Statistics 320 **Assignment 1** **Due: 2pm, March 21.**

1. Consider the following variation of the Monty Hall Problem, known as the Monty Crawl Problem. As in the original problem, once you have selected one of the three doors, the host then reveals one non-selected door which does not contain the car. However, the host is very tired, and crawls from his position (near door no. 1) to the door he is to open. In particular, if he has a choice of doors to open (i.e., if your original selection happened to be correct), then he opens the smallest number available door. For example, if you selected door no. 1 and the car was indeed behind door no. 1, then the host would always open door no. 2, never door no. 3.

   Will you then stick to the door you selected first or switch to the other one? Does your answer depends on the number of the opened door? Why? What is the probability that you will win the car if you follow an optimal strategy?

   In order to answer these questions, you will simulate the game using an approach very similar to that described in class. The R code for simulating the original Monty Hall game is available on CANVAS. You are welcome to use it as a starting point to implement a modified version of the game. You will add your code in an appendix of your assignment.

2. Suppose X and Y have joint pdf

$$f_{XY}(x,y) = \begin{cases} c(x^2y) & x > 0, \quad y > 0, \quad x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$

   (a) Show that $c = 60$.

   (b) Draw the distribution function of the joint distribution, i.e draw $F_{X,Y}(x,y)$.

   (c) Determine the marginal distributions of X and Y.

   (d) Calculate the mean and variance of X and Y, $\mathrm{E}(XY)$, $\mathrm{Cov}(X,Y)$, and $\rho_{X,Y}$.

   (e) Are X and Y uncorrelated? Are they independent? Justify your answers.

3. A drunk person is standing at the pub's doorstep intending to leave and go home. Fortunately, the house is only 20 steps away. However, walking in this condition is not an easy task, and the direction of each step taken is random, and it can be either forward (towards the house) or backwards (towards the pub). Specifically, the probability that the direction of the next step is forward is 0.7 and the probability that it is backwards is 0.3. Based on simulation, what is the probability that the person will be home before returning to the pub?

4. A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after one hour of travel. The second door leads to a tunnel that return him to the mine after two hours of travel. The third door leads to a tunnel that returns him to his mine after three hours. Assuming that the miner is at all times equally likely to choose any one of the doors, based on simulation, what is the expected length of time until the miner reaches safety?