Jason McCoy_Extra Credit Problem #3 (15 points)

Bootstrapping will be used to estimate confidence intervals for the variance of a non-normal distribution. Earthquake magnitude data will be used. Results will be compared to confidence intervals constructed using the traditional chi-square method that assumes normality.

The Percentile Bootstrap Method will be used with earthquake magnitude data. The earthquake magnitude data are right-skewed and are not derived from a normal distribution.

The following code ‘chunk’ defines the vector “mag” with the earthquake magnitudes.

```
mag <- c(0.70, 0.74, 0.64, 0.39, 0.70, 2.20, 1.98, 0.64, 1.22, 0.20, 1.64, 1.02,
         2.95, 0.90, 1.76, 1.00, 1.05, 0.10, 3.45, 1.56, 1.62, 1.83, 0.99, 1.56,
         0.40, 1.28, 0.83, 1.24, 0.54, 1.44, 0.92, 1.00, 0.79, 0.79, 1.54, 1.00,
         2.24, 2.50, 1.79, 1.25, 1.49, 0.84, 1.42, 1.00, 1.25, 1.42, 1.15, 0.93,
         0.40, 1.39)
```

There are two steps to this problem. First, compute the confidence interval for the population variance using the traditional chi-square method which assumes normality.

Step #1 (3 points)

The sample variance point estimate is calculated using the *var()* function. You will need to add code calculating a 95% confidence interval for the sample variance estimate, based on a chi-square distribution. This assumes normality.

```
var(mag)
```

```
## [1] 0.4401215
```

```
df = length(mag) - 1
chilower = qchisq((1 - 0.95)/2, df)
chiupper = qchisq((1 - 0.95)/2, df, lower.tail = FALSE)
c(df * var(mag)/chiupper, df * var(mag)/chilower)
```

```
## [1] 0.3071092 0.6834419
```

There is an extensive literature on bootstrapping. The methods shown here give an indication of the possibilities for estimating confidence intervals for a wide range of parameters. Some literature citations will be mentioned.

Next, use the Percentile Bootstrap Method for estimating a 95% confidence interval for the variance. This requires drawing 1000 random samples of size *n* = 50 with replacement from the earthquake data. The sample variance will be computed for each, and the 2.5% (0.025) and 97.5% (0.975) quantiles computed for these data using *quantile()*. Present a histogram of the 1000 sample variances along with the confidence interval. Compare to the traditional chi-square method.

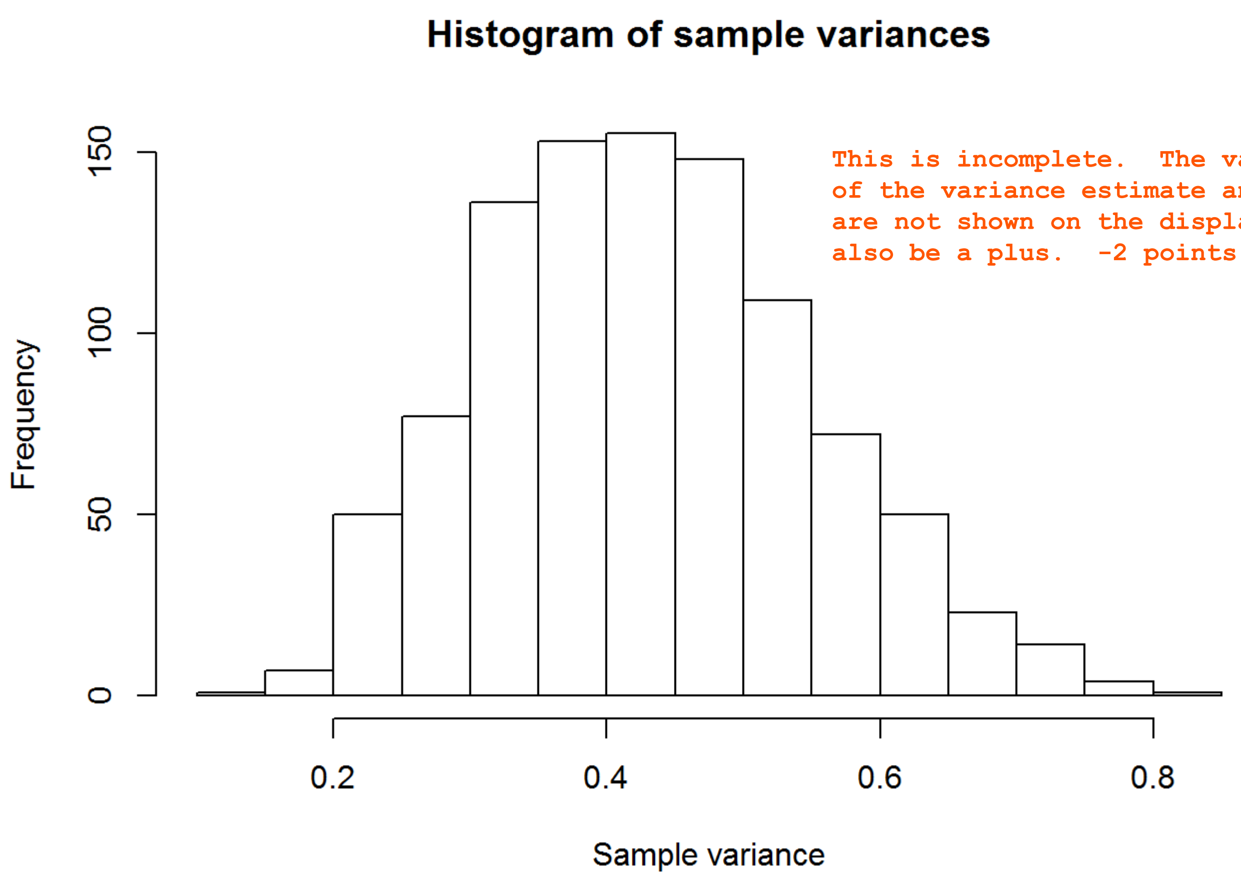
Step #2 (5 points)

