What is Calculus?

October 4, 2021

Approximating Length

Image

Approximating Length Attempt 1

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- Our second attempt is to use 19 lines.
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- The errors are less pronounced, but are still present.
- What if we could approximate the length with lines of zero length

The Notion of Zero Distance

$$\frac{x^3 - 2x^2}{x - 2} \qquad x \neq 2$$

$$\frac{x^2(x-2)}{(x-2)} \qquad x \neq 2$$

$$x^2 \qquad x \neq 2$$

What Does This Look Like

image

How Do We Describe This

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- "If $x = 2$ was in the domain then $f(2)$ would be 4"

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- "If $x = 2$ was in the domain then $f(2)$ would be 4"
- But $x = 2$ is not in the the domain, so this is a vacuous statement.

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- "The values of $f(x)$ around $x = 2$ have a value of 4"

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- "The values of $f(x)$ around $x = 2$ have a value of 4"
- 1.9 is near 2 but it has a value of $f(1.9)$ is 3.61 which isn't 4

How Do We Describe This

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- "As x get *arbitrarily* close to 2, the value of the $f(x)$ gets *arbitrarily* close to 4"

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- "As x get *arbitrarily* close to 2, the value of the $f(x)$ gets *arbitrarily* close to 4"
- ✓
- $$\lim_{x \to 2} \frac{x^3 - 2x^2}{x - 2} = 4$$

Revisiting the Length Problem

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- It turns out that we can look at the limit as the step size approaches zero.
- $$\lim_{\text{step size} \to 0} \text{approx len} = \text{true len}$$
- $\int_0^1 \sqrt{1 + 4\pi^2 cos^2(2\pi x)} dx$

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- As we have seen, calculus is all about trying to solve complex problems with infinitely many or infinitely small quantities through the use of clever manipulation and cancellation.

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- As we have seen, calculus is all about trying to solve complex problems with infinitely many or infinitely small quantities through the use of clever manipulation and cancellation.
- Some applications of calculus are: building accurate calculators, finding the orbits of planets, determining the volume and surface area of any object, modeling populations of animals in the wild, and much much more.