Business data science
Prediction & Forecasting

Assignment 1

Thao Le	523716tl@eur.nl
Yuanyuan Su	573197ys@eur.nl
Jason Wang	668796qw@eur.nl

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Q(a and the first part of c)

Gasoline Sales

Figure 1 is the observation points in 12 weeks of Gasoline Sales in the *GasolineSales1.csv*. Figure 2 has the observation points and the average line together. A measure of the average is $\bar{Y} = \frac{1}{T}\sum_{t=1}^{T} Y_t$.

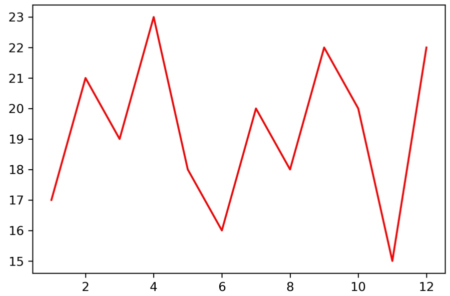

Figure 1: Example: 12 weeks of Gasoline Sales (page 6)

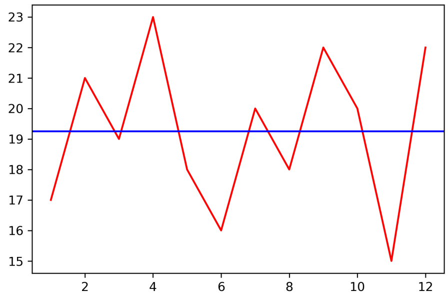

Figure 2: Average Gasoline Sales (page 8)

Figure 3 includes the observation points and the running average. The running average can be computed by $\bar{Y}_j = \frac{1}{J}\sum_{j=1}^{J} Y_j$ for the first J observations, for each $J = 1, \ldots T$. Figure 4 gives the Sales in red line, forecasts in blue line and errors in black index. The rea;-time forecasts for the collected observations are $\hat{Y}_t^F = \hat{Y}_{t-1}$, $t = 2$, 3, ... T. And the difference or the forecast error is calculated by $U_t = Y_t - \hat{Y}_t^F$, $t = 2$, 3, ... T.

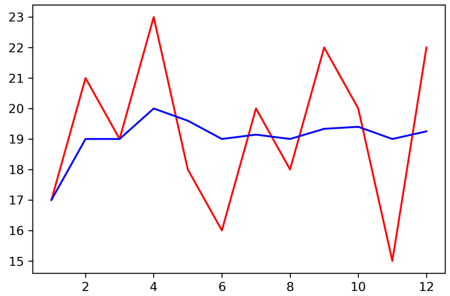

Figure 3: Running Average (page 11)

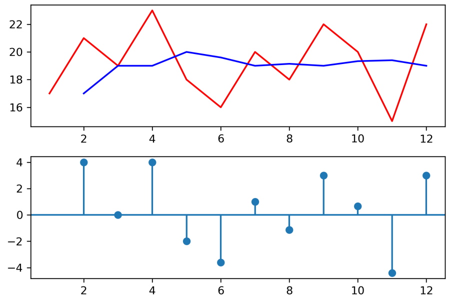

Figure 4: Forecasting: assessment (page 15)

Table 1 is the table of Y_t, $\hat{Y}_t^F$ and U_t. We also give the average of Y_t and the average of U_t at the bottom of the table. Table 2 adds mean absolute error, mean absolute percentage error and mean squared error.

	Y_t	$\hat{Y}_t^F$	U_t
2	21	17.00	4.00
3	19	19.00	0.00
4	23	19.00	4.00
5	18	20.00	-2.00
6	16	19.60	-3.60
7	20	19.00	1.00
8	18	19.14	-1.14
9	22	19.00	3.00
10	20	19.33	0.67
11	15	19.40	-4.40
12	22	19.00	3.00
Avg	19.46		0.41

Table 1: Forecasting Report For Sales (page 16)

	Y_t	$\hat{Y}_t^F$	U_t	$\|U_t\|$	$\%\|U_t\|$	U_t^2
2	21	17.00	4.00	4.00	19.05	16.00
3	19	19.00	0.00	0.00	0.00	0.00
4	23	19.00	4.00	4.00	17.39	16.00
5	18	20.00	-2.00	2.00	11.11	4.00
6	16	19.60	-3.60	3.60	22.50	12.96
7	20	19.00	1.00	1.00	5.00	1.00
8	18	19.14	-1.14	1.14	6.35	1.31
9	22	19.00	3.00	3.00	13.64	9.00
10	20	19.33	0.67	0.67	3.33	0.44
11	15	19.40	-4.40	4.40	29.33	19.36
12	22	19.00	3.00	3.00	13.64	9.00
			ME	MAE	MAPE	MSE
Avg			0.41	2.44	12.85	8.10

Table 2: Forecasting Report For Sales (page 21)

Table 3 compares the results of 4 methods (ME, MAE, MAPE and MSE) between running average and random walk for all 11 forecasts. Table 4 compares the results of 4 methods (ME, MAE, MAPE and MSE) between running average and random walk for last 6 forecasts.

	ME	MAE	MAPE	MSE
Running Avg	0.41	2.44	12.85	8.10
Random Walk	0.45	3.73	19.24	16.27

Table 3: Forecasting Comparisons, All 11 Forecasts (page 23)

	ME	MAE	MAPE	MSE
Running Avg	0.35	2.2	11.88	6.69
Random Walk	1.00	4.0	20.74	19.00

Table 4: Forecasting Comparisons, Last 6 Forecasts (page 24)

Figure 5 shows the forecast weights of applying different forecasting methods. Running average is computed by averaging all the available data over a set of dataset. Moving average smooths the data by taking the average over a fixed-size window. Random walk represents future observations are random variations of current observations, which is $Y_t = Y_{t-1} + \epsilon$. Exponential smoothing is a weight-based method. The weights decay exponentially over time, with the most recent observations usually having greater weights, which can be $\hat{Y}_{t+1}^F = \alpha Y_t + (1-\alpha)\hat{Y}_t^F$. Figure 6 illustrates the outcomes when applying various α to the exponential smoothing methods. Figure 7 shows the scaled results and memory index. The memory index is defined as the largest index $m_\alpha = J$ for which $(1-\alpha)^J > 0.10$, we can have $m_\alpha = \frac{\log(0.1)}{\log(1-\alpha)}$. The memory index serves a quantitative measure to asses the extent to which past observations influence the current forecast within the context of scaled exponential smoothing.

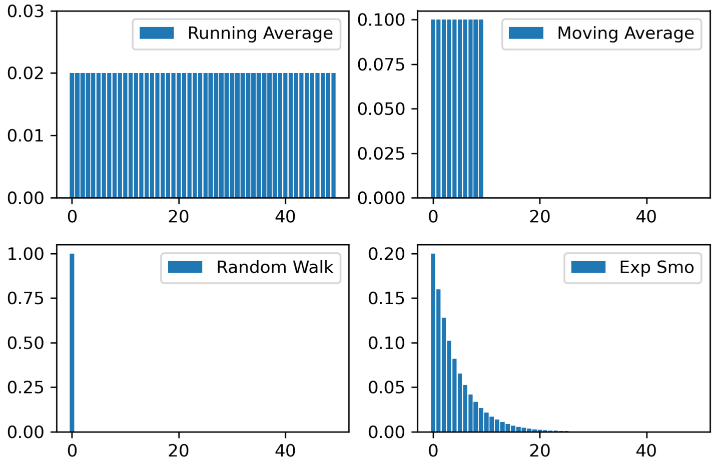

Figure 5: Forecast weights for 50 past observations (page 37)

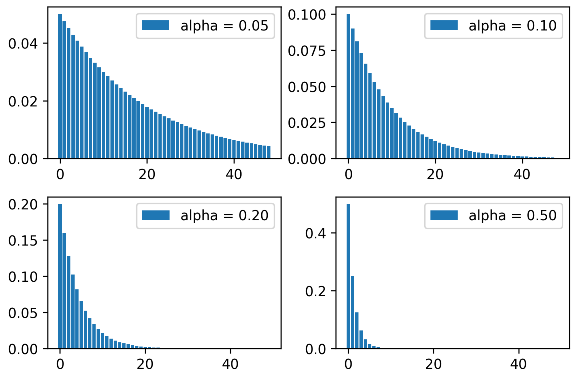

Figure 6: Exponential Smoothing Weights (page 38)

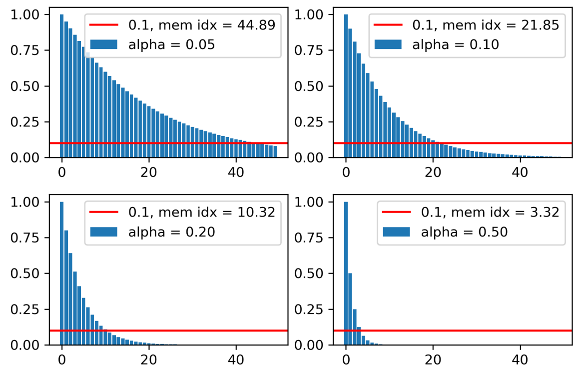

Figure 7: Exponential Smoothing Weights (scaled page 39 & 41)

Figure 8 and figure 9 respectively represent the forecasting result by exponential smoothing with $\alpha = 0.2$ and $\alpha = 0.8$.

Table 5 shows the results of different method.

Method	ME	MAE	MAPE	MSE
Running Avg	0.35	2.20	11.88	6.69
Random Walk	1.00	4.00	20.74	19.00
ExpSmo (0.2)	0.77	2.42	12.85	7.95
ExpSmo (0.8)	0.88	3.49	18.29	15.33

Table 5: Forecasting Comparisons, 6 Forecasts (page 46)

Figure 10 shows the "unstable world" when we add 10 more weeks of observations to the Gasoline Sales. Figure 11 illustrates the results with different methods and different parameters.

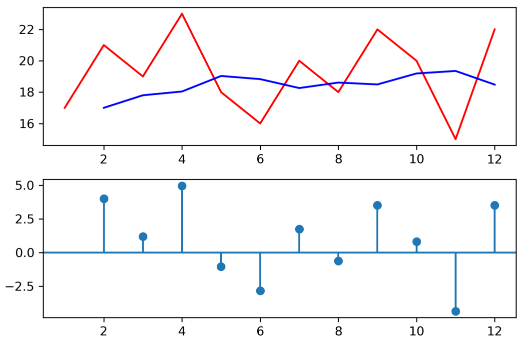

Figure 8: Forecasting by ES with $\alpha = 0.2$ (page 42)

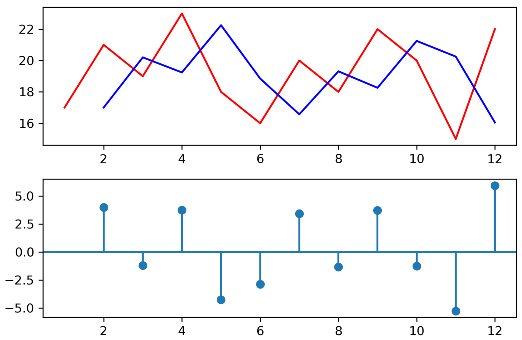

Figure 9: Forecasting by ES with $\alpha = 0.8$ (page 43)

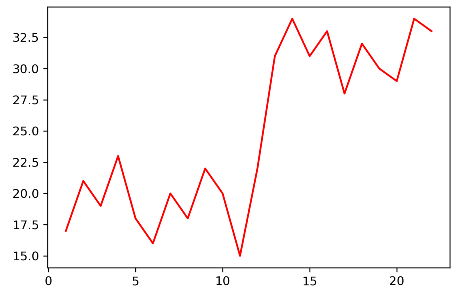

Figure 10: Forecasting (page 50)

