Homework #4

Complete the following problems and be sure to show all of your work. Please type-up your solutions and submit as a PDF file. This homework assignment is worth 100 points in total.

Problem #1 (20 points)

A large food chain owns a number of pharmacies that operate in a variety of settings. Some are situated in small towns and are open for only 8 hours a day, 5 days per week. Others are located in shopping malls and are open for longer hours. The analysts on the corporate staff would like to develop a model to show how a store's revenues depend on the number of hours that it is open. They have collected the following information from a sample of stores.

Hours of Operation	Average Revenue ($)
40	5958
44	6662
48	6004
48	6011
60	7250
70	8632
72	6964
90	11097
100	9107
168	11498

a) Use a linear function to represent the relationship between revenue and operating hours and find the values of the parameters using the Standard GRG Nonlinear solver that provide the **best fit** to the given data. What revenue does your model predict for 120 hours?

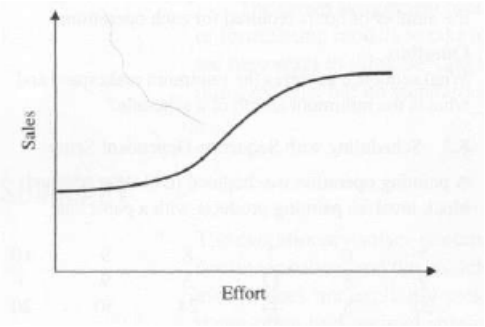
b) Suggest a two-parameter nonlinear model for the same relationship and find the parameters using the Standard GRG Nonlinear solver that provide the **best fit**. What revenue does your model predict for 120 hours? Which if the models in (a) and (b) do you prefer and why?

Your solutions for (a) and (b) should contain a detailed spreadsheet model (where the decision variables, parameters, objective function and constraints are identified and explained), as well as answers to the questions posed. You should use Microsoft Excel with ASPE to solve.

Problem #2 (20 points)

An organization has increased its profitability dramatically by using models to allocate its sales force among the company's major drugs. To guide that effort, a non-linear sales response curve is estimated for each drug. A sales-response curve relates the sale of a product to the effort expended to sell it, which in this case is measured by the number of calls made by the sales force. The following figure

shows the typical shape of such a curve:



For low levels of effort, sales rise rapidly with increased effort, but eventually the sales response levels out. Two elements are needed to develop a sales-response curve for a particular product: (1) some data or managerial judgments on the sales at various levels of effort and (2) a family of S-shaped curves to fit the data. A suitable family of curves for the organization is given by the function:

$$S = a + \frac{(b - a)E^c}{(d + E^c)}$$

where *S* is sales, *E* is effort, and *a, b, c,* and *d* are parameters that determine the shape and location of the curve. At the organization, sales of a particular drug were around 200 cases with a sales effort of 500 calls. Management was then asked to estimate sales at other levels of effort. Their estimates are as follows:

Effort (%)	**Actual Sales (% of Current)**
0	50
25	53
50	55
75	75
100	100
125	120
150	127
175	132
200	135

Using the Standard Evolutionary solver with ASPE in Microsoft Excel, solve both questions (a) and (b) below. Your solution should contain a detailed spreadsheet model (where the decision variables, parameters, objective function and constraints are identified and explained), as well as answers to the questions posed.

(a) Fit a sales-response curve to the data above using a least-squares criterion. Use percentages relative to the base case for the effort and sales measures. What are the best values of the parameters *a, b, c,* and *d*?

(b) What sales does the model in part (a) predict for an effort of 115%?

Problem #3 (20 points)

One of the examples in Ch. 12 of the textbook dealt with determining the optimal reorder point for a computer monitor sold by Millennium Computer Corp (MCC). Suppose that it costs MCC $0.30 per day in holding costs for each monitor in beginning inventory, and it costs $20 to place an order. Each monitor sold generates a profit of $45, and each lost sale results in an opportunity cost of $65 (including the lost profit of $45 and $20 in lost goodwill). **Modify** the spreadsheet shown in Figure 12.23 of the textbook to determine the reorder point and order quantity that maximize the average monthly profit associated with this monitor. *Be sure to only model the first 25 days (not 30 days as in Figure 12.23) using 1000 trials.* You should use Microsoft Excel with ASPE to solve.

