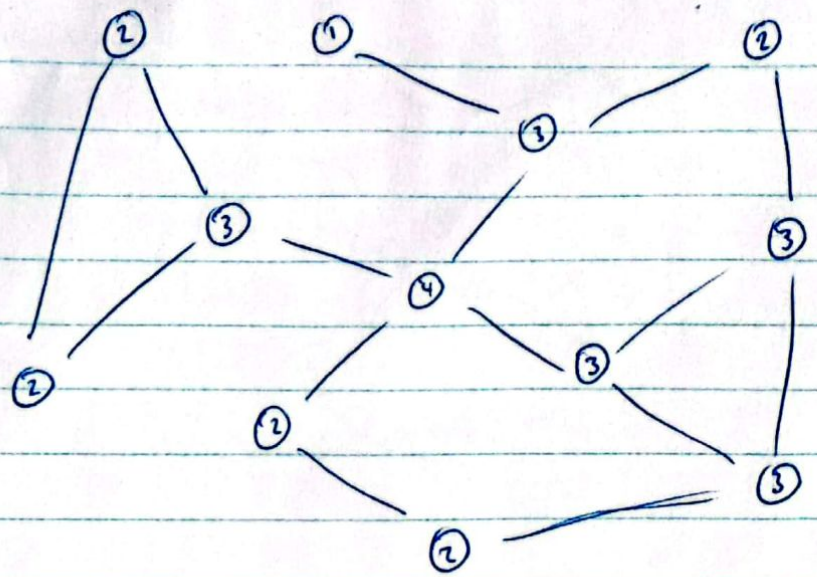
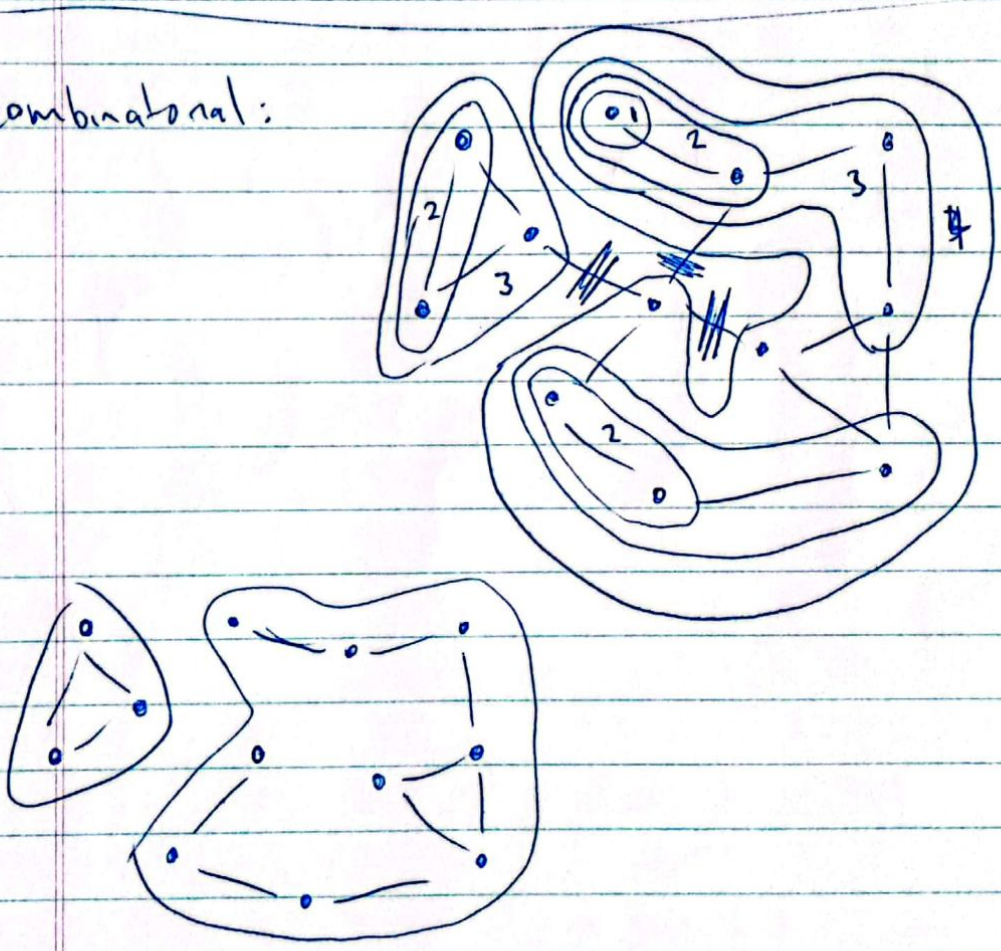
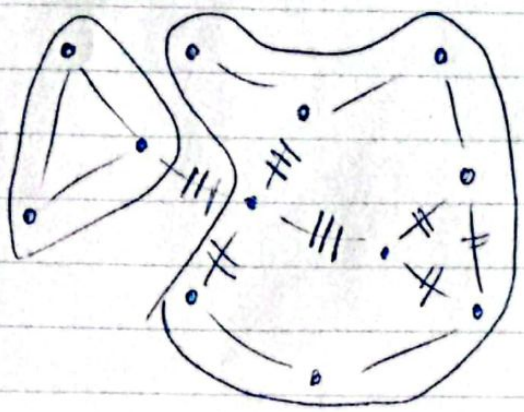
example network:



combinatonal:



divisive



generally;