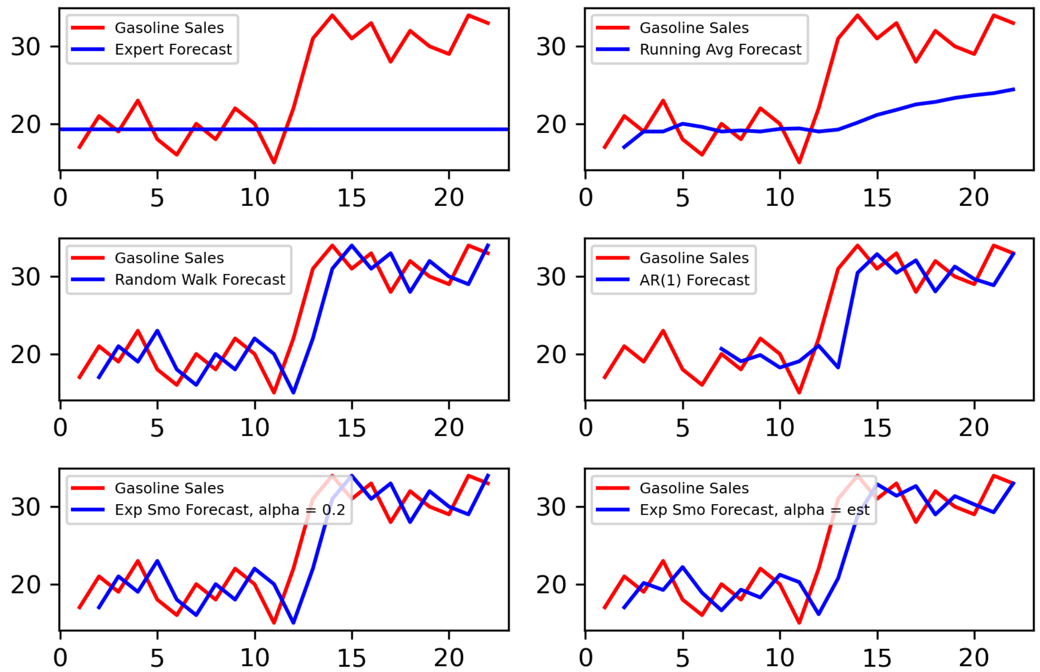

Figure 11: Forecasting with different methods (page 51)

Table 6 shows the results of different methods.

Method	ME	MAE	MAPE	MSE
Running Avg	5.88	6.57	22.50	59.63
Random Walk	1.06	3.69	14.77	18.06
ExpSmo (0.2)	3.78	4.40	15.63	32.29
ExpSmo (0.8)	1.28	3.43	13.63	17.82

Table 6: Forecasting Comparisons, 16 Forecasts (page 52)

Bicycle Sales

Figure 12 demonstrates the bicycle sales, and figure 13 adds the running average. However, we can see the running average is not adequate for the forecast.

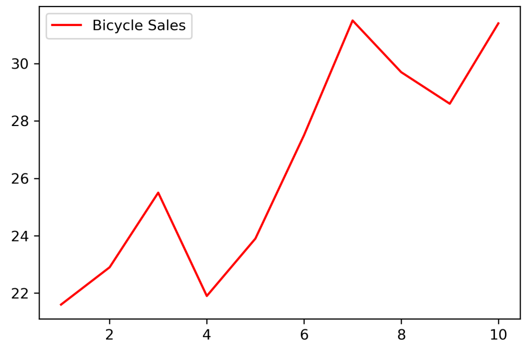

Figure 12: Bicycle Sales (page 54)

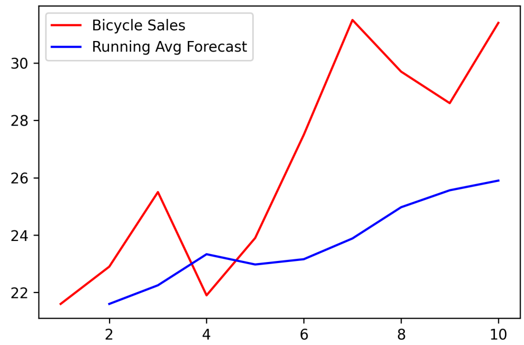

Figure 13: Bicycle Sales: Running Average (page 55)

Figure 14 shows the result of running trend method. For the running trend, it is based on the time trend model $Y_t = \mu + \beta t + \epsilon_t, E(\epsilon_t) = 0$. In the figure 15, we compute the forecast by $\hat{Y}_t^F = a_{t-1} + b_{t-1} \times t$, where a_{t-1} estimates of μ and b_{t-1} estimates of β with the data from $Y_1, ..., Y_{t-1}$.

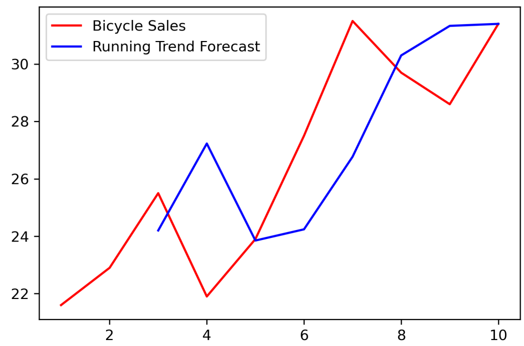

Figure 14: Bicycle Sales: Running Trend Forecasts (page 61)

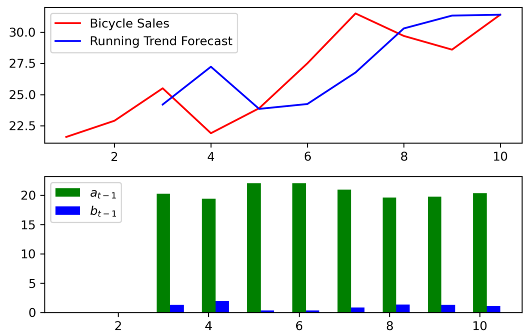

Figure 15: Bicycle Sales: Running Trend Forecasts (page 62)

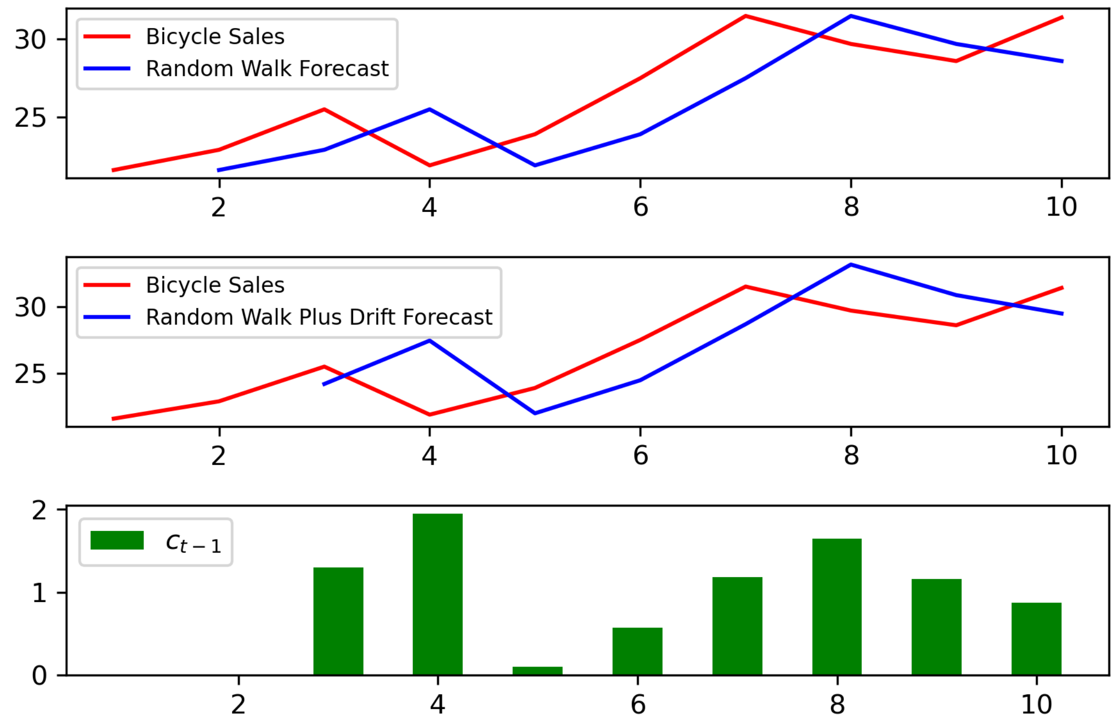

Figure 16: Bicycle Sales: Random Walk/ - Plus Drift Forecast (page 65)

Figure 16 gives the result of two methods. For the random walk method, it is based on the model $Y_t = Y_{t-1} + \epsilon_t, E(\epsilon_t) = 0, \hat{Y}_t^F = Y_{t-1}$. Random walk plus drift is based on the itime trend model $Y_t = \mu + Y_{t-1} + \epsilon_t, E(\epsilon_t) = 0, \hat{Y}_t^F = c_{t-1} + Y_{t-1}$. $c_t = \frac{1}{t-1}\sum_{j=0}^{t-2}(Y_{t-j} - Y_{t-1-j}) = \frac{1}{t-1}\sum_{j=0}^{t-2}\Delta Y_{t-j}$ for t= 3,...,T. Table 7 shows the results when we apply various methods with 6 forecasts. Table 8 shows the results when we tried different parameters and methods. The reason that we have a little difference with the slide is that we use all the forecast rather 6 forecast.

Method	ME	MAE	MAPE	MSE
Running Avg	3.25	3.57	12.54	17.09
Running Trend	0.08	2.25	8.51	8.86
Random Walk	1.09	2.53	9.48	7.41
Random Walk Plus Drift	-0.04	2.78	10.50	9.24

Table 7: Forecasting Comparisons, 6 Forecasts (page 66)

We apply grid search on parameter values and use MSE as our assessment. The final result is slightly different from the one from slides.

Method	ME	MAE	MAPE	MSE
Exp Smo (0.2)	3.33	3.48	12.22	16.63
Exp Smo (0.8)	1.29	2.43	9.02	7.52
Exp Smo (est)	1.13	2.52	9.39	7.41
Double Exp Smo (0.2, 0.1)	-0.64	1.90	7.27	5.15
Double Exp Smo (0.2, 0.3)	-0.46	1.86	7.16	5.28
Double Exp Smo (est)	-0.34	2.21	8.30	6.58

Table 8: Forecasting Comparisons (page 72)

Umbrella Sales for the first part of Q(c)

Figure 17 shows the 20 observation points and figure 18 gives other 8 forecasting points by Holt-Winters Additive Seasonal Method. The Holt-Winters Additive Seasonal Method is given by

$$L_t = \alpha(Y_t - H_{t-S}) + (1-\alpha)(L_{t-1} + G_{t-1})$$
$$G_t = \beta(Lt - L_{t-1}) + (1-\beta)G_{t-1}$$
$$H_t = \gamma(Y_t - L_t) + (1-\gamma)H_{t-S}$$
$$\hat{Y}^F_{t+1} = L_t + G_t + H_{t-S+1}$$

for $t = S+1, ..., T$, with *Level* L_t, *Growth* G_t and *Seasonal correction* H_t. Based on the equations, we used the *Holt-Winters Exponential Smoothing (HWES) in the statsmodels package.* Figure 19 illustrates the decomposition of the *umbrella sales* with the first subplot showing the observation values and the level, the second subplot showing the seasonal correction and the third subplot showing the growth. We can have *level* and *seasonal* directly and get the *growth* from Growth = Umbrella Sales Value - Level - Seasonal.

