EEEE2059
Practical Engineering Design Solutions and Project Development

Electronic Project: Individual Coursework
Bandpass Filter Design

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Submission Date: 4 December 2020

Task 1: Filter characteristic study

Butterworth filter

The Butterworth filter (i.e. maximally flat magnitude filter) is a type of signal processing filter designed to have a frequency response as flat as possible in the passband. The frequency response of the Butterworth filter is maximally flat (i.e. has no ripples) in the passband and rolls off towards zero in the stopband. When viewed on a logarithmic Bode plot, the response slopes off linearly towards negative infinity. A first-order filter's response rolls off at -6 dB per octave (-20 dB per decade) (all first-order lowpass filters have the same normalized frequency response). A second-order filter decreases at -12 dB per octave, a third-order at -18 dB and so on.

The various advantages and disadvantages of Butterworth filters were listed below.

Advantages:

1. Butterworth filters have a monotonically charging magnitude function with ω, unlike other filter types that have non-monotonic ripple in the passband and/or the stopband. In which case, the Butterworth filters have monotonic amplitude response in both passband and stopband.
2. Butterworth filters have a more linear phase response in the pass-band than Chebyshev Type I/Type II can achieve.
3. The passband is maximally flat indicating a smooth and monotonically decreasing frequency response.
4. Quick roll-off (i.e. steeper attenuation slope) around the cutoff frequency, which improves with increasing order.
5. Butterworth filters have no passband ripple.
6. A good compromise between sharp amplitude response and approximate linearity of phase response can be obtained with the Butterworth filter.

Disadvantages:

1. Compared with a Chebyshev Type I/Type II filter, the Butterworth filter has a slower roll-off, and thus will require a higher order to implement a particular stopband specification.
2. Considerable overshoot and ringing in step response, which worsens with increasing order in which case the quick roll-off and the unsignificant overshoot and ringing cannot be achieved simultaneously.
3. Slightly non-linear phase response.
4. Group delay largely frequency-dependent.
5. To achieve better performance, a higher order design of Butterworth filter was needed which means adding more components to the circuit.
6. Accuracy of approximation cannot be uniformly distributed in PB and SB regions.

Chebyshev filter

Chebyshev filters are analog or digital filters having a steeper roll-off than Butterworth filters, and have passband ripple (type I) or stopband ripple (type II). The type I Chebyshev filters are called usually as just “Chebyshev filters”, the type II ones are usually called “inverse Chebyshev filters”. Because of the passband ripple inherent in the Chebyshev filters, the ones that have a smoother response in the passband but a more irregular response in the stopband are preferred for some applications.

The various advantages and disadvantages of Chebyshev filters were listed below.

Advantages:

1. Chebyshev filters have the property that they minimize the error between the idealized and the actual filter characteristic over the range of the filter.
2. The attenuation between the passband and stopband is much steeper than a Butterworth filter if both filters are of the same filter order, in short, Chebyshev filters are sharper than the Butterworth filters.
3. Chebyshev Type II filters have flat passbands (no ripple), making them a good choice for DC and low frequency measurement applications, such as bridge sensors (e.g. loadcells).
4. The Chebyshev Type I filters roll-off faster than Butterworth and Chebyshev Type II.
5. The Chebyshev Type I filters have maximally flat stopband.
6. The Chebyshev Type I filter is a good compromise between Elliptic and Butterworth.
7. Although filters designed using the Type II method are slower to roll-off than those designed with the Chebyshev Type I as method, the roll-off is faster than those designed with the Butterworth method.

Disadvantages:

1. Chebyshev filters have ripples in the passband.
2. The step response offers more ringing than the Butterworth filter.
3. The desirable property of Chebyshev Type II filter comes at the expense of wider transition bands, resulting in low passband to stopband transition (slow roll-off).
4. The Chebyshev Type I filters have passband ripple and very non-linear passband phase characteristics.

Bessel filter

In electronics and signal processing, a Bessel filter is a type of analogue linear filter with a maximally flat group/phase delay (maximally linear phase response). Which preserves the wave shape of filtered signals in the passband. Bessel filters are often used in audio crossover systems.

The various advantages and disadvantages of Bessel filters were listed below.

Advantages:

1. The Bessel filter preserves the wave shape of filtered signals in the passband.
2. The Bessel filter tends towards the same shape as filter order increases.
3. Though the Bessel filter has a small amount of overshoot, but still much less than common frequency domain filters.
4. The maximally flat group delay of the Bessel filter means that it equally exhibits a maximally linear phase response.
5. A direct result of the maximally flat group delay of the Bessel filter is giving an output for a square wave input with no overshoot because all the frequencies are delayed by the same amount.

Disadvantages:

1. The transition from the pass band to the stop band for the Bessel filter is much slower or shallower than for other filters.
2. The slow roll-offs of Bessel filters result in wide transition regions.
3. The cutoff frequency of a Bessel filter changes as function of n.

Task 2: High pass Butterworth filter design

Design calculations

The order of filter that can meet the specification was determined first. Since the filter to design was a unity filter with filter gain of 1, the maximum gain in the passband ϵ was set to 1.

