Q1. Given:

$$T_1 = 1 \text{ day}$$
$$T_2 - T_1 = 2 \text{ weeks} = 14 \text{ days}$$
$$R_0 = 2.4$$

Then $T_2 = 14 + 1 = 15$ days

Note that:

- $R_0 := vc[T_2 - T_1]$

  $2.4 = vc[14]$

  $\therefore vc = \frac{2.4}{14}$
- $T_d := \frac{\ln(2)}{r}$ with r to be calculated

Assume incidence : $i(t) = Ke^{rt}$

and with with t in days, then $i(t)$ is the new cases per day or $I'(t)$

Then $i(t)$ may be thought of as:

$$i(t) = vc\int_{T_1}^{T_2} i(t-\tau)\,d\tau$$

$$Ke^{rt} = vc\int_{T_1}^{T_2} Ke^{r(t-\tau)}\,d\tau$$

$$\cancel{Ke^{rt}} = vc\int_{T_1}^{T_2} \cancel{Ke^{rt}} \cdot e^{-r\tau}\,d\tau$$

$$1 = vc\int_{T_1}^{T_2} e^{-r\tau}\,d\tau$$

If the RHS is a function then $f(r) = 1$

But

$$f(0) = vc\int_{T_1}^{T_2} e^{0}\,d\tau$$
$$= vc(T_2 - T_1),$$ which is defined to be R_0 when $I(t) = 0$

- Note that this implies R_0 will be the y-int. on the chart of $f(r)$

Then these functions intersect at some value of r

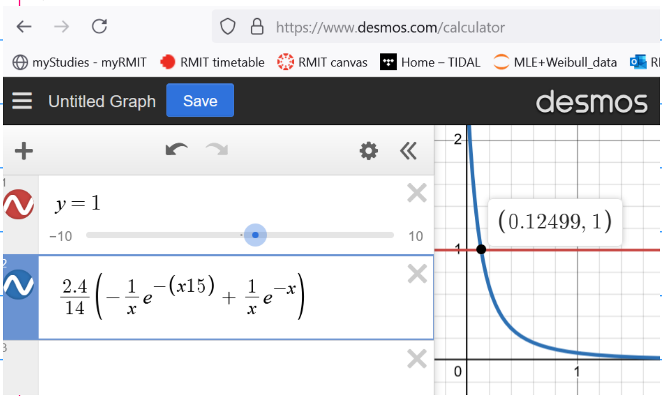
We can solve :

$$f(r) = vc\int_{T_1}^{T_2} e^{-r\tau}\, d\tau$$

$$= \frac{2.4}{14}\left[\frac{-1}{r} e^{-r\tau} + C\right]_1^{15}$$

$$f(r) = \frac{2.4}{14}\left[\frac{-1}{r} e^{-r15} + \frac{1}{r} e^{-r}\right]$$

We could use an iterative technique but a graphics calculator will suffice. Using the previous result that $f(r) = 1$



so, $r \simeq 0.12499$

and

$$T_d \simeq \frac{\ln(2)}{0.12499} \simeq 5.55 \text{ days}$$

Q2

Now let $R_0 = 1.1$

$T_1 = 7$ days

$T_2 - T_1 = 28$ days

Then $T_2 = 35$ days

$$R_0 = vc\,[T_2 - T_1]$$

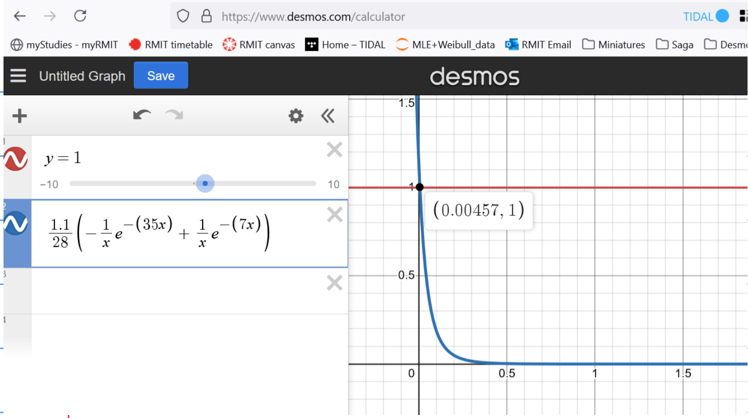
$$1.1 = vc\,[28]$$

$$vc = \frac{1.1}{28}$$

Again,

$$1 = vc\int_{T_1}^{T_2} e^{-r\tau}\, d\tau$$

$$f(r) = \frac{1.1}{28}\left[\frac{-1}{r} e^{-r35} + \frac{1}{r} e^{-r7}\right]$$



so, $r \simeq 0.00457$

$$T_d \simeq \frac{\ln(2)}{0.00457} \simeq 151.67 \text{ days}$$

So, these new values of R_0, T_2, T_1 result in a slower real time growth, and this leads to an extended doubling time.

