

3d Rotation with Quaternions

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Why Calculating Rotation in 3d is Valuable:

- Physics Simulations.
- 3d Animation.
- Navigation.
- And MUCH MORE!

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- Physics Simulations.
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- And MUCH MORE!
- Rotation Around Axis.
- Gimbals.

Why They Fail: Rotation Around Axis

Left rotation

Why They Fail: Rotation Around Axis

Left rotation

Right rotation

Why They Fail: Gimbals

Normal Gimbal

Why They Fail: Gimbals

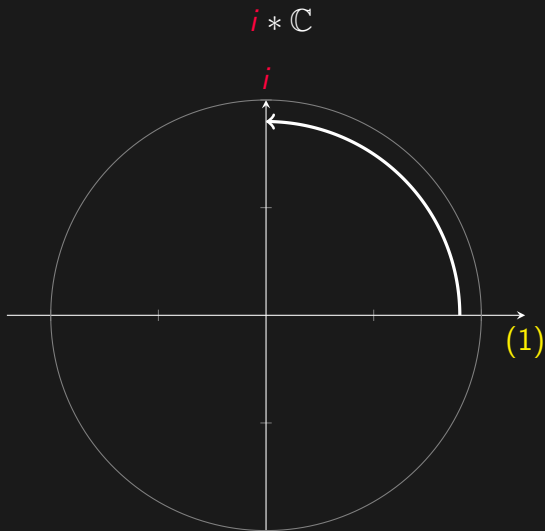
Normal Gimbal

Gimbal Lock

What we would like

Stick in ball

Complex Numbers



Complex Number Angles

Show briefly the angle formula for complex numbers

Introduce Quaternions

Complex Numbers

Quaternions

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$$c_0(1) + c_1 i$$

Quaternions

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Quaternions

$$c_0(1) + c_1 i + c_2 j + c_3 k$$

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$$i^2 = -1$$

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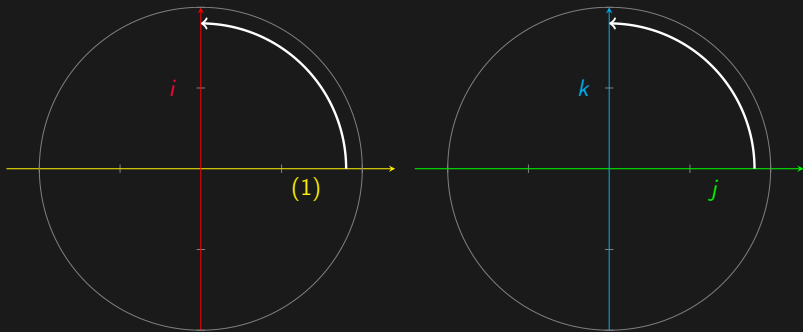
$$i * j = -j * i = k$$

Times Tables

*	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

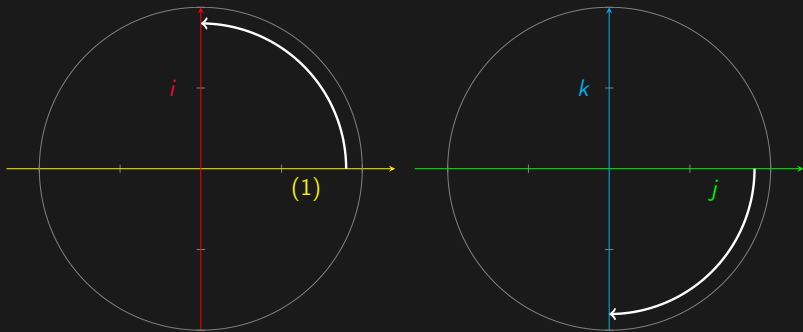
But What About Rotation

$$i * \mathbb{H}$$



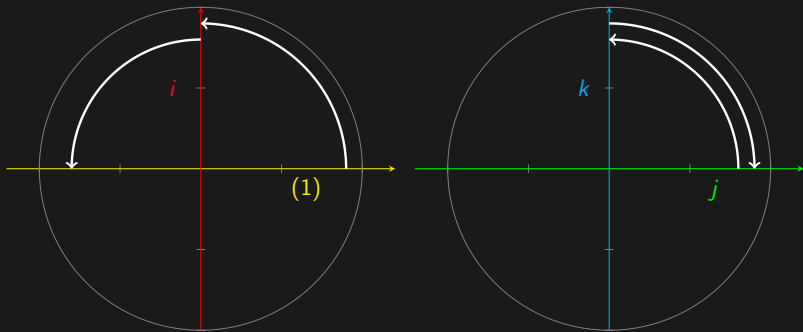
But What About Rotation

$$\mathbb{H} * i$$



The Big Idea

$$i * \mathbb{H} * i$$



Rotation!

$$i * \mathbb{H} * i$$

1 i graph

j k graph