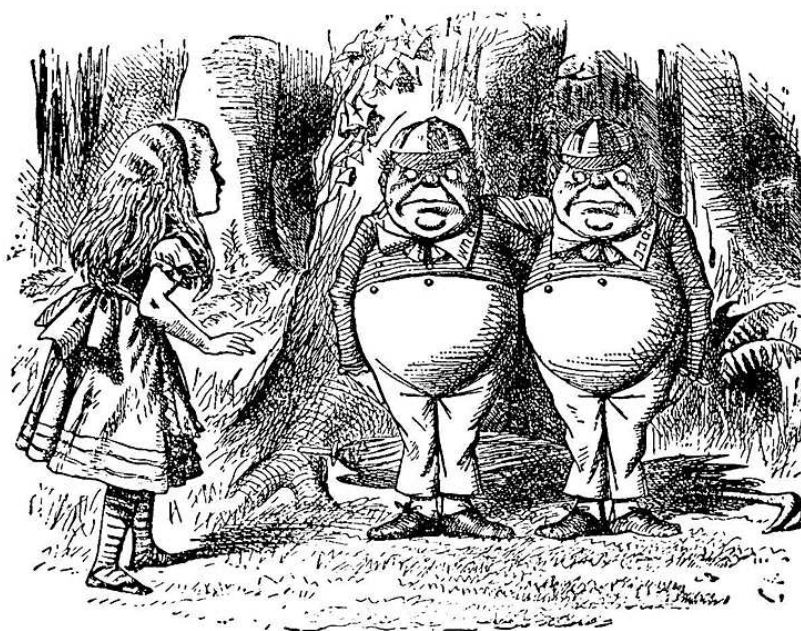# What Kinds of Liars and Truth-tellers Exist, and How Can They Be Identified Through Interaction?

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Philosophy 315

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Abstract

Truth-telling and lying come in many different forms - we intend to develop a taxonomy through the exploration of some intuitive and counterintuitive types, and determine how an agents category can be identified through interaction.

A Note on Belief

Its worth noting that the way in which one determines belief can have drastic impacts on logical systems of liars and truth tellers which consider belief.[3] If a Lockean conception of belief is used, τ being set to anything less than 0.5 can result in the speaker being both a liar and truth-teller simultaneously if using a system in which liars are those who say something they believe not to be true and truth-tellers are those that say something they believe to be true.[2]

Chapter 1

Basic Types

	Description	Logical formulation
1	Truth-telling as saying something which is true in reality	$(T_\bullet\psi \to \psi)$
2	Lying as saying something which is not the case in reality	$(F_\bullet\psi \to \neg\psi)$
3	Truth-telling as saying something sincerely	$(T_\bullet\psi \to B_\tau\psi)$
4	Lying as saying something insincerely	$(F_\bullet\psi \to \neg B_\tau\psi)$
5	Lying as saying something believed to be untrue	$(F_\bullet\psi \to B_\tau\neg\psi)$
6	Truth-telling as saying something that is both true in reality and sincere	$(T_\bullet\psi \to (\psi \wedge B_\tau\psi))$
7	Lying as saying something that is both untrue in reality and insincere	$(F_\bullet\psi \to (\psi \wedge \neg B_\tau\psi))$
8	Lying as saying something that is both untrue in reality and believed to be untrue	$(F_\bullet\psi \to (\psi \vee B_\tau\neg\psi))$
9	Truth-telling as imparting knowledge	$(T_\bullet\psi \to K_\tau\psi)$
10	Lying as imparting what one knows to be false	$(F_\bullet\psi \to K_\tau\neg\psi)$

1.1   Building on Omniscience

Based on situations 9 and 10, let's give our truth-tellers and liars names. Let an omniscient truth-teller be called Alan Truering, and an omniscient liar be called Ada Love-Lies.

1.1.1   Interaction with Alan Truering and Ada Love-Lies

If you know if a person is Alan Truering or Ada Love-Lies, you can identify the true state of ψ depending on what they say.

If an omniscient truth-teller states ψ, then you know ψ,due to:

$$(T_{a\spadesuit}\psi \to K_a\psi)$$
$$(K_x K_a\psi \to \psi)$$

The same is true for Ada Love-Lies by simply negating her statement.