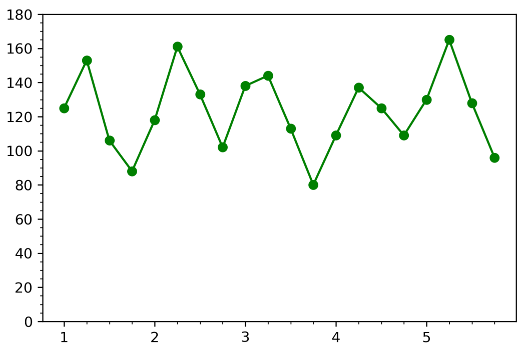

Figure 17: Umbrella Sales (page 84)

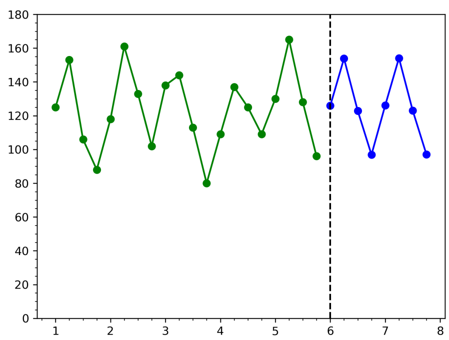

Figure 18: Umbrella Sales: forecasting (page 86)

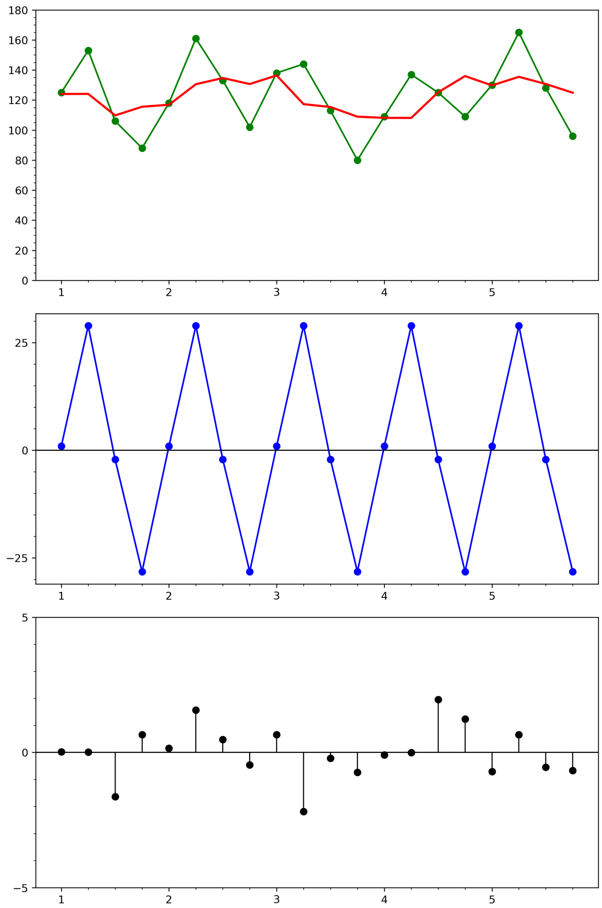

Figure 19: Umbrella Sales : decomposition (page 85)

Q(b)

For this question, we are asked to estimate the forecasts of 9 given time series with different methods. Then, we assess and compare the methods with graphs and tables of evaluation metrics. This process is first done using the first 40 observations, with the last 10 in the 40 observations sample used to measure the precision. Then, we repeat with all 50 observations, with the last 10 in the 50 observations used to measure the precision.

The methods we use here are: Running Average, Running Trends, Random Walk, Random Walk with Drift, Exponential Smoothing (ES), and Holt-Winters (HW or Double Exp Smo) methods. The evaluation metrics used are: ME, MAE, MAPE, MSE.

Since there are 9 time series given, we randomly chose 1 time series to elaborate on our forecasting steps. For the remaining 8 series, the steps are the same, so only the summary of the forecast and evaluation metrics are reported. The randomly chosen time series for detailed analysis is series 3 below:

Time series 3

Using the first 40 observations:
To evaluate the forecasts, we first look at the forcating plots of different methods. Figure 20 presents the running average and running trend forecasts next to each other, alongside the running trend coefficients. We can see that time trend coefficients b_{t-1} are very small while the mean coefficients a_{t-1} are much larger. This implies that the time trend effect is not large for our time series. However, the running trend still seems to perform better than the running average, as the forecast line seems to fit better with the observations.

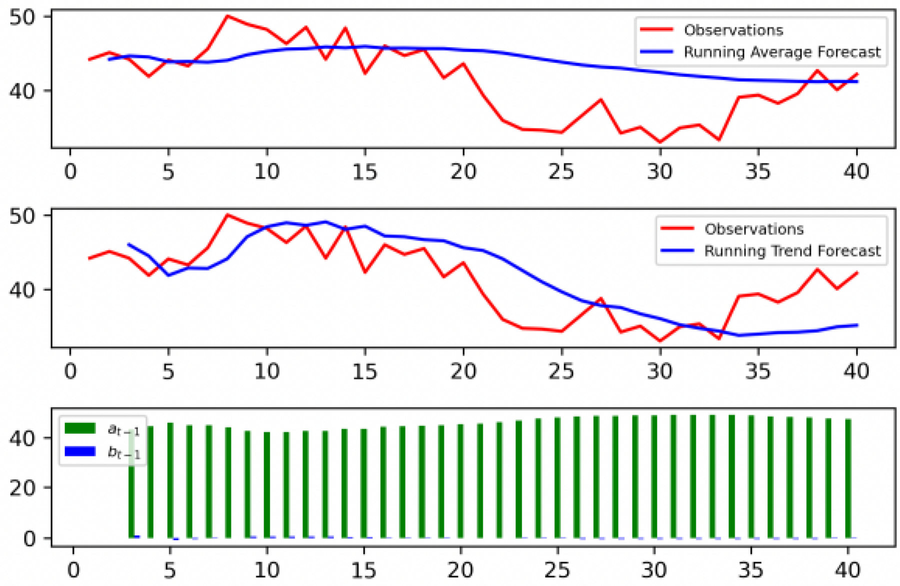

Figure 20: Running Average, Running Trend forecasts with Running Trend coefficients a_{t-1} and b_{t-1}.

Figures 21 and 22 offer a closer inspection of the forecasts by measuring their deviations from the original lines. The running average forecast seems to deviate more than the running trend forecast, as it fails to account for the dip in values from time periods 20 to 30 due to the lack of a time trend. The running trend forecast does account for the time trend. However, it does not respond fast enough to the decrease in times 20 to 25 and the increase in the last 5 time periods. Thus, the running trend forecast seems to perform worse in the last 10 observations.

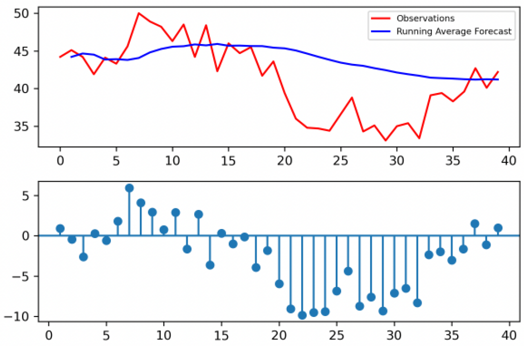

Figure 21: Running Average forecast

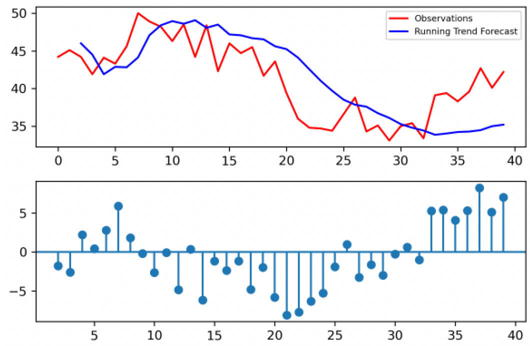

Figure 22: Running Trend forecast

To improve our forecasts, we want to make sure our forecasts responses quicker to changes in actual value of the previous time stamp. Thus, we construct *random walk* and *random walk plus drift forecasts*. In Figure 23, we can see that these two methods fit more closely to the lines of the actual values as their response quickly to changes in the previous value.

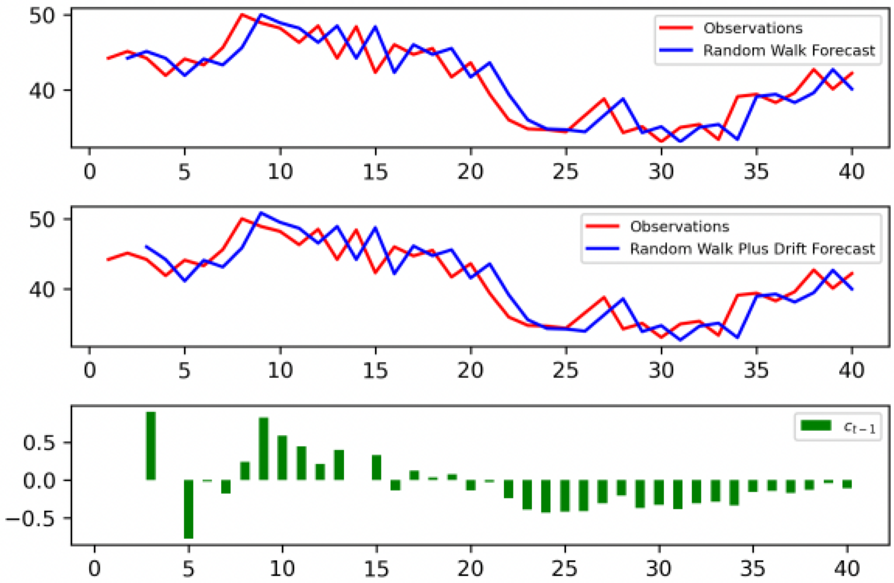

Figure 23: Random Walk, Random Walk w. Drift forecast, with drift coefficients c_{t-1}.

Last, we visually assess the forecasts made by Exponential Smoothing (ES) and Holt-Winters (HW) methods, with estimated alpha and beta. For this, we can look at Figure 24 below:

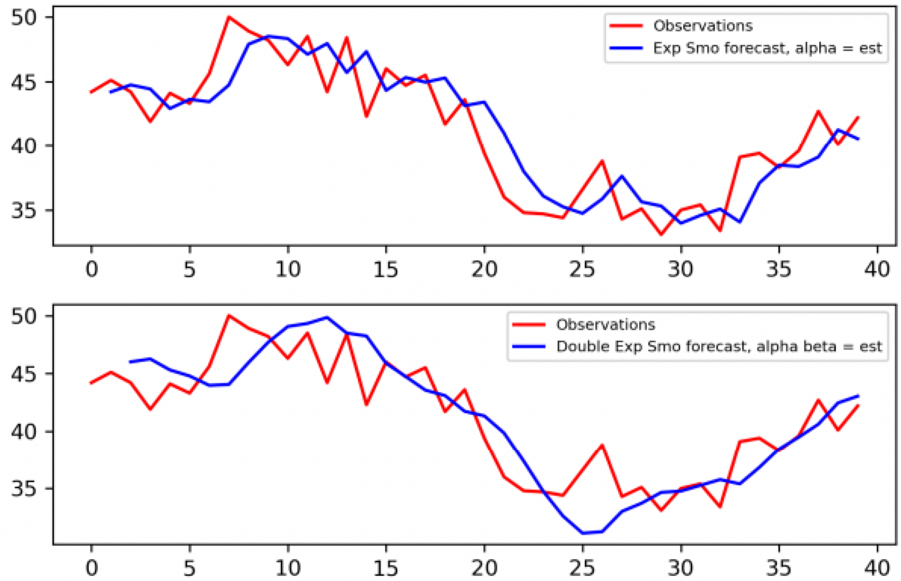

Figure 24: Exp Smo and Double Exp Smo (Holt-Winters) forecasts

Since visual evaluations can be misleading (due to different scaling and visual biases), we need to check evaluation metrics such as ME, MAE, MAPE, and MSE in Table 9 to choose the best method, using the last 10 observations in our sample. According to these metrics, the Holt-Winters (Double Exp Smo) methods with estimated alpha and beta perform the best.

Method	ME	MAE	MAPE	MSE
Running Avg	-2.95	3.45	9.53	18.77
Running Trend	3.96	4.22	10.52	24.51
Random Walk	0.91	2.05	5.28	6.40
Random Walk with Drift	1.12	2.16	5.57	7.03
Exp Smo (Est)	1.26	1.86	4.77	5.36
Double Exp Smo (Est)	**0.31**	**1.45**	**3.75**	**3.67**

Table 9: Forecast comparisons for Time Series 3 for T=31, ...,40
Note: the best value under each evaluation metric is in boldface.

Using all 50 observations:
We now re-evaluate the prediction using observations 41 to 50 to see whether our choice for the best method changes, in Table 9. Surprisingly, the method that performs the best now is the running average. ES and HW perform worse in the ranking. This might be because for the ES and HW to perform well, we should always re-estimate the alpha and beta parameters. The running average method performed worse initially due to its inability to account for abrupt and temporal changes in the trend. However, it is a good method to predict the long run stability, as shown in its ability to predict well the last 10 observations. We can also check this visually by looking at the last 10 observations of Figure 26.

