Homework 2

Problem #1

Decision variables

x_1, x_2, x_3

Where,

x_1 is the number of product A to be produced weekly

x_2 is the number of product B to be produced weekly

x_3 is the number of product C to be produced weekly

Objective function

Maximize weekly profit (in $)

$10\ x_1 + 10\ x_2 + 10\ x_3$

Calculations:

Profit per unit of product A = 101 - 7*3 – 5*2 - 15*4 = $10

Profit per unit of product B = 67 - 7*1 – 5*4 - 15*2 = $10

Profit per unit of product B = 97.50 - 7*5 – 5*0 - 15*3.5 = $10

Subject to Constraints

$3\ x_1 + \ x_2 + 5\ x_3 \leq 300$ (Each week there is 300 lbs. of Material 1)

$2\ x_1 + 4\ x_2 \leq 400$ (Each week there is 400 lbs. of Material 2)

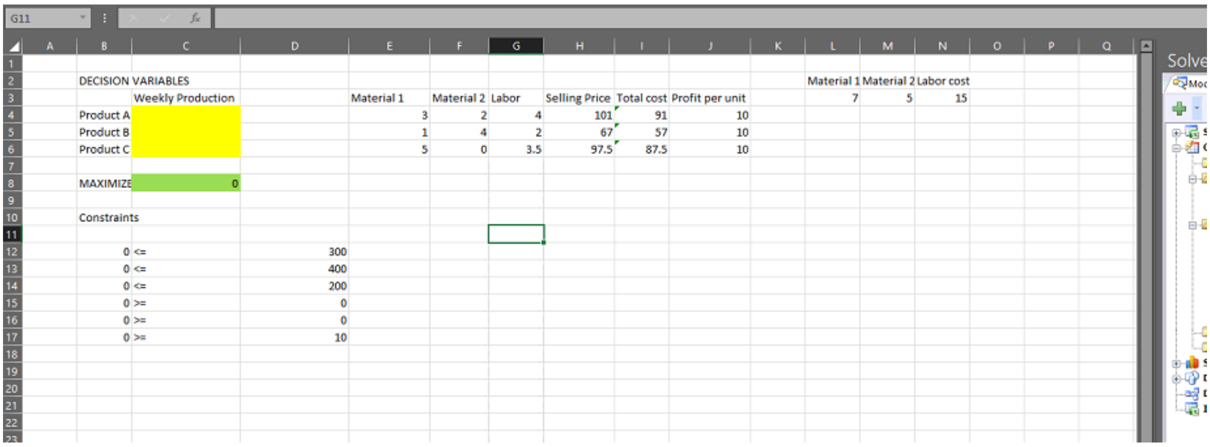
$4\ x_1 + 2\ x_2 + 3.5\ x_3 \leq 200$ (Each week there is 200 labor hours)

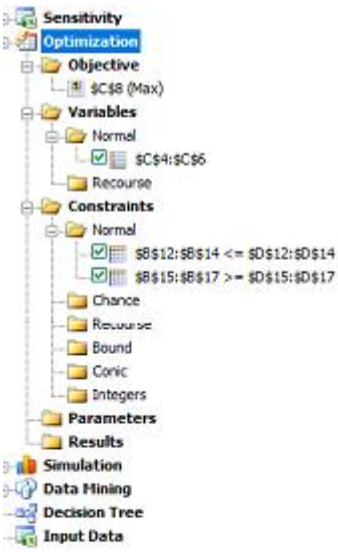
$x_1 \geq 0$ (Non-negativity constraint)

$x_2 \geq 0$ (Non-negativity constraint)

