

Analysis of Algorithms

CSC 402

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BRANCH AND BOUND

The Method

- Branch-and-Bound (BnB) algorithms are based on searches of an associated state space tree for goal states.
- In BnB algorithms, all the children of the E-node (the node currently being expanded) are generated before the next E-node is chosen. (unlike Backtracking where the tree is generated depth wise).
- The traversal is in BFS manner.

The Method

- When the children are generated, they become *live* nodes and are stored in a suitable data structure, say *LiveNodes*.
- Various versions of BnB (Branch and Bound) exist, each differing only in the manner the next E-Node amongst *LiveNodes* is selected
 - FIFO BnB □ LiveNode implemented as queue
 - LIFO BnB □ LiveNode implemented as stack
 - LC (Least Cost) BnB □ LiveNode implemented as priority queue i.e. the node with the best cost is expanded all others are discarded
 - We will be discussing LC BnB only

15 puzzle problem

- 15 numbered tiles on a single square frame of 16 tiles.
- Transform the initial arrangement of tiles into goal arrangement from a series of legal moves.

1	3	4	15
2		5	12
7	6	11	14
8	9	10	13

Initial Position



1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Goal Position

15 puzzle problem... contd

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

Initial Position



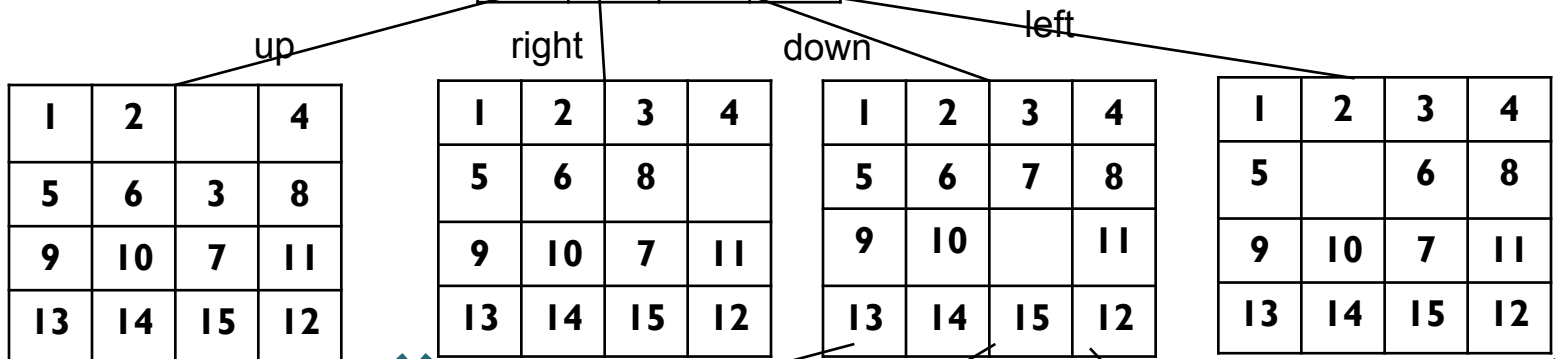
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

Goal Position

- Cost function $C^*(x) = f(x) + g^*(x)$
 - $C^*(x)$ = estimated cost to reach goal node
 - $f(x)$ = number of moves from initial state
 - $g^*(x)$ = estimate of cost of reaching from current state to answer state (number of non blank tiles not in their goal position)

1	2	3	4
5	6		8
9	10	7	11
13	14	15	12

$$C^{\wedge}(1) = 0 + 3 = 3$$



$$C^{\wedge}(2) = 1 + 4 = 5$$



$$C^{\wedge}(3) = 1 + 4 = 5$$

$$C^{\wedge}(4) = 1 + 2 = 3$$

$$C^{\wedge}(5) = 1 + 4 = 5$$



right

1	2	3	4
5	6	7	8
9	10	11	
13	14	15	12

$$C^{\wedge}(6) = 2 + 1 = 3$$

down

1	2	3	4
5	6	7	8
9	10	15	11
13	14		12

$$C^{\wedge}(7) = 2 + 2 = 4$$

left

1	2	3	4
5	6	7	8
9		10	11
13	14	15	12

$$C^{\wedge}(8) = 2 + 3 = 5$$



$$C^{\wedge}(9) = 3 + 2 = 5$$


up

1	2	3	4
5	6	7	
9	10	11	8
13	14	15	12

down

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

$$C^{\wedge}(10) = 3 + 0 = 3$$



TSP using branch and bound (refer Word document)