NORTHWESTERN UNIVERSITY | SCHOOL OF PROFESSIONAL STUDIES

Fall 2017 Midterm Exam

PREDICT 400: Math for Modelers

Points possible: 100

Description: The midterm exam will cover topics from sessions 1-4.

Resources: The exam is completely open book. You may use course textbooks, materials provided on Canvas, graphing calculators (such as TI 83 or 84); *but any more advanced calculators, Excel Solver, Web calculators, Web-graphic calculators, or simplex method calculators are not allowed. Programming languages other than Python are also not permitted*.

For questions that require calculations, all calculations should be shown, not just the final answer. This will allow for partial credit for those answers that might be set up correctly but have calculation errors. If variables are introduced, whether in a program or in a mathematical solution, you will need to explicitly state what the variables represent and clearly state the results based on the variable designations.

For questions that specifically require Python, your code and output should be included to constitute work shown along with solutions. For questions that require graphs, only use Python. Python can be used for all questions unless instructed otherwise. For any problem the uses Python, your code and output must be provided.

Restrictions: All answers are to be your work only. You are not to receive assistance from any other person.

To complete the exam:

1. Answer all questions on the exam thoroughly. Create a single Microsoft Word document, including the question number, the question, your typed answer, and graphs if required. You may use Word's equation editor to complete your answers.

2. Once you have completed your exam, save the file with a meaningful filename such as "Midterm" followed by your last name and return to the exam item where you downloaded the exam PDF, click View/Complete Assignment, and submit your document.

1. The cost to manufacture 1200 pairs of Nike's bestselling Crossfit shoe, the Metcon X, is $66,000 and the cost to manufacture 2500 pairs is $111,500. Determine the linear cost function and the price Nike should sell the shoes to break even at 267 shoes (round to the nearest dollar).

   Solution:
   Let "x" be the number of shoes and "y" is the price of the shoes

x	1200	2500
y	66000	111500

   Then we find the equation of the line.
   Gradient, m = change in y/change in x
   Y=mx+c
   m= (111500-66000)/(2500-1200)
   m= 35
   y= 35x+c
   Equating one of the points,
   111500= 35(1200) +c
   c=24000
   Therefore, y=35x+24000
   At 267 pairs
   y=35(267) +24000
   =$33345
   $33345/267 shoes = 124.88
   The value $33345 is the price that the 267 shoes should be sold for to break even, therefore…
   Rounding to the nearest dollar, the price Nike should sell the shoes to break even at 267 shoes is **$125**.

2. In an effort to replace the destruction of trees, five regions of the US have been tracking the annual rate of trees being planted (per thousand) in each region and the annual rate of trees being destroyed (per thousand). Additionally, the tree population (in thousands) by region has been recorded for five different years. The data is given in the tables below. **Use Python** to determine the number of trees planted and the number destroyed in each of the years listed.

	Rate of Trees Being Planted	Rate of Trees Being Destroyed
Northeast	0.0341	0.0115
Southeast	0.0174	0.0073

Midwest	0.0185	0.0056
Southwest	0.0131	0.0082
West	0.0096	0.0105

Tree Population by Region					
Year	Northeast	Southeast	Midwest	Southwest	West
1975	365	2036	285	226	460
1985	471	2494	361	251	485
1995	622	2976	441	278	499
2005	803	3435	523	314	514
2015	1013	3827	592	344	522

