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Mini-review

On Pierre Bézier's life and motivations

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Abstract

This paper presents two letters written by Pierre Bézier in October–November 1999 on the context and the motivations for the creation of the 'Bézier curves'. Some historical notes are added, as is a bibliography which includes some papers linked with the starting point of Bézier curves and tentatively all Pierre Bézier's papers.

1. Introduction

Interested in the genesis (date, motivation, references) of Bézier curves, I wrote to Pierre Bézier himself in late September 1999, asking him when he first developed 'Bézier curves', what were his main motivations and main ideas that led him to this, when was the first paper about it, and if he knew when the first paper giving his name to the method was published. He replied with a detailed letter on the context and the circumstances of his work at Renault in the 1960s, giving details on the way he developed the so-called Bézier curves; then, on receiving my thanks, he replied with a second letter, more focused on the scientific context.

His death very soon after (25 November 1999) adds a special light to these two letters, which are presented here with his son's permission. They are presented first as written, in French (Sections, 2.2, 2.3 and 4.2), and then in English (Sections 3.2, 3.3 and 5.2). Notes and comments are my own and are presented together in Section 6. A bibliography of the papers mentioned in Pierre Bézier's letters and in the notes, as well as an attempt at a complete bibliography of Pierre Bézier's papers conclude this paper. For the sake of completeness, the two letters I wrote to Pierre Bézier are also presented (in French: Sections 2.1 and 4.1, and in English: Sections 3.1 and 5.1).

People interested in how the idea was born will find other details in Ref. [44] and in its English (and shorter) version [28]. Many other details are given in the two books [1,5], translated into English [2,4], where Pierre Bézier gives a general presentation of this work. His thesis [3] provides even more details on the method (in French). The first English paper that presents the method and presently can easily be found is Ref. [62]; to introduce the curves, it first starts from the f_i functions (see §6 of the first letter, and note (9)), and then presents the form using Bernstein polynomials. Let us mention that Refs. [57,59] are the very first papers mentioning Bézier's work, Ref. [63] is the first book giving reference to Bézier's work (via [59]), and that Ref. [66] is probably the first textbook to include a full mathematical description of Bézier curves and surfaces, with examples and algorithms.

About the scientific context in the 1960s, during which time the development of Bézier curves occurred, the reader can find a survey written by Pierre Bézier [7,8]. Surveys of the first, say, 20 years of Computer Aided Geometric Design (CAGD) can be found in Refs. [68,73], and some other papers dealing with that period in Ref. [74]. The paper [33] is an overlook, as seen by Pierre Bézier, of some methods used in CAGD. The French book [72] deals with the French context of CAGD during the period 1959–1989.

About the context in Renault and the emergence of the method, and about Pierre Bézier's personality, the reader will find many interesting details in Refs. [71,72], and in various papers written by Pierre Bézier for nonscientists, such as Refs. [36–41], while the papers [42,43,45–47] show Pierre Bézier's vision for the possibilities of and the interest in CAGD. The recent French paper [76] tries to summarise for nonscientists the main steps of this invention.

Some words about the personality of Pierre Bézier, as I could discern it from various contacts with people knowing him (a careful reader will find most of them in the two following letters). He was always the perfect gentleman in the real sense of the word: kind, courteous, with a very warm and attractive contact to other people, he always showed a great interest in people, he was open-minded and had a great sense of humour, often sharp, especially when there was some opposition between power and

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competence. Obviously he had a strong personality which was greatly appreciated. He always had a concrete vision (and motivation) of the mathematics he handled, and always expressed them both verbally and with his hands.

He was grateful to the first mathematicians and scientists to recognise his work, who translated it into English and made it known. He was very concerned for Renault and for all changes which could improve the quality of the production, the communication between various departments, and the working conditions in the firm. He was really angry about the way the heads of Renault were reluctant for changes in the company and for the way they did not recognise the advantages of his way of thinking. He was amazed to be better known abroad than in France, and his deep regret was to have foreign car manufacturers using his work in car body design before, or even say better, than his own company. He was aware of, and took personal pleasure in, the originality and importance of his work, but always remained very modest about it.

When speaking as well as writing, he always wanted to use the right word in the right position, giving it the proper nuance. He was active in the scientific and literary community, even after he retired; he so animated the 'Club Pierre de Jumièges', a group of people inventing mathematical tricks (often geometrical ones), and the rubric 'maths et Bluettes' of the 'Arts et Métiers Magazine' where such tricks were published [51], which on numerous occasions gave him considerable pleasure… and malice. Let us mention here the last problem he posed: '*Three coplanar segments intersect in the middle of them in a single point. Prove they are the orthogonal projections of three orthogonal diameters of a single sphere*'. He also wrote some one page humoristic papers in 'Arts et Métiers Magazine' [50].

2. First letter

This section presents the first letter sent to Pierre Bézier and his reply, as written (notes are gathered together in Section 6).

2.1. Letter sent to Pierre Bézier

Christophe Rabut

Toulouse, le 20 septembre 1999
à Pierre Bézier

Cher Monsieur,

Peut-être vous rappelez-vous de moi, nous nous sommes rencontrés au congrès 'Courbes et Surfaces', à Chamonix, en 1990 (la dernière édition a eu lieu à Saint-Malo, en juillet dernier !).[1] Ce n'est cependant pas à propos de ce congrès que je vous écris, mais pour vous demander des précisions—qui pour moi ont leur importance—concernant l'histoire des courbes qui portent votre nom.

Tout d'abord, j'aimerais bien savoir en quelle année vous avez eu l'idée d'utiliser les polynômes de Bernstein pour tracer des courbes paramétriques, quelle est la référence de la première publication concernant les 'courbes de Bézier', ainsi que l'année et si possible la référence de l'article dans lequel on leur a donné pour la première fois le nom de 'polynômes de Bézier'. Enfin, quelle est l'année et si possible la reference de l'article où l'on a utilisé pour la première fois le nom de 'points de Bézier' pour les points de contrôle de ces courbes.

Au niveau de la genèse de cette méthode de génération de courbes, j'aimerais bien savoir si vous avez directement utilisé ces polynômes pour tracer des courbes paramétriques (en quelque sorte de façon géométrique et non de façon analytique), ou si au contraire vous avez d'abord utilisé cette base de polynômes pour calculer et tracer des polynômes.

J'aimerais bien aussi savoir ce qui vous a motivé pour définir ces courbes; je sais qu'il s'agit de la conception des formes de voitures, et je ne sais pourquoi, je crois qu'il s'agissait en fait des ailes de voiture, mais y avait-il une partie particulière de la voiture qui était plus particulièrement visée dans vos recherches ?

Votre problème était, me semble-t-il, en fait un problème de conception de surfaces et non de conception de courbes… Avez-vous d'abord travaillé sur les courbes (comme le laisse penser le nom de 'courbes de Bézier'), ou avez-vous directement travaillé sur la conception de surfaces, et dans ce cas avec quels outils exactement ('carreaux de Bézier', ou autres outils ?).

J'aimerais savoir enfin quels ont été les éléments qui vous ont guidés vers cette solution…

Pardonnez toutes ces questions, précises et souvent difficiles à répondre je pense, mais j'aimerais mieux connaître et comprendre les origines de cette méthode, et il me semble que vous êtes la personne la mieux placée pour m'apporter des éléments de réponse… Merci donc pour l'attention que vous porterez à cette lettre, et pour votre réponse, que j'espère aussi détaillée que possible, bien qu'il s'agisse de préoccupations sans doute éloignées des vôtres actuellement.

Christophe Rabut

2.2. Handwritten introduction to Pierre Bézier's first letter

Paris, 5 novembre 1999

Mon cher collègue,

Votre lettre du 20 septembre m'est parvenu peu de jours avant que je sois admis pour un mois à l'hôpital pour n'en sortir que récemment. Cependant j'avais eu le temps de rédiger une réponse, que vous trouverez sous ce pli (après mise au point).

De mon mieux, j'ai essayé de décrire la démarche d'une pensée qui m'a guidé, mécanicien chasseur de centièmes de mm, dans le monde de l'imprécision et de la subjectivité.

Sans doute la bibliothèque de l'INSA doit-elle contenir un exemplaire du livre publié par Hermès en 1986 sous le titre 'Courbes et Surfaces'. Il a treize ans, et quelques rides.

La thèse que j'ai soutenue en 1977, j'avais alors 67 ans, est déposée au CNRS, mais elle date ![2]

Si vous avez besoin d'autres renseignements je m'efforcerai de répondre à vos questions. On pourrait résumer mon apport par cette idée un peu bizarre : au lieu de déformer une courbe, ou une famille de courbes, il vaut mieux faire subir une distorsion générale à l'espace dans lequel on les a incluses.

La ligne droite n'est pas… etc.

Bien cordialement

P. Bézier

2.3. Pierre Bézier's first letter

Paris, 04 novembre 1999

Pierre Bézier

à Mr C. Rabut

INSA de Toulouse

Monsieur et cher collègue,

Le souvenir du congrès de Chamonix évoque pour moi une occasion qui m'a été donnée d'apprendre beaucoup de choses tout en faisant des rencontres aussi agréables qu'intéressantes.

Les questions que vous m'avez posées sont nombreuses et je vais essayer de ne pas me perdre dans des méandres superflus. Madame de Sévigné, qui n'était guère mathématicienne, écrivait un jour à sa fille '*pardonnez-moi ma toute bonne, mais aujourd'hui je n'ai pas eu le temps de faire court*'. C'est une phase que l'on devrait graver sur certains frontons, auprès de celles que l'on emprunte à Platon.[3]

D'abord, je dois préciser que j'ai été formé aux Arts et Métiers Promo 1927 en vue de devenir ingénieur mécanicien—c'était une vocation héréditaire—que j'ai passé ensuite un an à SupElec (Promo 1931)[4] et que mon comportement en est resté imprégné. Cela peut expliquer dans une certaine mesure ma façon de raisonner et de réagir.