Method	ME	MAE	MAPE	MSE
Running Avg	-0.46	**1.06**	**2.65**	**1.70**
Running Trend	4.19	4.19	10.20	19.89
Random Walk	**-0.33**	1.45	3.59	3.24
Random Walk with Drift	-0.26	1.45	3.58	3.26
Exp Smo (Est)	-0.35	1.28	3.19	2.23
Double Exp Smo (Est)	-0.82	1.53	3.80	4.12

Table 10: Forecast comparisons for Time Series 3 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

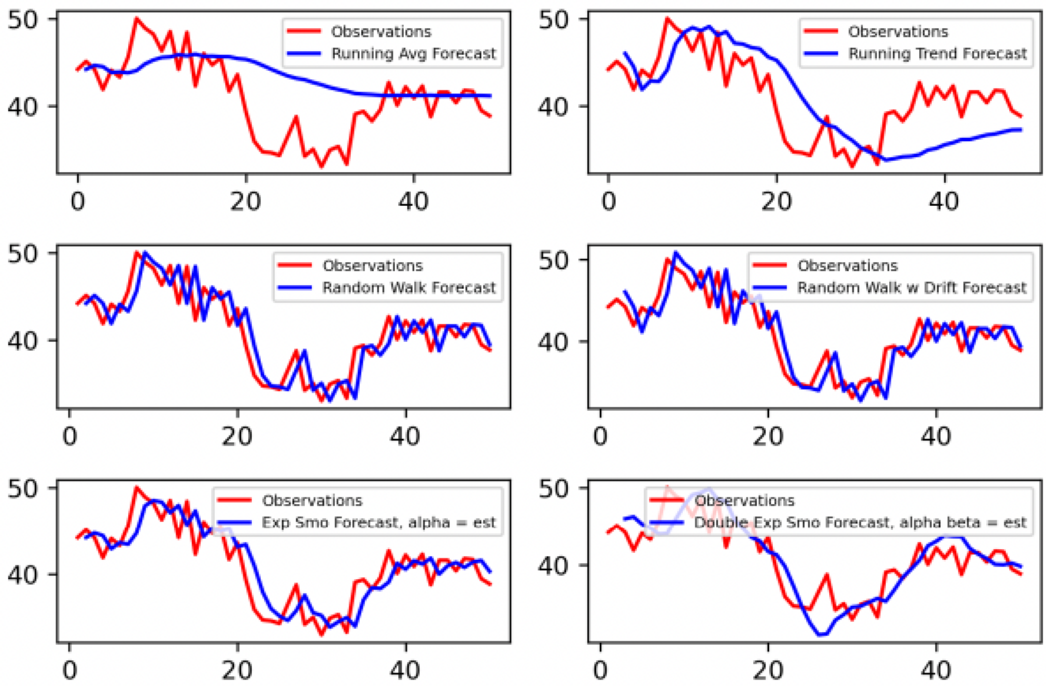

Figure 25: All forecasts methods for 50 observations

Repeating the same steps and analysis, we make the forecast for the other 8 time series below:

Time series 1

Using the first 40 observations:

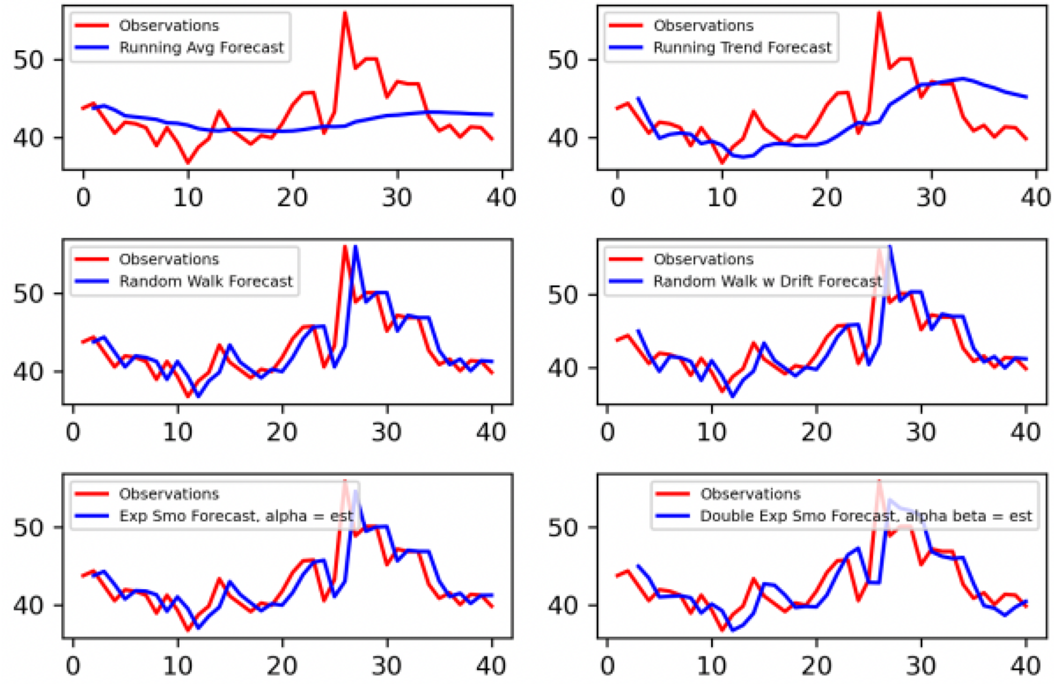

Figure 26: All forecasts methods, with 40 observations

Method	ME	MAE	MAPE	MSE
Running Avg	**-0.23**	2.60	5.99	8.02
Running Trend	-3.71	3.78	9.17	19.70
Random Walk	-0.53	1.33	3.14	3.14
Random Walk with Drift	-0.53	1.35	3.19	3.20
Exp Smo (Est)	-0.63	**1.26**	**2.99**	3.05
Double Exp Smo (Est)	**0.23**	1.41	3.35	**2.94**

Table 11: Forecast comparisons for Time Series 1 for T=31, ...,40

Note: the best value under each evaluation metric is in boldface.

Looking at the evaluation metrics and the plots, the preferred method is the exponential smoothing, having the best scores for 2 evaluation metrics (MAE and MAPE). Even though the HW also have the best score for 2 metrics (ME and MSE), the ME metrics can be misleading as it can average out negative and positive errors.

Using all 50 observations:

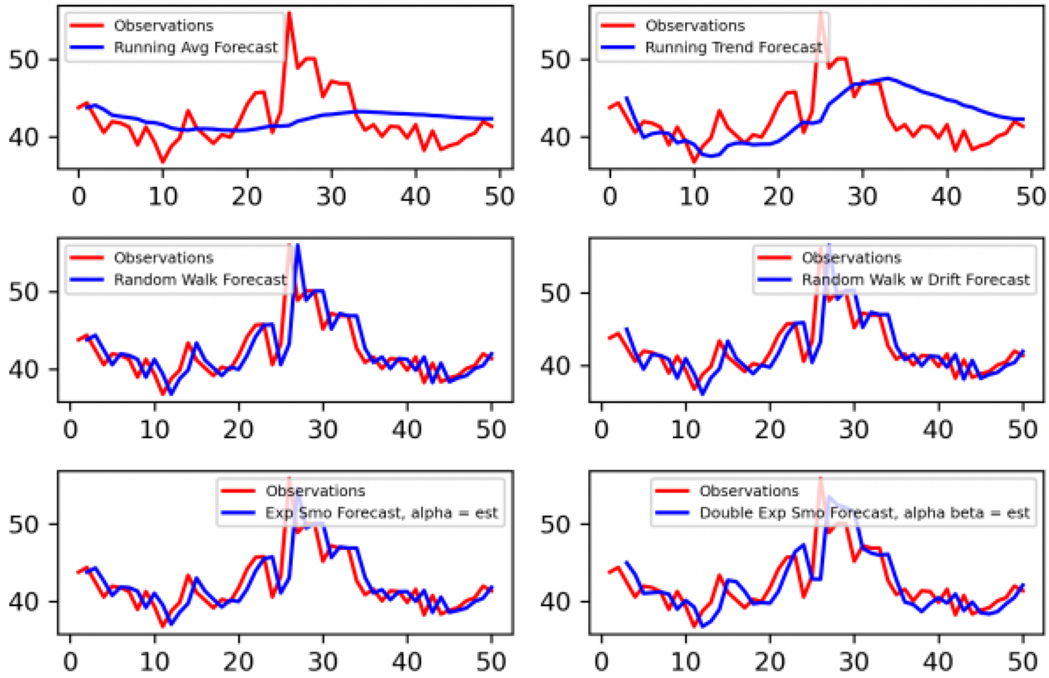

Figure 27: All forecasts methods, with 50 observations

Method	ME	MAE	MAPE	MSE
Running Avg	-2.49	2.49	6.32	8.08
Running Trend	-3.24	3.24	8.22	13.70
Random Walk	**0.15**	1.41	3.54	2.97
Random Walk with Drift	0.24	1.46	3.67	3.07
Exp Smo (Est)	0.16	**1.31**	**3.29**	2.57
Double Exp Smo (Est)	0.36	1.35	3.37	**2.26**

Table 12: Forecast comparisons for Time Series 1 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

Looking at the evaluation metrics and the plots, the preferred method is the exponential smoothing, our initial choice for best method does not change.

Time series 2

Using the first 40 observations:

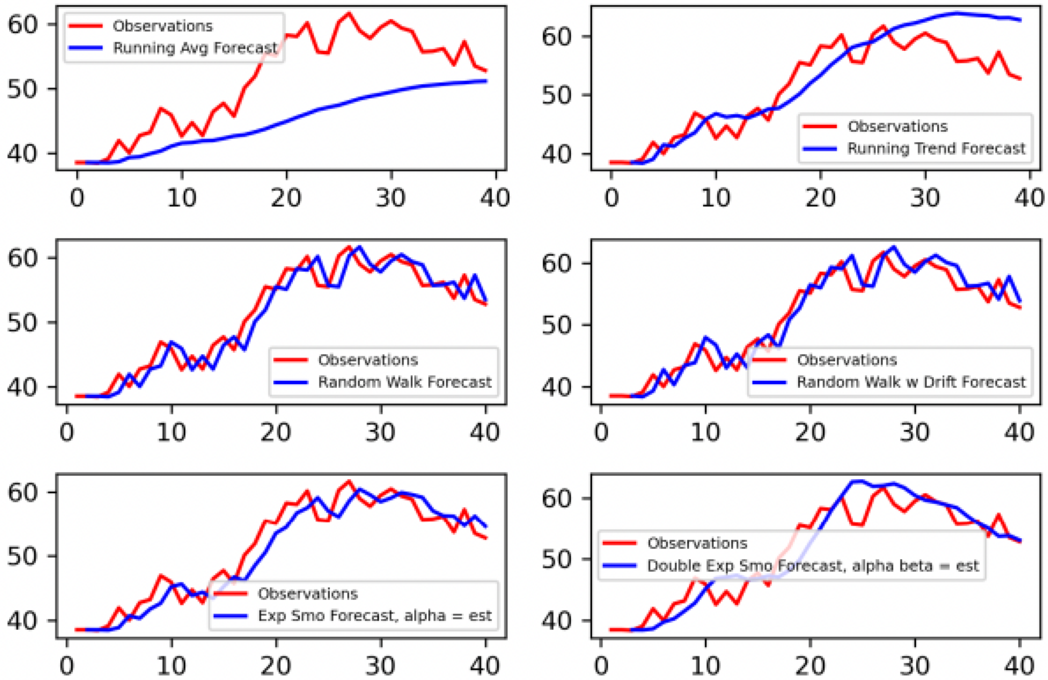

Figure 28: All forecasts methods, with 40 observations

Method	ME	MAE	MAPE	MSE
Running Avg	5.89	5.89	10.23	43.54
Running Trend	-6.95	6.95	12.54	54.74
Random Walk	-0.67	1.69	3.04	4.70
Random Walk with Drift	-1.23	1.92	3.46	5.87
Exp Smo (Est)	-0.93	1.73	3.11	4.07
Double Exp Smo (Est)	**-0.10**	**1.07**	**1.91**	**2.48**

Table 13: Forecast comparisons for Time Series 1 for T=31, ...,40
Note: the best value under each evaluation metric is in boldface.