Solution using Python:

```
import numpy as np


class CalculateRatio(object):
    def __init__(self, tree_being_planted_ratio, tree_being_destroyed_ratio, trees,
years, display_print):
        self.tree_being_planted_ratio = tree_being_planted_ratio
        self.tree_being_destroyed_ratio = tree_being_destroyed_ratio
        self.trees = trees
        self.years = years
        self.display_print = display_print

    def calculation(self):
        total_trees_being_planted = np.array(self.tree_being_planted_ratio) *
np.array(self.trees)
        total_trees_being_destroyed = np.array(self.tree_being_destroyed_ratio) *
np.array(self.trees)

        total_number_of_trees_being_planted_by_year =
np.sum(total_trees_being_planted, axis=1)
        total_number_of_trees_being_destroyed_by_year =
np.sum(total_trees_being_destroyed, axis=1)

        years = np.array(self.years)
        result = np.vstack([years, total_number_of_trees_being_planted_by_year,
total_number_of_trees_being_destroyed_by_year])
        answer_print = np.transpose(result)

        if self.display_print:
            return answer_print
        else:
            return result

def main():
    rate_of_trees_being_planted = [.0341, .0174, .0185, .0131, .0096]

    rate_of_trees_being_destroyed = [.0115, .0073, .0056, .0082, .0105]

    tree_population_by_region_per_thousands = [[365, 2036, 285, 226, 460],
                                               [471, 2494, 361, 251, 485],
```

```
                                                            [622, 2976, 441, 278, 499],
                                                            [803, 3435, 523, 314, 514],
                                                            [1013, 3827, 592, 344, 522]]

    years = [1975, 1985, 1995, 2005, 2015]

    display_print = True

    if len(rate_of_trees_being_planted) != len(rate_of_trees_being_destroyed):
        raise AssertionError("Rate of Trees Being Planted and the Rate of Trees Being
Destroyed must have the same total entries")

    calculation_obj =
CalculateRatio(tree_being_planted_ratio=rate_of_trees_being_planted,
tree_being_destroyed_ratio=rate_of_trees_being_destroyed,
                                             trees=tree_population_by_region_per_thousands,
years=years, display_print=display_print)

    output = calculation_obj.calculation()

    print(["Year", "Rate of Trees Being Planted (Thousands)", "Rate of Trees Being
Destroyed (Thousands)"])

    print(output)

if __name__ == '__main__':
    main()
```

```
/Users/jasonmccoy/anaconda/bin/python "/Users/jasonmccoy/Downloads/Question 3.py"
['Year', 'Rate of Trees Being Planted (Thousands)', 'Rate of Trees Being Destroyed (Thousands)']
[[ 1975.         60.522      27.3395]
 [ 1985.         74.0793     32.795 ]
 [ 1995.         89.5833     38.8665]
 [ 2005.        105.8746     45.2106]
 [ 2015.        121.6027     51.2036]]
```

Solution:

Let the annual rate of trees being planted (thousands) be represented by ‘’a’’ and the annual rate of trees being destroyed (thousands) be “b”.

	northeast	southeast	Midwest	southwest	west
year	$x10^3$	$x10^3$	$x10^3$	$x10^3$	$x10^3$
1975 a b	 0.0341x365 =12.4465 0.0115x365	 0.0174x2036 =35.4264 0.0073x2036	 0.0185x285 =5.2725 0.0056x285	 0.0131x226 =2.9606 0.0082x226	 0.0096x460 =4.416 0.0105x460

	=4.1975	=14.868	=1.596	=1.8532	=4.83
1985 a b	 0.0341x471 =16.0611 0.0115x471 =5.4165	 0.0174x2494 =43.3956 0.0073x2494 =18.2062	 0.0185x361 =6.6785 0.0056x361 =2.0216	 0.0131x251 =3.2881 0.0082x251 =2.0582	 0.0096x485 =4.656 0.0105x485 =5.0925
1995 a b	 0.0341x622 =21.2102 0.0115x622 =7.153	 0.0174x2976 =51.7824 0.0073x2976 =21.7248	 0.0185x441 =8.1588 0.0056x441 =2.4696	 0.0131x278 =3.6696 0.0082x278 =2.2796	 0.0096x499 =4.7904 0.0105x499 =5.2395
2005 a b	 0.0341x803 =27.3823 0.0115x803 =1.2045	 0.0174x3435 =59.769 0.0073x3435 =25.0755	 0.0185x523 =9.6755 0.0056x523 =2.9288	 0.0131x314 =4.1134 0.0082x314 =2.5748	 0.0096x514 =4.9344 0.0105x514 =5.397
2015 a b	 0.0341x1013 =34.5433 0.0115x1013 =11.6495	 0.0174x3827 =66.5898 0.0073x3827 =27.9371	 0.0185x592 =10.952 0.0056x592 =3.3152	 0.0131x344 =4.5064 0.0082x344 =2.8208	 0.0096x522 =5.0112 0.0105x522 =5.481

3. Lucinda's boutique sells three types of aromatherapy mixtures that each use essential oils, lavender, and chamomile. Scent A requires 3 oz. of essential oils, 2 oz. of lavender, and 1 oz. of chamomile. Scent B requires 3 oz. of essential oils, 1 oz. of lavender, and 2 oz. of chamomile. Scent C requires 2 oz. of essential oils, 1 oz. of lavender, and 2 oz. of chamomile. Determine how many of each scent can be made with 915 oz. of essential oils, 465 oz. of lavender, and 525 oz. of chamomile.

Solution:

Let "x" be essential

"y" be lavender

"z" be chamomile

A= 3x+2y+z

B= 3x+y+2z

C= 2x+y+2z

x=915, y=465 and z=525

Therefore,

3A+3B+2C=915 … (i)

2A+B+C=465 … (ii)

2A+2B+2C=525 … (iii)

Using elimination method, subtract equation (iii) from equation (i)

3A+3B+2C=915

A+2B+2C=525

We get 2A+B=390

Using equation (i) and (ii)

3A+3B+2C=915

2(2A+B+C=465)

We get, A-B=15

adding;

2A+B=390

A-B=15

3A=405

A=135

From equation above,

135-B=15

B=120

And;

A+2B+2C=525

135+240+2C=525

2C=150

C=75

135oz of scent A, 120oz of scent B and 75oz of scent C can be made with 915oz. of essential oils, 465oz. of lavender, and 525oz. of chamomile.

4. Ed's lawn supply store has 165kg of rye grass seed and 93kg of bluegrass seed. He plans to sell two different grass seed mixes. One mix will contain half rye grass seed and half bluegrass seed and will sell for $8.25 per kg. The other mix will contain ¾ rye grass seed and ¼ bluegrass seed and will sell for $7.50 per kg. **Use Python** to determine how many kilograms of each seed mixture he should prepare for the maximum revenue, as well as that revenue.

Solution using Python:

```
from pulp import *

# Stack variables
variable1 = LpVariable("variable1", 0, 165)
variable2 = LpVariable("variable2", 0, 165)
variable3 = LpVariable("variable3", 0, 165)

# kg of each seed problem
inequality = LpProblem("kg of each seed problem", LpMinimize)

# Remove the inequality
inequality += 0.5 * variable1 + 0.25 * variable2 + variable3 == 165
inequality += 0.5 * variable1 - 0.25 * variable2 + variable3 == 93
```

```
# Z-8.25a-7.50b=0
inequality += 8.25 * variable1 - 7.50 * variable2

# Solve the problem via GLPK/PuLP
inequality.solve(solver=GLPK("/Users/jasonmccoy/anaconda/pkgs/glpk-4.62-0/bin/glpsol
"))

Mix1 = value(variable3 - 15)
Mix2 = value(variable2)

print("A = ", Mix1)
print("B = ", Mix2)
print("=%dA+%dB" % (Mix1, Mix2))

total_revenue = 1048.50
print("Total Revenue is $%d" % total_revenue)
```

Solution:

Let rye grass be “x”

Let blue grass be “y”

Mix a= 0.5x+0.5y =8.25/kg

Mix b=0.75x+0.25y=7.50/kg

Let revenue be “z”

Max z=8.25a+7.50b

Such that:

0.5a+0.75b<=165: rye grass

0.5a=0.25<=93; blue grass

Z-8.25a-7.50b=0

Add stack variable to remove the inequality, represented as “s”

$0.5a+0.25b+s_1=165$

$0.5a-0.25b+s_2=93$

	z	a	b	S_1	S_2	results	Basic variable	ratio
R_0	1	-8.25	-7.50	0	0	0	Z=0	

R_1	0	0.5	0.75	1	0	165	S_1=165	330
R_2	0	0.5	0.25	0	1	93	S_2=93	186

Check most -ve in ratio

Check the smallest ratio

R_0’	0	-3.375	0	0	16.5	1534.5		
R_1’	0	0.5	1	0	-1	72	S_1=72	144
R_2’	1	0.5	0	1	2	186	A=186	372

R_0’’	1	0	0.75	13.125	1048.5	
R_1’’	0	1	2	2	144	B=144
R_2’’	0	0	-1	-1	114	A=114

=114A+144B

Revenue (z) =1048.5

Mix A should contain 114kg of seed whereas, Mix B contains 144kg of seed.