1.1.2   Identification of Alan Truering and Ada Love-Lies

To know if you are dealing with Alan Truering or Ada Love-Lies, all you need to know is the actual state of ψ. For example if an unknown agent a is Omniscient and states ψ, and you know ψ, you can identify that a is Alan Truering. Conversely, if an if an unknown agent a is Omniscient and states ψ, and you know $\neg\psi$, you can identify that a is Ada Love-Lies. The proof of this is as follows:

1. $(KK_a\psi \to \psi)$ Hypothesis
2. $(K_a\psi \to \psi)$ Facticity
3. $(K(K_a\psi \to \psi))$ Necessitation
4. $(KK_a\psi \to K_a\psi)$ Translates across implication
5. $(K\psi \to \psi)$ Facticity
6. $(((p \to q) \land (q \to r)) \to (p \to r))$ Tautology
7. $((KK_a\psi \to K\psi) \land (K\psi \to \psi)) \to (KK_a\psi \to \psi))$ Substitution
8. $((KK_a\psi \to K\psi) \land (K\psi \to \psi)$ 4,5, Conjunction
9. $(KK_a\psi \to \psi)$ 7,8, Modus Ponens
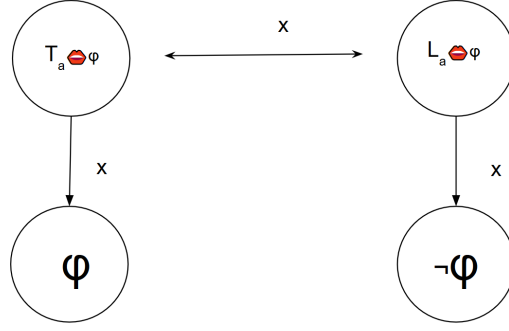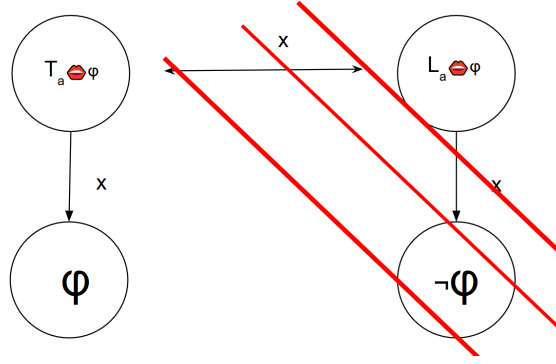
This entails the following model:

Figure 1.1: Initial space

Figure 1.2: After updating with ψ

1.2   Sincere and Insincere Men

Now instead of assuming that you must know ψ in order to be a truth teller, consider instead if all that is required to be a truth teller is to believe ψ, as per types 3,4, and 5, with liars being defined equivalently.

Unlike omniscient liars and truth-tellers, you cannot identify if someone is sincere or insincere based purely off whether their statement is true in reality.

Omniscient Truth-teller:

$$((T_{a}\psi \to K_{a}\psi) \to \psi)$$

Sincere Men:

$$((T_{a}\psi \to B_{a}\psi) \nrightarrow \psi)$$

Simply put, it is possible for people to have incorrect beliefs.

1.2.1   Identification of Sincerity

Although you are able to use the true state of ψ to identify if an agent a is a sincere of insincere man. Assuming a knows all updates are true updates, we have

$$K_a <!\psi >$$

We must also assume that an agent can only exist as a sincere of insincere man and cannot change.
Once the agent a makes their claim

$$a \spadesuit \psi$$

Then if you know ψ, all you need to do is update a to ψ.
If a knows that all updates are true updates $K_a <!\psi >$

$$K_a \psi$$
$$K_a \psi \rightarrow B_a \psi$$
$$B_a \psi$$

Knowing they now have a belief in ψ, if you ask them to repeat their statement they will have to respond in specific ways.
A sincere man will always state $a \spadesuit \psi$ as they now believe ψ
An insincere man will always state $a \spadesuit \neg \psi$ as they now believe ψ with $\tau = \infty$ and as such unless they have a threshold of 0, they will $\neg B_a \neg \psi$

