Homework #3

1. Let

$$p(x) = 1 + x, \quad q(x) = x.$$

Use linear finite element method to solve the following differential equation

$$-(pu')' + qu = f, \quad 0 < x < 2\pi,$$

with the Dirichlet boundary condition $u(0) = u(2\pi) = 0$. You may test your code by using $u(x) = \sin x$ to determine $f(x)$.

2. Derive a weak form for the following problem

$$\begin{aligned} -a^2\frac{\partial^2 u(x,y)}{\partial x^2} - b^2\frac{\partial^2 u(x,y)}{\partial y^2} + q(x,y)u(x,y) &= f(x,y), \\ (x,y) \in \Omega = \{(x,y), x^2 + y^2 \le 1\}, \qquad & u(x,y)|_{\partial\Omega} = 0, \end{aligned}$$

where $q(x,y) \ge 0$.

Hint: Consider the coordinates transform: $\bar{x} = x/a$; $\bar{y} = y/b$; and let $\bar{u}(\bar{x},\bar{y}) = u(a\bar{x}, b\bar{y})$; $\bar{q}(\bar{x},\bar{y}) = q(a\bar{x}, b\bar{y})$; $\bar{f}(\bar{x},\bar{y}) = f(a\bar{x}, b\bar{y})$.