The transfer function of an N^{th} order high pass Butterworth filter was given as:

$$H(j\omega) = \frac{-1}{\sqrt{1+\epsilon^2\left(\frac{\omega p}{\omega}\right)^{2N}}} \qquad \text{Eq. 1}$$

where $\omega = 2\pi f$ and the ω_p was the pass band frequency of 100 Hz.

From the design specifications, when at the stop band frequency of 50 Hz, the stop band attenuation was to be 30 dB. By manipulating this formula and substituting the known values of the pass band frequency, the stop band frequency, and the response of the filter at the stop band frequency into the Equation 1 above, the order of the filter to be designed was obtained:

$$-30\ dB = A = 20\log|H(j\omega)| = 20\log\left|\frac{-1}{\sqrt{1+1^2\times\left(\frac{2\pi\times100\ Hz}{2\pi\times50\ Hz}\right)^{2N}}}\right| \quad \text{Eq. 2}$$

Solving the Equation 2 above, N was calculated as 4.98 in which case N=5 was taken by approximation; therefore, a 5th order Butterworth filter was to be designed.

Table 1. Butterworth Polynomials

Order	Butterworth Polynomials in Factored Form
2	$(1+1.414s+s^2)$
3	$(1+s)(1+s+s^2)$
4	$(1+0.765s+s^2)(1+1.848s+s^2)$
5	$(1+s)(1+0.618s+s^2)(1+1.618s+s^2)$

According to the table of Butterworth Polynomials in Factored Form for different orders as shown above, the 5th order Butterworth filter can be created from two stages of 2nd order filter circuits and one 1st order filter circuit, with different component values in each stage to produce the desired response. Therefore, it was obtained that Q1=0.618 and Q2=1.618.

The transfer function of a second order high pass Sallen-Key filter was given by:

$$H(s) = \frac{G}{\frac{1}{s^2R_1R_2C_1C_2}+\left(\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{1-G}{R_1C_1}\right)+1} \quad \text{Eq. 3}$$

where G was the gain in the pass band.

Setting G=1 and with some simple manipulation:

$$H(s) = \frac{s^2R_1R_2C_1C_2}{s^2R_1R_2C_1C_2+s(R_1C_2+R_1C_1)+1} \quad \text{Eq. 4}$$

By setting $s = j\omega$:

$$\omega_c = \frac{1}{\sqrt{R_1R_2C_1C_2}} \quad \text{Eq. 5}$$

$$Q = \frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_2+C_1)} \quad \text{Eq. 6}$$

Normally resistor values are chosen to be at least 1 kΩ to avoid too much current being drawn from the op-amp. An upper limit of 100 kΩ is usually used to avoid Johnson noise becoming too large. Capacitor values larger than 1 nF are used; smaller capacitor values than this are comparable to the transistor parasitic capacitor values, which then affect the frequency response of the filter. An upper limit of several μF are used – capacitors that are too large become bulky.

As shown in the Equation 6, only one resistor value was in the denominator. Therefore, C1=C2=100 nF were chosen, then R1 and R2 were to be solved. Also, $\omega_c = 2\pi(100)\ rad/s$ which was the corner frequency between the pass band and stop band.

2nd order Butterworth filter with Q=0.618 design:

From Equation 5 above:

$$2\pi(100)\ {rad}/{s} = \frac{1}{100\ nF \times \sqrt{R_1 R_2}} \qquad \text{Eq. 7}$$

From Equation 6 above:

$$0.618 = \frac{\sqrt{R_1 R_2}}{2R_1} \qquad \text{Eq. 8}$$

Therefore, from Equation 7 above:

$$\sqrt{R_1 R_2} = \frac{1}{100\ nF \times 2\pi(100)\ {rad}/{s}} = 15915.5\ \Omega \qquad \text{Eq. 9}$$

Substituting Equation 9 into Equation 8:

$$R_1 = \frac{\sqrt{R_1 R_2}}{2 \times 0.618} = \frac{15915.5\ \Omega}{2 \times 0.618} = 12876.6\ \Omega \qquad \text{Eq. 10}$$

Substituting R1 into Equation 9:

$$R_2 = 19671.6\ \Omega \qquad \text{Eq. 11}$$

Similarly, 2nd order Butterworth filter with Q=1.618 design:

From Equation 5 above:

$$2\pi(100)\ {rad}/{s} = \frac{1}{100\ nF \times \sqrt{R_1 R_2}} \qquad \text{Eq. 12}$$

From Equation 6 above:

$$1.618 = \frac{\sqrt{R_1 R_2}}{2R_1} \qquad \text{Eq. 13}$$

Therefore, from Equation 12 above:

$$\sqrt{R_1 R_2} = \frac{1}{100\ nF \times 2\pi(100)\ {rad}/{s}} = 15915.5\ \Omega \qquad \text{Eq. 14}$$

Substituting Equation 14 into Equation 13:

$$R_1 = \frac{\sqrt{R_1 R_2}}{2 \times 1.618} = \frac{15915.5\ \Omega}{2 \times 1.618} = 4918.3\ \Omega \qquad \text{Eq. 15}$$

Substituting R1 into Equation 14:

$$R_2 = 51502.2\ \Omega \qquad \text{Eq. 16}$$

Lastly, 1st order Butterworth filter design:

The transfer function for 1st order Butterworth high pass filter:

$$H(s) = \frac{G\omega_0}{s+\omega_0} \qquad \text{Eq. 17}$$

where G=1 and $\omega_0 = \frac{1}{RC}$ which was the cutoff frequency (i.e. pass band frequency of 100 Hz).

