$S_3 = \langle (123), (12) \rangle$

$H \leq S_3$, $H = \{(123), (132), e\}$

$(123)H = H$

$(132)H = H$

$eH = H$

take (12)

$(12)H = \{(12)(123), (12)(132), (12)e\}$

$\{(23), (13), (12)\}$

The equivalence classes partition S_3, and all elements of S_3 are in a left coset, $\therefore$ $(12)H$ and H are the left cosets of H

ex ①

Now take $H = \langle (12) \rangle = \{(12), e\}$

$(12)H = \{(12), e\}$

But $(13)H = \{(13)(12), (13)e\}$

$= \{(123), (13)\}$

$(23)H = \{(23)(12), (23)e\}$

$= \{(132), (23)\}$

$\therefore$ the left cosets of $H = \langle (12) \rangle$ are H, $(13)H$, $(23)H$

relationship:

- if n = number of cosets, $n|H| = |S_3|$

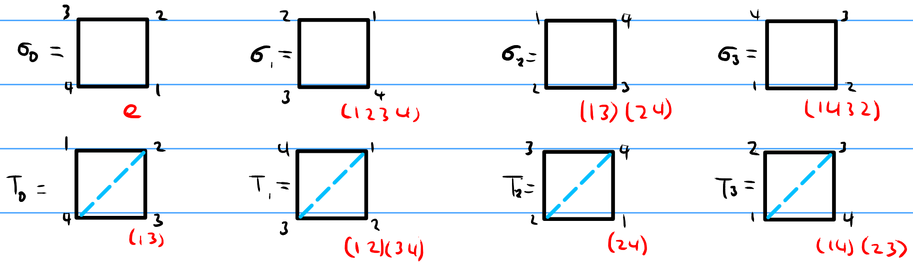
Or, if the size of H decreases, then we need more cosets to partition the group ex 2 For $n = 2$ then $H = n\mathbb{Z} = 2\mathbb{Z} = \{\dots, -2, 0, 2, \dots\}$

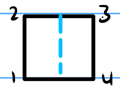
Then the quotient set is $\mathbb{Z}/n\mathbb{Z} = \mathbb{Z}/2\mathbb{Z}$

Then . $2\mathbb{Z}$

WIP

ex ③ Consider the dihedral group D_4




redundant since it's $= (13)(1234)^3$

Then $D_4 = \langle \sigma_1, \tau_0 \rangle = \langle (1234), (13) \rangle$

- let $H = \langle (1234) \rangle = \{ e, (1234), (13)(24), (1432) \rangle$

  for any $a \in H$, aH is the same coset

  $(13)H = \{ (13), (12)(34), (24), (14)(23) \}$

  $\therefore$ H, $(13)H$ are the left cosets when $H = \langle (1234) \rangle$

- let $H = \langle (13) \rangle = \{ (13), e \}$

  $(1234)H = \{ (1234)(13), (1234)e \}$

  $= \{ (14)(23), (1234) \}$

  $(13)(24)H = \{ (24), (13)(24) \}$

  $(1432)H = \{ (12)(34), (1432) \}$

  $\therefore$ H, $(1234)H$, $(13)(24)H$, $(1432)H$ are the left cosets where $H = \langle (13) \rangle$