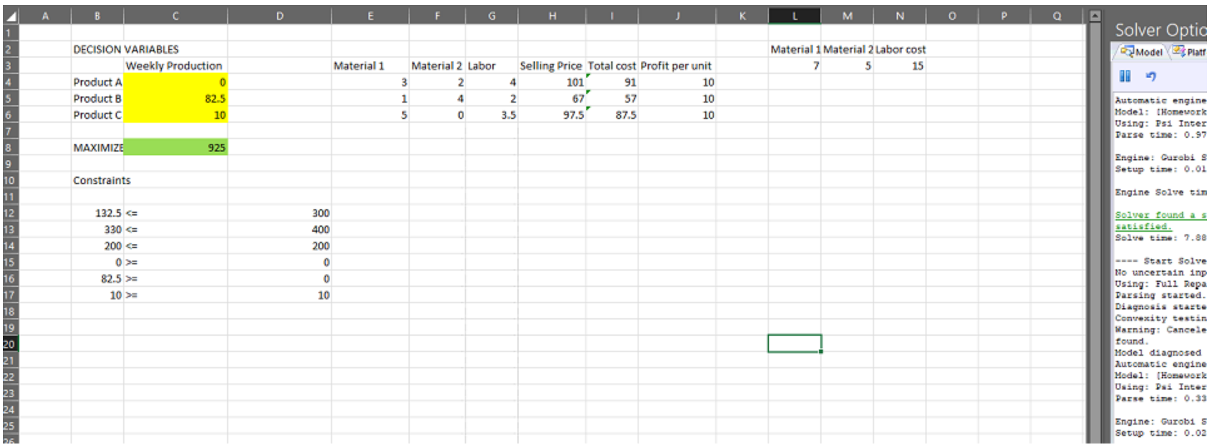
$x_3 \geq 10$ (Demand of at least 10 of product C each week)

Set up





Solution



The number of units of Product A, B and C to be produced weekly to maximize profits is 0, 82.5 and 10 respectively. The maximum profit is $925.

Note that fraction of units have been allowed. If we do not want units in fraction, additional 'integer' constraints will have to be specified.

DECISION VARIABLES										Material 1	Material 2	Labor cost
	Weekly Production		Material 1	Material 2	Labor	Selling Price	Total cost	Profit per unit		7	5	15
Product A	0		3	2	4	101	91	10				
Product B	82		1	4	2	67	57	10				
Product C	10		5	0	3.5	97.5	87.5	10				
MAXIMIZE	920											
Constraints												
132	<=	300										
328	<=	400										
199	<=	200										
0	>=	0										
82	>=	0										
10	>=	10										

With integer constraint:

The number of units of Product A, B and C to be produced weekly to maximize profits is 0, 82 and 10 respectively. The maximum profit is $920.

Problem #2

Decision variables

x_1, x_2, x_3, x_4

Where,

x_1 is the number of footballs made by the company per month in morning shift

x_2 is the number of baseballs made by the company per month in morning shift

x_3 is the number of footballs made by the company per month in evening shift

x_4 is the number of baseballs made by the company per month in evening shift

Objective function

Minimize cost (in $)

$20\,x_1 + 20\,x_2 + 25\,x_1 + 25\,x_2$

Subject to Constraints

$0.75\,x_1 + 2\,x_2 \leq 5000$

$0.75\,x_3 + \;2\,x_4 \leq 2000$

$7\,x_1 + \;15\,x_2 \leq 15000$

$7\,x_3 + 15\,x_4 \leq 14000$

$0.5\,x_1 + 2\,x_2 \leq 2000$

$0.5\,x_3 + 2\,x_4 \leq 1500$

$x_1 + x_3 \geq 1500$

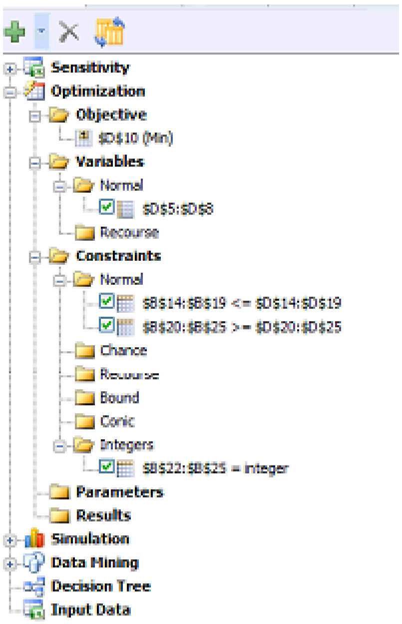
$x_2 + x_4 \geq 1200$

$x_1, x_2, x_3, x_4 \geq 0$ (Non-negativity constraint)

We will also use integer constraints for all 4 decision variables.