Therefore, using C=100 nF, the component value of the resistor was obtained:

$$R = \frac{1}{\omega_0 C} = \frac{1}{2\pi(100)\ rad/s \times 100\ nF} = 15915.5\ \Omega \qquad \text{Eq. 18}$$

In summary, the component values for the 5th order Butterworth high pass filter design were listed in the Table 2 below.

Table 2. Component values for 5th order Butterworth high pass filter design

Stage	Component	Value
2nd order Butterworth high pass filtering Q=0.618	C1	100 nF
	C2	100 nF
	R1	12876.6 Ω
	R2	19671.6 Ω
2nd order Butterworth high pass filtering Q=1.618	C3	100 nF
	C4	100 nF
	R3	4918.3 Ω
	R4	51502.2 Ω
1st order Butterworth high pass filtering	C5	100 nF
	R5	15915.5 Ω

The general form of a unity gain, second order Sallen-Key filter was as shown in the Figure 1 below. The circuits contained two capacitors and two resistors. The generalized Z impedances were replaced with resistors or capacitors in the arrangement as shown in the Figure 2 below to produce a high pass circuit behavior.

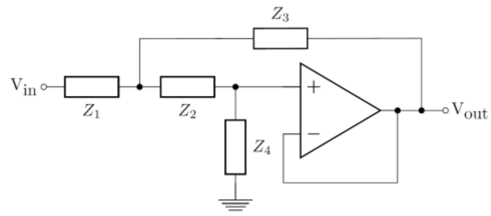

Figure 1. 2nd order Sallen-Key filter topology

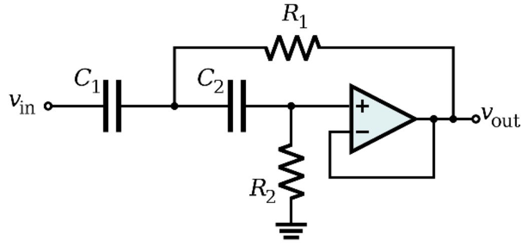

Figure 2. High pass arrangement of 2nd order Sallen-Key filter topology

LTspice implementation and simulations

The 5th order Butterworth high pass filter design was implemented in LTspice. The exact component values from calculation were applied and the LT1498 op-amp was used.

The schematic of the 5th order Butterworth high pass filter was as shown in the Figure 3 below.

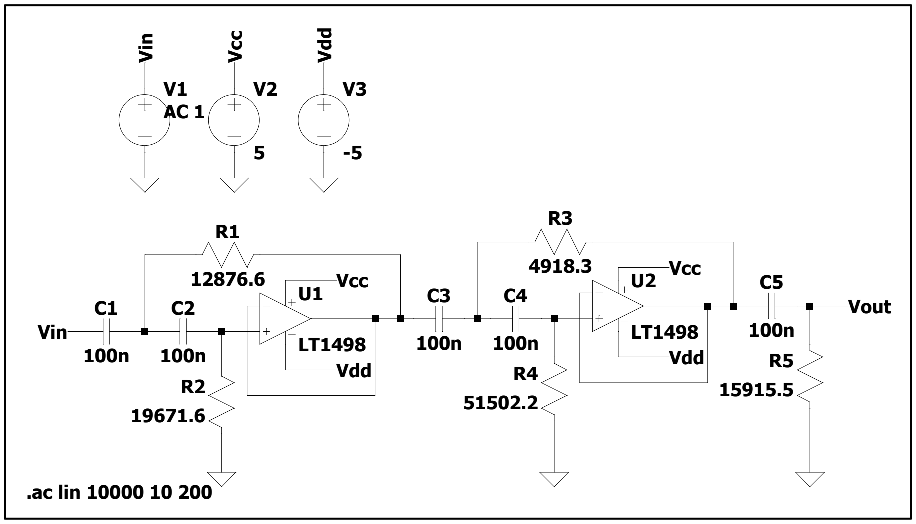

Figure 3. Schematic of the 5th order Butterworth high pass filter design

First, an AC simulation was run to determine the amplitude response of the circuit. An AC input was used, and the output of the complete design was plotted as shown in the Figure 4 below.