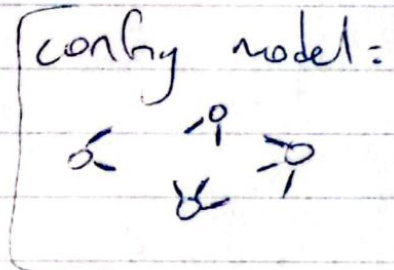
- Ravasz algorithm (combinatorial)
  - count the degree of every node and record the degree distribution
  - for every edge ~~include the edge(s) with the~~ find the edge(s) with the total lowest weights
  - include those nodes in a group together. Continue combining nodes until all nodes are in a group. There should be one edge that is cut
- Girvan-Newman Method (divisive)
  - essentially the inverse

- **Modularity**: given a network this is a ~~measure~~ measure (single value).

- $Q_g = \frac{1}{2m} \sum_{i,j \in g} A_{ij} - E_{ij}$ for each $g \in B$
  - m is the number of ~~links~~ links in B, not g
  - A_{ij} is the adjacency matrix
  - E_{ij} is the probability (given config model) ~~that~~ nds i,j will be connected.

$$E_{ij} = \frac{k_i k_j}{2m}$$

config model:



  - $Q \in [-1, 1]$ (or ~~is it open, can't~~

- B is the set of all components, g.

- For the entire graph, we take

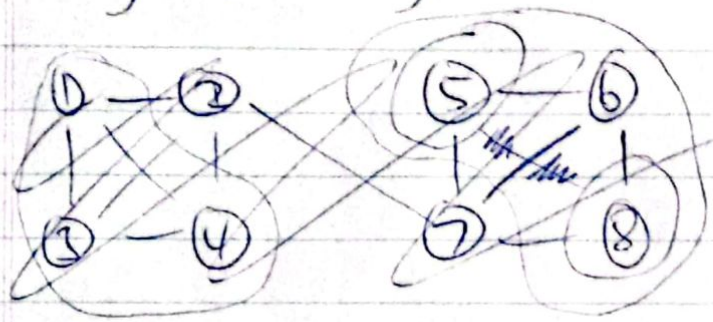
$$Q = \frac{1}{2m} \sum_{i,j \in g} A_{ij} - E_{ij}\, \delta(\gamma(i), \gamma(j))$$

  - γ returns the index $g \in B$,
  - $\delta(a,b) = \begin{cases} 1 \text{ if } a = b \\ 0 \text{ if } a \neq b \end{cases}$

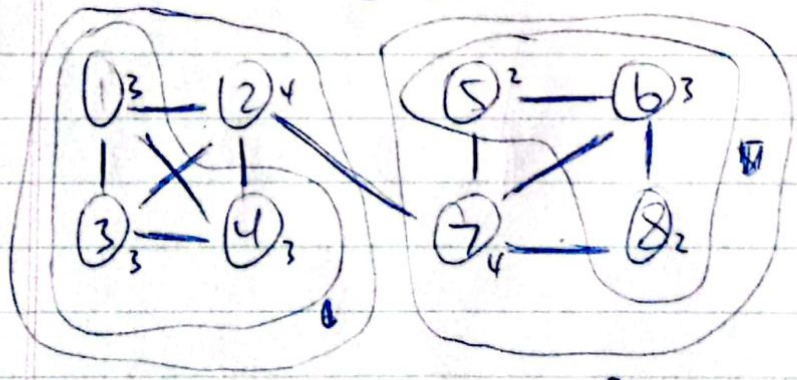
Modularity example:

- you would do this after determining the communities in a net. (divisive or combinatorial methods)

eg net using combinatorial method



so then the communities are $\{\{1,2,3,4\},\{5,6,7,8\}\}$



$B = \{\{1,2,3,4\},\{5,6,7,8\}\}$

$E_{ij} = \frac{k_i k_j}{2m}$

$Q_1 = \frac{1}{24}\left(\left(1-\frac{9}{24}\right)3 + \left(1-\frac{12}{24}\right)3\right)$

(3 = # of edges)

$Q_2 = \frac{1}{24}\left(\left(1-\frac{6}{24}\right)2 + \left(1-\frac{8}{24}\right)2 + \left(1-\frac{12}{24}\right)\frac{3}{2} + \left(0-\frac{4}{24}\right)\right)$