Nested Sampling and the Evaluation of the 'Evidence' for Bayesian Model Selection

Paul D. Baines, Nicholas Ulle
University of California, Davis

February 7, 2014

1 Introduction

- Explanation of Nested Sampling
- Intuitive explanation of the algorithm
- Basics of computing the evidence and Bayesian model selection

2 Example

Here we take a look at the classic mixture of normals:

$$Y_i = \sum_{j=1}^{K} I_{ij} Z_{ij}, \qquad i = 1, \ldots, n,$$

where:

$$I_i = (I_{i1}, \ldots, I_{iK}) \sim \text{Multinomial}\,(1, p)\,,$$
$$Z_{ij} \stackrel{iid}{\sim} N\,(\mu_j, 1)\,.$$

The parameters in the model are the mixture proportions $p = (p_1, \ldots, p_K)$ (with $\sum_j p_j = 1$) and the mixture locations $\mu = (\mu_1, \ldots, \mu_k)$. The number of mixture components K will be fixed for a given model, and we will use the evidence to motivate a model selection procedure to select the appropriate K. For convenience we choose conditionally conjugate priors for μ and p:

$$\mu \sim N\,(\mu_0, V_0)\,, \qquad p \sim \text{Multinomial}\,(\alpha)\,,$$

where μ_0, V_0 and α are fixed hyperparameters chosen by the analyst.

2.1 Posterior Distributions

TODO

2.2 Evaluating the Evidence: Analytically

TODO

2.3 Evaluating the Evidence: Nested Sampling

TODO

2.4 Evaluating the Evidence: Other Methods

TODO