

Math 124 Notes

Jason Miller

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Beginning Topology by Sue E. Goodman will be closely followed, most resources are on their webpage, homework maybe physically submitted.

- Chapter 1: Point set topology - lecture notes
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- Chapter 3: Euler characteristic, fundamental group, knot theory

1.1 Chapter 1, Point Set Topology

Def: A metric space X is a non empty set with a distance function $d, X * X \rightarrow \mathbb{R}_0^+$.

- There is a notion of symmetry, $d(x, y) = d(y, x)$
- Triangle inequality, $d(x, z) \leq d(x, y) + d(y, z)$
- $d(x, y) \longleftrightarrow x = y$

Some examples:

- Euclidean metric, aka \mathbb{R}^n with the distance being the magnitude of the line between the points.
- p-metric, \mathbb{R}^n , but distance is $(\sum_i |x_i - y_i|^p)^{\frac{1}{p}}$. 2-metric is the same as Euclidean metric. Called a norm in Numerical Analysis.
- Discrete metric, for any set $d(x, y) = 1 \longleftrightarrow x \neq y$. Like the path length in a un-weighted graph.

ε -neighborhood in a metric space (X, d) . The set of points $y \in X$, called $B_\varepsilon(x)$ near x such that $d(y, x) < \varepsilon$ and $0 < \varepsilon$.

2-metric gives circle neighborhoods, 1-metric gives diamond, ∞ -metric gives a square.

For any $\emptyset \neq Y \subset X$ and (X, d) is a metric space then (Y, d) is a metric space.

A point $x \in X$ is called an interior point of $A \subseteq X$ if $\exists \varepsilon > 0$ such that $B_\varepsilon(x) \subseteq A$. The set of all interior points is A°

A point $x \in X$ is called a limit point (cluster point, accumulation point) of A if for $\forall \varepsilon > 0, \exists y \neq x$ such that $y \in B_\varepsilon(x) \cap A$, the set of all points is called A' .

A set is open if every point is interior, a set is closed if every limit point of the set belongs to A .

The closure of a set $\overline{A} = A \cup A'$

$$A \subseteq \overline{A}, A^\circ \subseteq A$$

2 Lecture 2 9/27

For (x, y) all points in the set are interior points, therefore it is a open set.

The set of points $A = \{\frac{1}{n} | n \in \mathbb{N}\}$ is a peculiar example. 0 is a limit point of the set. The set is not closed because $A' \not\subseteq A$. A also has no interior points because you can always pick a $\varepsilon < \frac{1}{n} - \frac{1}{n+1}$ around the point $\frac{1}{n}$.

If $0 \in A$ then the set would be closed.

$$\mathbb{Q}^\circ = \emptyset, \mathbb{Q}' = \mathbb{R}$$