Question 2.

Cholera is a water-borne disease of humans that has had devastating consequences for some regions of the world and is associated with poverty and poor sanitation. The etiological agent of modern cholera is *V. cholerae* O139, which is a bacteria that is spread via contaminated water and food. People that survive an episode of cholera have long-lasting immunity.

Consider the following compartment model for cholera that focuses on water-borne transmission:

$$\begin{aligned}
\frac{dS}{dt} &= \mu N - a\lambda(B)S - \mu S\,,\\
\frac{dI}{dt} &= a\lambda(B)S - \gamma I - \mu I\,,\\
\frac{dR}{dt} &= \gamma I - \mu R\,,\\
\frac{dB}{dt} &= \epsilon I - \alpha B\,,
\end{aligned}$$

where

$$\lambda(B) = \frac{B}{K+B}$$

with B representing the amount of bacteria in the water supply (measured as concentration; with units being millions of bateria per cubic ml) and $K = 1 \times 10^6$ is the concentration of *V. cholerae* bacteria that yields a 50% chance of catching cholera. Note that the parameter a is a contact rate (the frequency with which humans come into contact with contaminated water).

Investigate the behaviour of this model by setting $\mu = 0.0001$ per day, $a = 1$ per day, $\gamma = 1/14$ per day, while allowing the shedding rate, ϵ, and the decay rate, α, to vary. This is an open question (that you can take in any direction you like) but you should at least investigate the steady-states of the model, numerically solve the system of differential equations and try to find an expression for R_0.

Question 3.

Carefully consider the following set of differential equations (with $0 < \kappa < 1$) that represent a well known infection of humans:

$$\begin{aligned}
\frac{dS}{dt} &= b + \sigma W - \beta' \frac{SI}{N} - \mu S \,, \\
\frac{dI}{dt} &= \beta' \frac{SI}{N} - \gamma I - \mu I \,, \\
\frac{dR}{dt} &= \gamma I + \kappa \beta' \frac{WI}{N} - \sigma R - \mu R \,, \\
\frac{dW}{dt} &= \sigma R - \kappa \beta' \frac{WI}{N} - \sigma W - \mu W \,.
\end{aligned}$$

(i) Create a state flow diagram showing how individuals can progress through the possible disease states according to the system of differential equations.

(ii) Describe the progression of the disease and make a list of the biological assumptions implicit in the compartment model.

(iii) What do you think the pathogen being modelled might be? While the model was inspired by a specific disease system, the model is relatively simple and there are several possible answers that I will consider correct.

Question 4.

Use a next-generation matrix approach to investigate the simplified scenario in which a sexually-transmitted infection, such as monkey pox, is occurring amongst two well-defined host groups: men who have sex with men (MSM) and sex workers (SW). Consider these two host groups as having sufficient epidemiological differences to warrant the system being considered as a multi-host system, and consider them as the only two host groups. Also assume that sex amongst sex workers doesn't happen so that one of the diagonal entries of the next-generation matrix will be zero (this is the key simplification).

There is no reason to expect symmetry such that $k_{12} = k_{21}$ and you are discouraged from making this assumption. The clients of sex workers that are not men who have sex with men are to be considered as a dead-end host for the purposes of this investigation.

At a practical level, a vaccination or awareness campaign against this sexually-transmitted infection would look very different if they were directed towards one group or the other. Derive expressions for the type reproduction number, T_1, first if sex workers are labeled as type 1, and then again, when men who have sex with men are labeled as type 1.

Use your analytical expressions as the basis to discuss the circumstances under which vaccination efforts should be directed at one group over another, i.e. should outreach programs target sex workers or the men in the wider populations that are at risk? For example, there will be values of the type-specific reproduction numbers for which targeting control at one type will never work (no matter how effective it is). The threshold behaviour of T_1 is discussed in the lecture notes.