Homework 3

Problem 1

Decision Variables

Xij which is the quantity manufactured in *i*th manufacturing plant and supplied to *j*th retail centre in units.

The 1st, 2nd and 3rd manufacturing plants are Atlanta, Tulsa, and Seattle OR Baltimore.

The 1st and 2nd retail store are LA and NY

For example X_{22} is the quantity manufactured in *Tulsa* manufacturing plant and supplied to *NY*

Note that this problem can also be set up as “binary” ILP, however, to keep it simple, I have done Seattle and Baltimore separately.

Objective function

Minimize total cost

If Seattle is chosen:

$8\ X_{11} + 5\ X_{12} + 4\ X_{21} + 7\ X_{22} + 5\ X_{31} + 6\ X_{32}$

If Baltimore is chosen:

$8\ X_{11} + 5\ X_{12} + 4\ X_{21} + 7\ X_{22} + 4\ X_{31} + 6\ X_{32}$

Subject to constraints

$X_{11} + X_{12} \leq 600$

$X_{21} + X_{22} \leq 900$

$X_{31} + X_{32} \leq 500$

$X_{11} + X_{21} + X_{31} \geq 800$

$X_{12} + X_{22} + X_{32} \geq 1200$

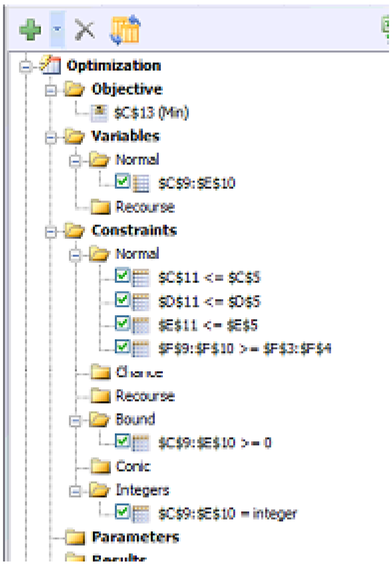
$Xij \geq 0$

Xij is an integer

Set Up and Solution

<u>Seattle</u>

	A	B	C	D	E	F
1						
2			Atlanta	Tulsa	Seattle	Demand
3		LA	8	4	5	800
4		NY	5	7	6	1200
5		Capacity	600	900	500	
6						
7		DECISION VARIABLES				
8			Atlanta	Tulsa	Seattle	
9		LA				0
10		NY				0
11			0	0	0	
12						
13		Cost	0	MINIMIZE		
14			Atlanta	Tulsa	Seattle	
15		LA	0	0	0	
16		NY	0	0	0	
17						
18						

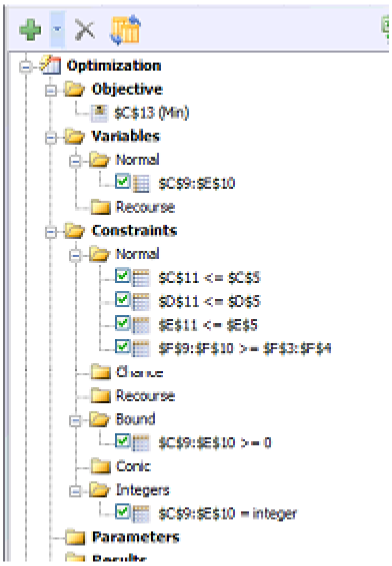


	A	B	C	D	E	F
1						
2			Atlanta	Tulsa	Seattle	Demand
3		LA	8	4	5	800
4		NY	5	7	6	1200
5		Capacity	600	900	500	
6						
7		DECISION VARIABLES				
8			Atlanta	Tulsa	Seattle	
9		LA	0	800	0	800
10		NY	600	100	500	1200
11			600	900	500	
12						
13		Cost	9900	MINIMIZE		
14			Atlanta	Tulsa	Seattle	
15		LA	0	3200	0	
16		NY	3000	700	3000	
17						

The minimum total cost is $9,900

<u>Baltimore</u>

	A	B	C	D	E	F
1						
2			Atlanta	Tulsa	Baltimore	Demand
3		LA	8	4	4	800
4		NY	5	7	6	1200
5		Capacity	600	900	500	
6						
7		DECISION VARIABLES				
8			Atlanta	Tulsa	Seattle	
9		LA				0
10		NY				0
11			0	0	0	
12						
13		Cost	0	MINIMIZE		
14			Atlanta	Tulsa	Seattle	
15		LA	0	0	0	
16		NY	0	0	0	
17						