Q3

model

$$\frac{dS}{dt} = -\lambda S$$

$$\frac{dE}{dt} = \lambda S - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

Then we need to find $\lambda = cvp$. With the assumption of frequency dependence, $\beta = cv$. The assumption of random mixing implies $p = \frac{I}{N}$. So, $cvp = \beta \frac{I}{N} = \lambda$

$$\frac{dS}{dt} = -\beta \frac{IS}{N}$$

$$\frac{dE}{dt} = \beta \frac{IS}{N} - \sigma E$$

$$\frac{dI}{dt} = \sigma E - \gamma I$$

$$\frac{dR}{dt} = \gamma I$$

i) $R_0 = cvp$

$p = T_2 - T_1$, but if we assume that $T_2 - T_1$ follows an exponential distribution then the average is $1/\gamma$

Note that the assumption of FDE implies $\beta = cv$

$R_0 = \frac{\beta}{\gamma}$

To check R_0, I will use mathematical reasoning to find the same result.

For the pathogen to cease, then $\frac{dE}{dt} + \frac{dI}{dt} \leq 0$

So, suppose that $\frac{dE}{dt} + \frac{dI}{dt} > 0$

$$\frac{dE}{dt} + \frac{dI}{dt} = \beta \frac{SI}{N} - \sigma E + \sigma E - \gamma I > 0$$
$$= \beta \frac{SI}{N} - \gamma I > 0$$
$$= \left(\beta \frac{S}{N} - \gamma\right) I > 0$$

Assume $I > 0$ (outbreak has started)

then $\beta \frac{S}{N} - \gamma > 0$

$$\beta \frac{S}{N} > \gamma$$

$$\beta \frac{S}{\gamma N} > 1$$

Also assume that $S \simeq N$ (outbreak is very small)

$$\beta \frac{1}{\gamma} > 1$$

so, $R_0 = \frac{\beta}{\gamma}$

ii) For fast mpox, $R_0 = 2.4 = \frac{\beta}{\gamma}$

$T_2 - T_1 = 14 \text{ days} = \frac{1}{\gamma}$

so $\gamma = 1/14$

$2.4 = \beta(14)$, $\beta = \frac{2.4}{14}$

Assume that the incubation period is exponentially distributed. Then for fast mpox, $1 = \frac{1}{\sigma}$, $\sigma = 1$

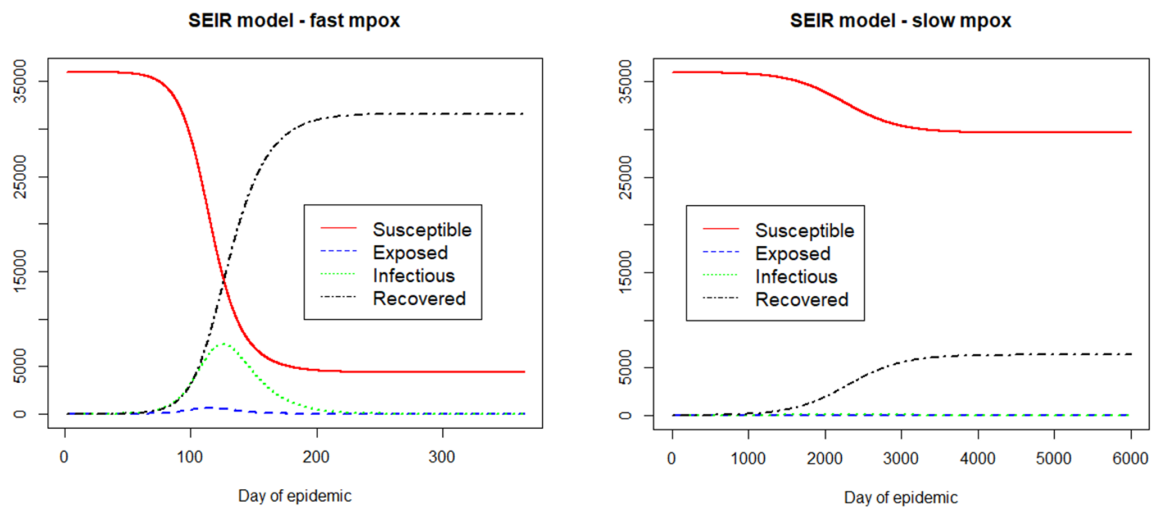
For slow mpox, $R_0 = 1.1 = \beta/\gamma$

$1/\gamma = 28$, $\gamma = 1/28$

$1.1 = \beta(28)$, $\beta = \frac{1.1}{28}$

$7 = \frac{1}{\sigma}$, $\sigma = 1/7$

iii)



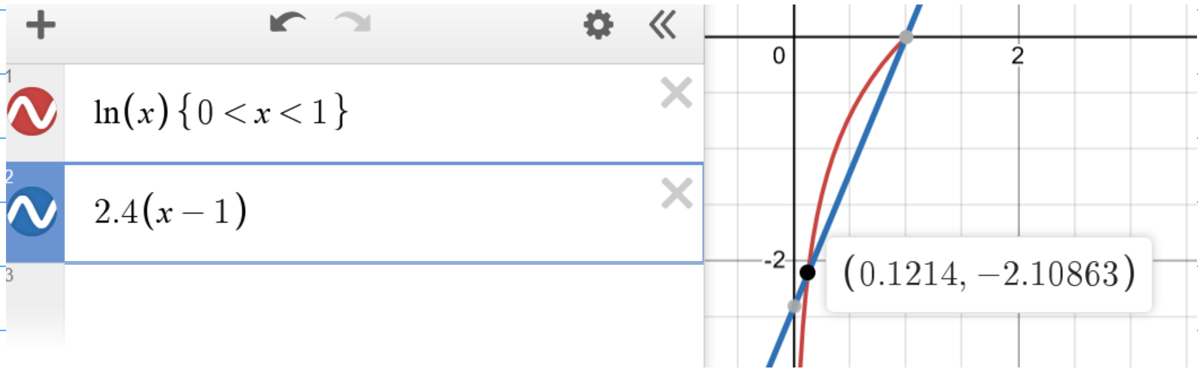
(please see final page for code)

The R script finds the total recovered population:

fast: 31,626

slow: 6,353

To check these, we can use the final size equation. That equation is $\ln(s(\infty)) = R_0(s(\infty) - 1)$ where $s(\infty)$ is the population who never get infected

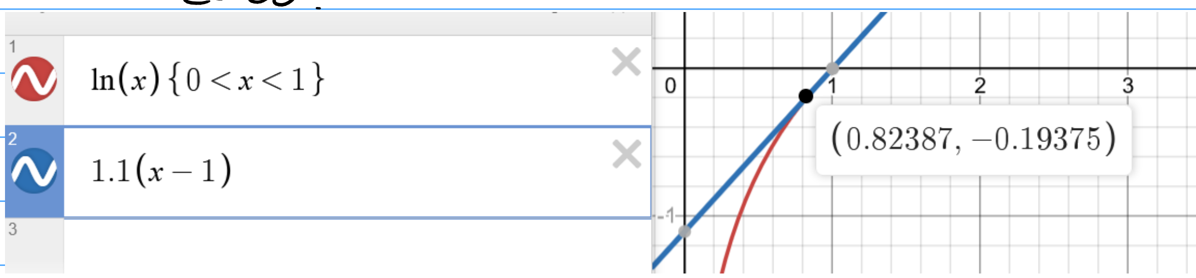


Then for fast Mpox, 0.1214 of MSM population would never be infected

The numerical result $s(\infty) = \dfrac{36{,}000 - 31{,}776}{36{,}000} = 0.1215$

So this is consistent with the final size equation

For slow Mpox, 0.8239 of MSM would never be infected.



The numerical $s(\infty) = \dfrac{36{,}000 - 6{,}353}{36{,}000} \simeq 0.8239$

This is very close to the final size equation
These results mean the numerical solutions are valid.

Q4 next page