1. (2 marks) Show that the set of connectives $\{\to\}$ is not complete. (See the slides for Chapter 1, near the end, for the definition of a complete set of connectives. They are posted on Canvas.)

2. The formal systems $\mathcal{F}_1$ and $\mathcal{F}_2$ are defined as follows.

Both have an alphabet consisting only of the symbol A. In both cases, the wff's are all strings and the single axiom is A.

(a) (1 marks) The single rule of $\mathcal{F}_1$ is: from x infer xAA. Describe the set of theorems of $\mathcal{F}_1$.

(b) (3 marks) The single rule of $\mathcal{F}_2$ is: from x infer xx. Describe the set of theorems of $\mathcal{F}_2$. Prove your claim.

3. (2 marks) Show that $\{\mathcal{A}\} \vdash_L \neg\neg\mathcal{A}$ for any wff $\mathcal{A}$, without actually giving a derivation in L. You can use any Theorem we proved in class.

4. (4 marks) Prove that the set of finite subsets of $\mathbb{Z}$ is countable. During the course of your proof state precisely which of the results in Section 2.6 you use.

5. (a) (1 marks) Show that if $\mathcal{A}$ is a tautology and $\Sigma \subseteq$ form(L) is a set such that $\Sigma \vdash_L \neg\mathcal{A}$, then Σ is inconsistent. (You can use theorems from class if you reference them properly.)

(b) (1 marks) Prove that for each set $\Sigma \subseteq$ form(L) and each statement form $\mathcal{A}$,

Σ does not entail $\mathcal{A}$ if and only if $\Sigma \cup \{\neg\mathcal{A}\}$ is satisfiable.

Do not use the adequacy or soundness theorems for L.

(c) (2 marks) Prove that for wff $\mathcal{A}, \mathcal{B}$ of the system L, if $\{\mathcal{A}\} \vdash_L \mathcal{B}$, then $(\mathcal{A} \to \mathcal{B})$ is a tautology. You can use theorems from class, if you reference them properly.