5. Camp Christopher wants to hire counselors and assistants to fill its seasonal staffing needs at a minimum cost. The average monthly salary of a counselor is $2400 and the average monthly salary of an assistant is $1100. The camp can hire up to 35 staff members and needs at least 20 to run properly. They must have at least 10 assistants and may have up to 3 assistants for every 2 counselors. **Using Python**, graph the feasible region and determine the number of counselors and assistants the campaign should hire.

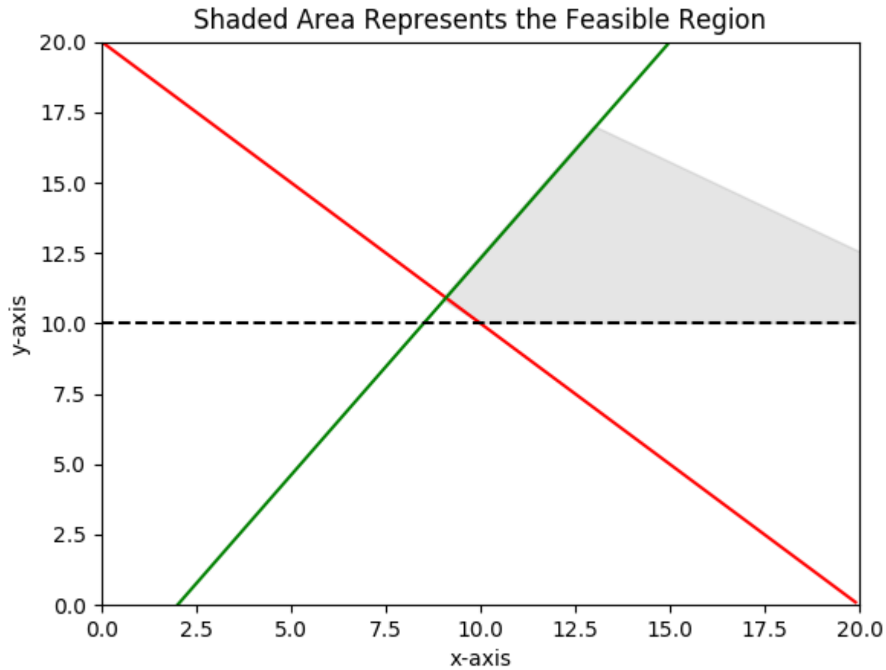
Solution using Python:

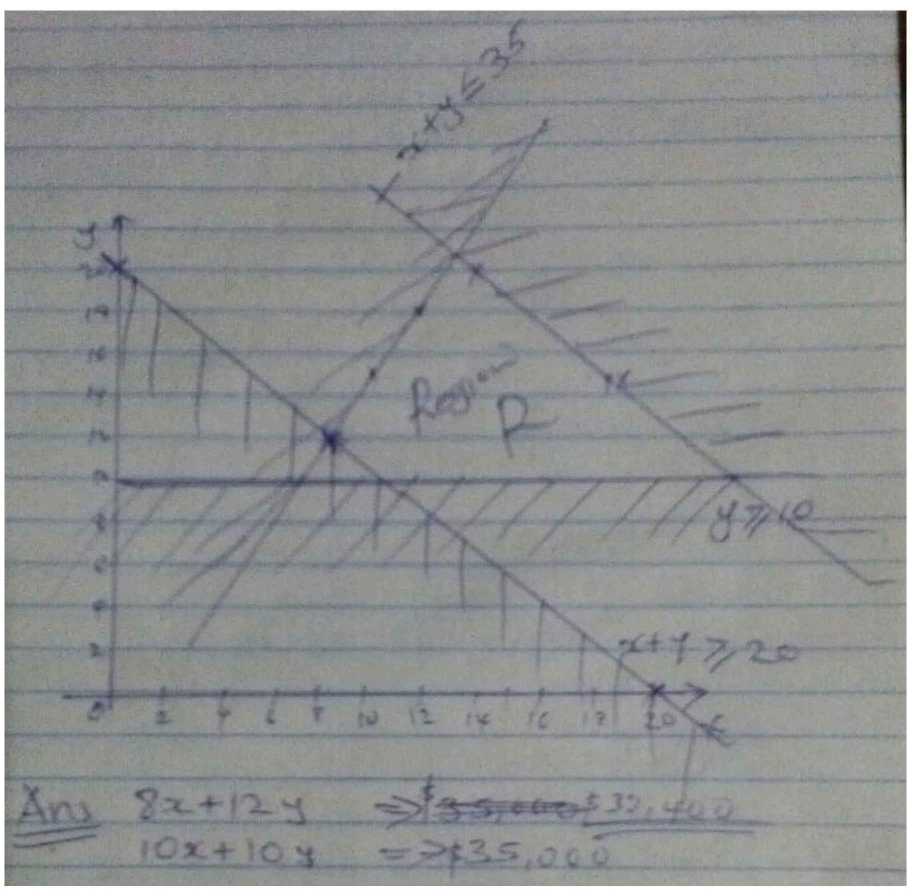
```
import pandas as pd
import matplotlib.pyplot
from matplotlib.pyplot import *
import numpy
from numpy import *

'''
Given:
Needs at least 20 staff members to run properly:
Monthly salary of a counselor is $2400
Monthly salary of an assistant is $1100
Must have at least 10 assistants
May have up to 3 assistants for every 2 counselors
'''

'''
8x + 12y
10x + 10y
x+y <= 35
y >= 10
x + y >= 20
'''

x = arange(0, 20, 0.1)
x0 = arange(40, -11, 0.1)
y0 = arange(0, 20, 0.1)

y = 10+0.0*x
y1 = 20.0-1.0*x  # red
y2 = 10.0-x0/4.0  # blue


# Vertical Line
x1 = 2 + .65 * y0


# Plotting limitation
xlim(0, 20)  # Left to Right
ylim(0, 20)  # Top to Bottom


# Titles
xlabel('x-axis')  # Bottom
ylabel('y-axis')  # Left Side
title('Shaded Area Represents the Feasible Region')  # Top Middle


# Plotting lines
plot(x, y1, 'r')  # Red Line
plot(x1, y0, 'g')  # Green Line
plot(x0, y2, 'b')  # Blue Line


#  Objective function
plot(x, y, 'k--')  # Black Dotted Line


# Shaded Area
x = [10, 9, 13, 24]
```

```python
y = [10, 10.85, 17, 10]
fill(x, y, color='grey', alpha=0.2)

show()

'''
8x + 12y
$32,400

10x + 10y
$35,000
'''

'''
Since assistants are cheaper to hire than counselors, we ideally want as many
assistants as possible, so we will have
exactly 3 for every 2 counselors. So the ratio of Assistants, A, to Counselors, C,
will be 3:2.

Also since 20 staff members is the minimum needed to run properly, and we want the
minimum cost, we will hire only
20 staff members.

Hence we have, 3A + 2A = 20, where 3A is the number of counselors, and 2A is the
number of counselors, hence maintaining
the 3:2 ratio. Here, we get A = 4, so we know there are 3 * 4 = 12 assistants, and
2 * 4 = 8 counselors.

So the answer is:
12 Assistants
8 Counselors
'''


table = [('x', 18, 12, 6),
         ('y', 12, 8, 4)
        ]
labels = [' ', ' ', ' ', ' ']

df = pd.DataFrame.from_records(table, columns=labels)

print(df)
```

```
Users/jasonmccoy/anaconda/bin/python /Users/jasonmccoy/Downloads/test4.py

  x  18  12  6
  y  12   8  4
```

8 counselors and 12 assistants should be hired and the combined minimum revenue will be $32,400.