1 Formation au Service des Méthodes Mécaniques de Renault

En 1933, la crise de 1929 n'était pas terminée ; après mon service militaire, j'ai été embauché par Renault comme ajusteur-outilleur ; je suis ensuite passé au bureau d'études des outillages, qui faisait partie du service des méthodes.

Ce service avait à choisir, à concevoir et à mettre en œuvre les moyens de production des pièces mécaniques ; toutes les surfaces qui nécessitaient une certaine précision étaient des plans, des cylindres ou des cônes, c'est à dire qu'il suffisait de droites et de cercles pour les définir ; seule exception : les flancs des dents des pignons, mais ils étaient taillés par des machines spécialisées qui les engendraient grâce à des combinaisons cinématiques appropriées. Les limites étaient exprimées en millièmes de mm car les tolérances étaient de l'ordre du centième, et parfois moins. Les contestations avec les contrôleurs portaient en général sur un ou deux millièmes et, dans l'argot de l'atelier, l'appareil de mesure, palmer ou comparateur, était appelé 'juge de paix'. Pas besoin de commentaire.

2 Les Méthodes de la carrosserie en 1960

Au contraire, pour la carrosserie, tout baignait dans un flou artistique ; le styliste était l'arbitre; son jugement ne pouvait être que subjectif et variait parfois avec le temps ; on ne demandait à personne d'avoir des connaissances mathématiques, exception faite des dessinateurs, qui étaient de vrais acrobates de la descriptive ; leurs instruments étaient des gabarits, des pistolets, des lattes flexibles, des compas à pointes sèches et des réglets gradués.

Les plans étaient médiocrement précis, et l'on citait les cas d'une voiture, pas plus laide qu'une autre d'ailleurs, dont les deux flancs différaient entre eux de plusieurs millimètres : pour l'esthétique et l'aérodynamisme, c'était sans importances, mais en cours de fabrication il n'en allait pas de même ; entre des pièces qui auraient dû s'assembler bord à bord il restait parfois des vides de plusieurs millimètres qu'il fallait combler avec de la soudure à l'étain, et cela coûtait cher.

Les définitions se transmettaient d'un service à l'autre sous forme de dessins dont la précision, médiocre dès l'origine, se dégradait à chaque étape car tout intervenant se sentait libre de procéder à des modifications supposées imperceptibles afin d'améliorer l'aspect extérieur ou de faciliter l'emboutissage, la soudure ou l'assemblage des éléments constitutifs. Les choses allaient ainsi depuis que nos aïeux avaient construit des chars à bœufs pour le Mérovingiens directs. Il en résultait des délais et des coûts, mais on était bien obligé de s'en contenter car si l'on avait voulu essayer d'employer la géométrie analytique la quantité de calculs à executer aurait été absolument prohibitive.

Schématiquement, lorsque l'on étudiait un nouveau véhicule, le procédé classique était d'abord de charger un styliste de tracer plusieurs croquis entre lesquels on faisait un choix, puis de modeler des maquettes en cire à l'échelle 1/8 au 1/10 ; ensuite, en plusieurs étapes, on en tirait un plâtre en grandeur nature qui était soumis au jugement d'un aréopage constitué par la Grande Direction, le Style, le Service Commercial et différents conseillers supposés qualifiés ; quand, au bout de plusieurs mois, et après maintes retouches et modifications, un accord était atteint, le bureau de dessin étudiait chacune des pièces intérieures de la caisse ; il fallait, pendant ce temps, tenir compte des impératifs de la fabrication : emboutissage, soudure, peinture, sellerie, fixation des organes mécaniques, assemblage général, entretien et réparation ; on construisait plus tard un maître-modèle dans un matériau assez stable, acajou ou résine organique, qui servait de référence pendant toute la production du véhicule, mais sa précision n'était pas parfaite et même, avec le temps, pouvait parfois subir une distorsion, ce qui est fâcheux pour un étalon.

3 Schéma d'un projet

Il y avait dans cet état de choses quelque chose de choquant pour un mécanicien habitué à une rigueur sans

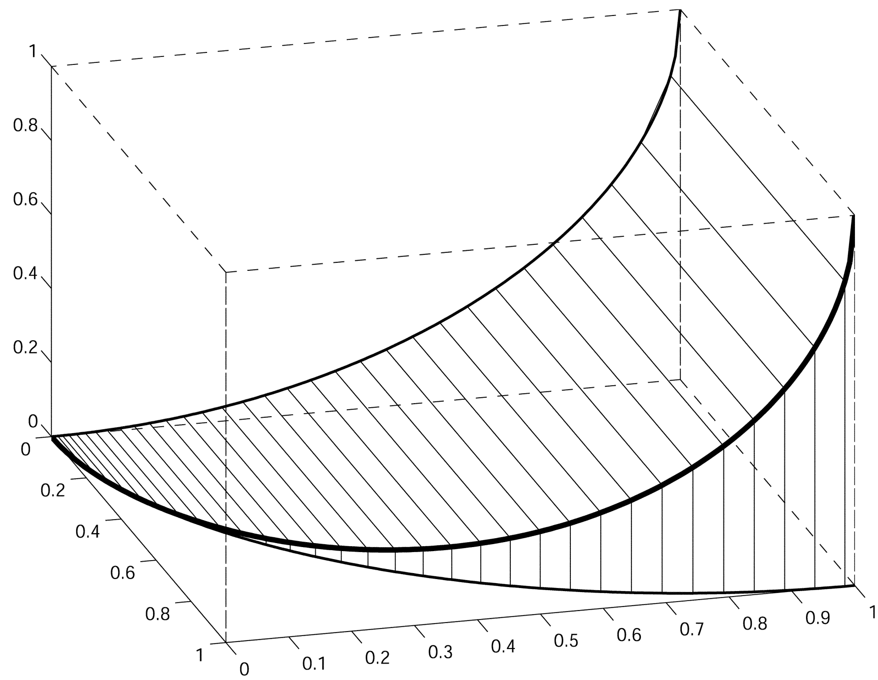

Fig. 1.

concession ; il me semblait qu'il faudrait parvenir à utiliser une définition indiscutable, exempte de distorsion et facile à communiquer, établie par le styliste lui-même et transmise ensuite sous forme numérique à tous les groupes, y compris les sous-traitants et les fournisseurs, intervenant dans les processus, depuis le styliste jusqu'au contrôleur opérant à la sortie de la chaîne de fabrication, et même aux ateliers d'entretien du réseau des agents et des concessionaires.

4 Emergence de l'ordinateur et des machines à commande numérique

L'ordinateur, apparu dans l'industrie vers 1950, travaillait naturellement en priorité pour les services administratifs ; quand il lui restait du temps, et c'était rare, il exécutait en mode différé quelques travaux à la demande des services scientifiques ou techniques.

Sa rapidité de calcul nous semblait fabuleuse; en 1955 sont apparues aux USA les premières machines-outils à commande numérique ; au début, c'était pour effectuer de point en point des perçages, des taraudages et des alésages ; plus tard on est passé au fraisage suivant des droites, puis des arcs de cercle; cela suffisait aux mécaniciens et l'on pouvait même placer bout à bout des arcs de cercles ou, graces à Chaïkin, de paraboles pour imiter d'autres courbes.[5] Bref, il n'était plus insensé de songer à s'attaquer au problème du tracé des carrosseries.

5 Tracé des courbes par déformation du référentiel

Le tracé de courbes était la première étape à franchir, car ce sont les courbes dites 'de construction' qui servent de guide pour représenter les surfaces ; les gens de métier les nomment ligne de ceinture, ligne de carre, ligne de bas de jupe, etc. ; ce sont des courbes gauches et il faut plusieurs projections pour les définir, en assurant leur compatibilité. Il n'aurait pas été bon de les constituer en mettant bout à bout beaucoup de petits arcs de cercle ou de paraboles parce que toute modification n'aurait pu être que locale alors qu'il fallait, au contraire, conserver l'allure générale de la courbe à corriger et que l'altération soit répartie progressivement sur toute sa longueur ; il était impératif de réduire au minimum le nombre des arcs à juxtaposer ; on a donc inscrit dans un cube une courbe dite 'de base', de forme bien adaptée, et l'on a pensé à déformer celui-ci pour en faire un parallélépipède (PPPD), autrement dit on lui a fait subir une transformation linéaire ;[6] pour définir celui-ci, au lieu de donner une origine commune aux trois vecteurs-unités du PPPD, on les a mis bout à bout ; la forme du polygone ainsi constitué évoque vaguement celle que prendra la courbe de base après avoir subi la même transformation.

A première vue, il semble moins logique de déformer tout un référentiel plutôt qu'une seule ligne, mais il faut considérer que l'on a besoin, dans la suite des travaux, de modifier l'ensemble d'un tracé composé de plusieurs arcs de courbes et qu'alors il sera plus simple de le faire d'un seul coup en agissant sur leur espace commun plutôt que sur chacun séparément.