1000 random samples will be drawn from “mag” and the sample variance calculated for each. The distribution of results will then be presented via histogram and quantiles determined. The quantiles provide a 95% bootstrap confidence interval. Please keep *set.seed(123)*. The *replicate()* function can be used to easily “replicate” the sampling and sample variance steps, with *n* = 1000.

```
set.seed(123)
quantile(replicate(1000, var(sample(mag, replace=TRUE))), probs=c(0.025, 0.975))
```

```
##      2.5%     97.5%
## 0.2364582 0.6920290
```

```
hist(replicate(1000, var(sample(mag, replace=TRUE))),main="Histogram of sample variances", xla
b="Sample variance")
```



This is incomplete. The values and location of the variance estimate and the quantiles are not shown on the display. Color would also be a plus. -2 points

Compare the traditional (i.e. chi-square) and bootstrap results. The traditional CI is (0.307, 0.683) while the bootstrap result CI is (0.236, 0.692). The 95% CI obtained using bootstrap results is wider.

Any rationale as to why they differ?
-1 point

Step 3, additional (7 points)

Additional credit (7 points) will be given for using the “boot” package discussed in Kabacoff Section 12.6, pages 292-298. Examples are given there.

The “boot” package requires a function written to return the sample variance values for each individual resample drawn.

Use the following function in *boot()* for the argument “statistic.”

This function is defined for you below.

```
f <- function(data, i){
  d <- data[i]
  return(var(d))
}
```

The user-defined function is passed to *boot()* with “mag” and the number of samples to be drawn with replacement. The confidence bounds follow. The “boot” package has a variety of options for determining confidence intervals. See *boot.ci()*, shown below, with the percentile option. The different computational options may produce slightly different results depending on the number of samples drawn during bootstrapping. For this problem, use 10,000 samples drawn with replacement. Again, please keep *set.seed(123)*.

I would encourage you to read the documentation pages for both; *?boot()* and *?boot.ci()* with the package loaded.

```
library(boot)  # install.packages("boot")
```

```
## Warning: package 'boot' was built under R version 3.4.3
```

```
set.seed(123)

# Here, you will need to add code defining an object created by boot() with data = mag, statis
tic = f,
# and R = 10000. For calculating the quantiles, you will need to refer to the vector of result
s via "$t".

boot_result = boot(data=mag, statistic=f,R=10000)
c(quantile(boot_result$t, 0.025), quantile(boot_result$t, 0.975))
```

```
##      2.5%     97.5%
## 0.2276694 0.6890742
```

Alternatively, we could use *boot.ci()*. To do this, we need to pass the object defined above by *boot()*, specifying “conf = 0.95” and “type =”perc“. *boot.ci()* calculates confidence intervals and stores them at”$percent" of the output.

```
boot.ci(boot_result,0.95,"perc")
```

```
## BOOTSTRAP CONFIDENCE INTERVAL CALCULATIONS
## Based on 10000 bootstrap replicates
##
## CALL :
## boot.ci(boot.out = boot_result, conf = 0.95, type = "perc")
##
## Intervals :
## Level     Percentile
## 95%   ( 0.2276,  0.6892 )
## Calculations and Intervals on Original Scale
```

It would be a plus to show the different estimates in a table as an aid to the reader.

The “boot” package provides a considerable capability for bootstrapping. Other discussions are given at http://www.ats.ucla.edu/stat/r/faq/boot.htm and http://www.statmethods.net/advstats/bootstrapping.html.

12 points Note my comments above.