

# Introduction to Mobile Robotics

## Wheeled Locomotion

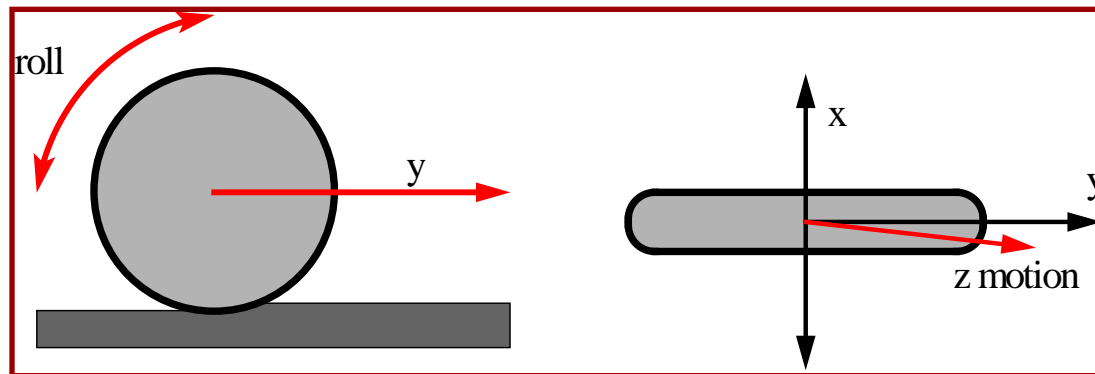
Wolfram Burgard



# Locomotion of Wheeled Robots

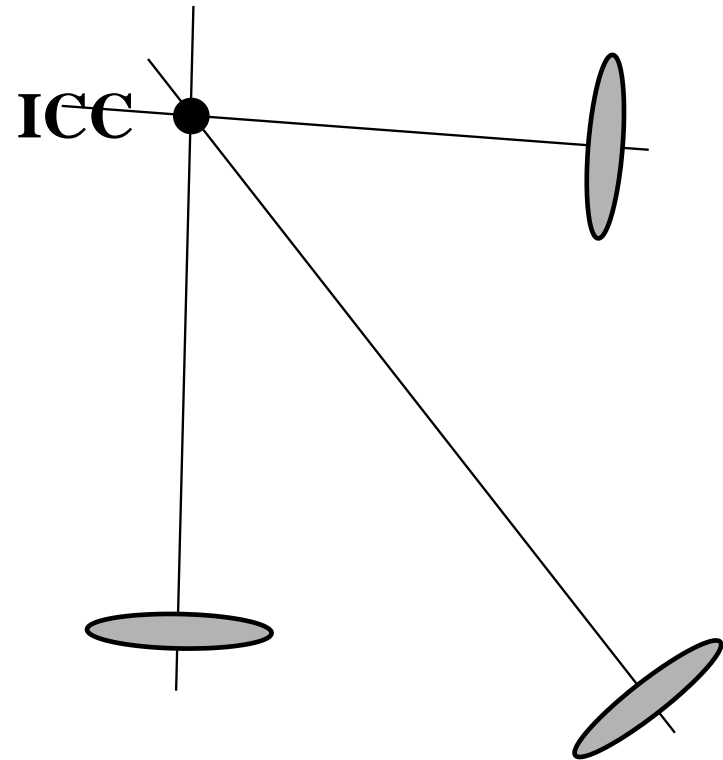
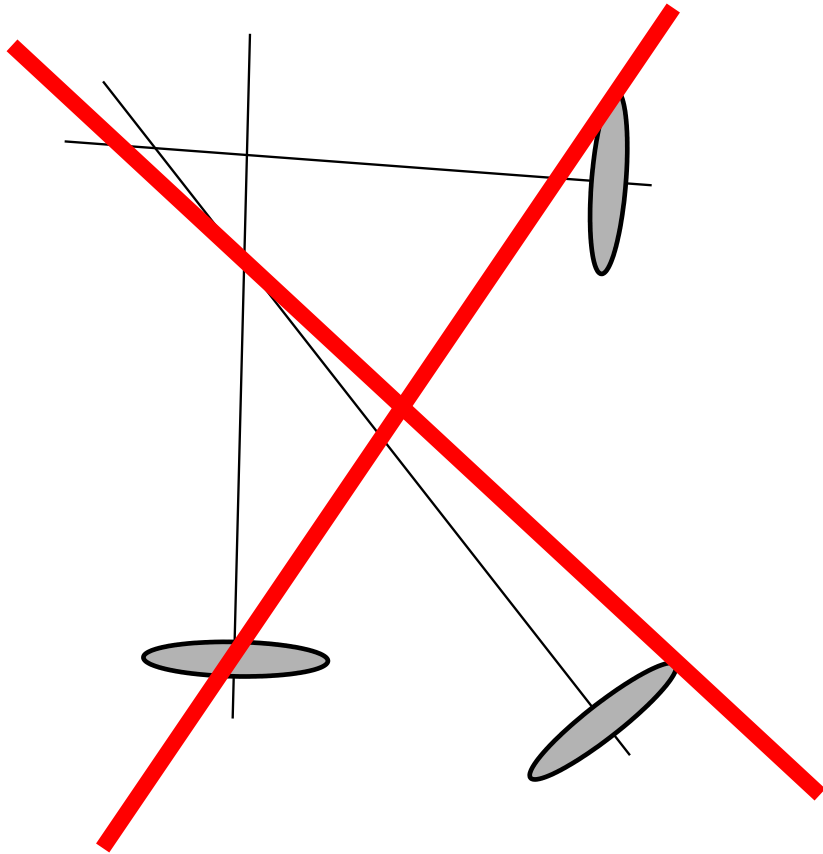
Locomotion (Oxford Dict.): Power of motion from place to place