Plus tard, on a pensé aussi qu'au lieu d'effectuer seulement une transformation linéaire, on pourrait imposer au cube une distorsion générale, au prix d'un accroissement de la quantité des calculs qu'entraînerait l'usage simultané de trois paramètres.[7]

6 Choix de la courbe de base. Fonctions f

J'avais choisi comme courbe de base, c'était une idée de

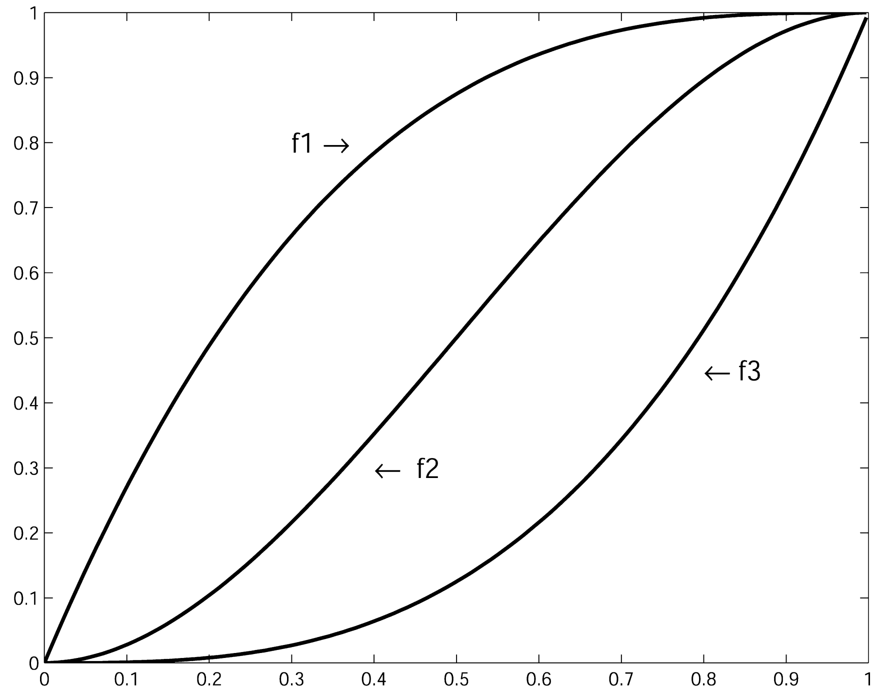

Fig. 2.

mécanicien, l'intersection de deux quarts de cylindres circulaires, (Fig. 1) ; l'ordinateur aurait développé les fonctions harmoniques pour calculer les points courants ; mais les opérations se sont compliquées dès que l'on a voulu utiliser des référentiels ayant plus de trois dimensions ; l'emploi des fonctions algébriques s'est alors naturellement imposé.

En son origine (0, 0, 0), la courbe de la Fig. 1 est tangente à Ox et osculatrice au plan xOy ; en (1, 1, 1), sa tangente est parallèle à Oz et son plan osculateur à yOz ; si l'on imagine qu'un point la parcourt à vitesse constante, l'on conclut que les vitesses de ses projections sur les trois arêtes a_1, a_2 et a_3 du pppd sont représentées par les diagrammes de la Fig. 2, la solution la plus simple étant constituée par trois fonctions cubiques f_1, f_2 et f_3, et la représentation du point courant a la forme[8,9]

$$\bar{P}(u) = \sum_{i=0}^{3} \overline{a_i} f_i(u).$$

Le livre de HERMES *Courbes et Surfaces* contient quelques détails sur les calculs correspondants, d'ailleurs bien élémentaires, ainsi que la forme générale des fonctions f pour toute valeur de i ce qui s'est vite montré nécessaire. Un exemplaire de la thèse soutenue en 1977 a été remis aux archives du CNRS ; peut-être même y en a-t-il un dans la bibliothèque de votre INSA.[2]

Il ne m'avait pas semblé nécessaire de donner un nom aux fonctions f, car j'étais persuadé qu'elles avaient déjà un état-civil ; or j'avais dû, pour des raisons diplomatiques plutôt que scientifiques, adresser quelques notes explicatives à différents personnages de rang élevé de la Régie[10] afin de les tenir par déférence au courant de mes activités, si peu qu'ils y aient pris intérêt.

7 Parrainage

Les choses prenaient un certain développement, mais les fonctions n'ayant pas à ma connaissance de patronyme officiel, j'ai cru judicieux de leur trouver au moins un parrain, ce qui leur conférerait une certaine respectabilité ; j'ai donc attribué leur invention à un professeur virtuel à qui j'ai donné pour nom Durand et pour prénom Onésime, afin d'éviter une homonymie susceptible d'engendrer une action contentieuse.

L'honorable Professeur Durand a donc été connu dès 1965 chez Renault ; ses fonctions ont été citées au CNAM et dans des congrès aux USA ou en Europe ; vers 1970 sur proposition de Mr Soubrier, de l'ADEPA,[11] le nom d'UNISURF a été adopté pour le procédé ; pas une seconde je n'aurais imaginé que mon patronyme figurerait un jour dans des textes sérieux, pas plus que je ne songe aujourd'hui à le voir un jour gravé sur le socle d'une statue, pour le moins équestre, érigée sur la place de mon village.

Les Américains m'ont fait le grand honneur, depuis les années 1975, d'employer mon nom dans leur communications, mais je ne saurais fournir une date plus précise.[12] Comme, en France, cela ne se décerne guère qu'à titre posthume, beaucoup de gens ont des raisons de penser que je suis définitivement décédé. Un peu de patience !

8 Fonctions de Bernstein

Comment est-on passé des fonctions f à celles de

Bernstein ? Tout simplement, mon camarade Riaux[13] m'a fait observer que les sommets du polygone, selon qu'ils étaient l'origine ou l'extrémité d'un des ses cotés, intervenaient deux fois dans le calcul du point courant ; les différences entre deux fonctions f successives sont les fonctions de Bernstein, dont les propriétés sont bien plus utiles que celles des fonctions f ; par exemple, la forme d'une courbe est invariante par rapport aux changements d'origine, c'est à dire aux translations et aux rotations.

A titre anecdotique, je vous signale que Bernstein était un ancien élève de SupElec,[14] où il m'a précédé exactement de trente ans, et qu'il a inventé ses fonctions pour établir des courbes d'espérance de vie pour une compagnie d'assurances.[15]

9 Généralisation

Tout ce que je vous ai écrit jusqu'ici concerne le tracé des courbes mais mon intention était, dès l'origine, d'aller bien au delà, et d'essayer de faire avancer un peu l'ensemble du problème de la conception et de la fabrication de la carrosserie sans laisser subsister la moindre part d'une méthode périmée. Ensuite, passer des courbes aux surfaces n'a été qu'un exercice d'algèbre élémentaire.

Il n'était pas besoin d'avoir beaucoup d'imagination pour penser que les pièces mécaniques seraient justiciables de la même méthode, tout comme les coques de bateaux et les voilures d'avions, mais c'étaient alors là des idées si hétérodoxes qu'il valait mieux ne pas les révéler ; j'avais déjà, en d'autres occasions, épouvanté de hauts dirigeants par des initiatives aux principes desquelles ils ne pouvaient guère rien comprendre, et dont la réussite n'avait pas calmé la frousse rétrospective ni absous mon non-conformisme ; je suis resté à jamais à leurs yeux l'affreux jojo anar ou le mouton noir, entouré de quelques énergumènes de son espèce.[16]

10 Doctrines

D'autres entreprises ont pensé, de façon plus raisonnable ou plus réaliste, c'est un peu la même chose, que ce serait déjà un grand progrès que de mesurer les coordonnées 3D d'une grosse quantité de points situés sur une maquette, puis de définir ensuite une surface qui les contiendrait ; cela rappelle les courbes de régression, les réseaux de Delaunay, la méthode de Bôse, etc.

Naïvement, j'ai cru au contraire qu'en mettant un moteur sur un char-à-bancs cela ne ferait jamais une automobile, mais qu'il fallait 'essuyer le tableau' et repartir de zéro. Pardonnez-moi si j'emploie sans modestie la première personne du singulier, mais je crois que si j'ai apporté quelque chose de valable dans le développement de la CFAO, c'est d'abord ce simple point de vue, dont l'hétérodoxie a convaincu à l'époque tous les gens dits sérieux, et prudents jusqu'à la couardise, que j'avais complètement déraillé de la voie tracée par leurs prédécesseurs. Je passe sur certains jugements qui auraient mérité son attribution si Monsieur Nobel avait prévu de créer, parmi les autres, un Prix de la Stupidité.

11 Equipment prototype

Dès 1965, les travaux théoriques étaient assez avancés et j'étais certain que la solution était valable mais, pour convaincre les tenants de la tradition, il aurait été indispensable de disposer d'une machine à dessiner de 8 m × 2 m, d'une machine à fraiser de faible puissance (0,5 kW) pour tailler des blocs de mousse de polystyrène, avec des courses de 1,5 m × 1,2 m × 0,8 m, et des avances de 2 m/min, ce qui semblait irréalisable à l'époque. De plus, pour travailler en mode conversationnel, il serait indispensable de disposer en permanence d'un ordinateur de puissance modeste, ce qui était contraire, en 1960, à la pratique admise ; il faudrait enfin bâtir un logiciel rudimentaire.

Le budget correspondant était évalué à 3MF et la Haute Direction montra les bornes de sa confiance en limitant son montant à 600 kF, à charge pour moi d'aller ailleurs tendre la sébile ; par chance, le projet inspira confiance à la DGRST[17] qui m'accorda 1,5 MF ; le reste fut prêté par un constructeur d'ordinateurs qui prêta 900 kF, car l'idée de multiplier les ordinateurs de petites dimensions lui parut bonne à encourager.

12 Bilan[18]

Les problèmes devenant plus complexes, nous sommes passes à des référentiels non-linéaires qui permettent de tracer une courbe de parametre w sur une surface définie par deux paramètres u et v, c'est à dire, par exemple, de tracer une échancrure de passage de roue dans une aile déjà déterminée, ou de modifier la totalité d'une caisse sans avoir à corriger séparément les surfaces élémentaires qui la composent.

Vous voyez, cher Monsieur, que tout cela est simple et sa ramène à quelques notions banales, sans aller au delà d'un peu de calcul vectoriel et matriciel. Pourquoi, en 1960 des chercheurs de l'industrie aéronautique ne l'ont pas ils pas trouvé de premier coup ? Je crois qu'ils ont été intoxiqués par l'idée de reproduire un modèle plutôt que de commencer en créant directement une forme et en l'affinant peu à peu ; je suppose que le problème posé par Citroën à Jean de la Boixière (SupElec) et à Paul de Casteljau (Norm Sup),[14] tous deux ingénieurs et mathématiciens, était aussi de traduire numériquement une maquette faite à la main.[19]

Si l'on veut considérer tout cela d'une façon plus générale, on peut dire que vers 1960, beaucoup de mécaniciens étaient encore peu renseignés sur tout ce que l'électricité pouvait leur apporter comme moyen de mesure, de calcul, de servo-commande, de distribution de force et de puissance. Quand j'ai fait mes premières armes dans l'industrie mécanique, c'était peu après 1930, son emploi dans les machines-outils se limitait à celui des moteurs asynchrones, des relais magnétiques et des interrupteurs de fin de course pour portes d'ascenseurs ; c'est vers 1935 que l'on m'a laissé, et avec réticence, utiliser l'automatisme séquentiel pour remplacer embrayages, crabotages, encliquetages et vérins hydrauliques.

Le cahier des charges que je m'étais proposé en 1960 avait pour avantage d'être fondé sur une experience que j'avais acquise en exerçant la plupart des metiers qui jouent un rôle dans la carrosserie: usinage, fonderie, électricité, électronique, tracé, soudure, dessin, ajustage, contrôle ; j'avais aussi conservé et développé quelques connaissances en mathématiques au delà de ce que l'on enseignait aux élèves des Arts et Métiers en 1930 ; la curiosité n'est pas toujours un péché capital.

Quand on veut dessiner une machine-outil, ce qui fut mon métier de base, on se forme d'abord une image de ce qu'elle devrait être ; ensuite, la définition finale s'élabore par permutation entre tracés, calculs et essais ; depuis vingt cinq ans, les mécaniciens et les électriciens collaborent dés le début de la période de conception ; plus tard, les philosophes, les psychologues et les organisateurs professionnels ont trouvé un nom pour cette pratique : c'est l'ingénierie simultanée.

La théorie des espaces paramétriques est maintenant largement connue et enseignée ; voir l'une des épreuves de math du concours des Grandes Ecoles en 1999.[20] La littérature est abondante ; les idées de base que j'ai essayé d'exprimer sont peut-être moins répandues, mais je crois qu'elles résisteront bien à l'érosion du temps.

Cela m'intéressait de savoir comment mes réponses ont répondu à votre demande car dans l'exercice de mon métier, qu'il ait été civil ou militaire, j'ai toujours attaché beaucoup d'importance au rôle de la pédagogie. Ne dit-on pas que notre époque est celle de la communication ?

Bien cordialement.

P. Bézier

3. English translation of the above first letter

This section presents the English translation of the first letter sent to Pierre Bézier and his reply (notes are gathered together in Section 6).

3.1. Translation of the first letter sent to Pierre Bézier

Christophe Rabut

20 September 1999
To Pierre Bézier

Dear Sir

You may remember me. We met in 1990 at the Chamonix 'Curves and Surfaces' conference (the last 'Curves and Surfaces' meeting took place in Saint-Malo, last July).[1] But that is not the purpose of this letter. I am writing to you to ask you for details, which seem important to me, regarding the curves named after you.

To start with, I would like to know the exact year in which you struck upon the idea of using the Bernstein polynomials to plot parametric curves, as well as what the reference for the first release regarding 'Bézier curves' is and also the year and, if possible, the reference for the paper in which the name 'Bézier curves' was used for the first time. Finally, I would like to know the year and, if possible, the reference for the paper in which 'Bézier points' were quoted as control points of these curves.

As for the development of the curve generating method, I would like to know if you started with polynomials as a basis for plotting parametric curves (in other words geometrically and not analytically), or on the contrary if you first used a polynomial basis to calculate and plot polynomials. I would also like to know what prompted you to define these curves. I know that it has to do with car shape design but it seems as if it was actually a matter of car wings. Did you focus on one specific car part in your research? I would think that the problem you encountered was connected to surface design rather than to curve design. Did you work on the curves at first, as the name Bézier curve would tend to suggest, or did you work directly on surface design, and if so, did you use 'Bézier patches', or other tools? Could you please tell me what led you to that solution?

Please forgive me for all these precise and difficult questions, but I would like to better know the method and its origins, and obviously you are the right person to answer them. I would be grateful for your reply to be as detailed as possible, even though I assume you probably have other concerns on your mind at the moment. I would like to thank you in advance for your attention.

Christophe Rabut

3.2. Translation of the handwritten introduction to Pierre Bézier's first letter

Paris, 5 November 1999

My dear colleague,

I received your letter of September 20th a few days before a one-month stay in hospital, from which I have returned only recently. However I had had time to write the enclosed answer (and finalise it).

As well as I could, I tried to describe the thought process that guided me, a mechanical engineer chasing after hundredths of a millimetre, in the world of inaccuracy and subjectivity.

The INSA library probably holds a copy of the book published by Hermès in 1986, with the title 'Courbes et Surfaces' ('Curves and Surfaces'). It was 13 years ago, and it is getting somewhat old-fashioned. The thesis which I defended in 1977 when I was 67, is at CNRS, but it is now outdated![2]

If you need further information, I will do my best to answer your questions. My contribution could be summarised by this somewhat strange idea: instead of modifying the shape of a curve, or of a family of curves, it is better to have the space into which we put them globally deformed.

The straight line is not … and so on.

Best wishes,

P. Bézier

3.3. Translation of Pierre Bézier's first letter

Paris, 04.11.99

Pierre Bézier

Mr C. Rabut

INSA de Toulouse

Dear Sir and Colleague,

The memory of the Chamonix conference comes back to me as an opportunity given to me to learn a lot of things while meeting interesting and pleasant people.

You asked me many questions, which I will try to answer without losing myself in details. Madame de Sévigné, who was far from being a mathematician, once wrote to her daughter: '*Forgive me my dear, but today I did not have enough time to be brief*'. This quote should be carved on some façades, besides those from Plato.[3]

Firstly I have to tell you that I was educated at Les Arts et Métiers, 1927 class, to become a mechanical engineer—a matter of family vocation. Then I spent one year at SupElec, 1931 class.[4] I am still full of this, and it can explain, to a certain extent, my way of thinking and reacting.

1 Training in the mechanical process planning department at Renault

In 1933, the country was still marked by the Crash of 1929 and that is the year in which Renault hired me as a tool-and-die maker, after my military service. Then I moved to the tool research department, which was part of the process planning department.

The department had to choose, design, and implement the production facilities for mechanical components. All the surfaces that required some precision were planes, cylinders or cones, i.e. to define them we only needed straight lines and circles. The only exception were the gear tooth sides of the pinions but they were generated by specific machines, which cut them by using appropriate kinematic combinations. The limits were in thousandths of a millimetre because the tolerance was a matter of hundredths, and even less sometimes. Generally, arguments with controllers were over one or two thousandths, and the measurement instrument, known as screw gauge or comparator, was referred to in the workshop jargon as 'the magistrate'. No comment.

2 Car body process planning department in 1960

As for the car body, it was just the opposite: an artistic air prevailed. The stylist was the referee: his judgement could only be subjective and sometimes would vary with time. There was no requirement for mathematical knowledge for anybody except for designers, who were real descriptive acrobats. They used camber boards, French curves, wooden splines, dividers, and steel rulers.

The drawings were not particularly accurate, and for example, once, a car, which was not aesthetically worse than some others, ended up with its two sides differing by several millimetres. In terms of aesthetics and aerodynamics, it did not matter. But during manufacturing, it turned out to be problematic: sometimes there were gaps of several millimetres between parts that should have abutted, and these gaps had to be filled in with tin welding, which was expensive.

The definitions were transmitted from one department to another in the form of drawings, whose precision, already bad, worsened at each stage because every person involved in the process felt free to make supposedly imperceptible changes in order to improve the external look or help to make the die stamping, welding or assembling of the components easier. That was the way things were ever since our forefathers had built oxcarts for the Merovingians. It resulted in extended deadlines and additional costs but we had to do with it for we could not even dream of using analytic geometry, given the prohibitive amount of calculations that would have been involved.

Briefly, when a new vehicle was being designed, it was common practice to put a designer in charge of making several sketches, among which one would be chosen, and then a wax model on a scale of 1/8 to 1/10 was moulded. Then, in several stages, a full-size plaster model was made from it and was then assessed by a committee of the wise including Top Management, Industrial Design Department, Sales Department and various supposedly qualified consultants. When, after several months and numerous alterations and changes, an agreement was reached, the Design Department could start designing each of the internal parts of the body shell. In the meantime manufacturing requirements had to be fulfilled. These were die stamping, welding, painting, leather upholstery, fixing mechanical components, overall assembling and servicing. Then a master cast was made out of a relatively stable material, either mahogany or organic resin, which was the preference throughout the manufacturing process of the vehicle. The problem though, was that its accuracy was not perfect, and it could even undergo changes with time, which is problematic for a standard.

3 Project development

This state of things tends to offend a mechanic used to uncompromising rigour, and I felt that an unquestionable definition, which would remain the same and would be easy to pass on, was required; the definition should be fixed by the industrial designer himself and then passed on as numerical data to every department, including subcontractors and suppliers involved in the process, ranging from the industrial designer to the controller operating at the end of the manufacturing line, and also to the maintenance shops of the network of agents and main dealers.

4 Emergence of the computer and numerically-controlled machines

The computer appeared in industry approximately in 1950 and naturally was used mainly by administrative services. When it was available, i.e. hardly ever, it performed some batch jobs at the request of scientific or technical services.