Set up

Decision Variables						
		Units	Production cost	Resource	Football	Baseball
Football, morning			20	Labor	0.75	2
Baseball, morning			20	Leather	7	15
Football, evening			25	Inner Plas	0.5	2
Baseball, evening			25	Total dem	1500	1200
Minimize cost		0				
Constraints						
0	<=	5000				
0	<=	2000				
0	<=	15000				
0	<=	14000				
0	<=	2000				
0	<=	1500				
0	>=	1500				
0	>=	1200				
0	>=	0				
0	>=	0				
0	>=	0				
0	>=	0				



Solution

Decision Variables							
	Units		Production cost		Resource	Football	Baseball
Football, morning	805		20		Labor	0.75	2
Baseball, morning	624		20		Leather	7	15
Football, evening	695		25		Inner Plas	0.5	2
Baseball, evening	576		25		Total dem	1500	1200
Minimize cost	60355						
Constraints							
1851.75 <=	5000						
1673.25 <=	2000						
14995 <=	15000						
13505 <=	14000						
1650.5 <=	2000						
1499.5 <=	1500						
1500 >=	1500						
1200 >=	1200						
805 >=	0						
624 >=	0						
695 >=	0						
576 >=	0						

Number of footballs made by the company is 805 in the morning and 695 in the evening. The number of baseballs made by the company is 624 in the morning and 576 in the evening. The minimized cost is $60,355.

Problem #3

Decision variables

$x_1, x_2, x_3, x_4, x_5, x_6$

Where,

x_1 is the number of fire fighters joining at beginning of shift midnight-4am

x_2 is the number of fire fighters joining at beginning of shift 4am – 8am

x_3 is the number of fire fighters joining at beginning of shift 8am - noon

x_4 is the number of fire fighters joining at beginning of shift noon – 4 pm

x_5 is the number of fire fighters joining at beginning of shift 4pm - 8pm

x_6 is the number of fire fighters joining at beginning of shift 8pm - midnight

Objective function

Minimize total number of fire fighters

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

Subject to Constraints

$x_6 + x_1 \geq 5$

$x_1 + x_2 \geq 6$

$x_2 + x_3 \geq 10$

$x_3 + x_4 \geq 12$

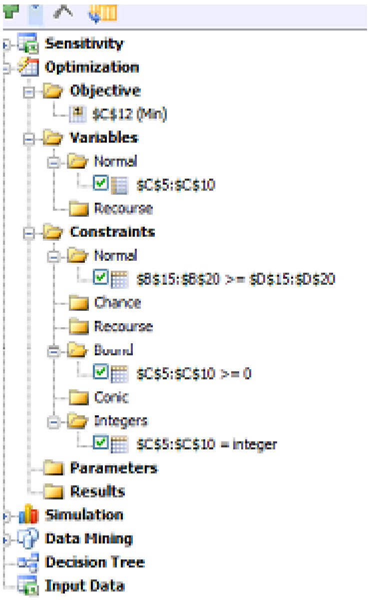
$x_4 + x_5 \geq 8$

$x_5 + x_6 \geq 5$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$ (Non-negativity constraint)

Set up

Decision variables			
Shift	No. of fire fighters joining at beginning of the shift		Requirement
Midnight - 4am			5
4am-8am			6
8am-noon			10
noon-4pm			12
4pm-8pm			8
8pm -midnight			5
MINIMIZE	0		
Constraints			
0	>=	5	
0	>=	6	
0	>=	10	
0	>=	12	
0	>=	8	
0	>=	5	
Non-negativity and integer constraints			



Solution

Decision variables			
Shift	No. of fire fighters joining at beginning of the shift		Requirement
Midnight - 4am	0		5
4am-8am	6		6
8am-noon	4		10
noon-4pm	8		12
4pm-8pm	0		8
8pm -midnight	5		5
MINIMIZE	23		
Constraints			
5	>=	5	
6	>=	6	
10	>=	10	
12	>=	12	
8	>=	8	
5	>=	5	
Non-negativity and integer constraints			

Minimum number of fire fighters needed to be join at beginning of the shifts midnight-4am, 4am-8am, 8am-noon, noon-4pm, 4pm-8pm, 8pm-midnight are 0, 6, 4, 8, 0 and 5 respectively. Overall, minimum of 23 fire fighters are needed to meet the shifts' requirements.

