3d Rotation with Quaternions

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Why Calculating Rotation in 3d is Valuable:

- Physics Simulations.
- 3d Animation.
- Navigation.
- And MUCH MORE!

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- Rotation Around Axis.
- Gimbals.

Why They Fail: Rotation Around Axis

Left rotation

Why They Fail: Rotation Around Axis

Left rotation

Right rotation

Why They Fail: Gimbals

Normal Gimbal

Why They Fail: Gimbals

Normal Gimbal

Gimbal Lock

What we would like

Stick in ball

Complex Numbers

Graph overview, unit multiplication.

Complex Number Angles

Show briefly the angle formula for complex numbers

Introduce Quaternions

Complex Numbers

Quaternions

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Complex Numbers

$$c_0(1) + c_1 i$$

Quaternions

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Quaternions

$$c_0(1) + c_1 i + c_2 j + c_3 k$$

Introduce Quaternions

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$$i^2 = -1$$

Quaternions

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Introduce Quaternions

Complex Numbers	Quaternions
$c_0(1) + c_1 i$	$c_0(1) + c_1 i + c_2 j + c_3 k$
$i^2 = -1$	$i^2 = j^2 = k^2 = -1$

Introduce Quaternions

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The product of any 2 different complex parts gives the third and they **anti-commute**

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Quaternions

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$$i^2 = j^2 = k^2 = -1$$

The product of any 2 different complex parts gives the third and they **anti-commute**

$$i * j = -j * i = k$$

Times Tables

*	1	i	j	k
1	1	i	j	k
i	i	-1	k	$-j$
j	j	$-k$	-1	i
k	k	j	$-i$	-1

But What About Rotation

$i * \mathbb{H}$

1 i graph

j k graph

But What About Rotation

$$\mathbb{H} * i$$

1 i graph

j k graph

The Big Idea

$$i * \mathbb{H} * i$$

1 i graph

j k graph

Rotation!

$$i * \mathbb{H} * i$$

1 i graph

j k graph