Chapter 2

Gettier Types

	Description	Logic
11	Gettier Truth-tellers announce truth, for the wrong reasons	$(T_{\downarrow}\psi \rightarrow ((\neg\chi \wedge \psi) \wedge B_{\tau}(\chi \rightarrow \psi) \wedge B_{\tau}\chi))$
12	Gettier Liars announce falsity, for the wrong reasons	$(F_{\downarrow}\neg\psi \rightarrow ((\neg\chi \wedge \psi) \wedge B_{\tau}(\chi \rightarrow \psi) \wedge B_{\tau}\chi))$

Based on Van Bentham's notion of a Gettier Truth-teller[1], consider a Gettier Truth-teller defined as such; Gettier Truth-tellers announce truth, sincerely, but based on false evidence. They announce ψ, which is true in reality, based on some evidence χ, which is false in reality. They make this based on a modus ponens implication of $(\chi \rightarrow \psi)$, the only falsity being their belief in χ, which is false.

Gettier Liars exist in a similar manner, the primary difference being that they seek to deceive, negating the truth, which they succeed in, but based on evidence that is false.

2.1 Gettier Interactions

From their announcement of ψ as well as your knowledge that they believe χ falsely, then due to the nature of implication, $(\chi \rightarrow \psi)$ holds regardless of the truth value of χ, due to ψ being true. Therefore you can't immediately conclude that they are a Gettier Truth-teller - they may be entirely generic.

In order to ascertain them as Gettier, truthfully announce that their (false, but not necessarily known to you) antecedent χ leads to the negation of a different (true, but not known to them as true) consequent ρ, giving the common knowledge $(\chi \rightarrow \neg\rho)$,

and they should then be able to make the announcement of their belief in the negation of the (true) consequent ρ, i.e. $\neg\rho$, which you know to be a falsehood. This can be modelled as:
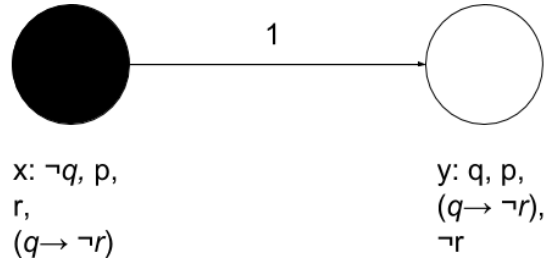

Figure 2.1: Plausibility model

Gettier Liars Interaction is symmetrical to Gettier Truth-tellers - this is not a unique property of Gettier announcers, however it is worth noting that symmetry isn't common to all types, such as 3,4,5
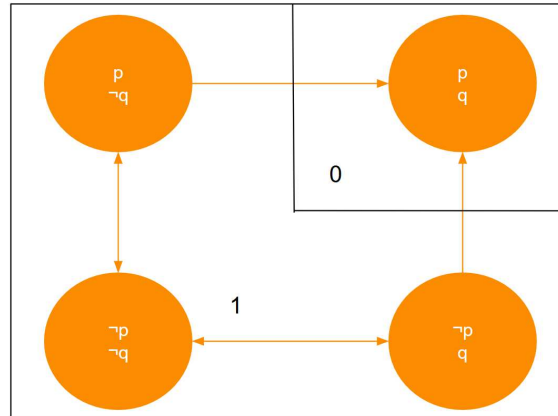
2.1.1 State Plausibility Models

Figure 2.2: Before $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

Figure 2.3: After $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

Chapter 3

Variations on Gettier Types

What if you aren't a Gettier Truth-teller? Negating the necessary conditions of Gettier Truth-tellers gives rise to 3 other possible cases when the state of the actual world remains the same. These cases give insight on what other kinds of truth tellers or liars may exist and what might differentiate them.

3.1 The Beliefless Truth-teller

$$((\neg\chi \wedge \psi) \wedge \neg B(\chi \rightarrow \psi) \wedge \neg B\chi)$$

In this case, the speaker has no belief in any of the Gettier conditions. The speaker may very well have other types of belief or knowledge that contribute to the announcement of ψ, but these are further sub-cases to be examined. If there is no other belief or knowledge that pertains to ψ, a speaker satisfying these conditions announces ψ with no rationale or justification. The announcement happens to be true in this world, but that is pure happenstance (when discarding other possible beliefs or knowledge).

