1. Let A be a predicate symbol with arity 3, and let B be a binary predicate symbol. Determine whether or not each predicate form is logically valid. Give either a proof or a counter example as appropriate.

(a) $(\forall x\, \forall y\, \forall z\, Axyz \rightarrow \forall y\, \exists x\, \exists z\, Axyz)$

(b) $(\forall x\, \exists y\, Bxy \rightarrow \forall y\, Byy)$

2. Show that, for any predicate form $\mathcal{A}$:

(a) $\mathcal{A} \Rightarrow \exists x\, \mathcal{A}$, and

(b) $\exists x\, Ax \not\Rightarrow Ax$.

3. Consider the following predicate forms:

$$\exists z\, \forall x\, (Axy \rightarrow Bz) \vee \exists z\, Axz \tag{1}$$

$$(\exists x\, \forall y\, Axy \rightarrow Ayy) \rightarrow \forall y\, (Afxyz \vee Bz) \tag{2}$$

(a) Perform the substitution fzy for x (i.e. substitute fzy for x) in each of these predicate forms.

(b) Complete the following table indicating which substitutions are free and which are not free in each of these two predicate forms.

	(1)	(2)
z for x	$\times$	
x for y		
y for z		
z for y		$\surd$
fxz for x		
$fygz$ for x		

4. Suppose that $\mathcal{A}$ and $\mathcal{B}$ are wffs of $K_{\mathcal{L}}$.

(a) Write a derivation to show that $\{(\mathcal{B} \rightarrow \mathcal{A})\} \vdash_{K_{\mathcal{L}}} \forall x((\neg\mathcal{A} \rightarrow \mathcal{B}) \rightarrow \mathcal{A})$.
[You will probably use a tautology instance.]

(b) Can we deduce that $((\mathcal{B} \rightarrow \mathcal{A}) \rightarrow \forall x((\neg\mathcal{A} \rightarrow \mathcal{B}) \rightarrow \mathcal{A}))$. Give a reason for your answer.