Math 124 Notes

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September 29, 2021

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Beginning Topology by Sue E. Goodman will be closely followed, most resources are on their webpage, homework maybe physically submitted.

- Chapter 1: Point set topology - lecture notes
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- Chapter 3: Euler characteristic, fundamental group, knot theory

1.1 Chapter 1, Point Set Topology

Def: A metric space X is a non empty set with a distance function $d, X * X \to \mathbb{R}_0^+$.

- There is a notion of symmetry, $d(x,y) = d(y,x)$
- Triangle inequality, $d(x,z) \leq d(x,y) + d(y,z)$
- $d(x,y) \longleftrightarrow x = y$

Some examples:

- Euclidean metric, aka $\mathbb{R}^n$ with the distance being the magnitude of the line between the points.
- p-metric, $\mathbb{R}^n$, but distance is $(\sum_i |x_i - y_i|^p)^{\frac{1}{p}}$. 2-metric is the same as Euclidean metric. Called a norm in Numerical Analysis.
- Discrete metric, for any set $d(x,y) = 1 \longleftrightarrow x \neq y$. Like the path length in a unweighted graph.

ε-neighborhood in a metric space (X,d). The set of points $y \in X$, called $B_\varepsilon(x)$ near x such that $d(y,x) < \varepsilon$ and $0 < \varepsilon$.

2-metric gives circle neighborhoods, 1-metric gives diamond, ∞-metric gives a square.

For any $\emptyset \neq Y \subset X$ and (X,d) is a metric space then (Y,d) is a metric space.

A point $x \in X$ is called an interior point of $A \subseteq X$ if $\exists \varepsilon > 0$ such that $B_\varepsilon(x) \subseteq A$. The set of all interior points is A°

A point $x \in X$ is called a limit point (cluster point, accumulation point) of A if for $\forall \varepsilon > 0, \exists y \neq x$ such that $y \in B_\varepsilon(x) \cap A$, the set of all points is called A'.

A set is open if every point is interior, a set is closed if every limit point of the set belongs to A.

The closure of a set $\overline{A} = A \cup A'$

$$A \subseteq \overline{A}, A^{\circ} \subseteq A$$

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For (x, y) all points in the set are interior points, therefore it is a open set.

The set of points $A = \{\frac{1}{n} | n \in \mathbb{N}\}$ is a peculiar example. 0 is a limit point of the set. The set is not closed because $A' \not\subseteq A$. A also has no interior points because you can always pick a $\varepsilon < \frac{1}{n} - \frac{1}{n+1}$ around the point $\frac{1}{n}$.

If $0 \in A$ then the set would be closed.

$$\mathbb{Q}^\circ = \emptyset, \mathbb{Q}' = \mathbb{R}$$

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The product of closed sets are closed and the product of open sets are open.

Proof, suppose we have open points in S_1 and S_2. This implies that there are ε_1 and ε_2 neighborhoods around P_1, P_2. Then the point $P_1 \times P_2$ would have a ε of the smaller of $\varepsilon_1, \varepsilon_2$.

Proof2, we have two closed sets S_1, S_2 and we want to know if $S_1 \times S_2$ is closed. Consider a limit point of $S_1 \times S_2$. This means that there is a $P_1 \times P_2$ that has a point y that is arbitrarily close. Well that means that the components of y are arbitrarily close to P_1 and P_2 in their respective sets. This means that P_1 and P_2 are in their sets from the closure and thusly $P_1 \times P_2$ is in $S_1 \times S_2$.

The ε neighborhood around a open point is a open set. This can be done with triangle inequality to find a epsilon for each of the points.