	A	B	C	D	E	F
1						
2			Atlanta	Tulsa	Baltimore	Demand
3		LA	8	4	4	800
4		NY	5	7	6	1200
5		Capacity	600	900	500	
6						
7		DECISION VARIABLES				
8			Atlanta	Tulsa	Seattle	
9		LA	0	800	0	800
10		NY	600	100	500	1200
11			600	900	500	
12						
13		Cost	9900	MINIMIZE		
14			Atlanta	Tulsa	Seattle	
15		LA	0	3200	0	
16		NY	3000	700	3000	

The minimum total cost is $9,900

The minimum cost is same for both Seattle and Baltimore which is $9,900. Hence, the toy manufacturer can select either of the two locations to establish the plant.

<u>Problem 2</u>

Decision Variables

B, C, P and T be the weights of beef, chicken, pork and turkey respectively in pounds.

Objective function

Minimize total cost

0.76B + 0.82P + 0.64C + 0.58T

Subject to constraints

B + P + C + T = 0.125 (Note that 2 ounce = 0.125 pounds)

32.5B + 54P + 25.6C + 6.4T <= 6 (fat constraint)

210B + 205P + 220C + 172T <= 27 (cholesterol constraint)

640B + 1055P + 780C + 528T <= 100 (calorie constraint)
B >= 0.25 (B+P+C+T) (min percentage of beef in hotdog)
P >= 0.25 (B+P+C+T) (min percentage of pork in hotdog)
B, P, C and T >= 0 (non-negativity constraint)

Set Up and Solution

	A	B	C	D	E	F	G	H	I	J	K	L	M
1								DECISION VARIABLE (Pound of meat in hotdog of 2 ounce)					
2			Cost	Calories	Fat	Cholestrol							
3		Beef	0.76	640	32.5	210							
4		Pork	0.82	1055	54	205							
5		Chicken	0.64	780	25.6	220							
6		Turkey	0.58	528	6.4	172							
7													
8		COST	0	MINIMIZE TOTAL COST									
9													
10		0	<=	6									
11		0	<=	27									
12		0	<=	100									
13		0	equals	0.125									
14		0	>=	0									
15		0	>=	0									

- Optimization
  - Objective
    - $C$8 (Min)
  - Variables
    - Normal
      - $H$3:$H$6
    - Recourse
  - Constraints
    - Normal
      - $B$10:$B$12 <= $D$10:$D$12
      - $B$13 = $D$13
      - $B$14:$B$15 >= $D$14:$D$15
    - Chance
    - Recourse
    - Bound
      - $H$3:$H$6 >= 0
    - Conic
    - Integers
  - Parameters

	A	B	C	D	E	F	G	H	I	J	K	L	M
1								DECISION VARIABLE (Pound of meat in hotdog of 2 ounce)					
2			Cost	Calories	Fat	Cholestrol							
3		Beef	0.76	640	32.5	210		0.03125					
4		Pork	0.82	1055	54	205		0.03125					
5		Chicken	0.64	780	25.6	220		0					
6		Turkey	0.58	528	6.4	172		0.0625					
7													
8		COST	0.085625	MINIMIZE TOTAL COST									
9													
10		3.103125	<=	6									
11		23.71875	<=	27									
12		85.96875	<=	100									
13		0.125	equals	0.125									
14		0.03125	>=	0.03125									
15		0.03125	>=	0.03125									
16													

Hot dog is comprised of 0.03125 pounds of beef, 0.03125 pounds of pork and 0.0625 pounds of turkey. The final cost is $0.0856.

Problem 3

Let $X = \{x_{ij}: i, j \in N, i \neq j\}$ represent the set of decision variables
Let $S = \{s_{ij}: i, j \in N, i \neq j\}$ represent the matrix of surgical setup times.