This case does allow for sub-cases in which the speaker does in fact have a justified true belief in ψ however, as it allows for subcases such as:

$$((\neg\chi \wedge \psi) \wedge \neg B(\chi \rightarrow \psi) \wedge B\neg\chi)$$
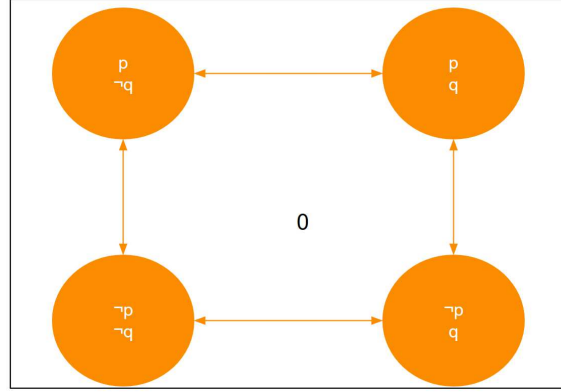
3.1.1 State Plausibility Models

Figure 3.1: Before $(\chi \rightarrow \neg\rho)$ announcement, with the derived Spohn rank of states
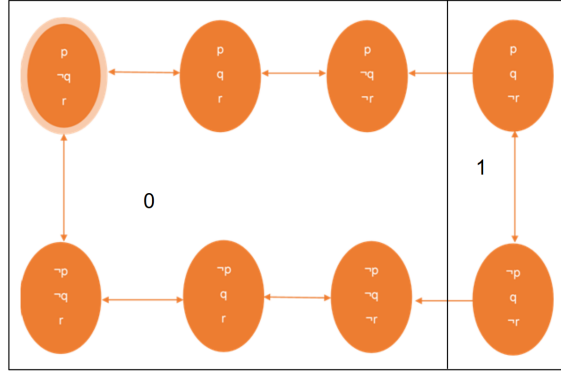

Figure 3.2: After $(\chi \rightarrow \neg\rho)$ announcement, with the derived Spohn rank of states

3.2 The Uninformed Accidental Truth-teller

$$((\neg\chi \wedge \psi) \wedge \neg B(\chi \rightarrow \psi) \wedge B\chi)$$

In this case, one has an untrue belief about the world ($B\chi$ when $\neg\chi$), but also doesnt have a belief in the implication ($\chi \rightarrow \psi$) that Gettier Truth Tellers have (a belief which holds true in reality in this state of the world). If one announces ψ with these conditions, they are announcing a true thing but have very little rationale in doing so without considering additional beliefs. They are indifferent on ψ if no other beliefs are held, and therefore announcing ψ seems to be announcing something insincerely.

One possible subcase:

$$((\neg\chi \land \psi) \land \neg B(\chi \to \psi) \land B\chi \land B\neg(\chi \to \psi))$$
$$\leftrightarrow((\neg\chi \land \psi) \land \neg B(\chi \to \psi) \land B\chi \land B(\chi \land \neg\psi))$$
$$\leftrightarrow((\neg\chi \land \psi) \land \neg B(\chi \to \psi) \land B\chi \land B\neg\psi)$$

This sub-case is one in which the speaker announcing ψ announces something true about the world, but does so by saying something she believes to be untrue - implying an intent to deceive and a type of lie, but announcing a true statement due to her incorrect beliefs.
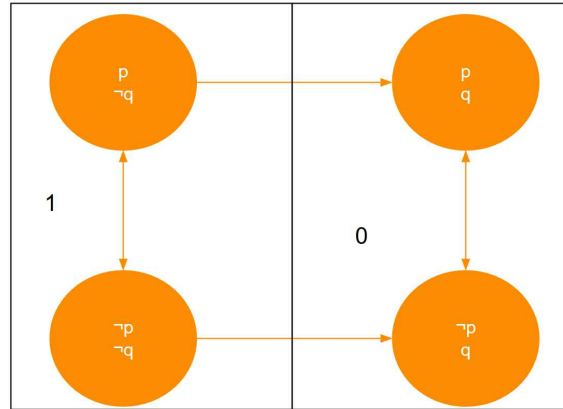
3.2.1   State Plausibility Models