- Differential drive (AmigoBot, Pioneer 2-DX)
- Car drive (Ackerman steering)
- Synchronous drive (B21)
- XR4000
- Mecanum wheels



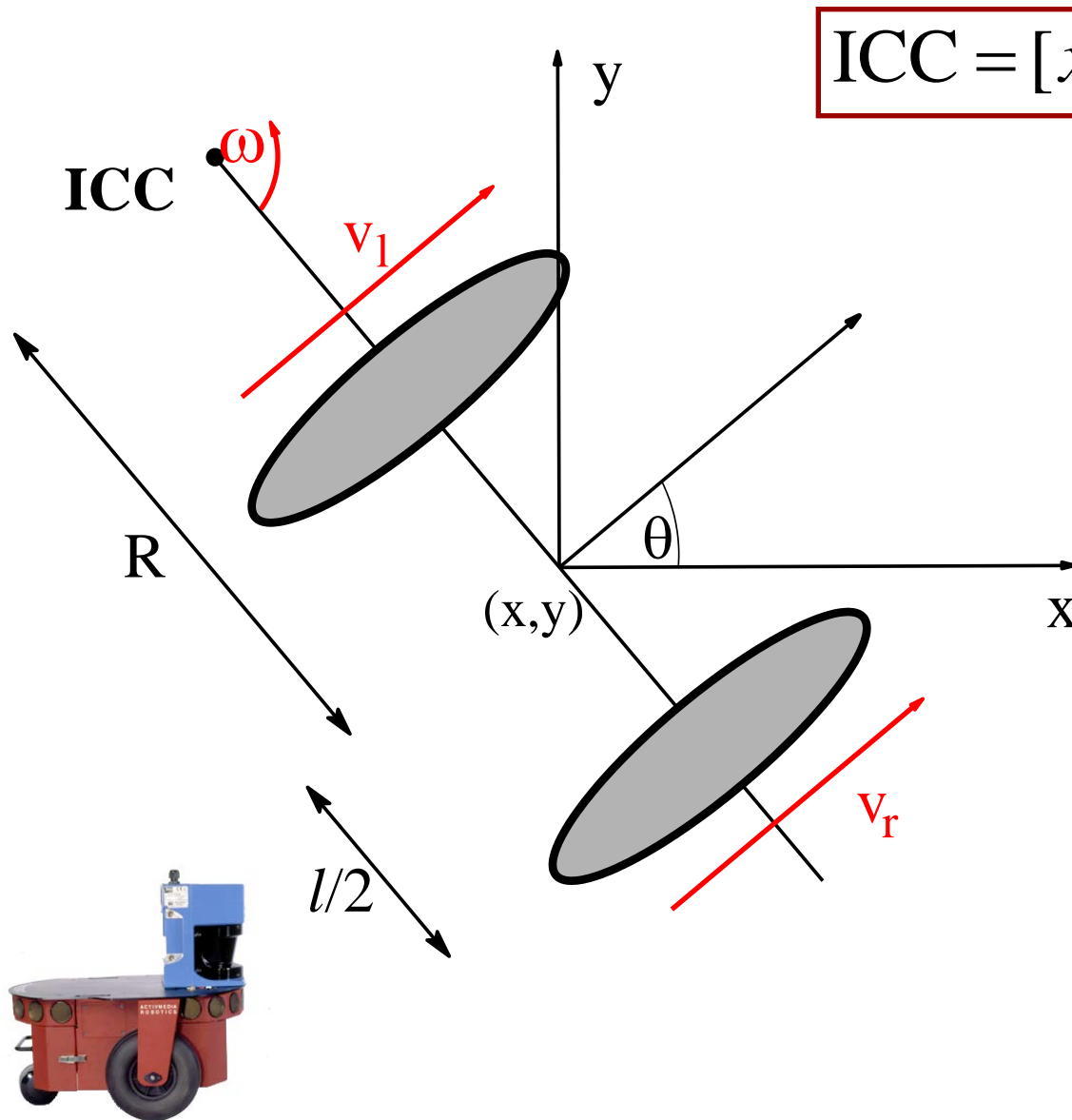
we also allow wheels to rotate around the z axis