Looking at the evaluation metrics and the plots, the preferred method is the Holt-Winters.

Using all 50 observations:

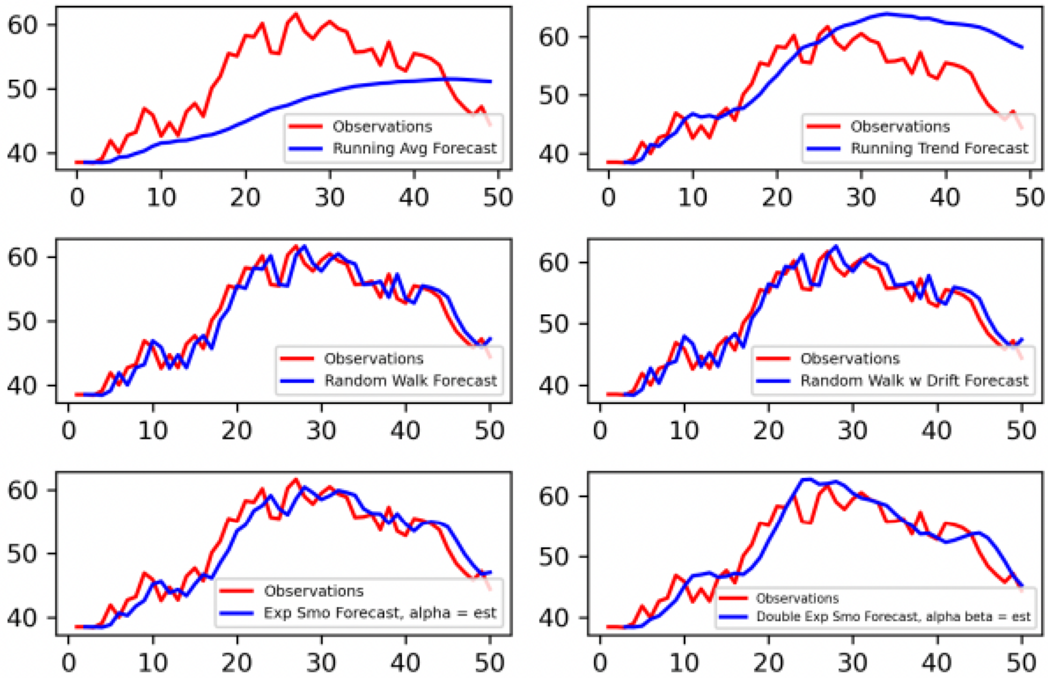

Figure 29: All forecasts methods, with 50 observations.

Method	ME	MAE	MAPE	MSE
Running Avg	-1.08	3.84	7.83	17.18
Running Trend	-10.57	10.57	21.60	119.44
Random Walk	**-0.84**	**1.66**	**3.37**	**3.61**
Random Walk with Drift	-1.14	1.85	3.75	4.15
Exp Smo (Est)	-1.35	1.93	3.96	5.18
Double Exp Smo (Est)	-0.87	2.42	4.86	8.20

Table 14: Forecast comparisons for Time Series 1 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

Looking at the evaluation metrics and the plots, the best method is the random walk, our initial method recommendation changes.

Time series 4

Using the first 40 observations:

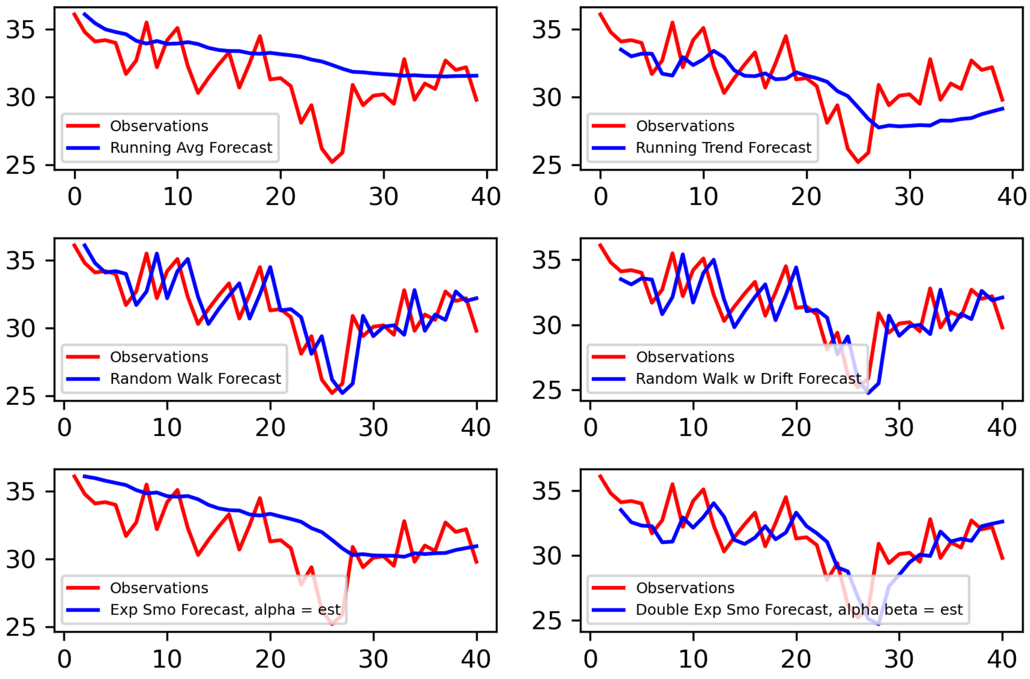

Figure 30: All forecasts methods, with 40 observations

Method	ME	MAE	MAPE	MSE
Running Avg	-0.52	1.22	3.99	**1.80**
Running Trend	2.67	2.67	8.47	8.64
Random Walk	**-0.03**	1.41	4.53	3.27
Random Walk with Drift	0.12	1.43	4.60	3.39
Exp Smo (Est)	0.58	**1.10**	**3.46**	1.84
Double Exp Smo (Est)	-0.15	1.18	3.81	2.41

Table 15: Forecast comparisons for Time Series 4 for T=31, ...,40
Note: the best value under each evaluation metric is in boldface.

Looking at the evaluation metrics and the plots, the preferred method is the exponential smoothing, having the best scores for 2 evaluation metrics (MAE and MAPE).

Using all 50 observations:

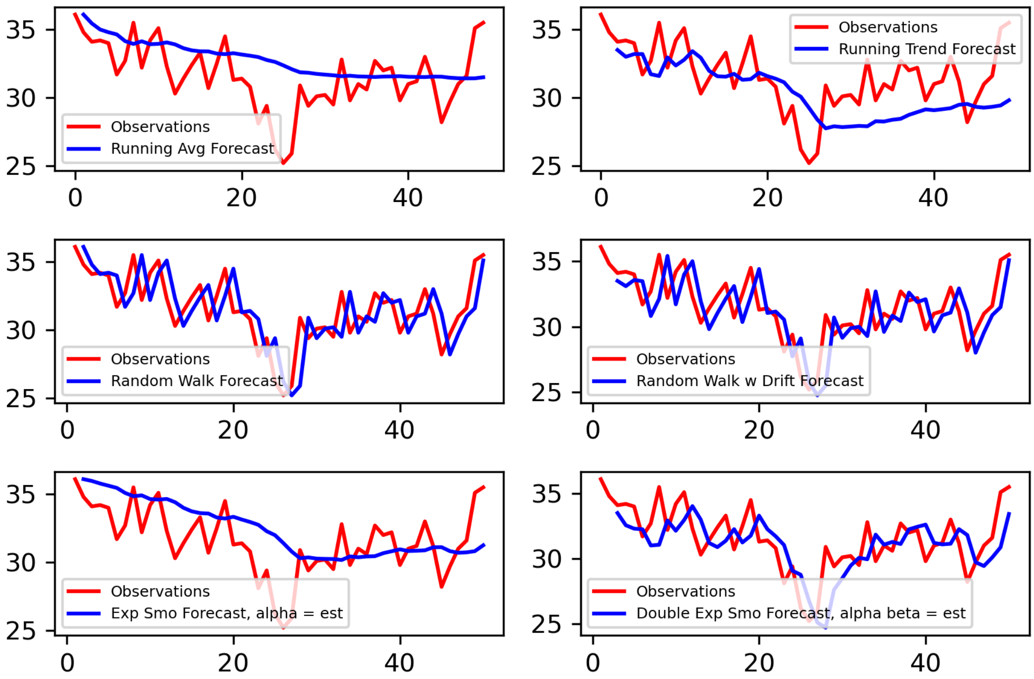

Figure 31: All forecasts methods, with 50 observations

Method	ME	MAE	MAPE	MSE
Running Avg	**0.26**	1.61	4.99	4.68
Running Trend	2.39	2.65	8.06	9.97
Random Walk	0.57	**1.53**	**4.86**	**3.37**
Random Walk with Drift	0.68	1.61	5.10	3.55
Exp Smo (Est)	0.84	1.65	5.04	5.16
Double Exp Smo (Est)	0.65	1.62	5.07	4.46

Table 16: Forecast comparisons for Time Series 1 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

Here the preferred method is the random walk which has three best score in MAE, MAPE and MSE. It is different from the method we apply methods with 40 observations.

Time series 5

Using the first 40 observations:

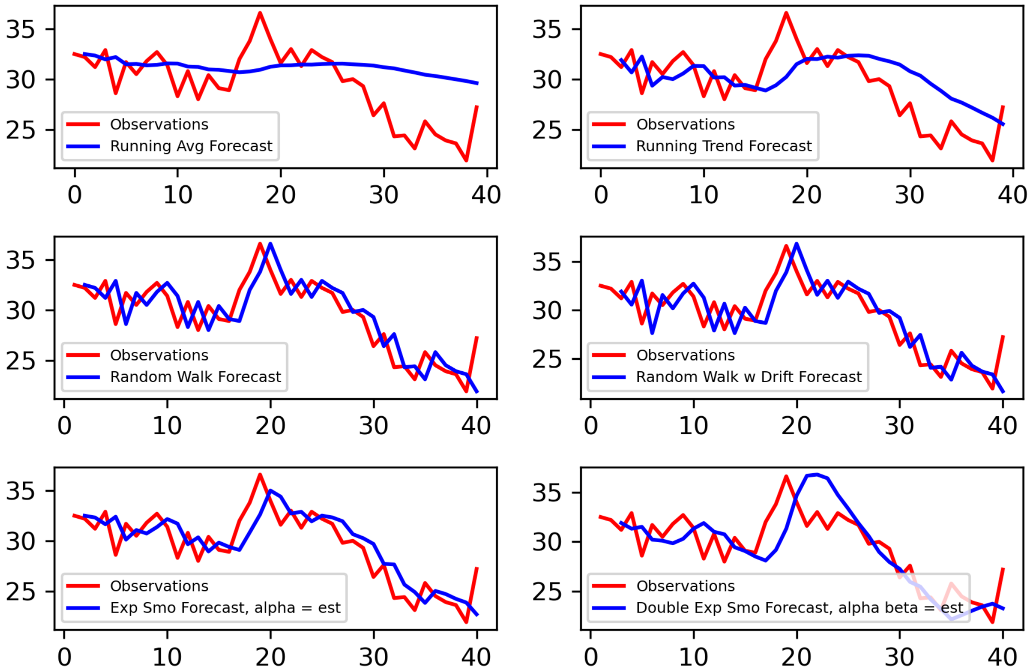

Figure 32: All forecasts methods, with 40 observations

Method	ME	MAE	MAPE	MSE
Running Avg	-5.78	5.78	23.99	36.10
Running Trend	-3.45	3.78	15.63	16.23
Random Walk	**0.08**	1.78	7.08	5.44
Random Walk with Drift	0.31	1.75	6.91	5.66
Exp Smo (Est)	-0.39	1.69	6.83	4.54
Double Exp Smo (Est)	0.91	**1.54**	**6.10**	**4.03**

Table 17: Forecast comparisons for Time Series 5 for T=31, ...,40
Note: the best value under each evaluation metric is in boldface.

