Statistics 320 **Assignment 2** **Due: 2pm, 18 April 2018.**

1. Show that
$$\pi_i = 2^{-N}\binom{N}{i}$$
for all i in $[0, N]$, satisfies the equality $\pi = \pi\mathbf{P}$, where $\mathbf{P}$ is the 1-step transition probability matrix for the Erhenfest process with N particles.

2. What is the variance of the time to fixation of a bi-allelic gene in a Wright-Fisher population given that the population size is $N = 100$ and the number of alleles of one type is originally equal to 50. You will describe your approach in plain English and write a computer program that simulates the evolution of the number of A alleles in a Wright-Fisher population (add the code of your program, with comments, in the appendix of your report).

3. A professor has 5 students. Each day, each of them sends her an email with probability 0.2 and independently of the other students. Each afternoon she checks her inbox and if there are more than five new emails she replies to all of them immediately. However, if there are five or less, she replies to all the emails with probability 1/3 and leaves them for the next day with probability 2/3. Consider the number of new emails in her inbox just before she checks it each day. Is it reasonable to model this as a Markov chain? If so, what is the state space and the 1-step transition probability matrix?

4. Write a computer program that calculates the PageRank score for the graph below. You will provide a brief outline of the method and discuss the results obtained (e.g., do you think the ranking provided by the algorithm is relevant or not). You will also describe way(s) to "cheat" the PageRank algorithm, i.e., how can you make sure a given page receives a high score (assuming you have freedom to create new pages).