# Instantaneous Center of Curvature



- For rolling motion to occur, each wheel has to move along its y-axis

# Differential Drive



$$\text{ICC} = [x - R \sin \theta, y + R \cos \theta]$$

$$\omega(R + l/2) = v_r$$

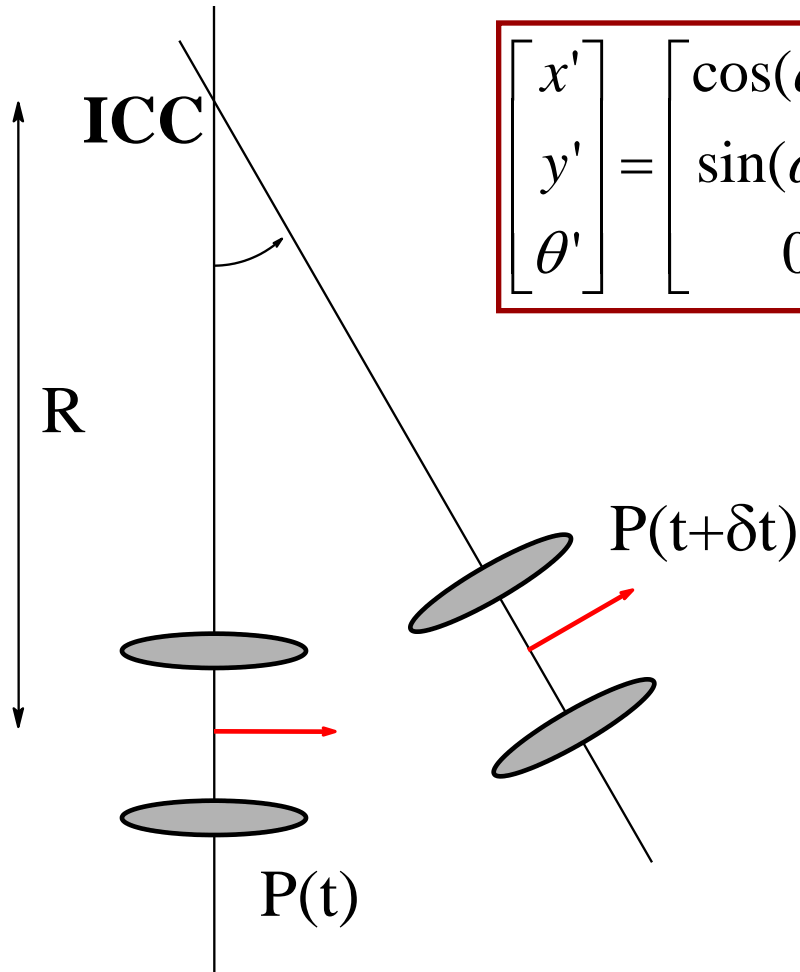
$$\omega(R - l/2) = v_l$$

$$R = \frac{l}{2} \frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

$$V = \frac{v_r + v_l}{2}$$

# Differential Drive: Forward Kinematics



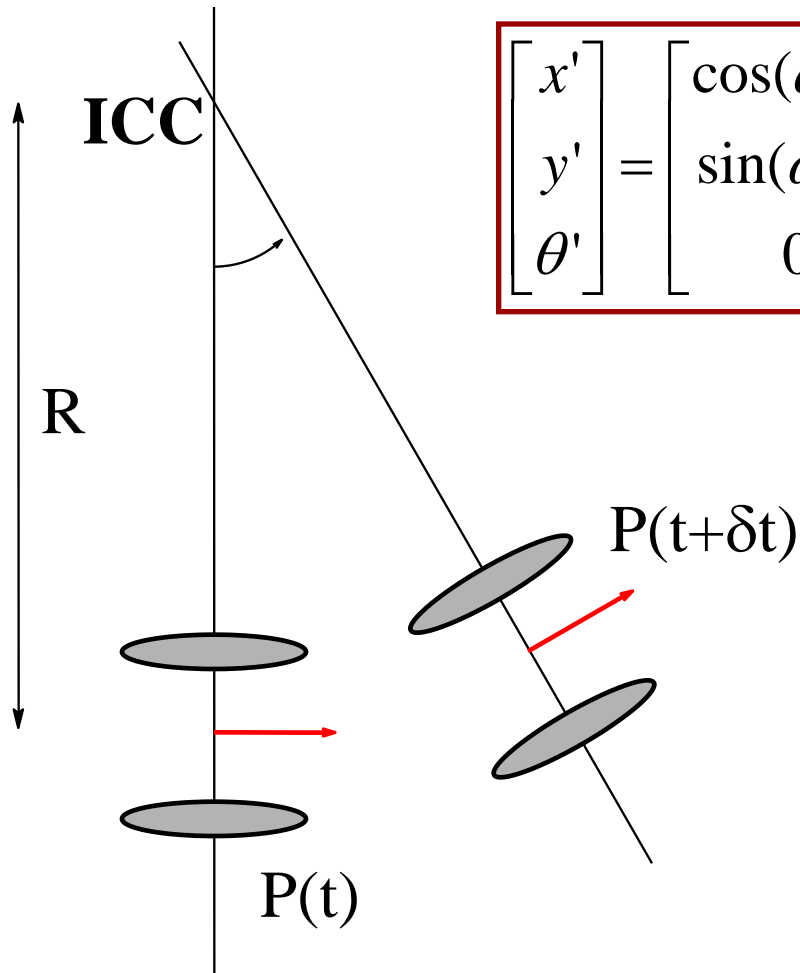
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - ICC_x \\ y - ICC_y \\ \theta \end{bmatrix} + \begin{bmatrix} ICC_x \\ ICC_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') dt'$$

# Differential Drive: Forward Kinematics



$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - \text{ICC}_x \\ y - \text{ICC}_y \\ \theta \end{bmatrix} + \begin{bmatrix} \text{ICC}_x \\ \text{ICC}_y \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \cos[\theta(t')] dt'$$

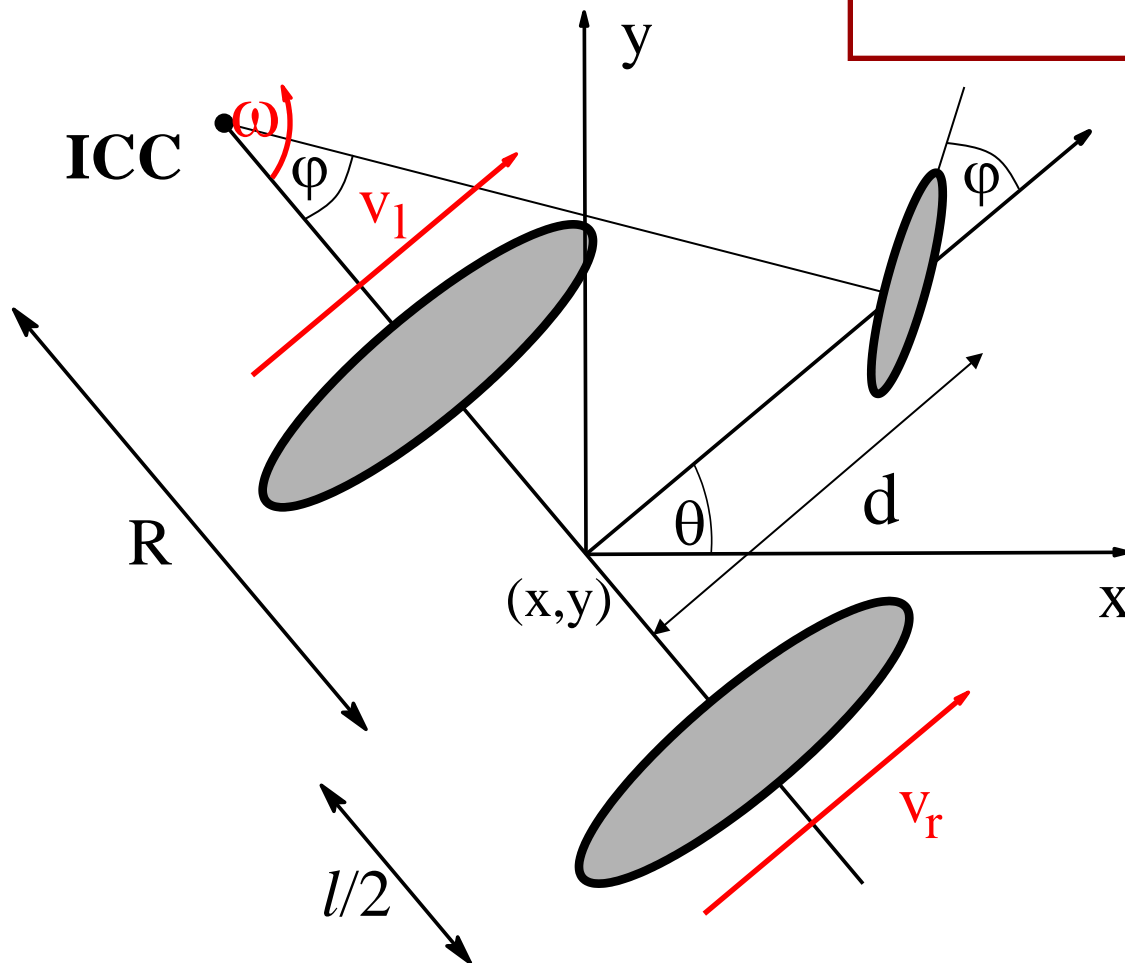
$$y(t) = \frac{1}{2} \int_0^t [v_r(t') + v_l(t')] \sin[\theta(t')] dt'$$

$$\theta(t) = \frac{1}{l} \int_0^t [v_r(t') - v_l(t')] dt'$$

# Ackermann Drive

$$\text{ICC} = [x - R \sin \theta, y + R \cos \theta]$$

$$R = \frac{d}{\tan \varphi}$$



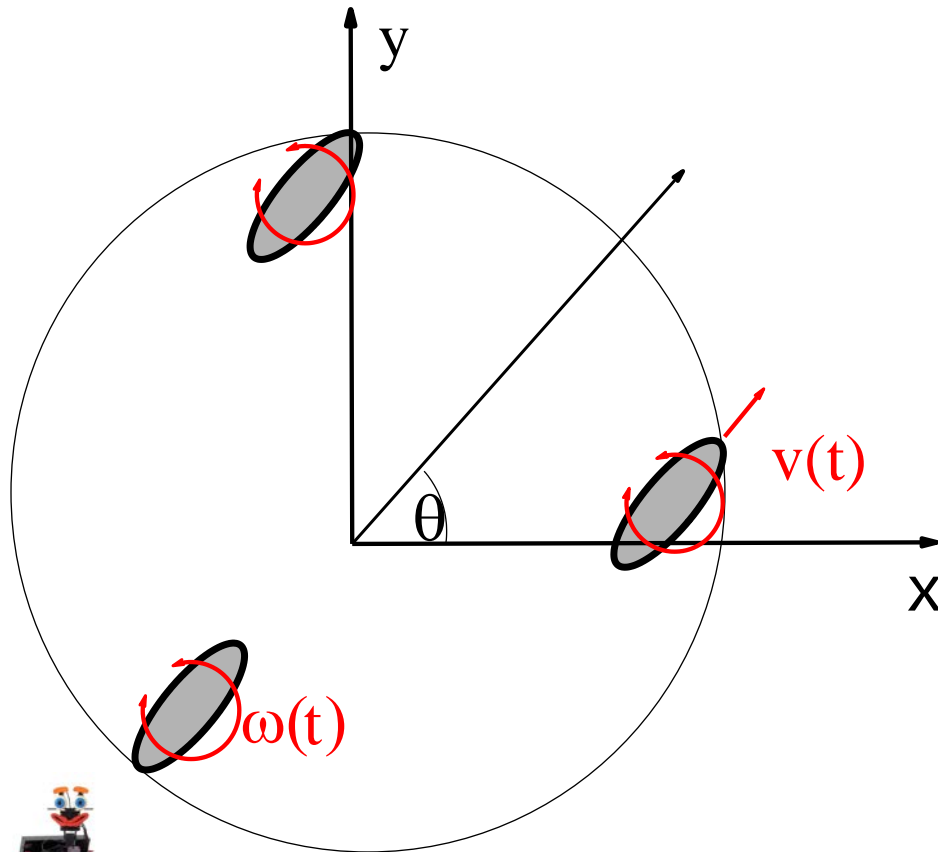
$$\omega(R + l / 2) = v_r$$

$$\omega(R - l / 2) = v_l$$