The preferred method is the double exponential smoothing for the 40 observations who includes three best score in MAE, MAPE and MSE.

Using all 50 observations:

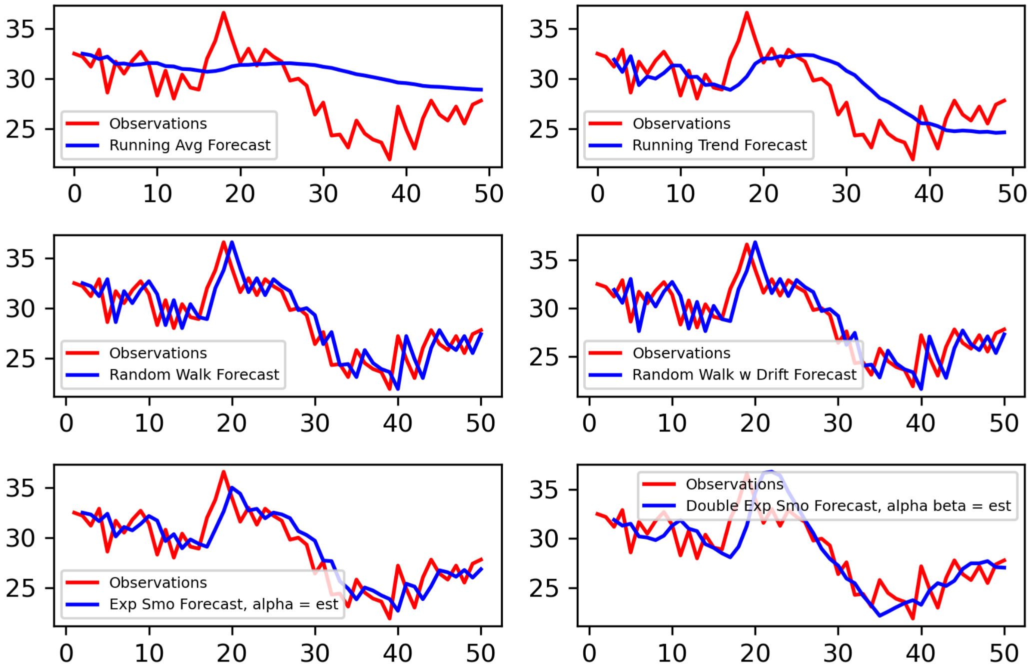

Figure 33: All forecasts methods, with 50 observations

Method	ME	MAE	MAPE	MSE
Running Avg	-2.98	2.98	11.76	11.36
Running Trend	1.34	1.91	7.22	4.51
Random Walk	**0.06**	1.64	6.33	3.21
Random Walk with Drift	0.21	1.65	6.34	3.31
Exp Smo (Est)	0.34	1.32	5.06	2.27
Double Exp Smo (Est)	-0.32	**1.13**	**4.41**	**2.00**

Table 18: Forecast comparisons for Time Series 1 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

The preferred method is also the double exponential smoothing for the 50 observations who includes three best score in MAE, MAPE and MSE, which is the same method that we applied for the 40 observations.

Time series 6

Using the first 40 observations:

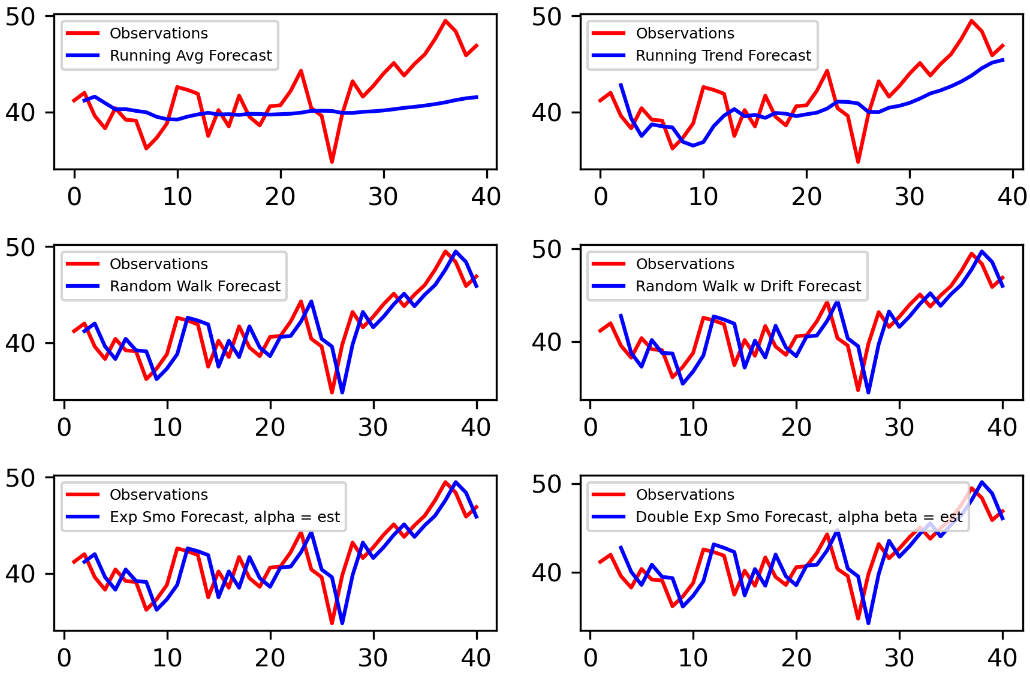

Figure 34: All forecasts methods, with 40 observations

Method	ME	MAE	MAPE	MSE
Running Avg	5.41	5.41	11.60	31.57
Running Trend	3.09	3.09	6.64	11.49
Random Walk	0.42	1.40	3.03	2.17
Random Walk with Drift	0.29	1.38	2.98	**2.15**
Exp Smo (Est)	0.42	1.40	3.03	2.17
Double Exp Smo (Est)	**0.01**	**1.32**	**2.85**	2.19

Table 19: Forecast comparisons for Time Series 5 for T=31, ...,40
Note: the best value under each evaluation metric is in boldface.

The preferred method is the double exponential smoothing for the 40 observations who contains three best score in ME, MAE and MAPE. It should be noted that the random walk and the exponential smoothing have the same results for the *Time series 6*.

Using all 50 observations:

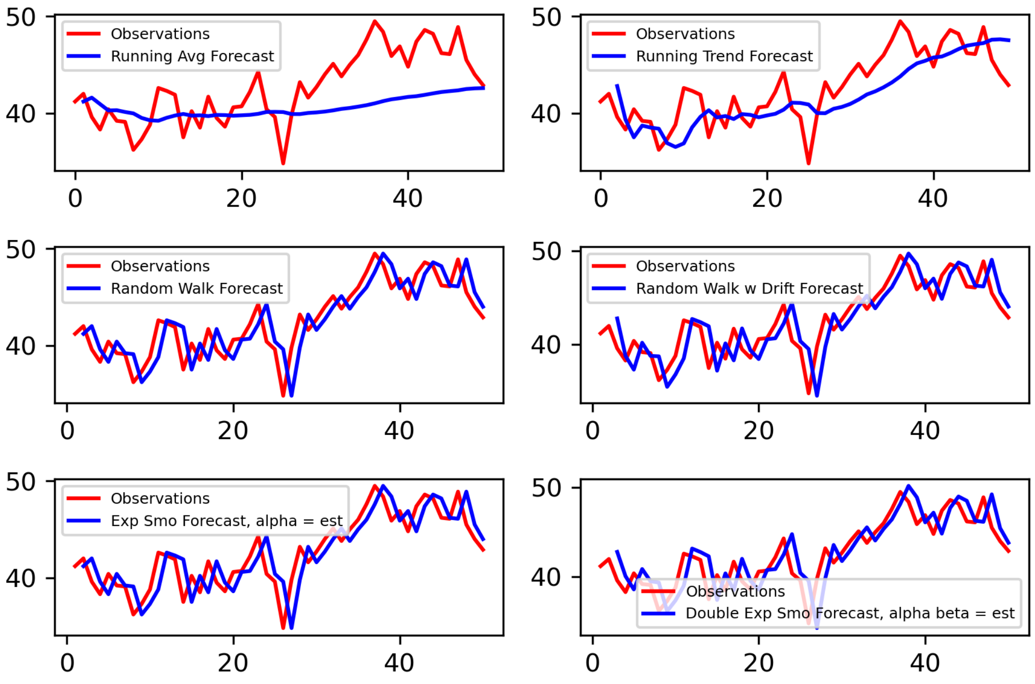

Figure 35: All forecasts methods, with 50 observations

Method	ME	MAE	MAPE	MSE
Running Avg	4.08	4.08	8.65	20.96
Running Trend	-0.59	2.03	4.45	5.50
Random Walk	**-0.40**	**1.72**	**3.72**	**3.96**
Random Walk with Drift	-0.53	1.78	3.85	4.14
Exp Smo (Est)	**-0.40**	**1.72**	**3.72**	**3.96**
Double Exp Smo (Est)	-0.56	1.80	3.89	4.34

Table 20: Forecast comparisons for Time Series 1 for T=41, ...,50
Note: the best value under each evaluation metric is in boldface.

The preferred method shown by Table 20 are the random walk and the exponential smoothing for the 50 observations who contains all four best score in ME, MAE and MAPE and MSE. Their assessment errors are the same. This is because the estimated $\hat{\alpha} = 1$ so that the exponential smoothing method becomes random walk method.

Time series 7

Using the first 40 observations:

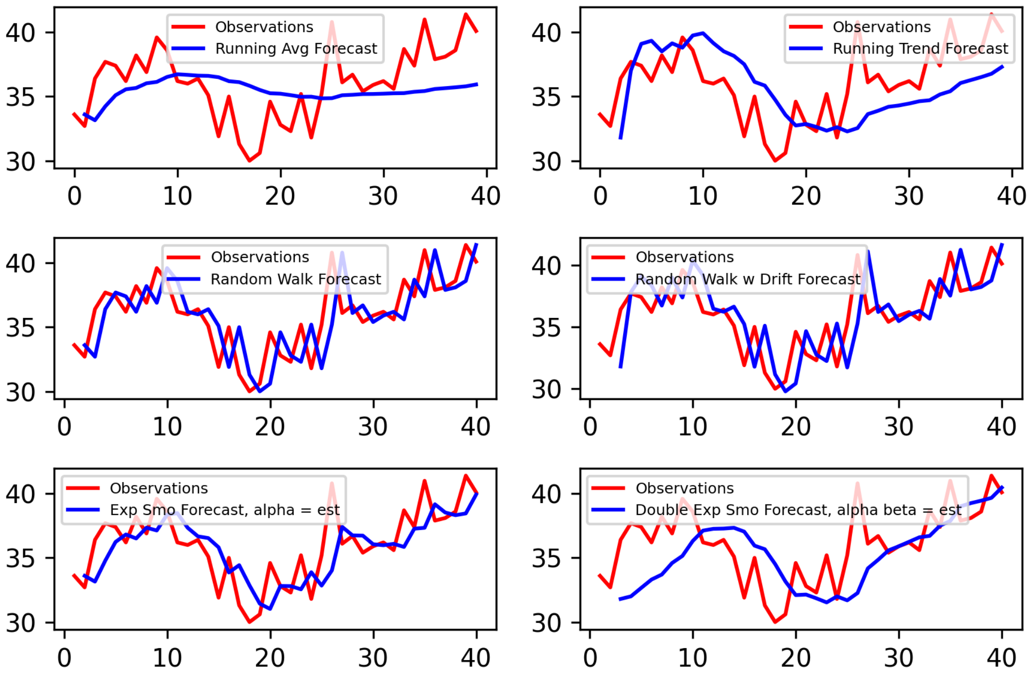

Figure 36: All forecasting methods, with first 40 observations

Method	ME	MAE	MAPE	MSE
Random walk	0.42	1.68	4.28	4.41
Running average	2.98	2.98	7.55	11.66
Running trend	2.77	2.77	7.07	9.66
Random Walk with Drift	0.29	1.68	4.29	4.45
Exp Smo (Est)	0.81	1.24	3.14	3.24
Double Exp Smo (Est)	**0.22**	**1.15**	**2.93**	**2.12**

Table 21: Forecast comparisons for Time Series 7 for T = 31, ..., 40
Note: the best value under each evaluation metric is in boldface.