As mentioned in the paper:

x_{ij} – the decision variable, equals 1 if surgery i is performed immediately before surgery j, or 0 otherwise.

s_{ij} – the setup time for surgery j when it immediately follows surgery i. Since the s_{ij} are sequence-dependent, the setup time matrix S is asymmetric ($s_{ij} \neq s_{ji}$).

$$\min \quad \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} s_{ij} x_{ij} \tag{1}$$

$$\text{subject to} \quad \sum_{i=1}^{n} x_{ij} = 1 \quad \forall j \in N \tag{2}$$

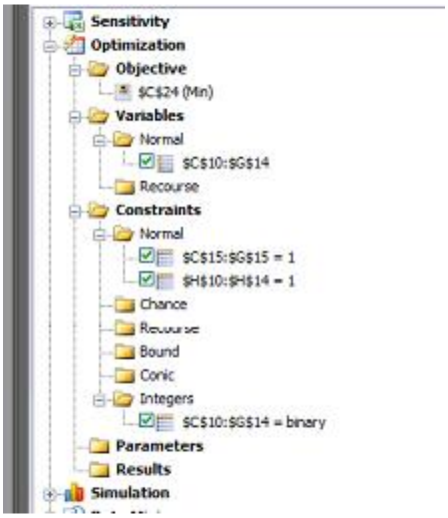
$$\sum_{j=1}^{n} x_{ij} = 1 \quad \forall i \in N \tag{3}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N \tag{4}$$

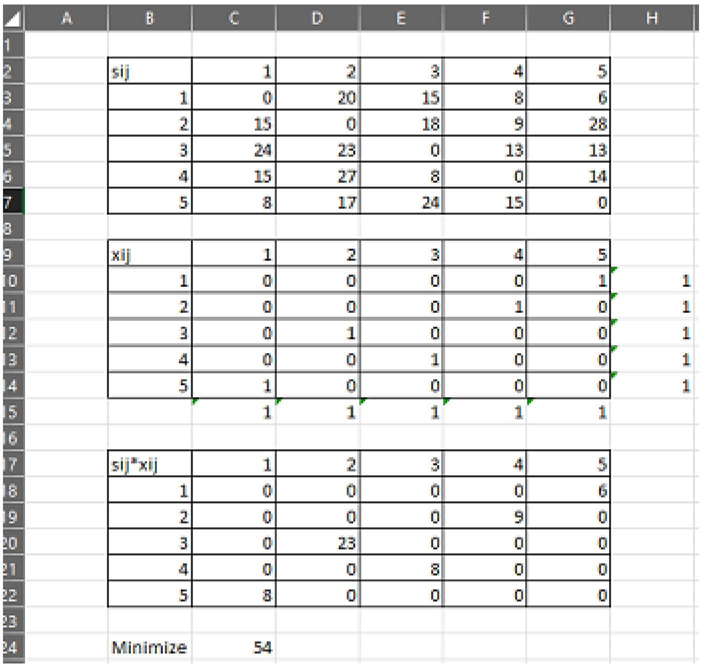
Instance: 5 patients

Setup

	A	B	C	D	E	F	G	H	I
1									
2		sij	1	2	3	4	5		
3		1	0	20	15	8	6		
4		2	15	0	18	9	28		
5		3	24	23	0	13	13		
6		4	15	27	8	0	14		
7		5	8	17	24	15	0		
8									
9		xij	1	2	3	4	5		
10		1						0	
11		2						0	
12		3						0	
13		4						0	
14		5						0	
15			0	0	0	0	0		
16									
17		sij*xij	1	2	3	4	5		
18		1	0	0	0	0	0		
19		2	0	0	0	0	0		
20		3	0	0	0	0	0		
21		4	0	0	0	0	0		
22		5	0	0	0	0	0		
23									
24		Minimize	0						



Solution



	A	B	C	D	E	F	G	H
1								
2		sij	1	2	3	4	5	
3		1	0	20	15	8	6	
4		2	15	0	18	9	28	
5		3	24	23	0	13	13	
6		4	15	27	8	0	14	
7		5	8	17	24	15	0	
8								
9		xij	1	2	3	4	5	
10		1	0	0	0	0	1	1
11		2	0	0	0	1	0	1
12		3	0	1	0	0	0	1
13		4	0	0	1	0	0	1
14		5	1	0	0	0	0	1
15			1	1	1	1	1	
16								
17		sij*xij	1	2	3	4	5	
18		1	0	0	0	0	6	
19		2	0	0	0	9	0	
20		3	0	23	0	0	0	
21		4	0	0	8	0	0	
22		5	8	0	0	0	0	
23								
24		Minimize	54					

The minimized total surgical set up time is 54
Optimal surgical schedule with minimum makespan is seen in the table below:

Xij	1	2	3	4	5
1	0	0	0	0	1
2	0	0	0	1	0
3	0	1	0	0	0
4	0	0	1	0	0
5	1	0	0	0	0

Schedules are:
2-4-3-2
1-5-1

Similar setup for other two instances.

