# EGR 111
# Heat Transfer

The purpose of this lab is to use MATLAB to determine the heat transfer in a 1-dimensional system.

New MATLAB commands: (none)

## 1. Heat Transfer 101

Heat Transfer is thermal energy in transit due to a temperature difference. One experiences heat transfer every day. Make a cup of hot cocoa or tea. Is it too hot? Just wait a bit. Thermal energy is in transit due to the difference in temperature between the lava-hot beverage and the surrounding air. In Figure 1 below, Object A has a higher temperature than Object B. If Objects A and B come into contact, heat transfers from Object A to Object B.

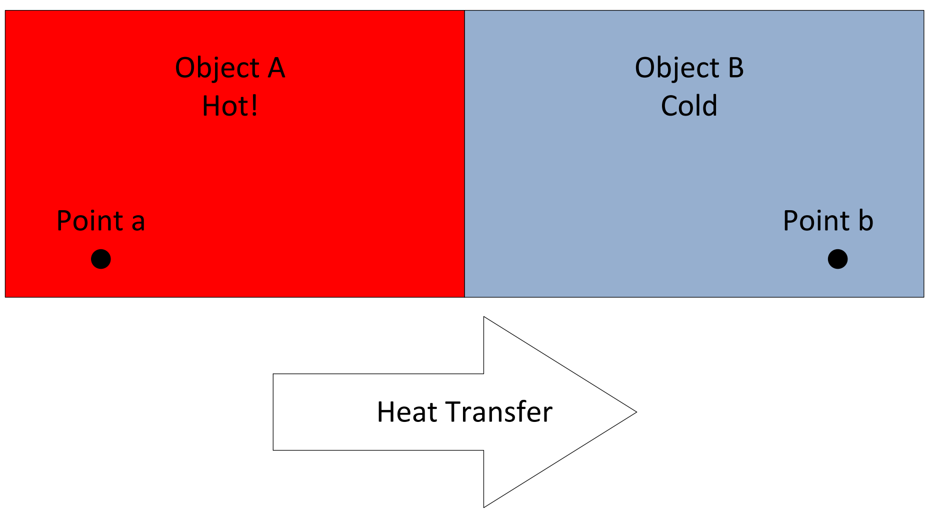


Figure 1. Heat transfer between a hot object A and a cold object B.

In the figure above, the type of heat transfer process that is taking place is called conduction. In conduction, heat transfer occurs across a stationary medium. When performing conduction analysis, engineers want to determine the temperature field in a medium. That is, they want to know the temperature distribution which represents how temperature varies with position in the medium. For example, in the figure above, in the initial moment that Object A comes into contact with Object B, the temperature at Point a is much greater than the temperature at Point b. However, if left in contact for a few minutes or a few hours, the temperatures at Points a and b would likely be equal.

In an engineering context, conduction analysis is important for many reasons. From a structural perspective, a temperature distribution can say something about the thermal stresses in a material. For example, ovens should not be built from materials susceptible to thermal stresses. Moreover, a temperature distribution can offer insight into the thickness of an insulating material that is required in an winter jacket.

To help with conduction analysis, engineers use Fourier's Law which states that heat transfer through a material is proportional to the negative gradient in the temperature and to the area through which the heat is flowing. This means that for a three dimensional object, one must consider the temperature gradient in the x, y, and z axes as well as the volume of the object. This analysis can get tricky, and so here we will only consider a 1-dimensional object.

**2. Analyzing an Aluminum Bar**

Analyzing a 1-dimensional object gives some insight into heat transfer as well as how to use MATLAB to perform a conduction analysis. Consider Figure 2 below which shows a 0.2 m long, pure aluminum bar, perfectly insulated at the top and bottom. This means that heat transfer may occur only in the horizontal direction. Also observe that it has five nodes, $T_1$ through $T_5$ which are reference points for the analysis.

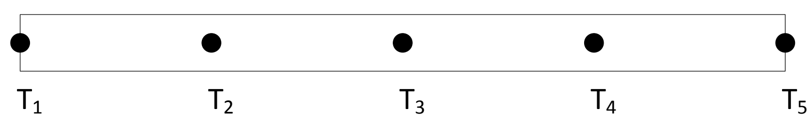


Figure 2. A 0.2 m long, pure aluminum bar, perfectly insulated at the top and bottom.

Fourier's law and partial differential equations yields the following equation which governs how heat transfer takes place over time in a 1-dimensional object.

$$T_i^{t+1} = \frac{\alpha \Delta t}{(\Delta x)^2}\left[T_{i+1}^t - 2T_i^t + T_{i-1}^t\right] + T_i^t$$

Simply stated, the temperature T at node i and at the new time increment t+1 may be calculated based on the temperature from the previous time increment t, and depends in particular on the following terms:

a) The thermal diffusivity of the material α.
b) The change in time Δt divided by the square of the change in distance Δx.
c) The temperature at the next node, $T_{i+1}$ in the previous timestep, t.
d) Two times the temperature in the node $T_i$ in the previous timestep, t.
e) The temperature in the previous node $T_{i-1}$, in the previous timestep, t.
f) The temperature in the node $T_i$ in the previous timestep, t.