$$R = \frac{l}{2} \frac{(v_l + v_r)}{(v_r - v_l)}$$

$$\omega = \frac{v_r - v_l}{l}$$

# Synchronous Drive

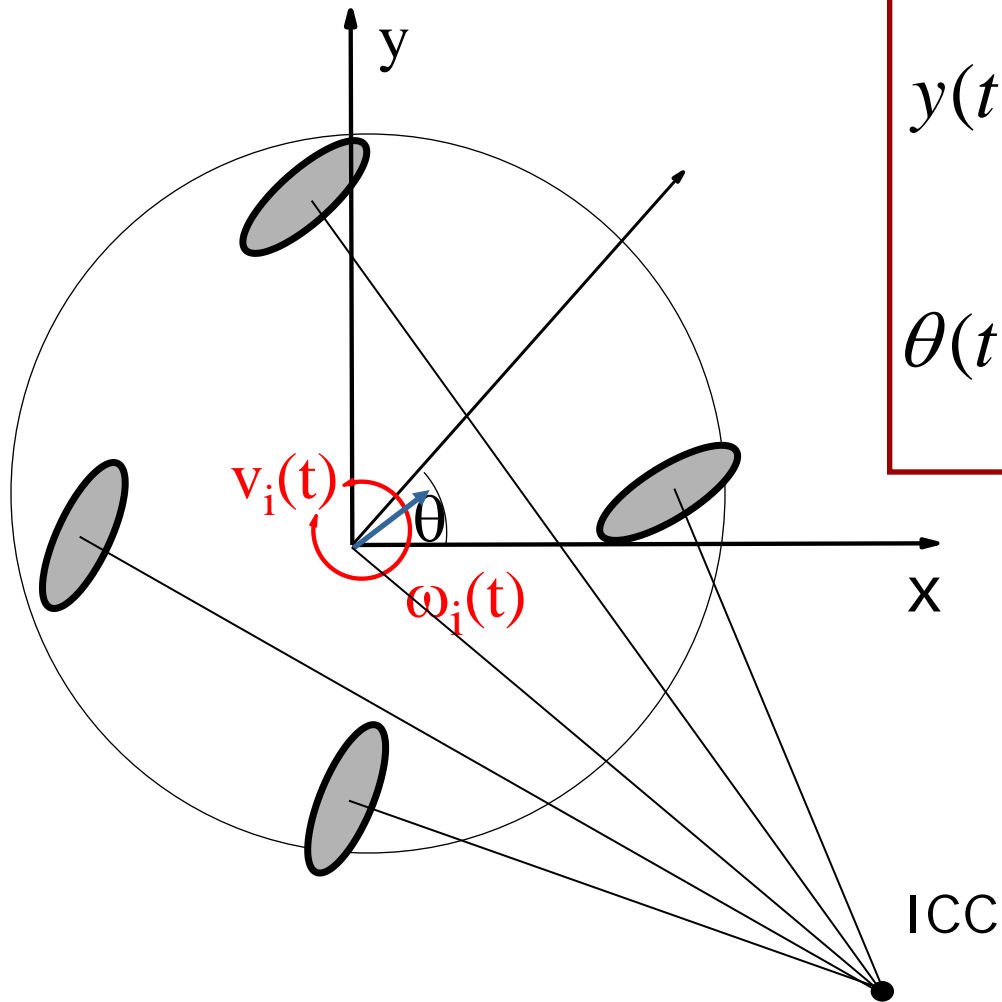


$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$
$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$
$$\theta(t) = \int_0^t \omega(t') dt'$$