Problem #4 (20 points)

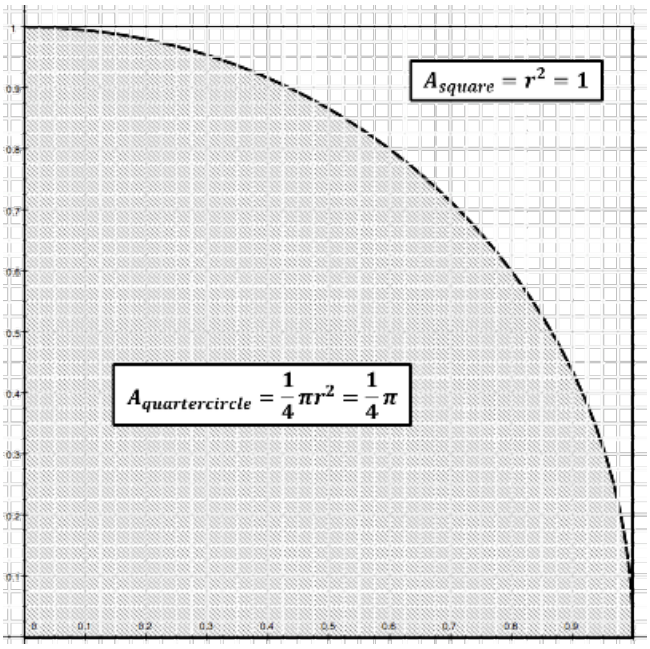
The Harriet Hotel in downtown Boston has 100 rooms that rent for $150 per night. It costs the hotel $30 per room in variable costs (cleaning, bathroom items, etc.) each night a room is occupied. For each reservation accepted, there is a 5% chance that the guest will not arrive. If the hotel overbooks, it costs $200 to compensate guests whose reservations cannot be honored. How many reservations should the hotel accept if it wants to maximize the average daily profit? You may use Microsoft Excel with ASPE or R statistical programming to solve.

Problem #5 (20 points)

Acme Equipment Company is considering the development of a new machine that would be marketed to tire manufacturers. Research and development costs for the project as expected to be about $4 million but could vary between $3 and $6 million. The market life for the product is estimated to be 3 to 8 years with all intervening possibilities being equally likely. The company thinks it will sell 250 units per year, but acknowledges that this figure could be as low as 50 or as high as 350. The company will sell the machine for about $23,000. Finally, the cost of manufacturing the machine is expected to be $14,000 but could be as low as $12,000 or as high as $18,000. The company's costs of capital is 15%. Use appropriate RNGs to create a model in Microsoft Excel with ASPE or R statistical programming to calculate the possible net present values (NPVs) that could result from taking on this project. What is the expected NPV for this project? What is the probability of this project generating a positive NPV for the company?

Extra Credit Problem (10 points)

Perform Monte Carlo integration using R statistical programming to estimate the value of π. To summarize the approach, consider the unit quarter circle illustrated in the figure below:



Generate N pairs of uniform random numbers (x, y), where $x \sim U(0,1)$ and $y \sim U(0,1)$, and each (x, y) pair represents a point in the unit square. To obtain an estimate of π, count the fraction of points that fall inside the unit quarter circle and multiply by 4. Note that the fraction of points that fall inside the quarter circle should tend to the ratio between the area of the unit quarter circle (i.e., ¼ π) as compared to area of the unit square (i.e., 1). We proceed step-by-step:

a) Create a function `insidecircle` that takes two inputs between 0 and 1 and returns 1 if these points fall within the unit circle.

b) Create a function `estimatepi` that takes a single input N, generates N pairs of uniform random numbers and uses `insidecircle` to produce an estimate of π as described above. In addition to the estimate of π, `estimatepi` should also return the standard error of this estimate, and a 95% confidence interval for the estimate.

c) Use `estimatepi` to estimate π for N = 1000 to 10000 in increments of 500 and record the estimate, its standard error and the upper and lower bounds of the 95% CI. How large must N be in order to ensure that your estimate of π is within 0.1 of the true value?

d) Using the value of N you determined in part c), run `estimatepi` 500 times and collect 500 different estimates of π. Produce a histogram of the estimates and note the shape of this distribution. Calculate the standard deviation of the estimates – does it match the standard error you obtained in part c)? What percentage of the estimates lies within the 95% CI you obtained in part c)?