Problem #4

Decision variables

Let PSS, PSO, PSN and PSD be the production schedule of Standard for Sep, Oct, Nov and Dec respectively

Let PHS, PHO, PHN and PHD be the production schedule of Heavy Duty for Sep, Oct, Nov and Dec respectively

The following are derived from decision variables

Let ISS, ISO, ISN and ISD be the inventory of Standard for Sep, Oct, Nov and Dec respectively

Let IHS, IHO, IHN and IHD be the inventory of Heavy Duty for Sep, Oct, Nov and Dec respectively

Derived using these:

Balance equations for inventories

$0 + PSS - 650 = ISS$
$0 + PHS - 900 = IHS$

$ISS + PSO - 875 = ISO$
$IHS + PHO - 350 = IHO$

$ISO + PSN - 790 = ISN$
$IHO + PHN - 1200 = IHN$

$ISN + PSD - 1100 = ISD$
$IHN + PHD - 1300 = IHD$

Objective function

Minimum total cost = Production cost + Inventory cost

$125\ PSS + 131.25\ PSO + 137.81\ PSN + 144.70\ PSD + 135\ PHS + 141.75\ PHO + 148.84\ PHN + 156.28\ PHD + 5\ (ISS + ISO + ISN + ISD + IHS + IHO + IHN + IHD)$

Total production cost

$= 125\ PSS + 131.25\ PSO + 137.81\ PSN + 144.70\ PSD$

$+ 135\ PHS + 141.75\ PHO + 148.84\ PHN + 156.28\ PHD$

Total Inventory cost

$= 5\ (ISS + ISO + ISN + ISD + IHS + IHO + IHN + IHD)$

Note that inventory cost can be expressed in terms of decision variables using balance equations of inventories described above.

Subject to Constraints

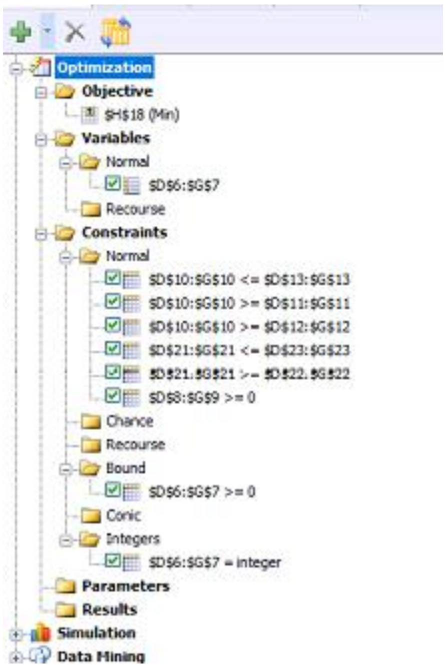
$ISD \geq 800$
$IHD \geq 850$
$ISS + IHS \leq 1800$
$ISO + IHO \leq 1800$
$ISN + IHN \leq 1800$
$ISD + IHD \leq 1800$

0.45 PSS + 0.52 PHS >= 1000
0.45 PSS + 0.52 PHS <= 1200
0.45 PSO + 0.52 PHO >= 1000
0.45 PSO + 0.52 PHO <= 1200
0.45 PSN + 0.52 PHN >= 1000
0.45 PSN + 0.52 PHN <= 1200
0.45 PSD + 0.52 PHD>= 1000
0.45 PSD + 0.52 PHD <= 1100

Non-negativity and integer constraints for all production decisions and inventories