<u>Instance: 10 patients</u>

The minimized total surgical set up time is 98
Optimal surgical schedule with minimum makespan is seen in the table below:

xij	1	2	3	4	5	6	7	8	9	10
1	0	0	1	0	0	0	0	0	0	0
2	0	0	0	0	0	1	0	0	0	0
3	0	0	0	0	0	0	0	1	0	0
4	0	1	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0
7	0	0	0	0	0	0	0	0	1	0
8	0	0	0	0	0	0	0	0	0	1
9	1	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0

Schedules are:
1-3-8-10-7-9-1
2-6-5-4-2

<u>Instance: 15 patients</u>

Solver has a limit of 200 decision variables. Here we have 210 decision variables. Hence unable to solve using Solver. But the process to solve is the same. We use open solver.

Set up

17																	
18	xij	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
19	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
20	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
21	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
22	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
23	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
24	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
25	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
26	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
27	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
28	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
29	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
30	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
31	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
32	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
33	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
34		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
35																	
36	xij*rij	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
37	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
38	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
39	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
40	4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
41	5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
42	6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
43	7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
44	8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
45	9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
46	10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
47	11	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
48	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
49	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
50	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
51	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	
52																	
53		MINIMIZE	0														

The minimized total surgical set up time is 111

Optimal surgical schedule with minimum makespan is seen in the table below:

xij	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
2	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
6	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
7	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
10	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
12	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
14	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
15	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Schedules are:

1-10-12-4-7-3-2-8-5-15-1
6-14-6
9-11-13-9

Problem 4

Decision Variables

X1, X2 and X3 which are the number of Tiny Tanks, Tiny Trucks and Tiny Turtles produced respectively.

Goals

The question says:
The company allows no more than 40 hours a week for production (priority #1). Thus, there cannot be overutilization of labor hours. But then under priority #3 it says 'maximize available labor hour usage. Hence, minimizing of underutilization of labor hours is priority 3 and hence goal 6 below.

	Goals	Description	Rank	Weight
a	Goal 1	Minimize overutilization of plastic	4	0.3636
a	Goal 2	Minimize overutilization of rubber	2	0.1818
a	Goal 3	Minimize overutilization of metal	2	0.1818
b	Goal 4	Minimize overutilization of budget	1	0.0909
b	Goal 5	Minimize underutilization of budget	1	0.0909
b	Goal 6	Maximize available hours usage i.e. Minimize underutilization.	1	0.0909
		Total:	11	1

Calculation: 4/11 = 0.3636 and so on…

Let Oi be the excess of right side of the goal w.r.t. the goal 'i'
Let Ui be the deficit of right side of the goal w.r.t. the goal 'i'
Where i = 1, 2, 3, 4, 5, 6

Objective Function
Minimize
0.3636 O1 + 0.1818 O2 + 0.1818 O3 + 0.0909 O4 + 0.0909 U5 + 0.0909 U6

Subject to Constraints
1.5 X1 + 2 X2 + X3 + U1 - O1 = 16000 (Weekly availability of plastic constraint)
0.5 X1 + 0.5 X2 + X3 + U2 – O2 = 5000 (Weekly availability of rubber constraint)
0.3 X1 + 0.6 X2 + U3 – O3 = 9000 (Weekly availability of metal constraint)
7 X1 + 5 X2 + 4 X3 + U4 - O4 = 164000 (Budget constraint overutilization)
7 X1 + 5 X2 + 4 X3 + U5 - O5 = 164000 (Budget constraint underutilization)
2 X1 + 2 X2 + X3 + U6 – O6 = 40 (labor hour usage constraint, O6 = 0 as labor hours cannot be more than 40)
X1, X2, X3, Oi, Ui ≥ 0 (Non-negativity constraints)

<u>Problem 5</u>

Decision Variables
Let T be the advertising expenditure for TV in thousand dollars.
Let R be the advertising expenditure for radio in thousand dollars.

