HIGHER-ORDER FUNCTIONS 2

COMPUTER SCIENCE 61A

June 28, 2016

1 Higher-Order Functions

A **higher order function** (HOF) is a function that manipulates other functions by taking in functions as arguments, returning a function, or both.

1.1 Functions as Arguments

One way a higher order function can manipulate other functions is by taking functions as input (an argument). Consider this higher order function called `negate`.

```
def negate(f, x):
    return -f(x)
```

`negate` takes in a function `f` and a number `x`. It doesn't care what exactly `f` does, as long as `f` is a function, takes in a number and returns a number. Its job is simple: call `f` on `x` and return the negation of that value.

1.2 Questions

1. Here are some possible functions that can be passed through as `f`.

   ```
   def square(n):
       return n * n

   def double(n):
       return 2 * n
   ```

   What will the following Python statements display?

```
>>> negate(square, 5)
```

Solution:
```
-25
```

```
>>> negate(double, -19)
```

Solution:
```
38
```

```
>>> negate(double, negate(square, -4))
```

Solution:
```
32
```

1.3 Functions as Return Values

Often, we will need to write a function that returns another function. One way to do this is to define a function inside of a function:

```
def outer(x):
    def inner(y):
        ...
    return inner
```

The return value of `outer` is the function `inner`. This is a case of a function returning a function. In this example, `inner` is defined inside of `outer`. Although this is a common pattern, we can also define `inner` outside of `outer` and still use the same `return` statement. However, note that in this second example (unlike the first example), `inner` doesn't have access to variables defined within the `outer` function, like `x`.

```
def inner(y):
    ...
def outer(x):
    return inner
```

1.4 Questions

1. Use this definition of `outer` to fill in what Python would display when the following lines are evaluated.

```
def outer(n):
    def inner(m):
```

```
            return n - m
        return inner
>>> outer(61)
```

Solution:
```
<function outer.inner ...>
```

```
>>> f = outer(10)
>>> f(4)
```

Solution:
```
6
```

```
>>> outer(5)(4)
```

Solution:
```
1
```

2. Write a function `and_add` that takes a function `f` (such that `f` is a function of one argument) and a number `n` as arguments. It should return a function that takes one argument, and does the same thing as the function `f`, except also adds `n` to the result.

```
def and_add(f, n):
    """Return a new function. This new function takes an
    argument x and returns f(x) + n.

    >>> def square(x):
    ...     return x * x
    >>> new_square = and_add(square, 3)
    >>> new_square(4)   # 4 * 4 + 3
    19
    """
```

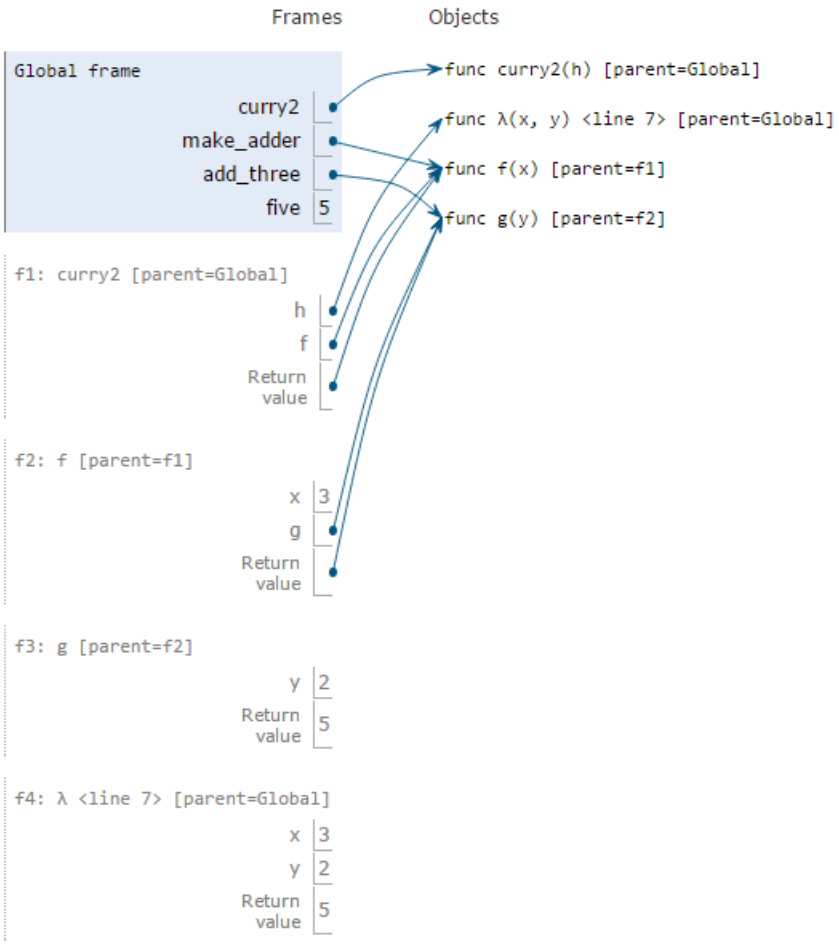
Solution:
```
    def g(x):
        return f(x) + n
    return g
```