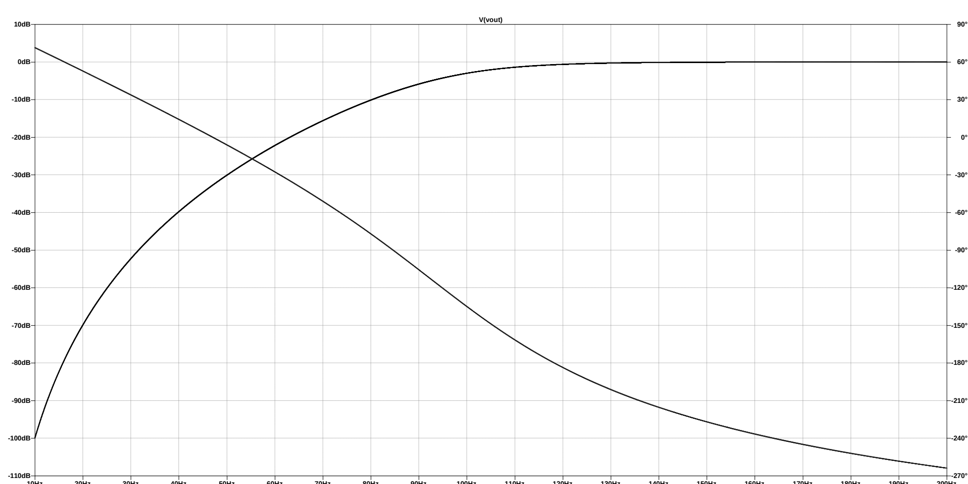

Figure 4. AC simulation: Amplitude response of the circuit

As shown in the Figure 5 below, the -3 dB low cut off frequency of the filter was 100 Hz which matched the design and calculations indicating a good design of the 5th order Butterworth filter.

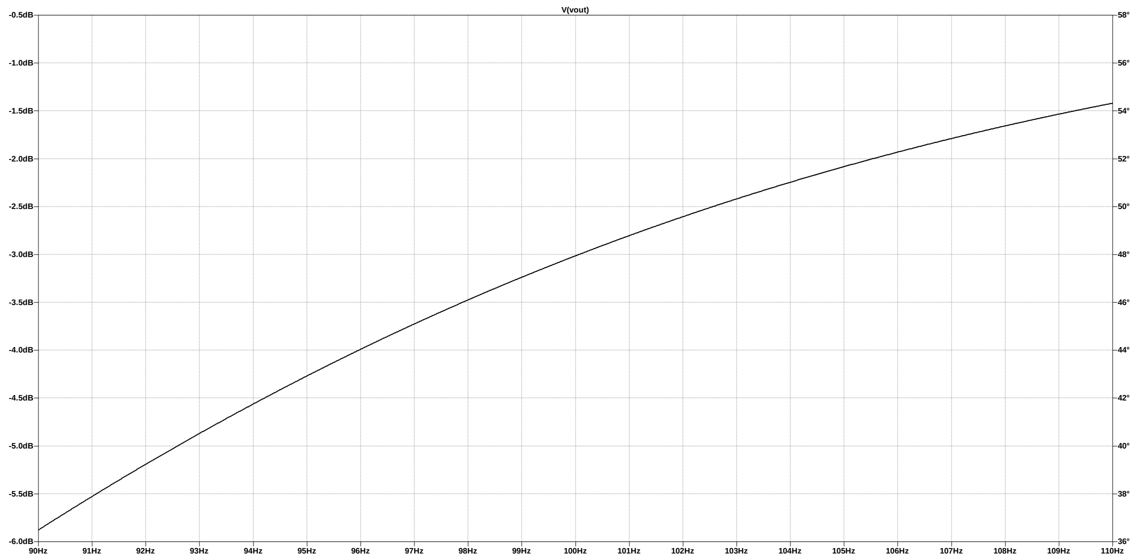

Figure 5. Zoomed in amplitude response of the filter at -3 dB low cutoff frequency

The second simulation was a transient simulation (i.e. in time). The input voltage source was configured to be 1 V amplitude with a frequency of 500 Hz for the first time and 10 Hz for the second time to check the operation of the circuit. For 500 Hz the simulation length was set to 0.02 s and it was set to 1 s so that around 10 cycles of the unput were able to be seen. The output plots were as shown in the Figure 6 and 7 below.

