

Exercise : show  $C_1 - C_3$

(C1) take  $g_1, g_1', g_1'' \in C_1$   
 $g_2, g_2', g_2'' \in C_2$

$$(((g_1, g_2), (g_1', g_2')), (g_1'', g_2'')) = (((g_1 *_{C_1} g_1') *_{C_1} g_1''), ((g_2 *_{C_2} g_2') *_{C_2} g_2''))$$

but since  $C_1$  is a group then its binary operator is associative, and so too is  $*_{C_2}$

$$\therefore (((g_1 *_{C_1} g_1') *_{C_1} g_1''), ((g_2 *_{C_2} g_2') *_{C_2} g_2'')) = (((g_1 *_{C_1} (g_1' *_{C_1} g_1'')), (g_2 *_{C_2} (g_2' *_{C_2} g_2''))))$$

so  $C_1$  holds

(C2) Since  $C_1, C_2$  are defined as groups then the identity is  $(e_{C_1}, e_{C_2})$

$$\text{so } (g_1, g_2) \times (e_{C_1}, e_{C_2}) = (g_1, g_2)$$

(C3) Similarly, since  $C_1, C_2$  are given as groups then  $\forall g_1 \in C_1, g_1^{-1} \in C_1$   
same argument for  $C_2$

Then  $C_1 \times C_2$

$$\begin{aligned} ((g_1, g_2), (g_1^{-1}, g_2^{-1})) &= (g_1, g_1^{-1}), (g_2, g_2^{-1}) \\ &= (e_{C_1}, e_{C_2}) \end{aligned}$$