Figure 3.3: Before $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

3.3   The Accurate Believer

$$(\neg\chi \land \psi) \land B(\chi \to \psi) \land \neg B\chi)$$

In this case, announcing ψ is again saying something true, but based on very little evidence without considering other beliefs the speaker may hold. They dont believe χ, but they do believe $(\chi \to \psi)$, and announcing ψ may be with the intent to deceive
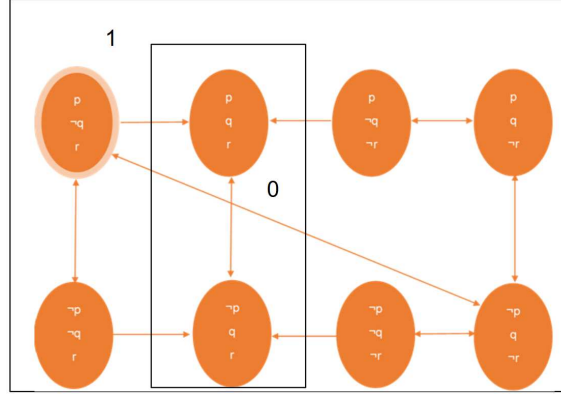
Figure 3.4: After $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

if they dont believe ψ. This case is one in which the speaker has the most accurate set of beliefs. It is the only case in which the speaker both believes the true statement $(\chi \to \psi)$ and does not falsely believe χ. Interestingly, this case is the only one in which Spohn Rank 0 contains only 5 cases rather than 6. This case allows for sub-cases in which the speaker has a justified true belief of ψ or a justified false belief in $\neg\psi$.

$$(\neg\chi \land \psi) \land B(\chi \to \psi) \land \neg B\chi \land B(\psi \to \chi))$$

In this case, one cannot be sincere in announcing ψ as it would then result in $B\chi$ but it is given that $\neg B\chi$
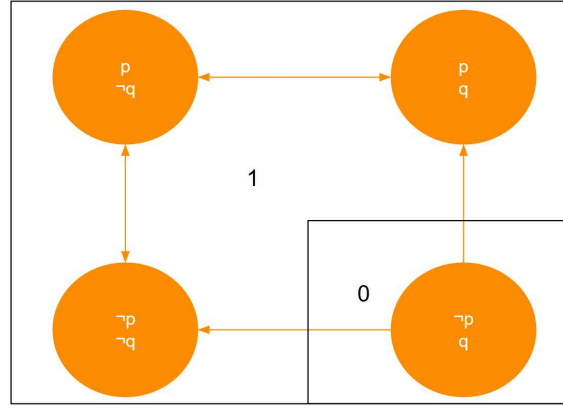
3.3.1   State Plausibility Models

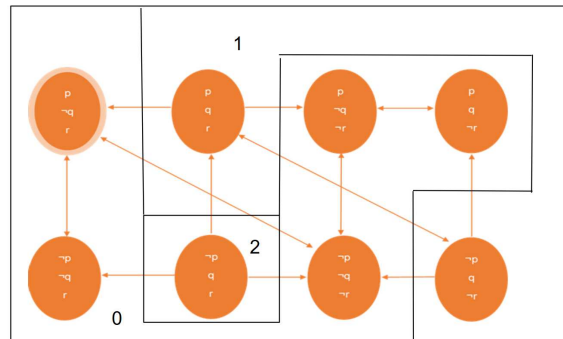Figure 3.5: Before $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

Figure 3.6: After $(\chi \to \neg\rho)$ announcement, with the derived Spohn rank of states

Chapter 4

Further Considerations

Meta:

- Is there an exhaustive list of liars and truth-tellers?

- Is it even possible to arrive at one at all?

Basic Types:

- Interactions between omniscient and insincere men

- How different thresholds affect what can be claimed by sincere and insincere men ($\tau \leq \frac{1}{2}$ and $\tau > \frac{1}{2}$)

Building on Gettier and non-Gettier Types:

- What sub-cases arise from the non-Gettier cases?

- What facets of interaction with these different sub-case agents will differ?

- Is this non-Gettier approach a useful approach ir is too much redundancy embedded in it?

Bibliography

[1] Johan van Benthem. *Modal logic for open minds*. CSLI Publications, Center for the Study of Language and Information, Leland Stanford Junior University, Stanford, Calif., 2010.

[2] Vincent Hendricks and John Symons. Epistemic logic. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, fall 2015 edition, 2015.

[3] Krister Segerberg, John-Jules Meyer, and Marcus Kracht. The logic of action. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.