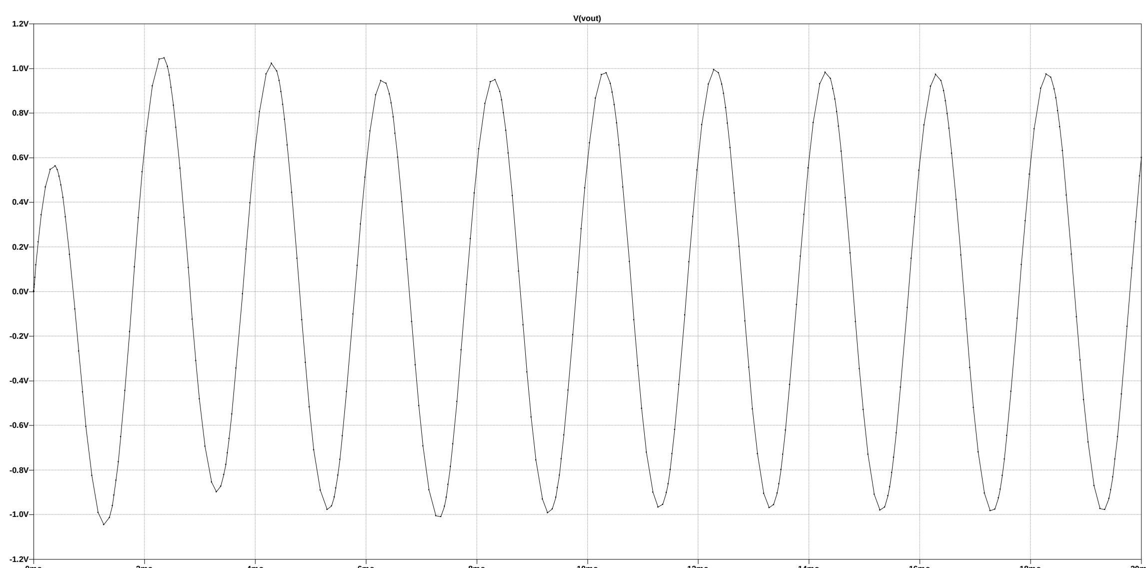

Figure 6. Transient simulation output with 500 Hz 1 V input

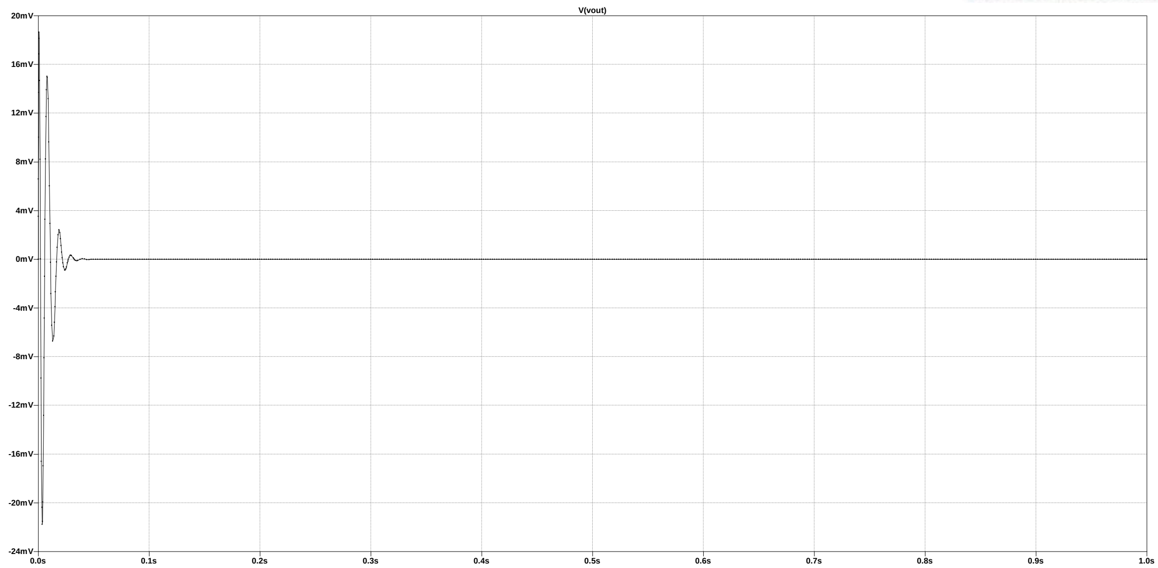

Figure 7. Transient simulation output with 10 Hz 1 V input

The filter was designed to filter out the frequencies lower than the pass band frequency of 100 Hz so that higher frequencies than 100 Hz were able to pass the filter, in which case it was expected that when using a 10 Hz 1 V voltage source as input to the filter, the output was to be a straight horizontal line indicating the input signal was filtered out; and for 500 Hz 1 V input, the output preserved the original signal amplitude indicating that the input signal passed the filter. As shown in the Figure 6 and 7 above, the transient simulation results matched the expectations. The 500 Hz high frequency was well into the pass band while the 10 Hz low frequency was stopped before the pass band. However, it was noticed that for 500 Hz simulation, the output had slight distortions of the signal shape – the amplitude of the output signal slightly varied around 1 V which was the input signal amplitude. Also, it was noticed that for both simulations, the circuits needed a short period of time to reach its stable status.