Its computation speed seemed amazing to us. In 1955 the first numerically-controlled machine tools appeared in the US. At the beginning, they were used to carry out point-to-point drillings, tappings and borings, Later they were capable of milling on the basis of straight lines and then arcs of a circle. That is all the mechanics needed and we could even put arcs of a circle end-to-end or, thanks to Chaïkin, parabolas to imitate other curves.[5] In short, it was not nonsense anymore to consider tackling the problem of car-body plotting.

5 Curve plotting by deformation of a reference frame

Curve plotting was the first step to take because the so-called ‘construction curves’ are the guidelines for plotting surfaces. Experts call them belt lines, edge lines, hemlines, etc. They are non-planar curves, defined by several projections, with conditions for their compatibility. It would not have been a good solution to form them by putting many small arcs of a circle or parabolas end-to-end because changes could have been only local whereas the curve to be modified had to keep its overall shape and the alteration had to be homogeneously distributed all along the curve. It was necessary to minimise the number of arcs to be juxtaposed. Consequently we inscribed a so-called ‘basic’ curve, with the appropriate shape, in a cube. We thought of deforming the cube into a parallelepiped (PPPD). In other words, the cube underwent a linear transformation.[6] Instead of giving a common origin to the three PPPD unit vectors, we put them end-to-end to define the parallelepiped. The shape of the resulting polygon gives a rough idea of what the basic curve shape will be after undergoing the same transformation.

At first sight, it seems less logical to deform the entire reference frame rather than one single line, but it should be kept in mind that we need, later on, to modify the whole plot made of several arcs of curves; so it was thought to be easier to do it at one go, by working on their common space rather than on each one separately.

Later, we thought that the cube could undergo an overall deformation, instead of a simple linear transformation, at the cost of an increase in the amount of computation, resulting from simultaneous use of three parameters.[7]

6 Choosing the basic curve. *f* functions

For the basic curve, I had chosen the intersection of two quarters of circular cylinders (Fig. 1); that was an idea of a mechanic. The computer would have evaluated the truncated Taylor series of trigonometric functions to calculate the points along it, but this would have become complicated when it came to using reference frames with more than three dimensions. Consequently, it became clear that algebraic functions had to be used.

At its origin (0, 0, 0), the curve (Fig. 1) is tangent to Ox and osculating to the xOy plane. At (1, 1, 1), its tangent line is parallel to Oz and its osculating plane is parallel to yOz. Starting from the hypothesis that a point traverses the curve at a constant speed, the conclusion is that the speed of its projections onto the three PPPD edges a_1, a_2 and a_3 are represented by the graphs of Fig. 2. The easiest solution is made up of the three cubic functions f_1, f_2 and f_3, with the following representation of the running point:[8,9]

$$\bar{P}(u) = \sum_{i=0}^{3} \overline{a_i} f_i(u).$$

The book published by HERMES entitled *Courbes et Surfaces* (*Curves and Surfaces*) contains some details on the calculations mentioned above, which actually are basic, and also contains the overall shape of the f functions, for any value of i, which quickly proved to be essential. A copy of my thesis submitted in 1977 is archived at the CNRS. You might even find one in the library of your INSA.[2]

I did not feel the need to give a name to the f functions because I was convinced they were already known. For diplomatic rather than scientific reasons, I had to report briefly to different highly-placed people in the Régie[10] to let them know what I was doing, if indeed they had any interest in it whatsoever.

7 Naming

Things were moving along, but the functions had as yet no official patronymic, as far as I knew, and that is why I thought it would be a good idea to find them at least one Godfather who would give them some respectability. So I credited a virtual professor with their invention. I gave him the surname of Durand and the first name of Onésime in order to avoid any possible homonymy resulting in any kind of legal action.

So the esteemed professor Durand was known at Renault from 1965 onwards. His functions were then quoted at the CNAM and in some American and European conferences. Around 1970, at the suggestion of Mr Soubrier, from ADEPA,[11] the name UNISURF was adopted for the process. It never occurred to me that my surname would ever appear in serious texts, neither can I possibly imagine that it could one day be engraved on the base of some, say equestrian, statue, in the middle of my village.

Americans have honoured me since 1975 by quoting my name in their papers, but I could not tell you the exact date.[12] As in France, this kind of thing is usually only awarded posthumously, many people may think that I am dead and buried. Just be patient!

8 Bernstein functions

How did we move from f functions to Bernstein functions? My friend Riaux[13] simply pointed out to me that the polygon vertices, depending on whether they were the origin or the end points of one of the sides, occurred twice in the running point calculation. Bernstein functions are the differences between two successive f functions and their properties are much more useful than those of the f functions. For example, the shape of a curve is invariant regarding origin changes, i.e. translations and rotations.

As an anecdote, Bernstein was a former student at SupElec,[14] exactly 30 years before me and he invented his functions to plot life expectancy curves for an insurance company.[15]

9 Generalisation

Everything I have written so far deals with curve plotting; what I intended, though, right from the start, was to go far beyond that and try to tackle further the entire problem of car-body designing and manufacturing without leaving any space for the slightest obsolete method. Then moving from curves to surfaces was an elementary algebraic exercise.

There was no need to have much imagination to think that the same method would be applicable to mechanical parts as well as boat hulls and rotor wings, but at the time, those ideas were so heterodox that it was better not to reveal them. I had already, on other occasions, scared senior officers with initiatives of which they could hardly grasp the concept, and the success of which had neither calmed retrospective fear nor atoned for my non-conformism. In their eyes, I will always remain the bad boy, the anarchist or the black sheep, keeping company with some other mad people as he himself was.[16]

10 Doctrines

Other companies thought, in a more sensible or realistic way, which comes to the same thing, that it would already be an achievement to be able to measure the 3D coordinates of a large amount of points located on a model, and then to define a surface that would contain them. It is reminiscent of regression curves, Delaunay networks, Bôse method, etc.

Naively, I thought that, on the contrary, we would never make a car just by putting an engine on a charabanc. I thought that we had to 'erase the blackboard', and start all over from scratch. Forgive me for not being modest when speaking in the first person but I think that if I contributed to the development of CAD/CAM in some way, it is mainly that point of view, whose heterodoxy convinced all the so-called serious, and cautious even cowardly, people that I had gone off the track of their predecessors. I skip some judgements, which would have deserved the Stupidity Prize, if Mr Nobel had planned to create it, alongside the other ones.

11 Prototype equipment

In 1965, the theoretical work was quite advanced and I was certain the solution was valid, but in order to convince traditionalists, it would have been essential to have an 8×2 m drawing machine, a low-power (0.5 kW) milling machine to cut polystyrene foam blocks with $1.5 \times 1.2 \times 0.8$ m courses and 2 m/min feeds at one's disposal, which did not seem realistic at the time. Moreover, in order to work in a conversational mode, we would have needed a computer of modest power entirely at our disposal, which went against the accepted practice in 1960. Finally, we would have needed to develop some rudimentary software.

The budget for all this was evaluated at 3,000,000 FF and Top Management showed the limits of its confidence by restricting the amount of 600,000 FF. So I had to go begging. Fortunately, the DGRST[17] placed its confidence in the project and they granted me 1,500,000 FF. As for the remaining amount, a computer company lent me 900,000 FF because they thought multiplying small dimension computers was an idea interesting to encourage.

12 Conclusion[18]

As the problems became more complex, we used nonlinear reference frames which permitted the plotting of a curve with parameter w on a surface, defined by two parameters u and v, permitting, for example, the plotting of a cut-out wheel arch in an already defined wing or to change a whole body without having to correct separately the elementary surfaces that form it.

As you can see, dear Sir, all that is simple and refers to some common notions and to some vectorial and matrix computations. Why, in 1960, did no research worker in aeronautics find this straight away? I think they were fixed on the idea of reproducing a model instead of directly creating a shape and progressively refining it. I assume that the problem set by Citroën to Jean de la Boixière (SupElec) and Paul de Casteljau (Norm Sup),[14] both engineers and mathematicians, was also to numerically translate a hand-made model.[19]

Generally speaking, it can be said that around 1960, many mechanical engineers still knew little about how helpful electricity could be for measurement, calculation, machine control, and the distribution of force and power. When I made my debut in the mechanics industry, i.e. around 1930, it was only used in machine tools as induction motors, magnetic relays and end-of-stroke switches for elevator doors. Around 1935 I was reluctantly allowed to use sequential automation to replace clutches, jaw clutchings and hydraulic jacks.

The book of specifications I had set for myself in 1960 had the advantage of being based on a certain experience I had gained by holding most of the positions involved in the car-body field, such as machining, foundry, electricity, electronics, plotting, welding, sketching, tight-fitting and control. Furthermore, I still had some knowledge of mathematics from my training, which I enhanced beyond what the students at the Ecole des Arts et Métiers were taught in 1930. Who said curiosity killed the cat?

When someone undertakes the designing of a machine tool, which was my first job, they have to first imagine what it should be like. Then, the final definition develops through permutation between plottings, calculations and assays. For 25 years, mechanical and electrical engineers have been working together right from the start of the design phase. Later, philosophers, psychologists and professional organisers found a name for that practice: 'ingénierie simultanée' (simultaneous engineering).

The theory of parametric spaces is now largely known and taught. See one of the maths examination papers of the 'Grandes Ecoles' in 1999.[20] Literature on that is plentiful. The basic ideas I tried to express may be less widespread but I think they will stand the test of time.

I would like to know to what extent my answers met your expectations because in the exercise of my profession, either in civil or military life, I have always attached high importance to the role of pedagogy. Is not our time said to be the time of communication?

Kind regards,

P. Bézier

4. Second letter

This section presents the second letter sent to Pierre Bézier, and his reply, as written.