# XR4000 Drive



$$x(t) = \int_0^t v(t') \cos[\theta(t')] dt'$$

$$y(t) = \int_0^t v(t') \sin[\theta(t')] dt'$$

$$\theta(t) = \int_0^t \omega(t') dt'$$



# Mecanum Wheels



$$v_y = (v_0 + v_1 + v_2 + v_3) / 4$$

$$v_x = (v_0 - v_1 + v_2 - v_3) / 4$$

$$v_\theta = (v_0 + v_1 - v_2 - v_3) / 4$$

$$v_{error} = (v_0 - v_1 - v_2 + v_3) / 4$$

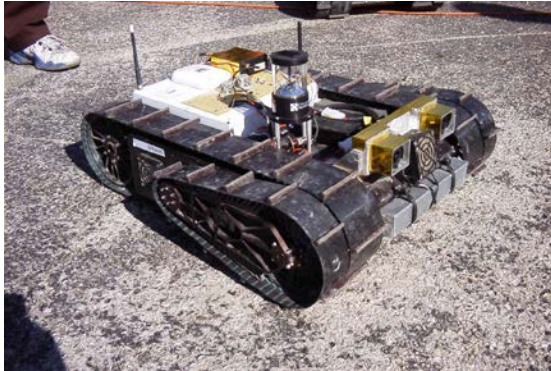
# The Kuka OmniRob Platform



# Example: KUKA youBot



# Tracked Vehicles





# Other Robots: OmniTread



[courtesy by Johann Borenstein]

# Non-Holonomic Constraints

- Non-holonomic constraints limit the possible incremental movements within the configuration space of the robot.
- Robots with differential drive or synchro-drive move on a circular trajectory and cannot move sideways.
- Mecanum-wheeled robots can move sideways (they have no non-holonomic constraints).

# Holonomic vs. Non-Holonomic

- Non-holonomic constraints reduce the **control space** with respect to the current configuration
  - E.g., moving sideways is impossible.
- Holonomic constraints reduce the **configuration space**.
  - E.g., a train on tracks (not all positions and orientations are possible)



# Drives with Non-Holonomic Constraints

- Synchro-drive
- Differential drive
- Ackermann drive



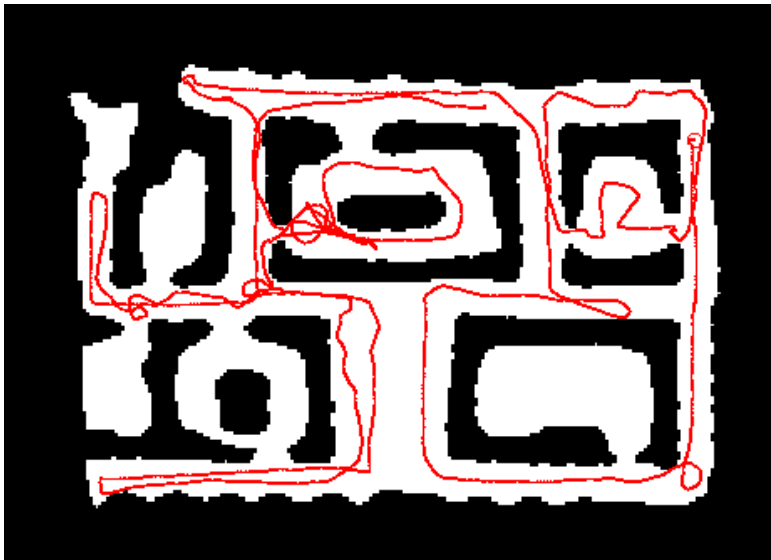
# Drives without Non-Holonomic Constraints

- Mecanum wheels



# Dead Reckoning and Odometry

- Estimating the motion based on the issued controls/wheel encoder readings
- Integrated over time



# Summary

- Introduced different types of drives for wheeled robots
- Math to describe the motion of the basic drives given the speed of the wheels
- Non-holonomic constraints
- Odometry and dead reckoning