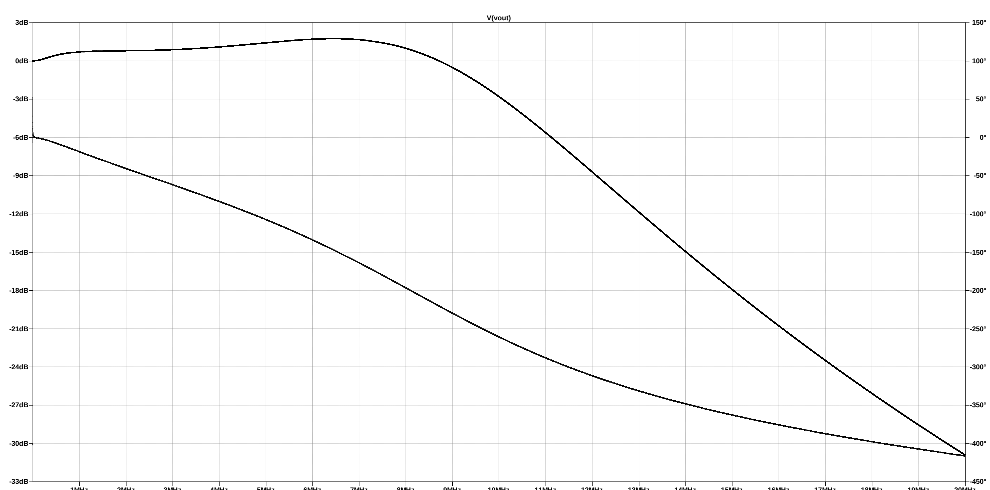

Figure 8. Frequency response of the filter at high frequencies

As shown in the Figure 8 above, it was obtained that the -3 dB high cut off frequency of the 5th order Butterworth high pass filter circuit was 10 MHz by approximation. The reason why the frequency response started attenuating rapidly when the frequency was greater than 10 MHz was that when the frequency was significantly high, the capacitance would produce parasitic effect, also, the bandwidth gain product of the op-amp was too small and the operational frequency range was determined by the bandwidth gain of an op-amp.

Task 3: Practical considerations

Resistor and capacitor values are typically available in so-called "E" ranges. The component values from the E24 range were used and a single resistor/capacitor was to be used in each place. The E24 component values chosen for each component of the 5th order Butterworth high pass filter were as listed in Table 3 below.

Table 3. E24 component values for 5th order Butterworth high pass filter design

Stage	Component	Value	E24 Value
2nd order Butterworth high pass filtering Q=0.618	C1	100 nF	100 nF
	C2	100 nF	100 nF
	R1	12876.6 Ω	13 kΩ
	R2	19671.6 Ω	20 kΩ
2nd order Butterworth high pass filtering Q=1.618	C3	100 nF	100 nF
	C4	100 nF	100 nF
	R3	4918.3 Ω	5.1 kΩ
	R4	51502.2 Ω	51 kΩ
1st order Butterworth high pass filtering	C5	100 nF	100 nF
	R5	15915.5 Ω	16 kΩ

After replacing the component values from calculation results to E24 values, the schematic of the 5th order Butterworth high pass filter was as shown in the Figure 9 below.

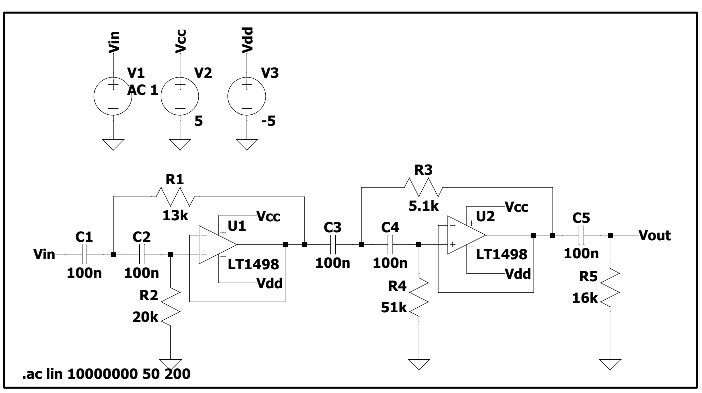

Figure 9. Schematic of the filter design with E24 component values

The AC simulation was run using the new schematic of the filter with E24 components, and the amplitude response of the new circuit was as shown in the Figure 10 below.

