

R5 1 wrt  $\times$  (identity for  $\times$ )

For this to be true, there must be some

$$(b,a) \in F \text{ s.t. } (b,a) \times (d,c) = (d,c) = (d,c) \times (b,a)$$

$(1,1)$  is that element, since

$$(1,1) \times (d,c) = (1d, 1c) \quad (\text{by defn of } \times \text{ in } D)$$

$$= (d,c) \quad (D \text{ is the integral domain with } 1,$$

so  $1$  is in  $D$ , and is the

multi. identity in  $D$ )

$$= (1d, 1c) \quad (D \text{ is commutative})$$

$$\text{so, } (d,c) \times (1,1) = (d,c)$$

R6 inverses wrt  $\times$

take  $(b,a), (d,c) \in F$

$$\text{s.t. } (b,a) \times (d,c) = e = (1,1)$$

$$(bd, ac) = (1,1)$$

$$\left. \begin{array}{l} bd = 1 \\ ac = 1 \end{array} \right\} \text{multiplication in } D$$

$$\text{so } bd = ac$$

$$\text{suppose } b = -c \\ d = -a$$

$$\text{then } (-c, a) \times (-a, c) = (ac, ac) \quad \text{defn of } \times \text{ in } F$$

$$\text{we require } (ac, ac) \sim (1,1) \iff ac \times 1 = ac \times 1$$

$$\text{which is true, so } (ac, ac) \sim (1,1)$$

$$\text{and } (b,a) = (-c, -d)$$

so, we have inverses in  $(F, \times)$