According to Table 21, the Holt-Winters forecasting method is preferred because it attains the lowest MAE, MAPE, and MSE.

Using all 50 observations:

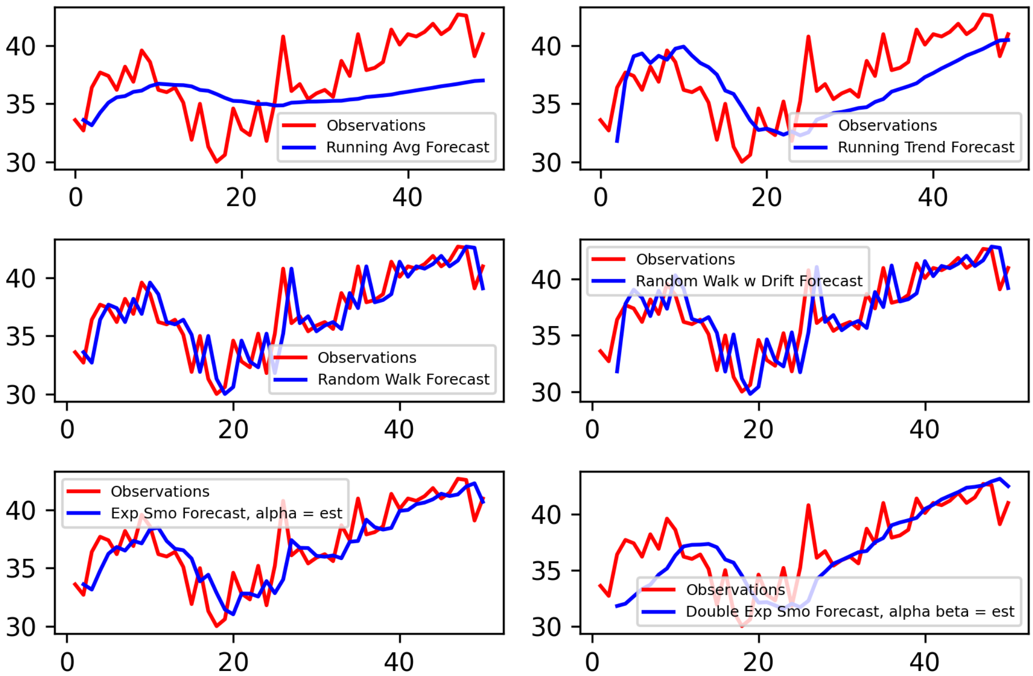

Figure 37: All forecasting methods, with first 50 observations

Method	ME	MAE	MAPE	MSE
Random walk	**0.09**	1.03	2.54	1.99
Running average	4.73	4.73	11.40	23.43
Running trend	2.05	2.32	5.61	6.11
Random Walk with Drift	$-$**0.09**	1.01	2.48	2.02
Exp Smo (Est)	0.17	**0.89**	**2.19**	**1.51**
Double Exp Smo (Est)	-0.89	0.95	2.36	2.21

Table 22: Forecast comparisons for Time Series 7 for T = 41, ...,50
Note: the best value under each evaluation metric is in boldface.

The Table 22 shows that the exponential smoothing is preferred because it attains the lowest MAE, MAPE, and MSE. Thus we need to adjust our initial recommendation.

Time series 8

Using the first 40 observations:

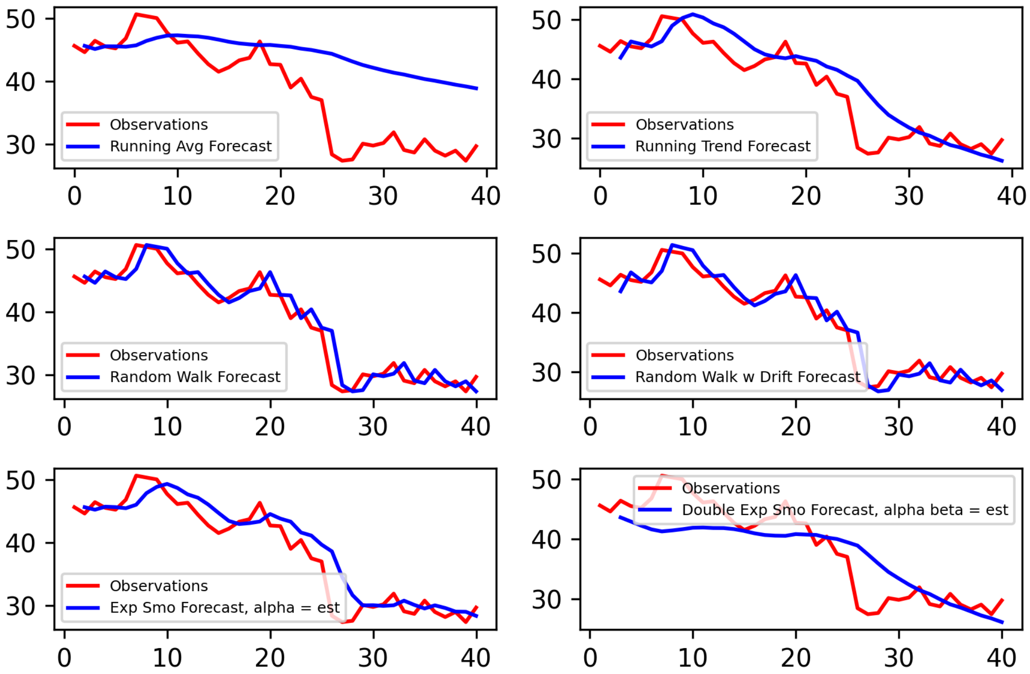

Figure 38: All forecasting methods, with first 40 observations

Method	ME	MAE	MAPE	MSE
Random walk	**−0.01**	1.47	4.99	2.78
Running average	-10.85	10.85	37.07	118.80
Running trend	0.58	1.35	4.56	2.59
Random Walk with Drift	0.47	1.51	5.09	3.08
Exp Smo (Est)	-0.26	**1.19**	**4.07**	**1.74**
Double Exp Smo (Est)	0.42	1.42	4.81	2.93

Table 23: Forecast comparisons for Time Series 1 for T = 31, ...,40
Note: the best value under each evaluation metric is in boldface.

The Table 23 shows that the exponential smoothing method is the best because it has the lowest MAE, MAPE, and MSE.

Using all 50 observations:

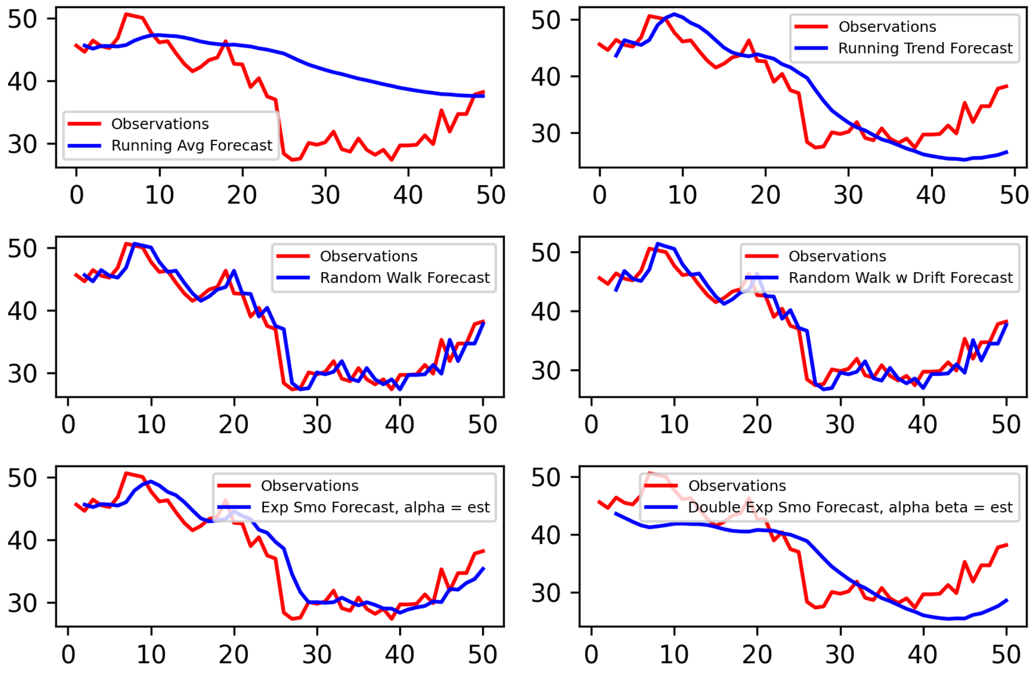

Figure 39: All forecasting methods, with first 50 observations

Method	ME	MAE	MAPE	MSE
Random walk	**0.85**	**1.81**	**5.31**	**6.25**
Running average	-4.62	4.79	15.36	32.83
Running trend	7.59	7.59	22.17	65.99
Random Walk with Drift	1.16	2.00	5.88	6.95
Exp Smo (Est)	1.90	2.01	5.73	6.56
Double Exp Smo (Est)	6.96	6.96	20.42	53.75

Table 24: Forecast comparisons for Time Series 1 for T = 41, ...,50
Note: the best value under each evaluation metric is in boldface.

The Table 24 shows that the random walk method is the best because it has the lowest MAE, MAPE, and MSE. Thus we need to adjust our initial recommendation.

Time series 9

Using the first 40 observations:

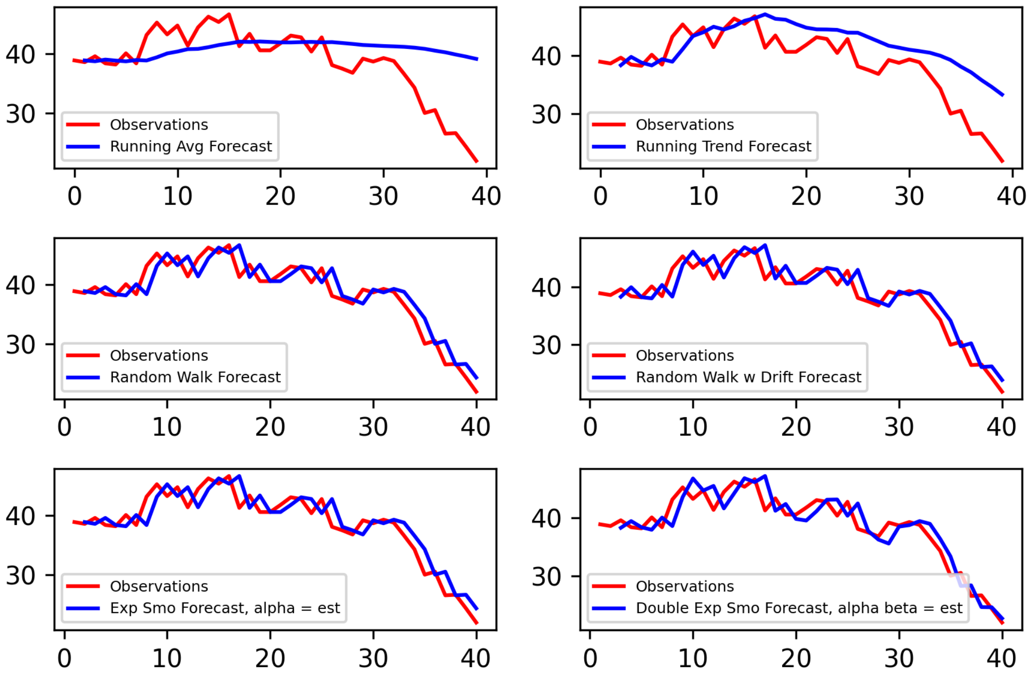

Figure 40: All forecasting methods, with first 40 observations

Method	ME	MAE	MAPE	MSE
Random walk	-1.68	1.92	6.74	5.65
Running average	-9.63	9.63	35.65	119.26
Running trend	-7.12	7.12	26.01	62.55
Random Walk with Drift	-1.50	1.87	6.49	5.06
Exp Smo (Est)	1.68	1.92	6.74	5.65
Double Exp Smo (Est)	**-0.64**	**1.61**	**5.30**	**3.51**

Table 25: Forecast comparisons for Time Series 1 for T = 31, ...,40

Note: the best value under each evaluation metric is in boldface.