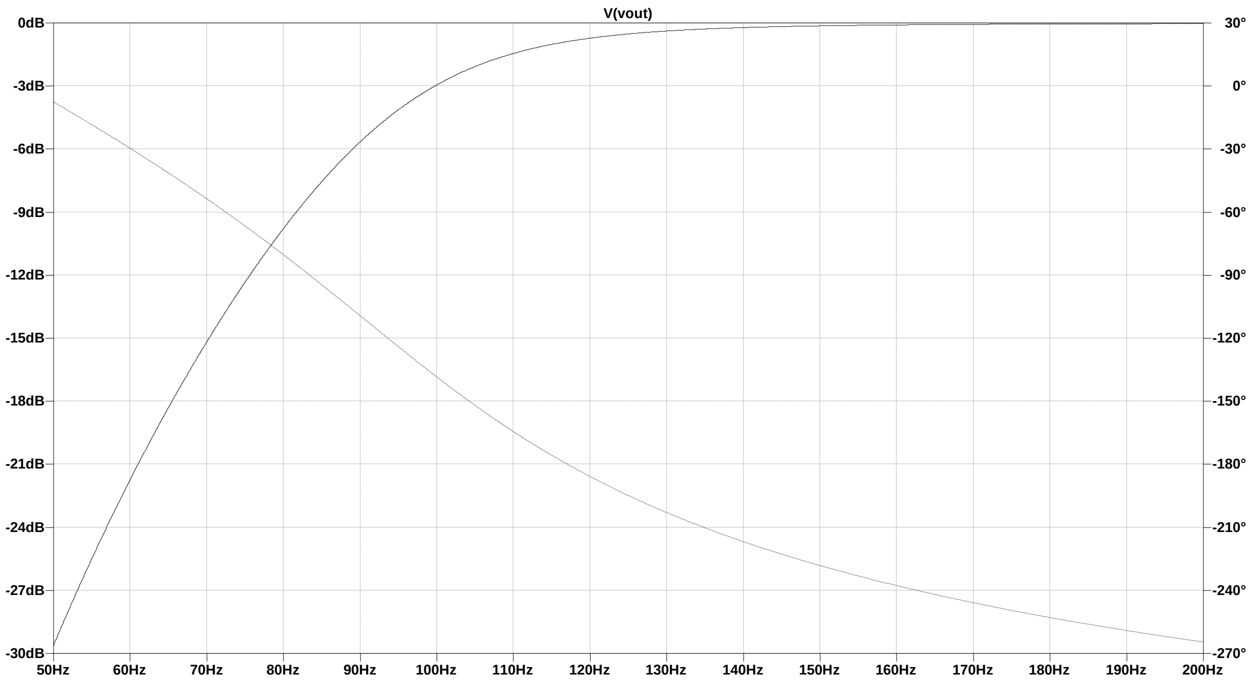

Figure 10. Amplitude response of the filter with E24 components

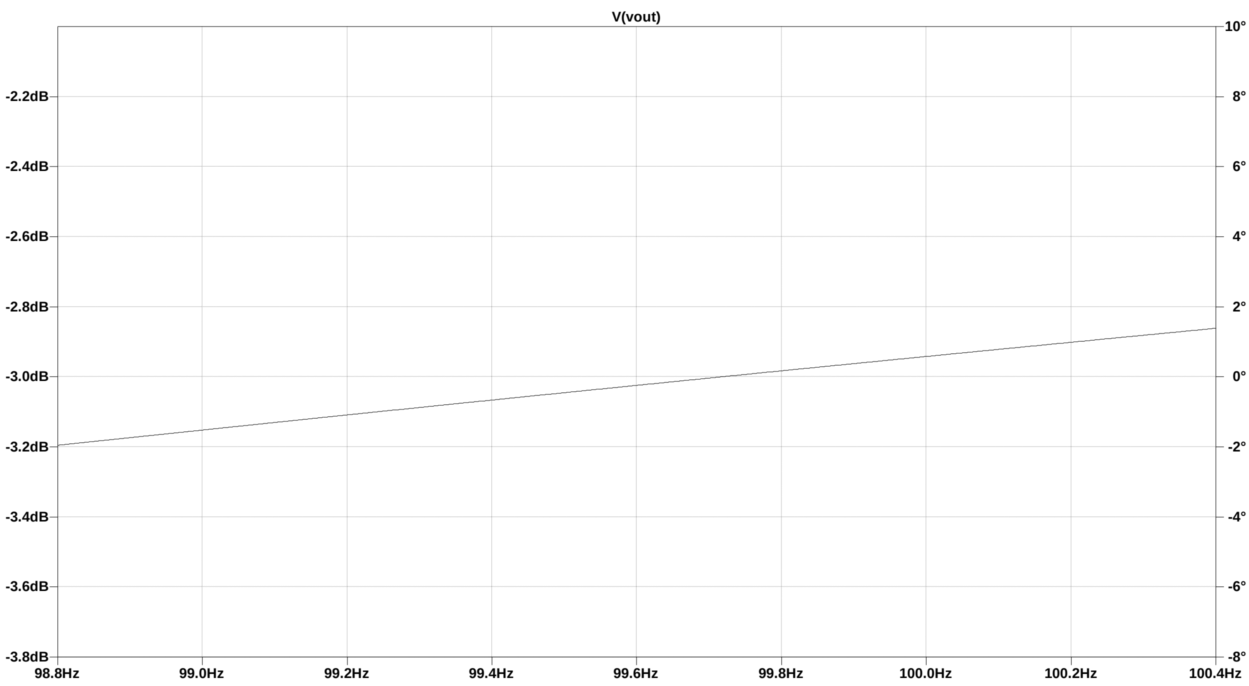

Figure 11. -3 dB low cut off frequency of the filter circuit with E24 components

As shown in the Figure 10 and 11 above, the -3 dB low cutoff frequency was now 99.7 Hz which was 100 Hz by approximation which matched the design specifications.

Lastly, if the specification of the filter design was changed to include amplification within the pass band, the change of the filter design was discussed below.