Goals

Goals	Description	Rank	Weight
Goal 1	Achieve total exposures of at least 750,000 persons, i.e. minimize underutilization of total exposures	5	w1 = 0.3226
Goal 2	Avoid expenditures of more than $100,000, i.e. minimize overutilization of budget	4	w2 = 0.2581
Goal 3	Avoid expenditures of more than $70,000 for television advertisements, i.e. minimize overutilization of television expenditures	3	w3 = 0.1935
Goal 4	Achieve at least 1 million total exposures, i.e. minimize underutilization of total exposures	2	w4 = 0.1290
Goal 5	Reach at least 250,000 persons in age group 25-30 years, i.e. minimize underutilization	1	w5 = 0.0645
Goal 6	Reach at least 250,000 persons in age group 18-21 years, i.e. minimize underutilization	0.5	w6 = 0.0323
	Total:	15.5	1

Note that there are other ways of assigning weights depending on the "relative importance" of the goals.

Let Oi be the excess of right side of the goal w.r.t. the goal 'i'
Let Ui be the deficit of right side of the goal w.r.t. the goal 'i'
Where i = 1, 2, 3, 4, 5, 6

Objective Function
Minimize
w1*U1 + w2*O2 + w3*O3 + w4*U4 + w5*U5 + w6*U6
Or,
Maximize
w1*O1 + w2*U2 + w3*U3 + w4*O4 + w5*O5 + w6*O6

Subject to Constraints
G1: 10,000 T+ 7,500 R + U1 - O1 = 750,000
G2: 1,000 T + 1,000 R + U2 – O2 = 100,000
G3: 1,000 T + U3 – O3 = 70,000
G4: 10,000 T+ 7,500 R + U4 - O4 = 1,000,000
G5: 3,000 T + 1,500 R + U6 - O6 = 250,000
G6: 2,500 T + 3,000 R + U5 - O5 = 250,000

T, R, Oi, Ui ≥ 0 (Non-negativity constraints)

Extra Credit Problem

There are 6 sensible patterns that can be used to cut the 10 ft. boards.

Patterns	3ft	4ft	5ft	Waste (in ft)
1	3	0	0	1
2	2	1	0	0
3	1	0	1	2
4	0	0	2	0
5	0	2	0	2
6	0	1	1	1

Decision variables

X1, X2, X3, X4, X5 and X6 are the number of 10 ft. boards of each of the six patterns used.

Objective Function

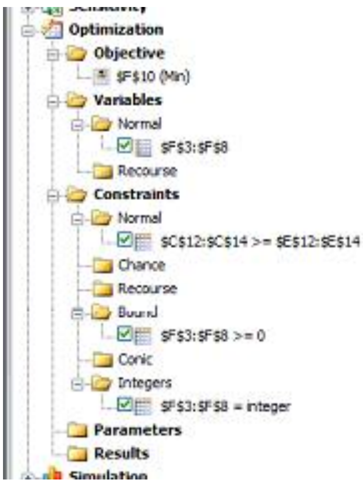
Minimize X1 + X2 + X3 + X4 + X5 + X6

Subject to Constraints

3 X1 + 2 X2 + X3 >= 90 (order for 3 ft. boards)
X2 + 2 X5 + X6 >= 60 (order for 4 ft. boards)
X3 + 2 X4 + X6 >= 60 (order for 5 ft. boards)
X1, X2, X3, X4, X5 and X6 >= 0 and integers (non-negativity constraint)

Set up and Solution

	A	B	C	D	E	F	G
1							
2		Patterns	3ft	4ft	5ft	Decision Variable	
3		1	3	0	0		
4		2	2	1	0		
5		3	1	0	1		
6		4	0	0	2		
7		5	0	2	0		
8		6	0	1	1		
9							
10					Minimize	0	
11							
12			0	>=	90		
13			0	>=	60		
14			0	>=	60		



	A	B	C	D	E	F	G
1							
2		Patterns	3ft	4ft	5ft	Decision Variable	
3		1	3	0	0	0	
4		2	2	1	0	45	
5		3	1	0	1	0	
6		4	0	0	2	30	
7		5	0	2	0	8	
8		6	0	1	1	0	
9							
10					Minimize	83	
11							
12			90	>=	90		
13			61	>=	60		
14			60	>=	60		
15							
16							

The optimal number of patterns is Forty Five Pattern 2, Thirty Pattern 4 and Eight Pattern 5. Total minimum number of boards to cut is 83.