Shaded Area Represents the Feasible Region
y-axis
x-axis


x+y≤35
Region R
y≥10
x+y≥20
Ans 8x+12y ⇒ ~~$35,000~~ $32,400
10x+10y ⇒ $35,000

Solution:

Let counselors be "x"

Let assistants be "y"

$20<=x+y<=35$

Minimize $z=2400x=1100y$

Such that;

$x+y<=35$

$x+y>=20$

Maximum: 3y for 2x

$y>=3/2x$

For $x+y>=20$

X	0	20
y	20	0

For $x+y<=35$

x	15	30
y	20	5

For $y<=3/2x$

x	8	12
y	12	18

6. Louise has decided to plan a monthly exercise program to include yoga, Crossfit, and swimming. She would like to limit her exercise routine to at most 28 hours per month. Her plan would be to spend at most 9 hours swimming, and limit Crossfit to no more than the total hours spent swimming and doing yoga. She can burn 200 calories per hour of yoga, 597 calories per hour of Crossfit, and 263 calories per hour of swimming. How many hours should be spent for each activity to maximize the number of calories burned and how many calories will she burn?

Solution:

Let exercise be "e"

Let swimming be "s"

Let crossfit be "c"

Let yoga be "y"

e+s+c=<28

s<=9

s+y>c

200y+597c+263s

Plotting of feasibility graph we obtain

S=9, y=6, c=13

Therefore;

(200x9) + (597x13) + (263x6) = 11139

9 hours in swimming, 13 hours in cross fit and 6 hours doing yoga should be spent for each activity to maximize the number of calories burned. The total calories burned will be 11139 for that month.

7. Goodyear Commercial Retread facility recycles worn or damaged tires to retread two sizes of tires, medium and large, for tractor trailers. To fill one client's order, they will use a combination of three machines to manufacture the retreads and will need to produce at least 54 medium tires and 59 large tires. In one hour, Machine I can produce 2.5 medium tires and 5 large tires, Machine II can produce 4.5 medium tires and 3 large tires, and Machine III can produce 5 medium tires and 10 large tires. The cost per hour to operate Machine I, II, and III are $8, $9, and $10, respectively. There are two combinations of using the three machines that produce the same minimum cost. What are those two combinations and what is the minimum cost?

Solution:

Let the medium tires be "m"

Let the large tires be "l"

Let the hours taken be "h"

h (2.5m + 4.5m + 5m) = 54m

h (5l + 3l + 10l) = 59l

Machine 1 = $8 (2.5m + 5l) = 20m + 40l

(2.5 x 54) + (5 x 59) = 430; (20 x 54) + (40 x 59) = 3440

Machine 2 = $9 (4.5m + 3l) = 40.5m + 27l

(4.5 x 54) + (3 x 59) = 420; (40.5 x 54) + (27 x 59) = 3780

Machine 3 = $10 (5m + 10l) = 50m + 100l

(5 x 54) + (10 x 59) = 860; (50 x 54) + (100 x 59) = 8600

The two combinations of using the three machines that produce the same minimum cost are Machine 1 & 2 and Machine 2 & 1 and the minimum cost is 850.

8. A company is creating a new fertilizer additive for lawn seed, which will be a mixture of nitrogen, phosphate, and potash. Based on their research, the total amount fertilizer added must be at least 14 oz. per 5 lb. bag of lawn seed, but should not exceed 20 oz. per 5 lb. bag. At least ¼ oz. of nitrogen must be used for every ounce of phosphate, and at least 1 oz. of potash must be used for every ounce of nitrogen. The costs per ounce of nitrogen, phosphate, and potash are $0.30, $0.18, and $0.54, respectively. Use Python to determine the mixture of the three ingredients in a 5 lb. bag that minimizes costs, as well as that cost.

