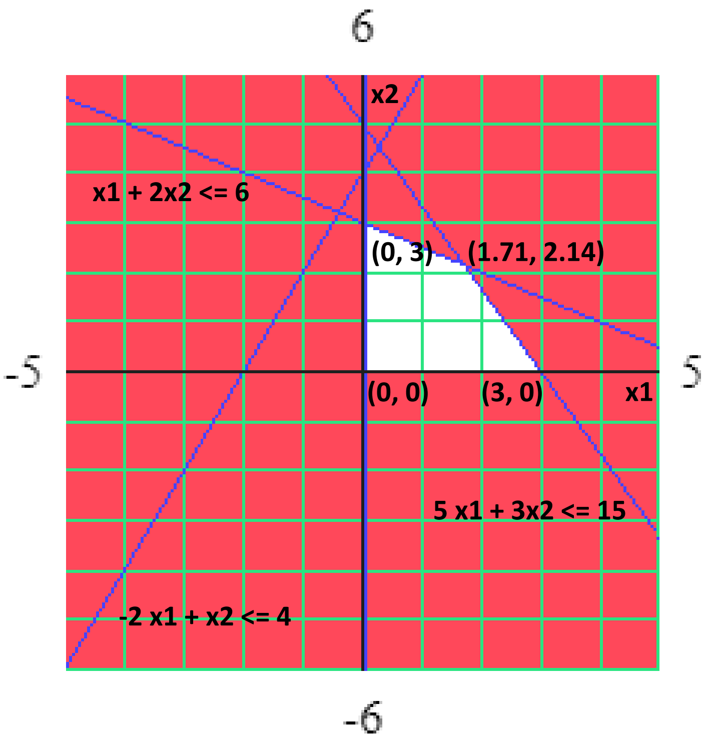
$y_1 - 2y_2 + y_3 + 2y_4 - 10y_0 \leq 0$

$y_2 - y_3 + 5y_4 - 12y_0 \leq 0$

$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0, y_4 \geq 0$ (Non-negativity constraints)

$y_0 > 0$ (This is because if y_0 is negative then the non-negativity constraints mentioned above won't necessarily hold true and $y_0 = 0$ is not possible because of the way y_0 is defined)

(b) The feasible region is shown in white.



The corner points of the feasible region are: (0, 0), (0, 3), (3, 0) and (1.71, 2.14)

The maximum value of $5\ x_1 + 4\ x_2$ is when $\boldsymbol{x_1 = 1.71\ and\ x_2 = 2.14}$

The maximum value is **17.14**

Problem #5

Decision variables

x_1, x_2, x_3, x_4

Where,

x_1 is the amount invested in Stock 1 (in dollars)

x_2 is the amount invested in Stock 2 (in dollars)

x_3 is the amount invested in Stock 3 (in dollars)

x_4 is the amount invested in Stock 4 (in dollars)

Risk Calculation

	Stock 1		Stock 2		Stock 3		Stock 4	
	Return	ABS	Return	ABS	Return	ABS	Return	ABS
Forecast 1	3	0.1	13	7.6	4	1.4	25	5
Forecast 2	1	1.9	4.5	0.9	0.6	2	15	5
Forecast 3	2.75	0.15	1.75	3.65	2.75	0.15	20	0
Forecast 4	4.5	1.6	5	0.4	1.9	0.7	5	15
Forecast 5	3.25	0.35	2.75	2.65	3.75	1.15	35	15
MAD	2.9	0.82	5.4	3.04	2.6	1.08	20	8

Objective function

Minimize RISK

$0.82\, x_1 + 3.04\, x_2 + 1.08\, x_3 + 8.00\, x_4$

Subject to Constraints

Constraint 1: $\boldsymbol{x_1 + x_2 + x_3 + x_4 \leq 100{,}000}$

Explanation: Investment of no more than $100,000

Constraint 2: $\boldsymbol{0.1\, x_1 + 0.1\, x_2 + 0.1\, x_3 - 0.9\, x_4 \leq 0}$

Explanation: At least 10% of total investment in Stock 4

$x_4 \geq 0.1(x_1 + x_2 + x_3 + x_4)$

Constraint 3: $\boldsymbol{2.8\, x_1 + 5.3\, x_2 + 2.5\, x_3 + 19.9\, x_4 \geq 0}$

Explanation: Expected return of at least 10% of total amount invested

$2.9\, x_1 + 5.4\, x_2 + 2.6\, x_3 + 20\, x_4 \geq 0.1(x_1 + x_2 + x_3 + x_4)$

Constraint 4: $\boldsymbol{x_i \geq 0}$ **For *i* = 1,2,3,4** (Explanation: Non-Negativity constraints)