3d Rotation with Quaternions

Jason Miller

Why Calculating Rotation in 3d is Valuable:

- Physics Simulations.
- 3d Animation.
- Navigation.
- And MUCH MORE!

Why Calculating Rotation in 3d is Valuable:

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- Rotation Around Axis.
- Gimbals.

Why They Fail: Rotation Around Axis

Left rotation

Why They Fail: Rotation Around Axis
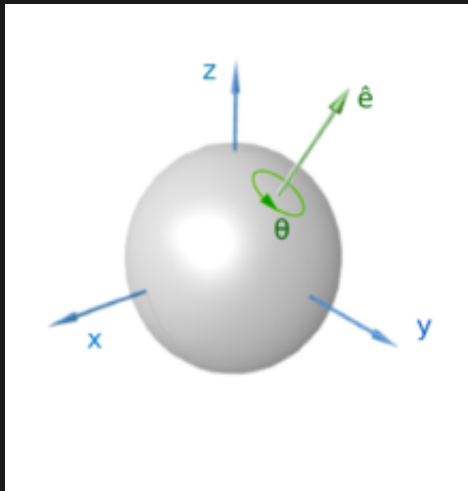
Left rotation

Right rotation

Normal Gimbal
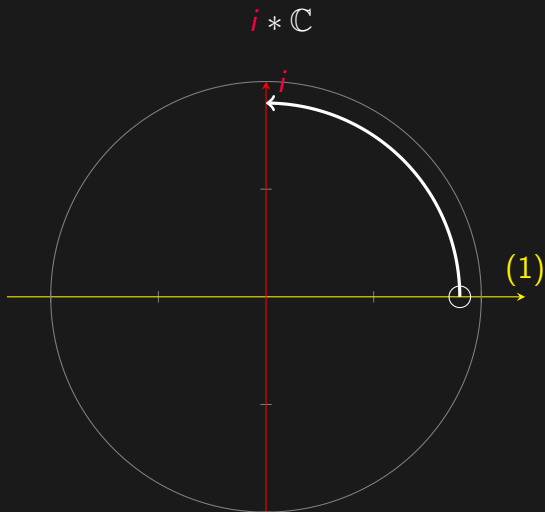
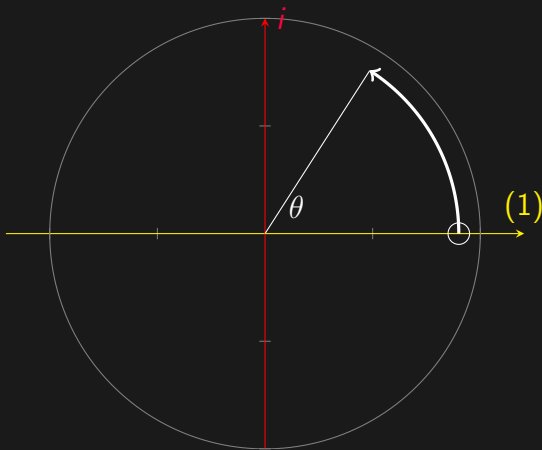# Why They Fail: Gimbals

Normal Gimbal

Gimbal Lock

What We Want

Complex Numbers

Complex Number Angles

$$(\cos \theta(1) + \sin \theta i) * \mathbb{C}$$

Introduce Quaternions

Complex Numbers

Quaternions

Complex Numbers

Quaternions

$$c_0(1) + c_1 i$$

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Quaternions

$$c_0(1) + c_1 i + c_2 j + c_3 k$$

Introduce Quaternions

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$$i^2 = -1$$

Quaternions

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Introduce Quaternions

Complex Numbers	Quaternions
$c_0(1) + c_1 i$	$c_0(1) + c_1 i + c_2 j + c_3 k$
$i^2 = -1$	$i^2 = j^2 = k^2 = -1$

Introduce Quaternions

Complex Numbers

$$c_0(1) + c_1 i$$

$$i^2 = -1$$

Quaternions

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The product of any 2 different complex parts gives the third and they **anti-commute**

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The product of any 2 different complex parts gives the third and they **anti-commute**
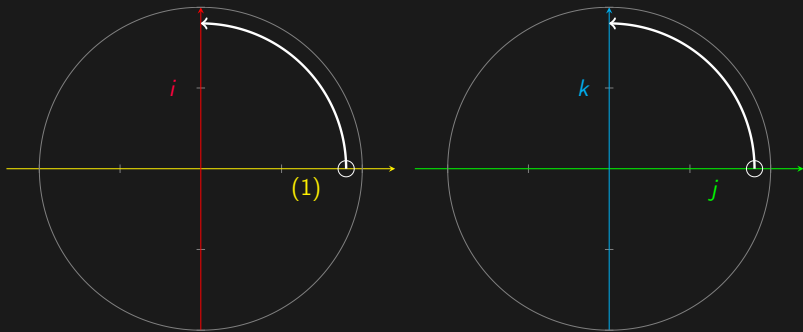
$$i * j = -j * i = k$$