The Table 25 shows that the Hold-Winters method is the best because it has the lowest MAE, MAPE, and MSE.

Using all 50 observations:

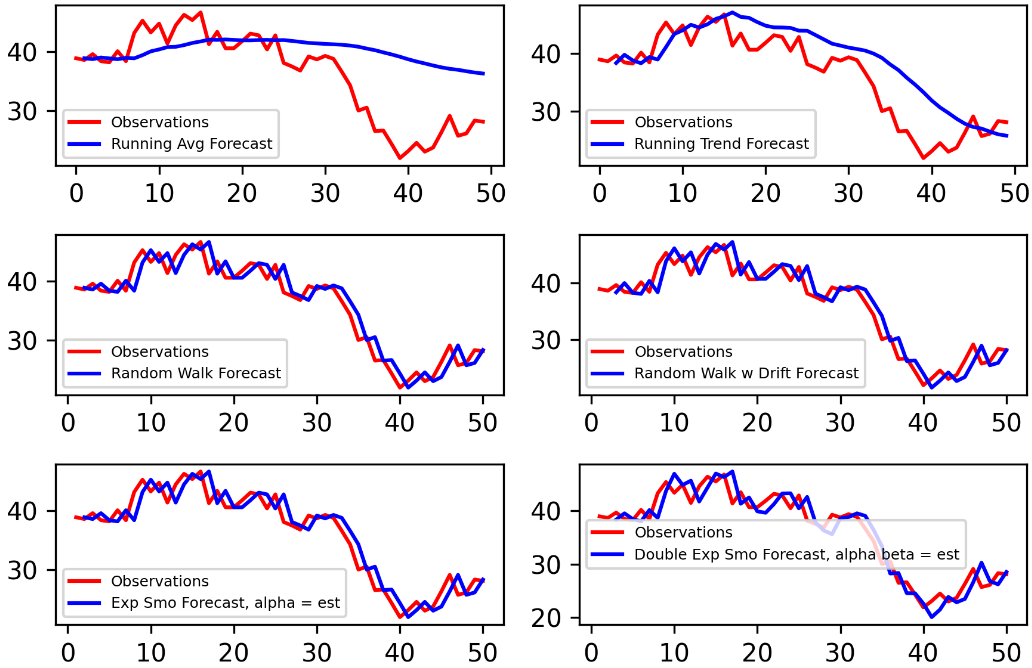

Figure 41: All forecasting methods, with first 50 observations

Method	ME	MAE	MAPE	MSE
Random walk	**0.62**	**1.64**	**6.31**	**3.73**
Running average	-11.57	11.57	46.02	141.45
Running trend	-2.33	3.62	14.81	20.22
Random Walk with Drift	0.94	1.81	6.97	4.34
Exp Smo (Est)	**0.62**	**1.64**	**6.31**	**3.73**
Double Exp Smo (Est)	0.77	2.08	8.13	5.96

Table 26: Forecast comparisons for Time Series 1 for T = 41, ...,50
Note: the best value under each evaluation metric is in boldface.

The Table 26 shows that the random walk method and exponential smoothing method are the best because it has the lowest MAE, MAPE, and MSE. Thus we need to adjust our initial recommendation. The assessment errors from the two methods are the same. This is because the estimated $\hat{\alpha} = 1$ so that the exponential smoothing method becomes random walk method.

Q(c)

For this part of Q(c), we apply the seasonal random walk with drift, running seasonal regression, Holt-Winters Multiplicative seasonal method, and Holt-Winters Additive seasonal method to forecast umbrella sales. the forecasting results are shown in Figure 42.

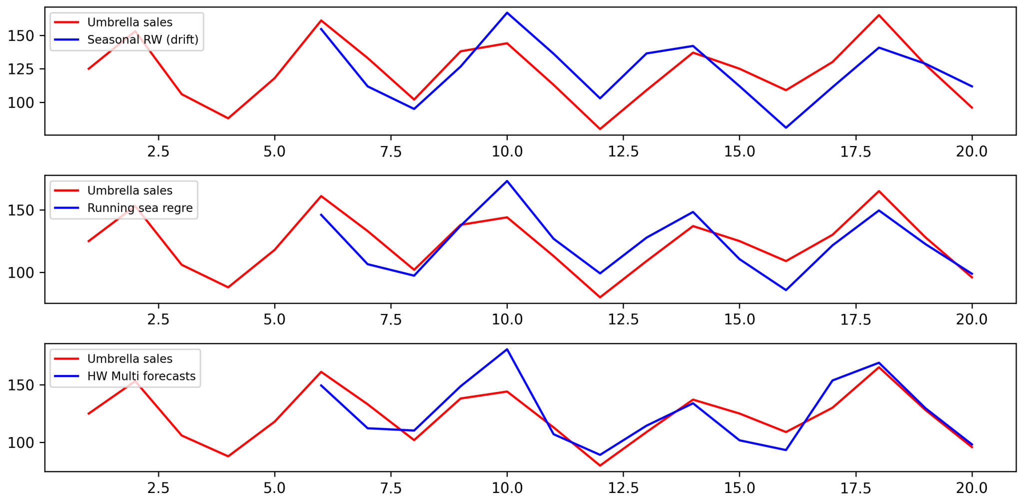

Figure 42: Umbrella sales forecasts

The forecasting precision is always measured using the last 12 observations. When each of the method is applied, the parameters of the method will be optimized according to the specified assessment error. For example, when we use Holt-Winters Multiplicative seasonal method to forecast and use MSE to measure the forecasting error, the parameters of this method have been optimized by selecting the parameter values with minimal MSE. The Table 27 shows the results forecast precision of the four methods.

Method	ME	MAE	MAPE	MSE
Seasonal random walk with drift	$\mathbf{-1.88}$	17.77	15.36	385.95
Running seasonal regression	-2.27	13.56	11.53	248.45
Holt-Winters Multiplicative seasonal method	-12.99	**11.72**	**9.41**	**247.10**
Holt-Winters Additive seasonal method	-5044.09	30.95	25.73	1559.02

Table 27: Forecast comparisons, 12 forecasts
Note: the best value under each evaluation metrics is in boldface.

The Table 27 shows that the Holt-Winters Multiplicative seasonal method has lowest MAE, MAPE, and MSE among all forecasting methods. The seasonal random walk with drift has lowest absolute ME error but ME is not reliable because it can be canceled out by different values of itself. Therefore, we can conclude that the Holt-Winters Multiplicative seasonal method is preferred in terms of forecasting umbrella sales.

Q(d)

Sunspots are phenomena on the Sun's photosphere that appear as temporary spots that are darker than the surrounding areas. The Sun undergoes a type of seasonal variability, with its activity waxing and waning over the course of nearly two years, according to a new study by a team of researchers led by the National Center for Atmospheric Research (NCAR). This behavior affects the peaks and valleys in the approximately 11-year solar cycle, sometimes amplifying and sometimes weakening the solar storms that can buffet Earth's atmosphere. Hence, it's meaningful to have a sufficient investigation to the seasonal effect shown by sunspotys. We collect the the data of sunspot [1] for over 80 years. Figure 43 shows a plot of the whole dataset. It is evident from the plotted figure that there is a monthly influence every year. In order to simplify our research, we select only the most recent 7 years (84 observations) as our dataset.

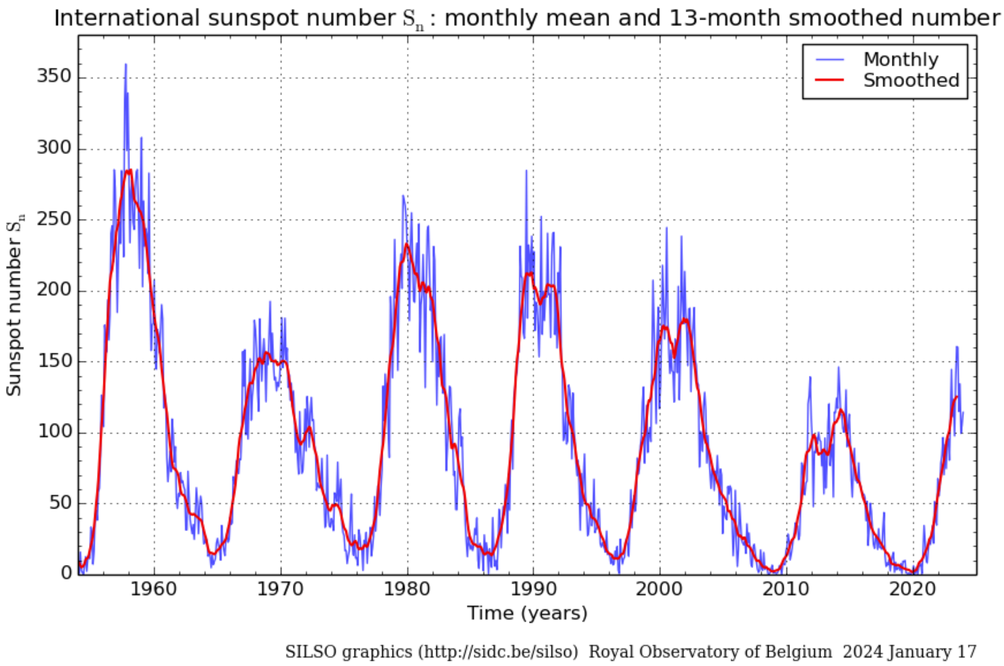

Figure 43: The number of sunspot time series

First of all, we split our dataset into two samples. The first 5 years' dataset is used as our in-sample dataset and the last 2 years' dataset is regarded as out-of-sample dataset. Next, we apply the seasonal random walk with drift, running seasonal regression, Holt-Winters Multiplicative seasonal method, and Holt-Winters Additive seasonal method with our in-sample dataset to forecast the number of sunspots for each month. The evaluation table is shown is Table 28.

Method	ME	MAE	MAPE	MSE
Seasonal random walk with drift	21.28	21.42	78.48	635.53
Running seasonal regression	24.04	24.04	83.68	758.12
Holt-Winters Multiplicative seasonal method	**0.02**	11.32	40.78	220.08
Holt-Winters Additive seasonal method	1.66	**11.11**	**38.97**	**201.21**

Table 28: In-sample forecast comparisons, 12 forecasts
Note: the best value under each evaluation metrics is in boldface.

The Figure 43 shows that the Holt-Winters Multiplicative seasonal method has the lowest mean error.

[1]https://www.sidc.be/SILSO/datafiles

However, because the mean error can be canceled out with each other, thus we do not rely on this single assessment to make decisions. Moreover, the Holt-Winters Additive seasonal method has lowest MAE, MAPE, and MSE values among all forecasting methods, which indicates the Holt-Winters Additive seasonal method is the optimal one in terms of forecasting. Meanwhile, it's also easy to spot that the assessment errors of Holt-Winters Multiplicative seasonal method are not significantly larger that those from Holt-Winters Additive seasonal method. This might comes from the similar derivations of both methods.

Furthermore, the out-of-sample dataset should be employed in case over-fitting issues. If Holt-Winters Additive seasonal method still has the lowest assessment errors, then it's optimality is approved empirically. The Table 29 presents the the out-of-sample assessment errors.

Method	ME	MAE	MAPE	MSE
Seasonal random walk with drift	28.11	32.29	23.90	1615.25
Running seasonal regression	48.01	48.01	36.30	3000.86
Holt-Winters Multiplicative seasonal method	**−7.70**	**23.72**	**20.11**	**1323.68**
Holt-Winters Additive seasonal method	28.34	28.99	21.04	1348.84

Table 29: Out-of-sample forecast comparisons, 12 forecasts
Note: the best value under each evaluation metrics is in boldface.

Unfortunately the Holt-Winters Additive seasonal method loses its optimality and Holt-Winters Multiplicative seasonal method attains the lowest assessment errors in terms of the out-of-sample dataset. But the assessment errors from Holt-Winters Additive seasonal method are not significantly larger than those from the multiplicative one. Thus, the priority of the Holt-Winters seasonal method is proved in both in-sample and out-of-sample datasets no matter the method is multiplicative or additive. Therefore, we can conclude that the Holt-Winters seasonal methods including the multiplicative season method and additive seasonal method are preferred.