4.1. Second letter sent to Pierre Bézier

Christophe Rabut

Toulouse, le 9 novembre 1999

à Pierre Bézier

Cher Monsieur,

Un grand merci pour votre longue lettre, pleine de détails et très chaleureuse. Je n'ai pas le temps matériel actuellement (et pour au moins 15 jours… sans doute même jusqu'en janvier prochain !) de travailler sur votre lettre, car les contraintes d'enseignement et de recherché prennent tout mon temps disponible. Mais je l'ai bien sûr lue avec intérêt et j'ai beaucoup apprécié tous les details que vous m'envoyez.

A vrai dire, je fais un cours d'analyse numérique pour l'ingénieur (j'essaie que ces termes ne soient pas contradictories !), et j'apprécie de pouvoir donner aux étudiants un minimum d'information historique : dates, et si possible motivation, concernant l'origine des méthodes présentées. Cela me paraît important pour mieux comprendre la science, son évolution et même son utilisation actuelle. Outre mon intérêt personnel sur le sujet (je travaille beaucoup sur les B-splines, la representation de courbes et surtout de surfaces), c'est pour cela que je m'étais permis de vous écrire. Oui, vous avez tout à fait répondu à mon attente, et je vous en remercie vivement.

Sur le fond, c'est tout à fait étonnant de constater à quel point la méthode qui porte votre nom n'a pas été formalisée au départ par un article précis la présentant. De plus en plus actuellement un tel article est presqu'indispensable, même si en général un tel article ne pouvait prévoir toutes ses propres conséquences (citons dans notre domaine par exemple l'article de Schoenberg, 1946, sur les splines, l'article de Duchon, 1976, sur les surfaces splines). Dans le cas des 'courbes de Bézier'—et curieusement aussi dans le cas de l'algorithme dit de de Casteljau—il n'y a pas de tel article, 'point de départ' de la méthode.[21] Très intéressant de voir aussi à quel point cette méthode est partie de façon très concrète, presque technique, pour être ensuite 'appropriée' par les mathématiciens (au service de l'industrie en général).[22]

J'espère que vos soucis de santé sont maintenant résorbés, et je vous remercie encore pour votre longue lettre pleine de précision.

Christophe Rabut

4.2. Pierre Bézier's second letter

14 novembre 1999

P. Bézier

à Mr le professeur C. Rabut

INSA Toulouse

Monsieur et cher collègue,

Puisque vous souhaitez avoir quelques détails supplémentaires à propos de ma participation aux débuts de la CFAO je vais tâcher d'exhumer quelques souvenirs d'un passé déjà presque quadragénaire ; j'y ajouterai quelques anecdotes, sans doute superflues, pour vous donner une idée de l'ambiance dans laquelle baignait la tentative ; si ma mémoire a quelques défaillances ou si elle enjolive le passé, je demande à bénéficier de la prescription.

Ma motivation, je vous l'ai dit, était de faire disparaître ce qu'il y avait de flou dans les pratiques d'une profession sans repères fermes ; le jugement du styliste était sans appel, souvent inconstant et parfois fondé sur un caprice ; pour vous donner un exemple, il nous a une fois demandé que la cambrure d'un capot soit modifiée de trois quarts de millimètre ; or l'emboutissage est une opération dont le résultat dépend des propriétés physiques de la tôle, et celles-ci varient un peu selon qu'elle est tirée du début ou de la fin d'une bobine livrée par l'aciérie après son recuit ; les différences du retrait, dit 'spring back' par les franglicisants, peuvent dépasser le millimètre. Je crois que l'on peut avouer aujourd'hui que l'on n'a effectué aucune correction sur le dessin et que le styliste, invité quarante-huit heures plus tard à venir sur place juger du résultat, a déclaré que c'était bien mieux et qu'il ne regrettait pas le surcroît de travail que cela avait nécessité. En fait, les deux jours avaient été consacrés à un tournoi de belote entre les dessinateurs. C'est la foi qui sauve.

Gagner quelques millimètres de précision globale est sans importance quant à l'esthétique, mais le prix de revient de l'assemblage s'en ressent ; quand une porte ne s'encastrait pas bien dans son logement, la seule ressource était de la contraindre à l'aide d'une barre à mine, et c'était si fréquemment nécessaire que cette retouche figurait dans le vocabulaire de l'atelier sous la désignation de 'bidouillage' et il en était tenu compte dans la gamme établie par le service de chronométrage.

La thèse soutenue en 1977 a été la première publication un peu consistante à propos du procédé mis en œuvre chez Renault ; en effet, jusqu'en 1975 j'y étais en service et je n'avais guère le temps de regrouper les notes successivement adressées à divers collègues ; j'avais de bonnes raisons de penser que la plupart n'y comprendraient rien ou ne verraient aucun intérêt à cette tentative ; et puis, si par grand hasard il y avait quelque chose à en attendre, il ne serait pas habile d'en faire profiter des concurrents ; d'ailleurs il n'existait pas à cette époque de revue technique francophone prête à accueillir ce genre de littérature. Celle d'HERMES n'a paru qu'après 1980.[2]

Enfin, si j'avais consacré du temps à publier quelque chose, la Direction de la Régie[10] y aurait vu une manifestation d'amateurisme improductif. Paul de Casteljau, de son côté, était tenu au silence en raison du secret fort strict imposé chez CITROEN.

Aux USA, l'industrie aéronautique avait réalisé des systèmes, mais ils étaient, en ces débuts, destinés à réaliser des maquettes de soufflerie ; leur géométrie était fondée sur l'usage de la droite et du cercle, à tel point que si les fuselages et les ailes étaient définies de cette façon, les raccords de von Karman étaient chaudronnés à la main et reproduits ensuite sur des outils de formage.

C'est en 1964, alors que nos études avaient déjà pris forme, que James Ferguson, de chez Boeing, a publié dans le bulletin de l'Association for Computing Machinery une étude sur les courbes paramétriques polynomiales du troisième degré ;[23] un arc étant défini par quatre conditions, $P(0)$, $P(1)$, $\mathrm{d}P/\mathrm{d}u(0)$ et $\mathrm{d}P/\mathrm{d}u(1)$; si l'on assimile l'intervalle (0, 1) à l'unité de temps, la dérivée est une vitesse, et si la forme de l'arc n'est pas trop biscornue, la longueur du vecteur qui la représente est voisine de celle de l'arc reliant $P(0)$ à $P(1)$. Il en résulte que l'image d'un carreau qui comporte en chaque coin les vecteurs représentant les deux dérivées partielles et la dérivée mixte est passablement embrouillée. Au contraire, pour les fonctions $f_1(0)$ et $f_3(1)$,[24] la dérivée est égale à 1/3 et la forme du polygone ressemble un peu à celle de l'arc qu'il engendre et c'est d'autant plus marqué que le degré est élevé. Je dois rendre grâces à la chance qui m'a été favorable en cette occurrence.

Les splines, auxquelles vous vous intéressez, ont été créées à l'origine pour faire passer un arc par des points donnés, et l'on connaît plusieurs solutions pour en déduire les vecteurs dérivés en ces points ou pour accepter une approximation. Quant aux NURBS, j'ai beaucoup d'admiration pour ceux qui les ont inventées mais je trouve que leur manipulation est moins instinctive que celle des polygones,[25] et beaucoup de dessinateurs sont du même avis ; au fond, les dérivés résultent d'un choix entre l'automatisme et l'arbitraire. Si j'ai gardé une vieille préférence pour l'usage des polygones,[25] je crains que ce ne soit qu'une manifestation de nostalgie ou, au pire, de sénilité.

Sur un point d'histoire que vous soulevez, je crois que le premier article publié en français à propos de courbes et des carreaux est paru dans la revue AUTOMATISME aux environs de 1973.[26] Par ailleurs, j'ai pris part en 1974, à Détroit, au congrès de la Society of Automotive Engineers ;[27] les textes des communications y sont publiés et remis aux participants plusieurs semaines avant l'ouverture afin qu'ils puissent préparer les questions à poser à la suite de l'exposé oral qui est un résumé. Je n'ai jamais utilisé mon nom pour désigner le système à base de polygones et de réseaux et je ne sais absolument pas à quelle date ni à quelle occasion il a été employé, mais je suppose que c'est aux USA ; INTERNET en sait plus que moi vous y trouverez, paraît-il, un bon nombre de références liées à mon patronyme, y compris celle d'un groupe d'amateurs de polygones dont le siège est en Californie; de quoi rivaliser à l'échelle $1/n$, avec Aznavour ou Johnny Haliday ; quand je donnerai un recital à Bercy ou à l'Olympia, je rassemblerai une foule d'admirateurs délirants et hystériques quand je sortirai, tout modestement, par la porte dérobée après que mon homme d'affaires ait pris soin de prévenir la Presse, la Radio et la Télévision.

En fin de compte, les idées favorables qui m'ont guidé, et dont je n'ai pas l'exclusivité, se résument ainsi :

1. Ne pas chercher à recopier un objet (2D ou 3D) ou à perfectionner une méthode existante.
2. Choisir une representation paramétrique polynomiale, ce qui permet de modifier la forme d'une courbe ou d'une collection de surfaces en faisant varier seulement celle de leur référentiel.
3. Représenter un référentiel en mettant bout à bout ses vecteurs unité, au lieu de leur donner une origine commune.

Tout cela n'est pas bien compliqué, et je suis surpris que d'autres n'en aient pas eu l'idée depuis longtemps. Sans doute ai-je bénéficié d'une formation dans laquelle la géométrie tenait une plus grande place que dans celle d'aujourd'hui et de conduire mon raisonnement en me représentant des figures plutôt qu'une famille d'équations. Je ne suis pas axiomaticien. Tant pis pour moi ! Quelles sont, par comparaison, les réactions de vos étudiants ?

Sans doute ai-je repris ici des notions que j'avais déjà exprimées dans ma précédente lettre ; elles ont été, pour moi, fondamentales.