Times Tables

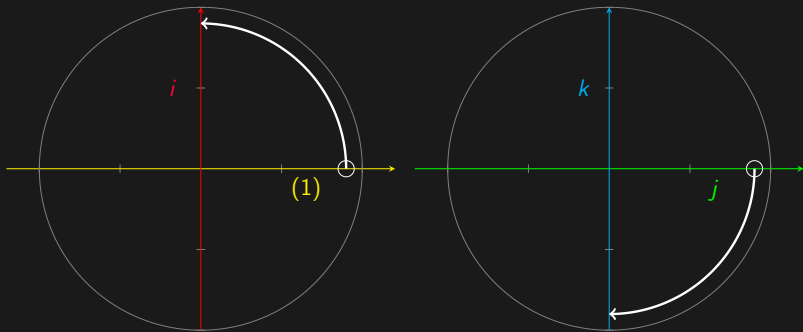| *	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

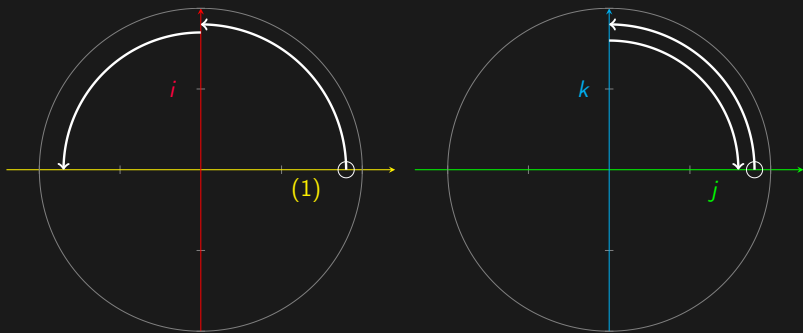# But What About Rotation

$$i * \mathbb{H}$$

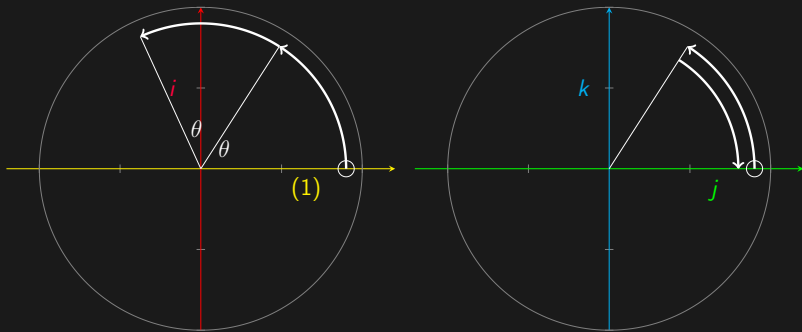# But What About Rotation

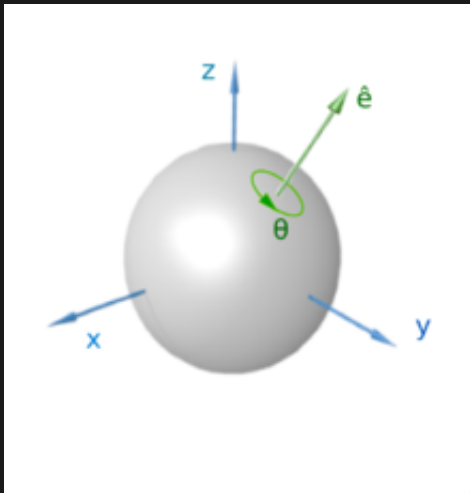

$$\mathbb{H} * i$$

The Big Idea

$$i * \mathbb{H} * i$$

Rotation!

$$(\cos\theta(1) + \sin\theta\, i) * \mathbb{H} * (\cos\theta(1) + \sin\theta\, i)$$

3d Rotation

$$\left(\cos\frac{\theta}{2}(1) + \sin\frac{\theta}{2}\overrightarrow{v}\right) * \mathbb{H} * \left(\cos\frac{-\theta}{2}(1) + \sin\frac{-\theta}{2}\overrightarrow{v}\right)$$

https://upload.wikimedia.org/wikipedia/commons/thumb/5/51/Eule
Euler_AxisAngle.png