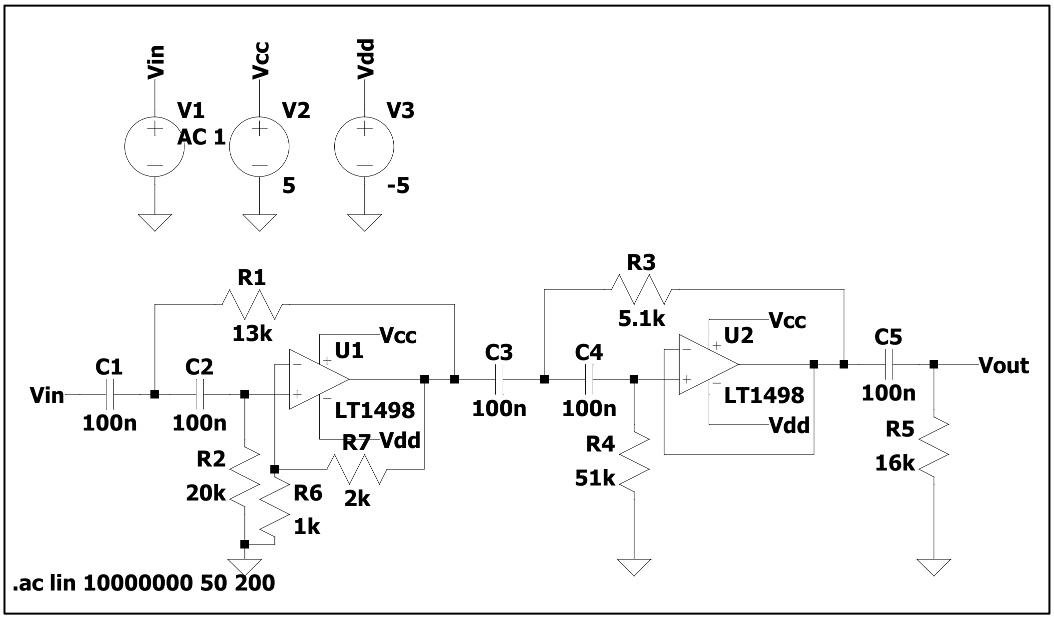

Figure 12. Schematic of the E24 filter with "desired" gain of 1.5 in passband

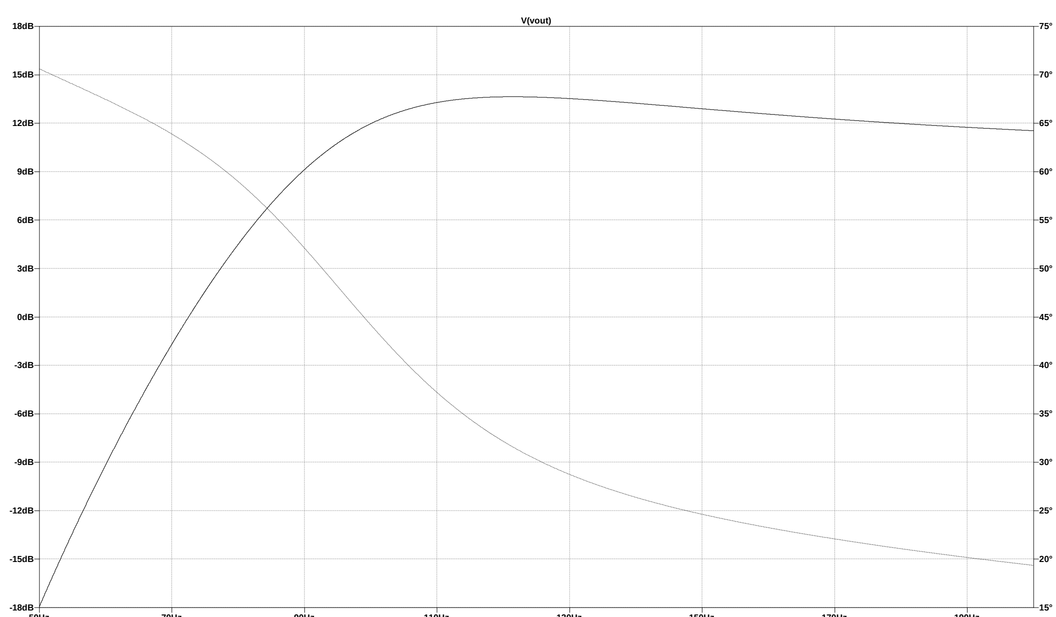

Figure 13. Amplitude response of the filter with passband gain

As shown in the Figure 12 above, two resistors were added onto the feedback of the op-amp in the first stage filtering. The "desired" passband gain was:

$$G = 1 + \frac{R_6}{R_7} = 1.5 \qquad \text{Eq. 19}$$

However, as shown in the Figure 13 above, it was obvious that the passband gain did not match the expectation. That was because the passband gain was determined by the characteristic of the op-amp:

$$G = 3 - \frac{1}{Q} \qquad \text{Eq. 20}$$

Therefore, the gain of the pass band was fixed using a specific op-amp in which case the resistors on the feedback were to be determined only to satisfy the passband gain. To design a filter that matches the passband specification, an appropriate op-amp was to be chosen at first.

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