Croyez, je vous prie, mon cher collègue, à mes sentiments bien cordiaux.

Pierre Bézier

5. English translation of the second letter

This section presents the translation of the second letter sent to Pierre Bézier, and his reply.

5.1. Second letter sent to Pierre Bézier

Christophe Rabut

Toulouse, 9 November 1999
to Pierre Bézier

Dear Sir,

Many, many thanks for your long letter, warm and full of details. Unfortunately I presently have no time to work on your letter, since I have strong constraints in teaching and research (this will be true for at least a fortnight, and even probably up to January 2000). But of course I read it with keen interest, and I greatly appreciate all the details you sent me.

To be complete, I am teaching a course on Numerical Analysis for engineers (I try to do such that there is no contradiction in these words!), and I am keen on giving a minimum of historical information to students: dates, and whenever possible motivation, about the origin of the presented methods. I think this is important to better understand science, its evolution and also its present use. Beyond my personal interest on the subject (I work a lot on B-splines, and on curve and surface modelling), this is the reason I took the liberty to write to you. Yes, of course, your answer completely meets my expectation, and I am warmly thankful to you for that.

Basically, it is amazing to note that the method which has your name has not been formalised from the start by a precise paper. More and more such a paper is essential, even if it cannot foresee all its own consequences (let us quote in our domain, for example, Schoenberg's paper, 1946, on spline functions, Duchon's paper, 1976, on surface splines). In the case of 'Bézier curves'—and amazingly for the so-called de Casteljau algorithm—there is no such paper given as the starting point for the method.[21] It is also very interesting to see that this method started in a very practical, quite technical way, and has been 'taken' by mathematicians (generally speaking working for industry).[22]

I hope that your health problems are now solved, and I thank you again for your so long and so detailed letter.

Christophe Rabut

5.2. Pierre Bézier's second letter

14 November 1999

P. Bézier

Mr Professor C. Rabut
INSA Toulouse

Dear Sir and Colleague,

Because you would like to have some further details concerning my contribution to the beginning of CAD/CAM, I will try to recollect some memories dating back almost 40 years. I will add some probably superfluous anecdotes to give you an idea of the atmosphere in which my efforts took place. If my memory suffers from weaknesses or embellishes the past, please forgive me.

As I told you, I intended to erase all the vagueness occurring in the practice of a profession without fixed benchmarks. The stylist's judgement was irrevocable, often irregular and sometimes based on an impulse. To give you an example, he once asked us to modify the hood camber by three-quarters of a millimetre; however, punching is a process, the result of which depends on the properties of sheet and these vary slightly according whether it is pulled out from the beginning or from the end of the roll delivered by steel works after annealing; 'spring-back' differences, as some French willingly call this, can amount to more than one millimetre. I think that, today, we can state that we made no change in the design and that the stylist, who was invited two days later to come and see the result for himself, declared that it was much better and that he didn't regret the extra work required. Actually, those two days had been devoted to a card game tournament among the designers. Blessed are the faithful!

Gaining a few millimetres of global accuracy is of no importance concerning the aesthetic aspect but the assembly price cost increases proportionally. When a door did not fit into its frame, the only solution was to force it with a steel rod, and this was so often necessary that this alteration was part of the workshop's vocabulary under the denomination 'fudging' and was taken into account in the range of tasks established by the time-and-motion department.

The thesis submitted in 1977 was the first well-grounded enough publication on the process implemented at Renault where I was working until 1975, and I did not have much time to collect the notes successively addressed to various colleagues. I had good reasons to think that most of them would not understand anything of it or would find this venture totally uninteresting. And if, by chance, something could come out of it, it would not be clever to let competitors benefit from it. Anyway, at that time, there was no French technical journal willing to publish such literature. HERMES' book was not printed until 1980.[2]

Finally, if I had taken the time to publish something, the Régie[10] would have considered it as counterproductive amateurism. As for Paul de Casteljau, he had to keep quiet on stringent secrecy grounds established by CITROEN.

In the US, the aeronautic industry completed systems but, at the early stage of their development, they were intended to make wind-tunnel models. Their geometry was based on straight lines and circles, so that if the fuselages and the wings were defined that way, the von Karman joints were hand-cast and then reproduced on forming tools.

In 1964, as our research was already well advanced, James Ferguson, from the Boeing company, published a study on third-degree polynomial parametric curves in the Association for Computing Machinery bulletin.[23] An arc is defined by four conditions, $P(0)$, $P(1)$, $dP/du(0)$ and $dP/du(1)$. By considering the interval (0, 1) as the time unit, the derivative is a speed and if the arc shape is not too irregular, the length of the vector that represents it is close to that of the arc linking $P(0)$ to $P(1)$. As a consequence of it, the image of a patch is rather unclear and presents, in each corner, the vectors representing the two partial derivatives and the composite derivatives. On the contrary, for the functions $f_1(0)$ and $f_3(1)$,[24] the derivative equals 1/3 and the shape of the polygon looks a bit like that of the arc that it generates and it is all the more obvious that the degree is high. I am thankful for the chance to have been so lucky on that occasion.

The splines you are interested in were originally created to make an arc join given points. There are several solutions to deduce the derivative vectors in these points from that or to make an acceptable approximation. As for NURBS, I admire the people who invented them but I think that their handling is less instinctive than the one of the polygons;[25] many designers agree. Basically, the derivatives result from a choice between something automatic and something arbitrary. I still prefer to use polygons,[25] but I fear this might just be the expression of nostalgia or even senility.

Concerning the historical point that you raised in your letter, I think that the first article in French on curves and patches was released in the AUTOMATISME journal around 1973.[26] In addition, in 1974, I took part in the Society of Automotive Engineers congress in Detroit;[27] the proceedings of it were given to the participants a few weeks before the opening so that they were able to prepare their questions following the talks which actually were abstracts. I never used my name to refer to the polygon and network-based system and I have no idea of the date or occasion on which it was used. I assume it was in the US. INTERNET knows it better than me and I have been told that you will find quite a lot of references concerning my last name, including some of a circle of polygon enthusiasts, whose headquarters are in California. This allows me to compete with Aznavour or Johnny Haliday on a scale of $1/n$. When I give a concert at Bercy or at Olympia, I will pull a crowd of frenzied hysterical fans as I leave through the stage door, very modestly, after my manager has carefully alerted Press, Radio and Television.

In the end, the positive ideas that have guided me and of which I am not sole owner are, in short, the following.

1. Do not try to make a copy of a 2D or 3D object or improve an existing method.
2. Choose a parametric polynomial representation, allowing the shape of a curve or of several surfaces to be modified by letting only their reference frame vary.
3. Represent a reference frame by putting end-to-end its unit-vectors instead of giving them a common origin.

All this is not very complicated, and I am surprised that others had not thought of it a long time before. I probably benefited from a training in which geometry enjoyed a larger place than in today's training, and from allowing myself to reason by imaging figures rather than a family of equations. I am not an axiomatician. Never mind! How do your students react, by comparison?

I probably repeated some notions I had already written about in my previous letter. They were fundamental to me.

Please accept, my dear colleague, my kindest regards.

Pierre Bézier

6. Notes and comments

[1] Pierre Bézier was an invited speaker in the first Curves and Surfaces conference, held in Chamonix, in June 1990.

[2] See Ref. [5] for the book, and Ref. [3] for the thesis.

[3] The leading sentence of Pierre Bézier's thesis [3] is 'Tout est nombre' (everything is number), for which he refers to Plato.

[4] This means that he entered 'Ecole des Arts et Métiers', a mechanical engineering school in Paris in 1927, and received his degree in 1930, then studied one year at the 'Ecole Supérieure d'Electricité', an electrical engineering school.

[5] The reference to Chaïkin is surprising here since the first paper from Chaïkin seems to be Ref. [65] and the first reference to Chaïkin's algorithm seems to be Ref. [64]. Probably people in Renault used this material before it had been formalised and published by Chaïkin.

[6] Explanations of the idea of space deformation, and of putting the unit vectors successively (so forming the cube) instead of giving them the same origin, are nicely given in Refs. [28,44] (cube becoming a parallelepiped by linear transformation, pantograph for space deformation, a rectangle becoming a rhombus, a circle becoming an ellipse).

[7] In some way, this is the same idea as the one introduced much later by Tom Sederberg to the graphic community under the name 'free-form deformation' (see for example Ref. [69]).

[8] The figures were hand-drawn. The versions reproduced here were generated using Matlab.

[9] We have to read $\bar{P}(u) = \sum_{i=1}^{3} \overline{a_i} f_i(u)$ instead of $\bar{P}(u) = \sum_{i=0}^{3} \overline{a_i} f_i(u)$. Some details to help the reader (more explanations are given in Refs. [3–5,28,44,62]). The starting idea was to use a control polygon, instead of interpolating points; let us call $(P_i)_{i=0,\dots,n-1}$ the vertices of this control polygon, and so $(P_iP_{i+1})_{i=0,\dots,n-1}$ are the sides of the

polygon; furthermore, let $P_{-1}=0$. The second idea ('space deformation', 'putting the vectors end-to-end'), is to use the edges of the polygon instead of the vertices of it. Last to write a curve in the form $P(t)=\sum_{i=1}^{n} P_{i-1}P_i f_{n,i}(t)$ where $f_{n,i}(t)$ are appropriate scalar functions, as those shown in Figs. 1 and 2.