Set up

Rotary Pump		Sep	Oct	Nov	Dec		
Standard (S)		650	875	790	1100		
Heavy Duty (H)		900	350	1200	1300		
Production Decision (S)							
Production Decision (H)							
Inventory (S)	0	-650	-1525	-2315	-3415		
Inventory (H)	0	-900	-1250	-2450	-3750		
Total Inventory		-1550	-2775	-4765	-7165		
Minimum Inventory (S)	>=	0	0	0	800		
Minimum Inventory (H)	>=	0	0	0	850		
Maximum Inventory	<=	1800	1800	1800	1800		
Production cost per unit (S)		125	131.25	137.8125	144.7031		
Production cost per unit (H)		135	141.75	148.8375	156.2794		
Total inventory cost						-81275	
Total production cost						0	
Total cost						-81275	MINIMIZE
Labor hours (S)		0	0	0	0		
Labor hours(H)		0	0	0	0		
Total Labor Hours		0	0	0	0		
Minimum Labor Hours	>=	1000	1000	1000	1000		
Maximum Labor Hours	<=	1200	1200	1200	1100		



Solution

Rotary Pump			Sep	Oct	Nov	Dec		
Standard (S)			650	875	790	1100		
Heavy Duty (H)			900	350	1200	1300		
Production Decision (S)			653	876	794	1098		
Production Decision (H)			1358	1165	1236	973		
Inventory (S)		0	3	4	8	6		
Inventory (H)		0	458	1273	1309	982		
Total Inventory			461	1277	1317	988		
Minimum Inventory (S)	>=		0	0	0	800		
Minimum Inventory (H)	>=		0	0	0	850		
Maximum Inventory	<=		1800	1800	1800	1800		
Production cost per unit (S)			125	131.25	137.8125	144.7031		
Production cost per unit (H)			135	141.75	148.8375	156.2794		
Total inventory cost							20215	
Total production cost							1149399	
Total cost							1169614	MINIMIZE
Labor hours (S)			293.85	394.2	357.3	494.1		
Labor hours(H)			706.16	605.8	642.72	505.96		
Total Labor Hours			1000.01	1000	1000.02	1000.06		
Minimum Labor Hours	>=		1000	1000	1000	1000		
Maximum Labor Hours	<=		1200	1200	1200	1100		

Production schedule is:

Standard: Sep: 653 units, Oct: 876 units, Nov: 794 units and Dec: 1098 units

Heavy Duty: Sep: 1358 units, Oct: 1165 units, Nov: 1236 units and Dec: 973 units

The total minimized cost is $1,169,614

Problem #5

a) The developer should build **3** small and **44** large offices.

b) The total optimal monthly revenue is 3*600 + 3*750 + 44*1000 = **$48,050**

c) Square footage unused = 100,000 – 48,200 = **51,800 square feet.**

d) The allowable increase for small offices is 400. The increase of rent for small office from $600 to $800 is an increase of $200 which is within the allowable limits. Hence, there will be **no impact on the optimal allocation of the offices.** The objective function value would increase by 3*$200 = **$600.** Hence, the optimal monthly revenue would now be $48,650.

e) No impact. This is because the optimal number of offices (50) allowed has already been exhausted within 100,000 square feet with some unused square footage left. So, increase in 52,800 sq. feet of additional footage will not impact optimal objective function value.

f) There will be **no change** in the current optimal allocation of offices/ current optimal solution. This is because an increase of $50 for small offices and a decrease of $200 for large offices is within the respective allowable increase of $400 for small offices and allowable decrease of $250 for large offices.
Change in objective function value = 3*50 – 44*200 = - 8650.
New optimal monthly revenue = 48,050 – 8650 = $39,400
Thus, the objective function value **decreases by $8650**.

Extra Credit

Writing the 2 inequalities as planes in 3 dimensional space:

x + y + z = 5
-x + y +2z = 6

Put x = 0 in the equations, gives
y = 4, z = 1

Also, the two normal vectors to the plane are (1, 1, 1) and (-1, 1, 2). The cross product is (1, -3, 2)

The symmetric form of line of intersection of two planes is:

$$\frac{x-0}{1} = \frac{y-4}{-3} = \frac{z-1}{2} = t \ \ where\ t \in R$$

This gives x = t, y = 4-3t and z = 1+2t

Since x, y and z are >= 0, it gives us 0 <= t <= (4/3)

The boundary/extreme point of the solid region obtained will be:

$$\{(t, 4-3t, 1+2t) | 0 \le t \le \frac{4}{3}\}$$