To perform a conduction analysis, one must find the temperature at the center node (in this case $T_3$) at some time, t, using a specific $\Delta t$ and a specific number of nodes.

Suppose that we want to analyze the bar using 5 nodes from t = 0 to t = 1 second, with time slices, $\Delta t = 0.1$ seconds. Note that the thermal diffusivity of aluminum is $\alpha = 97.1\text{x}10^6\ \text{m}^2/\text{s}$.

Open a new script file and use MATLAB to initialize the above information:

```
% Define thermal diffusivity for aluminum.
alpha = 97.1 * 10^6;
% Define the total time, in seconds.
totalTime = 1;
% Define delta t, in seconds.
dt = 0.1;
% Define number of nodes in the bar
nn = 5;
% Define the length of the bar, in meters
len = 0.2;
```

We can use a 2-dimensional matrix TM (Temperature Matrix) to store the temperature at each node for each time. Each row of the matrix TM will store the temperatures for each time step, and each column will store the temperatures for each of the nodes. Thus the number of rows in TM must be the same as the number of time steps, and the number of columns in TM must be equal to the number of nodes.

```
% Define the number of time slices to be analyzed.
slices = totalTime/dt + 1; % Add one, since time starts at 0.
% Define matrix TM with slices rows and nn columns.
TM = zeros(slices, nn);
```

Suppose that the bar initially starts at 20°C, but that it has two constant temperature sources of 200°C, one at each end. Initialize TM to reflect the initial temperature conditions.

```
% Initialize TM
% The first row is 20 degrees C, the initial temperature.
% The first and last columns are 200 degrees C, since the bar
% has constant heat sources.
TM(1, :) = 20; % set first row to 20 degrees C
TM(:, 1) = 200; % set first column to 200 degrees
TM(:, nn) = 200 % set last column to 200 degrees
```

```
TM =
 T1   T2  T3  T4   T5
200  20  20  20  200 <- temperatures at t=0.0 sec
200   0   0   0  200 <- temperatures at t=0.1 sec
200   0   0   0  200 <- temperatures at t=0.2 sec
200   0   0   0  200 <- temperatures at t=0.3 sec
200   0   0   0  200 <- temperatures at t=0.4 sec
200   0   0   0  200 <- temperatures at t=0.5 sec
200   0   0   0  200 <- temperatures at t=0.6 sec
200   0   0   0  200 <- temperatures at t=0.7 sec
200   0   0   0  200 <- temperatures at t=0.8 sec
200   0   0   0  200 <- temperatures at t=0.9 sec
200   0   0   0  200 <- temperatures at t=1.0 sec
```

With the initial temperature conditions set, the next step is to use Fourier's law to calculate the remaining zero cells in the matrix. Note that TM(2,2) represents the temperature of the bar at node $T_2$ at t = 0.1 seconds. Similarly, TM(5,4) represents the temperature at node $T_4$ at t = 0.4 seconds.

Using the equation above, we can calculate the cell TM(2,2) in the matrix. In the equation, t+1 indicates the current time step, so t refers to the previous time step (previous row in matrix TM). In the equation, i, indicates the node number, so i+1 is the next node (next column in TM) and i-1 is the previous node (previous column in TM). Let's calculate one term of the equation:

```
% Calculate the term delta x squared
dx2 = (len / (nn - 1))^2;

TM(2,2) = alpha*dt/dx2*(TM(1,3) - 2*TM(1,2) + TM(1,1)) + TM(1,2)
ans =
20.6991
```

**Exercise 1:** Write a program that uses nested `for` loops to compute the temperature for an aluminum bar whose initial temperature is 20°C at time t = 0 seconds. At time t = 0, both ends of the bar are instantly brought up to a temperature of 200°C and held constant for the rest of the time. The goal is to use conduction analysis to determine the temperature at the center of the bar at t = 300 seconds using $\Delta t = 0.1$ seconds and 5 nodes along the length of the bar. Use the above program to help you formulate your solution. Then plot the temperature of the center node as a function of time (in seconds).

***<u>Checkpoint1:</u>*** Show the instructor your plot from Exercise 1.

**Exercise 2:** Change your program to compute the temperature at t = 300 seconds using $\Delta t = 0.1$ seconds and 5 nodes for an aluminum bar whose initial temperature is 0°C. At time t = 0, the right side of the bar is instantly brought up to a temperature of 200°C and held constant for the rest of the time, while the left side is held at 0°C. Plot the temperature of all of the nodes on the same graph as a function of time (in seconds). Explain why the nodes do not all have the same final temperature.

***<u>Checkpoint1:</u>*** Show the instructor your plot from Exercise 2.