The chosen functions were required to meet the following properties:

$$\begin{cases} \text{For all } t \text{ in } [0,1],\ f_{n,0}(t)=1 \\ \text{For all } i=1,2,\dots,n: \\ \quad f_{n,i}(0)=0,\ f_{n,i}(1)=1 \\ \quad \text{for } j=1,2,\dots,i-1:\ \dfrac{d^j f_{n,i}}{dt^j}(0)=0 \\ \quad \text{for } j=1,2,\dots,n-i:\ \dfrac{d^j f_{n,i}}{dt^j}(1)=0 \end{cases}$$

Note that such functions are not necessarily polynomials (and, as said at the beginning of §6, and shown in Fig. 1, the first choice was not polynomials). Finally, Pierre Bézier chose degree n polynomials, which gives the following general expressions:

$$f_{n,i}(t)=\frac{(-t)^i}{(i-1)!}\frac{d^{i-1}\left(\dfrac{(1-t)^n-1}{t}\right)}{dt^{i-1}}$$

$$=\sum_{j=i}^{n}(-1)^{i+j}\binom{n}{j}\binom{j-1}{i-1}t^j$$

A little bit later, (see §8 of the letter), it was noticed that $P(t)$ can also be written (we take here $f_{n,n+1}(t)\equiv 0$):

$$P(t)=\sum_{i=0}^{n}(P_i-P_{i-1})f_{n,i}(t)=\sum_{i=0}^{n}P_i(f_{n,i}(t)-f_{n,i+1}(t))$$

from where we have the well known expression

$$P(t)=\sum_{i=0}^{n}P_iB_{n,i}(t)$$

where

$$B_{n,i}(t)=\binom{n}{i}t^i(1-t)^{n-i}$$

is the i-th Bernstein polynomial of degree n, since it can be easily checked that $f_{n,i}(t)-f_{n,i+1}(t)=B_{n,i}(t)$.

[10] Pierre Bézier means here 'Régie Renault' or 'Renault'.

[11] CNAM is for Conservatoire National des Arts et Métiers. ADEPA is for Association pour le Développement de la Production Automatisée, which means Association for the Development of Automatic Production.

[12] The very first papers mentioning Bézier come from the Cambridge University CAD Group: the first one mentioning Pierre Bézier's work is Ref. [59], while the first ones giving the curves the name 'Bézier curves' are Refs. [60–62].

[13] Claude Riaux was working at Renault with Pierre Bézier. This has been seen independently, and quite probably at the same time, by R. Forrest [59].

[14] SupElec: Ecole Supérieure d'Electricité, Paris; Normsup: Ecole Normale Supérieure, Paris.

[15] Whereas mathematicians take it for granted that the so-called Bernstein polynomials were introduced by Bernstein for the proof of the Weierstrass theorem (density of the space of polynomials in $C^0[a,b]$ for the $L^\infty[a,b]$ norm), since

$$\sum_{i=0}^{n}f\left(\frac{i}{n}\right)\binom{n}{i}\left(\frac{t-a}{b-a}\right)^i\left(\frac{b-t}{b-a}\right)^{n-i}$$

uniformly converges to f in $[a,b]$.

I can neither confirm nor deny what Pierre Bézier says here. Sergi Bernstein introduced the so-called Bernstein polynomials in the paper [52], republished in Bernstein's collected works [53], and reproduced in Ref. [67]. A strong link is made with probability (even in the title), as shown in the following sentence, taken from Ref. [52]:

> *Let us consider the event A, whose probability is equal to x. We assume that there are n samples and that we decide to pay a partner the amount*
>
> $$F\left(\frac{m}{n}\right)$$
>
> *if the event occurs m times. In that case, the 'mathematical expectation' E_n of profit for the partner has the value*
>
> $$E_n=\sum_{m=0}^{n}F\left(\frac{m}{n}\right)C_n^m x^m(1-x)^{n-m}.$$
>
> *From the continuity of F(x), we can choose a real number δ such that the inequality $|x-x_0|<\delta$ implies*
>
> $$|F(x)-F(x_0)|<\frac{\epsilon}{2}$$
>
> …

However, it seems that no paper of Bernstein mentions a connection between 'Bernstein polynomials' and any kind of data from an insurance company. But probability theory was one of Bernstein's main preoccupations (see http://www-history.mcs.st-and.ac.uk/history/Mathematicians/Bernstein_Sergi.html), which makes Pierre Bézier's anecdote at least believable.

Let us mention here that Schoenberg introduced B-splines [54] for the smoothing of ballistic tables.

So it seems probable that Bernstein polynomials as well as Bézier curves and B-spline functions, were invented with a truly applied motivation.

[16] Here Pierre Bézier refers to the reactions of the heads of Renault when he introduced inside Renault the 'machines de transfert' (transfer machines) after the Second World War: to numerous machines devoted each one to a single

action (and with a worker at each machine), he succeeded in realising multi-purpose machines, equipped with three different heads, easy to replace in case of necessity. He thus significantly improved the times for building a car, so augmenting the number of cars built in a day, and cutting down the costs. Actually Pierre Bézier is much more known for the 'Bézier curves', but he really was a precursor in tooling, and he could have been famous in this domain also. As mentioned in these lines and at end of §10, he encountered much resistance to his way of thinking of problems globally and trying to find global solutions starting from a new basis (what he calls 'essuyer le tableau et repartir de zéro' (erase the blackboard and start all over from scratch). This was thought to be excessive by the heads of Renault. Let us mention that de Casteljau encountered lots of difficulties—probably at an even higher level—in Citroen.

Obviously Pierre Bézier, who was very cautious over the impact of his work on the working conditions in the firm, was very sorry (even angry) about the unwillingness of Renault's management to fully recognise what he did. Let us however mention that, looking now at the situation, it seems that Renault finally gave to Pierre Bézier sufficient resources to develop and implement his ideas, including the authorisation of publishing his work, which has been decisive in that matter.

[17] DGRST: Délégation Générale à Recherche Scientifique et Technique (National Council for Scientific and Technical Research).

[18] This section title is **not** in the original letter. I added it here because the end of the letter is obviously not relevant for §11 *Equipement prototype*.

[19] Actually, at Citroën, Paul de Casteljau defined the 'Bézier curves' in a geometric and algorithmic way, by using the 'de Casteljau's algorithm', late 1958 and 1959; he used Bernstein polynomials soon afterwards, much before Pierre Bézier. This has been presented in internal Citroën reports, such as [55], was taught in the internal drawing school of Citroën from 1963, has been mentioned to '*Institut de la Propriété Industrielle*' in March 1959 and June 1963, and has been kept secret by Citroën for a long time.

Pierre Bézier was aware of that and used to fully recognise that his own work was posterior, though completely independent, to the one of Paul de Casteljau: '*Most likely, Citroën people are the first ones in Europe who worked on the numerical definition of surfaces, since their works began, if I am not wrong, in 1958. With pleasure, I do pay homage to their knowledge and to their initiative*', and further: '*…I understood* [in 1972] *that the conception of the type of curves and surfaces representation was born in the brain of mathematicians, namely, MM. de Casteljau and Vercelli, whose capacity I admire. Right from the start, they thought to use the properties of Bernstein functions, while I ignored their existence, instead of doing, as I did, a heavy analytic study of the properties of the functions I wanted to use for the curves and surfaces representation. Finally, I ended up at the same result, but by using a very bumpy way*' (translated from Ref. [38], p. 329).

For more details on this, see Ref. [72].

[20] 'Concours des Grandes Ecoles' are French competitions for entering various engineering schools.

[21] I was completely wrong: Refs. [9,11,62] can be considered as the starting papers on Bézier curves and Bézier surfaces.

[22] This is true if we consider the Bézier–Renault story of the method, but is false if we consider the de Casteljau–Citroën one, since de Casteljau was a mathematician and started with a mathematical point of view.[19]

[23] See [56], also explained in Section 1.4.3.1 of Ref. [3]. This is a parametric, C^1, piecewise cubic polynomial, expressed in the canonical basis (in the univariate and bivariate cases).

[24] Pierre Bézier probably means here '… for the functions f_1 (resp. f_3), the derivative in 0 (resp. in 1) equals 1/3, and the shape …'.

[25] Pierre Bézier probably means here '*polynomials*'.

[26] Actually in May 1968; see Ref. [12]. Ref. [7] does not present the Bézier method, but the state of the art in 1966 of curves and surfaces numerical modeling.

It is worthwhile to note that Bernstein polynomials are not used in this paper (only the so-called $f_{n,i}$ functions). The first papers using Bernstein polynomials are Refs. [60,62]. This paper (and later, in more detail, [3–5]) presents the 'system UNISURF'—see §7 of the first letter—which is what we presently call 'Bézier curves' and 'Bézier surfaces'. The curves are defined as in note (8), and the surfaces with 'Bézier patches'. A surface is presented as a curve that moves, and that changes while moving (a nice explanation and illustration of that is given in Refs. [28,44]); so each vertex of the characteristic polygon of the moving curve follows a line that is also a polygon. This gives form to a tensor product representation of surfaces.

[27] Here there is probably confusion with the Society of Automotive Engineers Congress in Detroit in 1968. The paper mentioned is Ref. [9].

7. Notes for references

Refs. [1–6] are books (on the thesis) written by Pierre Bézier.

Refs. [7–35] are Pierre Bézier's papers published in scientific books and journals.

Refs. [36–41] are Pierre Bézier's papers published in the Bulletin de la Section d'Histoire des Usines Renault.

These papers can be ordered from the 'Section d'Histoire des Usines Renault', 27 rue des Abondances, 92100 Boulogne Billancourt, France. They are also available on the Web page http://www.gmm.insa-tlse.fr/~rabut/bezier

Refs. [42–51] are Pierre Bézier's papers published in Arts et Métiers Magazine and other journals.

The papers published in Arts et Métiers Magazine can be ordered from Arts et Métiers Magazine, 9bis avenue d'Iéna, 75783 Paris Cédex 16, or at amm@arts-et-metiers.asso.fr. They are also available on the Web page http://www.gmm.insa-tlse.fr/~rabut/bezier

Refs. [52–76] are related papers and books.

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