Solution using Python:

```
from pulp import *

# Fertilizer additive for lawn seed variables
n = LpVariable("variable1", 0, 14)
p = LpVariable("variable2", 0, 14)
k = LpVariable("variable3", 0, 14)

# Mixture of the three ingredients in a 5 lb. bag that minimizes costs
inequality = LpProblem("mixture of the three ingredients in a 5 lb. bag that minimizes 
costs problem", LpMinimize)
```

```
# Cost is c=0.30n+0.18p+0.54k
inequality += (.30 * n) + (.18 * p) + (.54 * k) == 14

# Total amount fertilizer added (14oz), should not exceed (20oz) and ¼oz. of n must be used.
inequality += 14 * n + 20 * p + .25 * k

# Solve the problem via GLPK/PuLP
inequality.solve(solver=GLPK("/Users/jasonmccoy/anaconda/pkgs/glpk-4.62-0/bin/glpsol"))

region_of_total_mixture = value(k)

print("\nThe region of total mixture, which will minimize the cost is: ", region_of_total_mixture)
'''
n = 1/6x18 = 3oz
p = 4/6x18 = 8oz
k = 1/6x18 = 3oz
'''
cost_per_5lb_bag = (0.30 * 3) + (0.18 * 8) + (0.54 * 3)
print("The cost of the total mixture (which will minimize the cost) is: $", cost_per_5lb_bag)
```

Solution:

Let fertilizer be “f”

14=<f<=20

Let nitrogen be “n”

Let phosphate be “p”

Let potash be “k”

f=0.25n+p+0.25k

We can deduce that quantity of n and k must be the same in the fertilizer from the equation above.

Let the cost be ‘”c”

c=0.30n+0.18p+0.54k

Using ratios:

1n:4p:1k

Graphing we obtain the region as total mixture as 18 ounces which will minimize the cost. From the ratios:

n=1/6x18=3oz

p=4/6x18=8oz

k=1/6x18=3oz

c= (0.30x3) + (0.18x8) + (0.54x3)

c=$3.96 per 5lb bag

The mixture of the three ingredients in a 5 lb. bag that minimizes costs is 3oz of nitrogen, 8oz of phosphate and 3oz of potash. The cost is $3.96 per 5lb bag.

9. A survey of 4500 people was conducted to collect data on family history of high blood pressure (HBP), high cholesterol (HC), and heart disease (HD) among Caucasian women. The responses showed that 1621 had a family history of all three, 1829 had a family history of HBP and HC, 695 had a family history of HBP only, 2710 had a family history of HBP, 1779 had a family history of HD and HC, 2619 had a family history of HC, and 2537 had a family history of HD. How many of the women surveyed had no family history of any of these?

Solution:

If hbc+hc+hd=1621

hbp+hc=1829

then hbp+hc = (1829-1621)

=208

hbp only = 695.

To get the value for hbp+hd, take the value for all tested for hbp

Hbp = 2710; hbp+hd=2710-(695+208+1621) = 186

hd+hc = 1779

therefore, hd+hc only will be ; 1779-1621(value for all 3) = 158

value for hc only will be; 2619-(208+1621+158) = 632

to get hd only, 2537-(186+1621+158) = 572 to get the value for tested but with none of the diseases, take add the value for the different combinations of the sick, minus from the number of people tested.

4500-(695+208+1621+186+572+158+632)

=428; number of healthy tested people

Of the women surveyed, **428** had no family history of high blood pressure (HBP), high cholesterol (HC), and heart disease (HD) among Caucasian women.

10. A new type of test to determine the presence of lead in drinking water is successful 97.5% of the time when lead is present. However, the same test indicates lead is present 0.3% of the time when it is not present. One percent of the homes in Cleveland actually have lead present in the drinking water. Determine the probability that a home in Cleveland has lead in the drinking water given that the test indicates the presence of lead.

Solution:

Probability of lead being detected when present =0.975

Probability of test showing lead present when not in water = 0.003

Let the probability of lead in Cleveland water be “c”

c=0.01

Probability of lead in a Cleveland home= (0.975x0.01) - (0.003x0.01)

=0.00972