MATHS 315 Assignment 4

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(a)

1.	$(\forall x \neg Ax \to \neg Ax)$	K4
2.	$((\forall x \neg Ax \to \neg Ax) \to (Ax \to \neg \forall x \neg Ax))$	Tautology Instance
3.	Ax	Hypothesis
4.	$(Ax \to \neg \forall x \neg Ax)$	1,2 Modus Ponens
5.	$\neg \forall x \neg Ax$	3,4 Modus Ponens
6.	$\exists x Ax$	5, Abbreviation

(b)

We have shown that $\{Ax\} \vdash_{K_L} \exists x Ax$. Therefore we may add a hypothesis Bx such that $\{Ax, Bx\} \vdash_{K_L} \exists x Ax$. As x was not generalised over as a free variable (no generalisation was used at all in the derivation), we have by the deduction theorem that $\{Ax\} \vdash_{K_L} (Bx \to \exists x Ax)$

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As $\Sigma \cup \{(A \vee (B \wedge C)\}$ is a consistent set of closed wffs, by theorem 4.20, there is some model $\mathcal{I}$ such that $\mathcal{I} \vDash D$ for every $D \in \Sigma \cup \{(A \vee (B \wedge C)\}$, by definition 4.10.
Therefore the model $\mathcal{I} \vDash (A \vee (B \wedge C)$, this being an abbreviation for $\mathcal{I} \vDash_v (\neg A \to (B \wedge C)$ for every $\mathcal{I}$-assignment v. By Definition 3.10, 5, we have that $\mathcal{I} \nvDash_v \neg A$ or $\mathcal{I} \vDash_v (B \wedge C)$
iff $\mathcal{I} \vDash_v A$, due to A being derived from a closed, consistent set of wffs, or $\mathcal{I} \vDash_v B$ and $\mathcal{I} \vDash_v C$.
So either $\mathcal{I}$ is a model for $\Sigma \cup \{A\}$ or $\Sigma \cup \{B\}$.
As $\Sigma \cup \{A\}$ has a model, and is a closed set of wffs, it is consistent, by proposition 4.18. Conversely, the same is true of $\Sigma \cup \{B\}$. So either $\Sigma \cup \{A\}$ or $\Sigma \cup \{B\}$ is consistent.

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(a)

G1. $\forall x(Px \lor Lx)$
G2. $\neg\exists x(Px \land Lx)$

(b)

G3. $\exists x Px$
G4. $\exists y Ly$

(c)

G5. $((((((((Ll_1 \land Ll_2) \land \neg El_1l_2) \land Pp_1) \land Pp_2) \land Il_1p_1) \land Il_1p_2) \land Il_2p_1) \land Il_2p_2) \rightarrow (Ep_1p_2))$

(d)

G6. $(((Pp_1 \land Ll_2) \land \neg Ep_1p_2) \rightarrow \exists l((Ll \land Ilp_1) \land Ilp_2))$

(e)

If $\mathcal{I}$ is a normal model of S with domain D and p_1 and p_2 are points of $\mathcal{I}$ (in other words, $p_1, p_2 \in D$ and $P^{\mathcal{I}}p_1, P^{\mathcal{I}}p_2$ both hold and $p_1 \neq p_2$) then there is a unique $l \in D$ such that $L^{\mathcal{I}}, I^{\mathcal{I}}lp_1$ and $I^{\mathcal{I}}lp_2$ all hold.
As $\mathcal{I}$ is a normal model, it must satisfy axioms G1 - G6, as well as K1 - K5 and E1 - E3, with E interpreted as equality.
Given two unique points p_1 and p_2, by axiom G6 we must have under the normal model at least one line l incident with both points, such that $L^{\mathcal{I}}, I^{\mathcal{I}}lp_1$ and $I^{\mathcal{I}}lp_2$ all hold.