Homework #2

Complete the following problems and be sure to show all of your work. Please type-up your solutions and submit as a PDF file. This homework assignment is worth 100 points in total.

Problem #1 (20 points)

A production manager wants to determine how many units of each product to produce weekly to maximize weekly profits. Production requirements for the products are shown in the following table.

Product	Material 1 (lbs.)	Material 2 (lbs.)	Labor (hours)
A	3	2	4
B	1	4	2
C	5	none	3.5

Material 1 costs $7 a pound, material 2 costs $5 a pound, and labor costs $15 per hour. Product A sells for $101 a unit, product B sells for $67 a unit, and product C sells for $97.50 a unit. Each week there are 300 pounds of material 1; 400 pounds of material 2; and 200 hours of labor. Also, there is a weekly demand of at least 10 units of product C each week.

Formulate the given problem scenario as a linear program, and **solve** the problem using Microsoft Excel with ASPE. For the formulation, provide a complete description of the decision variables used along with their units and also label the constraints mentioned in the problem as completely as possible. Also, the **spreadsheet model setup** should provide clearly labeled values used for the decision variables, constraints, and objective function.

Problem #2 (20 points)

A sports manufacturer produces two products: footballs and baseballs. These products can be produced either during the morning shift or the evening shift. The cost of manufacturing the football and the baseball in the morning shift is $20 each, and the cost of manufacturing the football and the baseball in the evening shift is $25 each. The amounts of labor, leather, inner plastic lining, and demand requirements are given as follows:

Resource	Football	Baseball
Labor (hours/unit)	0.75	2
Leather (pounds/unit)	7	15
Inner plastic lining (pounds/unit)	0.5	2
Total demand (units)	1500	1200

Based on the information about the company, we know that the maximum labor hours available in the morning shift and evening shift are 5,000 hours and 2,000 hours, respectively, per month. The maximum amount of leather available for the morning shift is 15,000 pounds per month and 14,000 pounds per month for the evening shift. The maximum amount of inner plastic lining available for the morning shift is 2,000 pounds per month and 1,500 pounds per month for the evening shift.

Formulate the given problem scenario as a linear program so as to minimize the production cost and determine the number of footballs and baseballs made by this company. **Solve** the problem using Microsoft Excel with ASPE. For the formulation, provide a complete description of the decision variables used along with their units and also label the constraints mentioned in the problem as completely as possible. Also, the **spreadsheet model setup** should provide clearly labeled values used for the decision variables, constraints, and objective function.

Problem #3 (20 points)

In a small suburban town, firefighters work 8-hour shifts. Assume there are 6 shifts each day that are divided into six 4-hour periods. The minimum number of firefighters needed on each shift is illustrated below.

Shift	**Number of Firefighters Needed**
Midnight–4 a.m.	5
4 a.m.–8 a.m.	6
8 a.m.–Noon	10
Noon–4 p.m.	12
4 p.m.–8 p.m.	8
8 p.m.–Midnight	5

Firefighters must report to work at the beginning of the above time periods and must work eight consecutive hours.

Formulate the given problem scenario as a linear program so as to determine the minimum number of firefighters needed on each shift. **Solve** the problem using Microsoft Excel with ASPE. For the formulation, provide a complete description of the decision variables used along with their units and also label the constraints mentioned in the problem as completely as possible. Also, the **spreadsheet model setup** should provide clearly labeled values used for the decision variables, constraints, and objective function.

Problem #4 (20 points)

A manufacturer of rotary pumps is planning production for the next four months. The forecast demand for the rotary pumps is shown in the following table.

Rotary pump	SEP	OCT	NOV	DEC
Standard	650	875	790	1,100
Heavy duty	900	350	1,200	1,300

At the beginning of September, the warehouse is expected to be completely empty. There is room for no more than 1,800 rotary pumps to be stored. Holding costs for both types are $5 per unit per month. Because workers are given time off during the holidays, the manufacturer wants to have at least 800 standard rotary pumps and 850 heavy duty rotary pumps already in the warehouse at the beginning of January.

Production costs are $125 per unit for standard rotary pumps and $135 per unit for heavy duty rotary pumps. Because demand for raw materials is rising, production costs are expected to rise by 5% per month through the end of the year.

Labor to make the standard rotary pump is 0.45 hours per unit; making heavy duty rotary pumps takes 0.52 hours per unit of labor. Management has agreed to schedule at least 1,000 hours per month of labor. As many as 200 extra hours per month are available to management at the same cost, except during the month of December, when only 100 extra hours are possible.

Formulate the given problem scenario as a linear program so as to determine the production schedule for standard and heavy duty rotary pumps for the four months. **Solve** the problem using Microsoft Excel with ASPE. For the formulation, provide a complete description of the decision variables used along with their units and also label the constraints mentioned in the problem as completely as possible. Also, the **spreadsheet model setup** should provide clearly labeled values used for the decision variables, constraints, and objective function.

Problem #5 (20 points)

A real estate developer is planning to build an office complex. There are three office sizes currently under consideration: small, medium, and large. Small offices can be rented for $600 per month, medium offices can be rented for $750 per month, and large offices can be rented for $1,000 per month. Each small office requires 600 square feet, each medium office requires 800 square feet, and each large office requires 1,000 square feet. The current plot of land available to the developer is 100,000 square feet. The developer wants to ensure that the office complex has at least 3 units of each office size. Moreover, zoning restrictions limit the total number of offices to 50. The developer solved this problem such that he could accrue maximum rent from the small, medium, and large offices he builds. Your job is to analyze this sensitivity report and answer the following questions:

Sensitivity Report

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Optimal Values Small	3	0	600	400	1E+30
C4	Optimal Values Medium	3	0	750	250	1E+30
D4	Optimal Values Large	44	0	1000	1E+30	250

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease

E8	Square footage	48200	0	100000	1E+30	51800
E9	Minimum no. of small	3	-400	3	41	3
E10	Minimum no. of medium	3	-250	3	41	3
E11	Minimum no. of large	44	0	3	41	1E+30
E12	Total no. of offices	50	1000	50	51.8	41

a. (2 points) How many small and large offices should the developer build?

b. (4 points) What is the total optimal monthly revenue?

c. (2 points) If the developer implements the optimal solution, what amount of square footage would remain unused?

d. (5 points) What is the impact on the optimal allocation of offices and the objective function value if small offices can be rented for $800 per month rather than $600 per month?

e. (2 points) What impact would an increase in 52,800 sq. ft of additional footage have on the optimal objective function value?

f. (5 points) What impact will an increase in the monthly rental of small offices to $650 and simultaneous decrease to $800 in the monthly rental of large offices have on the current optimal solution and the objective function value?

Extra Credit Problem (5 points)

Find the extreme points of the region defined by the following inequalities:

$$
\begin{aligned}
x_1 + x_2 + x_3 &\leq 5 \\
-x_1 + x_2 + 2x_3 &\leq 6 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}
$$