1.5 Environment Diagrams

1. Draw the environment diagram for the following code:

```
def curry2(h):
    def f(x):
        def g(y):
            return h(x, y)
        return g
    return f

make_adder = curry2(lambda x, y: x + y)
add_three = make_adder(3)
five = add_three(2)
```

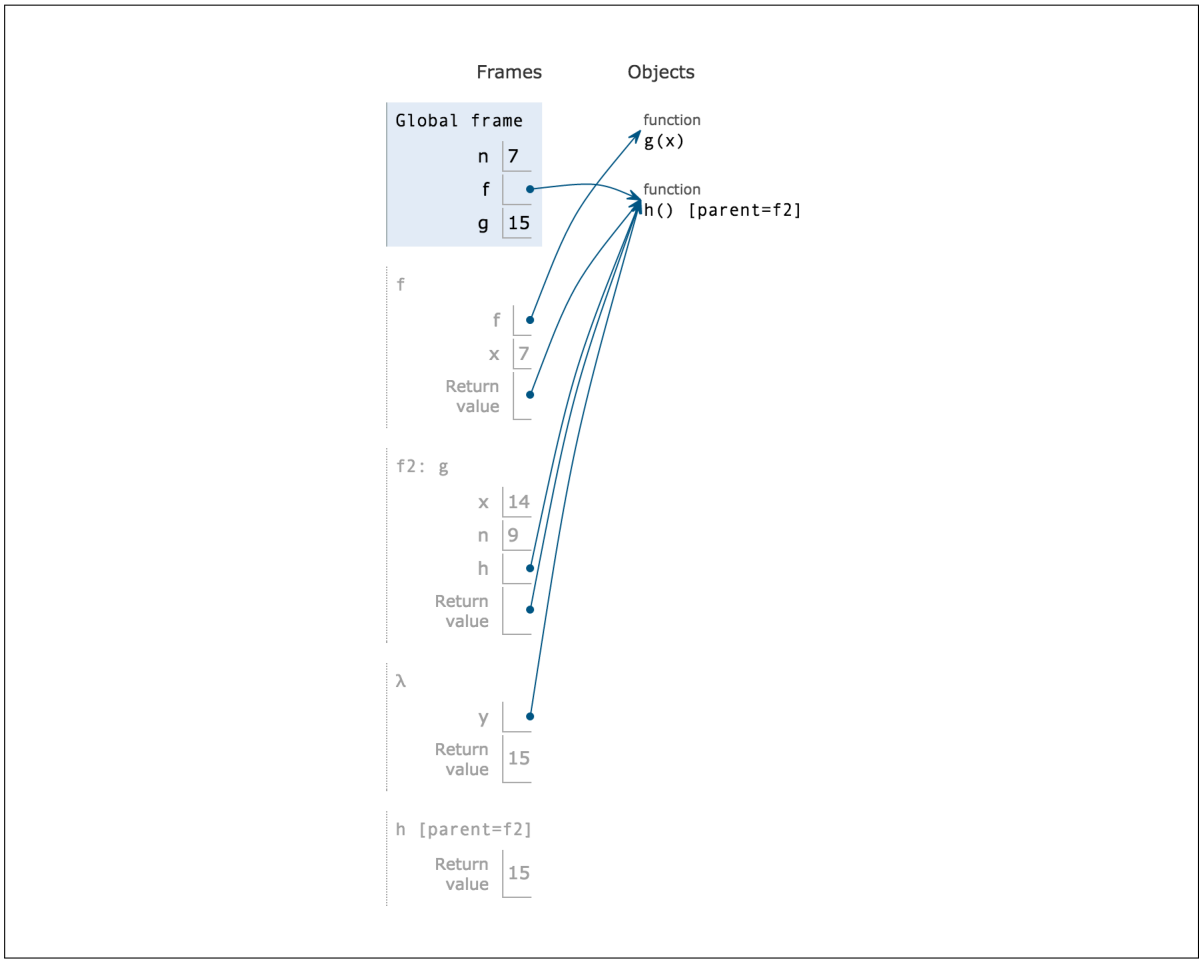
Solution:



2. Draw the environment diagram that results from running the following code:

```
n = 7

def f(x):
    n = 8
    return x + 1

def g(x):
    n = 9
    def h():
        return x + 1
    return h

def f(f, x):
    return f(x + n)

f = f(g, n)
g = (lambda y: y())(f)
```

Solution:


Frames
Objects
Global frame
n 7
f
g 15
function
g(x)
function
h() [parent=f2]
f
f
x 7
Return value
f2: g
x 14
n 9
h
Return value
λ
y
Return value 15
h [parent=f2]
Return value 15

3. **The following question is extremely difficult. Something like this would not appear on the exam. Nonetheless, it's a fun problem to try.**

   Draw the environment diagram for the following code: (Note that using the + operator with two strings results in the second string being appended to the first. For example `"C" + "S"` concatenates the two strings into one string `"CS"`)

```
y = "y"
h = y
def y(y):
    h = "h"
    if y == h:
        return y + "i"
    y = lambda y: y(h)
    return lambda h: y(h)
y = y(y)(y)
```

Solution: