EGR111
Projectile Project

Introduction

In this project we will use Newton's laws to simulate projectile motion. You will use MATLAB to simulate this motion under two conditions and optimize the distance traveled by varying the angle of departure. An experiment will then be performed to test the results.

Background

Consider a body under free flight acted upon only by gravity. Here let us consider the body as being released with an initial velocity, v_0, at an angle of θ from the horizontal, as shown in Figure 1.

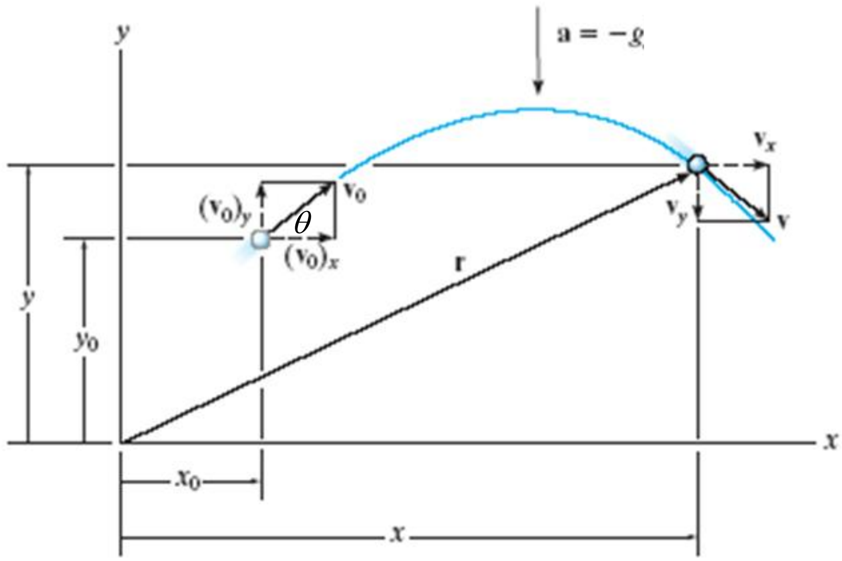

Figure 1: Projectile motion of a body.
*from Hibbeler, R.C., "Engineering Mechanics: Dynamics." 13^{th} Ed., Pearson, 2013.

Kinematics is the study of bodies in motion. Definitions of kinematic motion are given in Equation 1 where velocity, **v**, is defined as the time derivative of position, **r**. Similarly acceleration, **a**, is the time derivative of velocity. These relations should be familiar to you from Physics. When these vector relations are evaluated under constant acceleration, they produce a set of scalar relations. In the case of projectile motion, where a body travels in free flight, the acceleration is constant and equal to zero in the horizontal and -32.2 ft/s^2 in the vertical. In projectile motion, the motion of the body is governed by Equations 2-4.

In these equations the x is displacement in the horizontal direction, y is displacement in the vertical direction, t is time, and g is the gravity at the surface of the earth. The initial velocity, v_0, can be realized in its x and y, respectively v_{0_x} and v_{0_y}, directions using trigonometry and the angle of departure, θ. The path of the body will be parabolic and governed by these relations. Evaluating Equations 2 and 3 at any time t will yield the current x and y position. Similarly, evaluating the equations at a known x or y location can yield the time associated with travel to that point.

$$\boldsymbol{v} = \frac{d\boldsymbol{r}}{dt} \qquad \boldsymbol{a} = \frac{d\boldsymbol{v}}{dt} \tag{1}$$

$$x - x_0 = v_{0_x} t \tag{2}$$

$$y - y_0 = v_{0_y} t - \frac{gt^2}{2} \tag{3}$$

$$v_y - v_{0_y} = -gt \tag{4}$$

Exercise 1: To determine the total horizontal and vertical distances traveled by the body, these equations may be rearranged and used sequentially. You are encouraged to do the following:

1) Based on Equation 3, write a function "*t_of_f.m*" that will receive as inputs the initial and final heights of the trajectory, the initial velocity, v_0, and the angle of departure, θ. The function should then return as output the time of flight. In this application simply set y to the final height of the trajectory (e.g. $y_0 = y = 0$ if the initial and final position is ground level).

2) Based on Equation 2, write a function "*max_hor_dist.m*" that takes the time of flight, the initial velocity and the angle of departure, and returns the total horizontal distance traveled, $x - x_0$.

3) Based on Equation 4, write a function "*max_vert_dist.m*" that will determine the time, t_{max}, associated with a maximum height where v_y goes to zero. Then use t_{max} and Equation 3 to determine the maximum height, y_{max}. Return the maximum height. You will need to decide which inputs are required.

Now, for an assumed initial velocity of 200 ft/s and gravity known to be 32.2 ft/s^2, generate a plot of both total horizontal distance as a function of θ and maximum height as a function of θ. In each case treat the starting position as the origin $(x_0, y_0) = (0, 0)$ and let θ range from 0 to 90°.

Label the axes on your plot. Include units and a title. Annotate the plot using the pull down menu and clearly indicate the angles associated with the maximum horizontal distance and the maximum vertical distance.

Checkpoint 1: Show your instructor your plot from Exercise 1.

In association with Engineer's week each February, students from the University of Portland stand at the entrance to Franz Hall and launch water balloons into the quad with a three person surgical tube slingshot. These shots are typically aimed at Engineering Club Presidents, or other volunteers. In the course of the last four years hundreds of shots have been attempted and only one hit the target.

To enhance the modeling of this effort, we will introduce a new relationship. We will relate the maximum allowable stretch of the surgical tubing to the velocity imparted on the body when it leaves the slingshot. Using the Conservation of Energy, we can equate the initial and final energies of the body while in contact with the slingshot. We assume the body is placed in the sling and pulled back some distance *d*, and then released from rest. Following the Conservation of Energy we have:

$$\frac{1}{2}kd^2 = \frac{1}{2}m{v_0}^2 + mgh \qquad (5)$$

Here *k* is the stiffness of the tubing, *d* is the distance stretched, *m* is the mass of the body, and *h* is the vertical distance the body is displaced in the stretch. See Figure 2 for a model of the slingshot in use. The body starts from rest at x_0 and leaves the slingshot at an angle of θ with a velocity v_0 at the position where the slingshot is held.

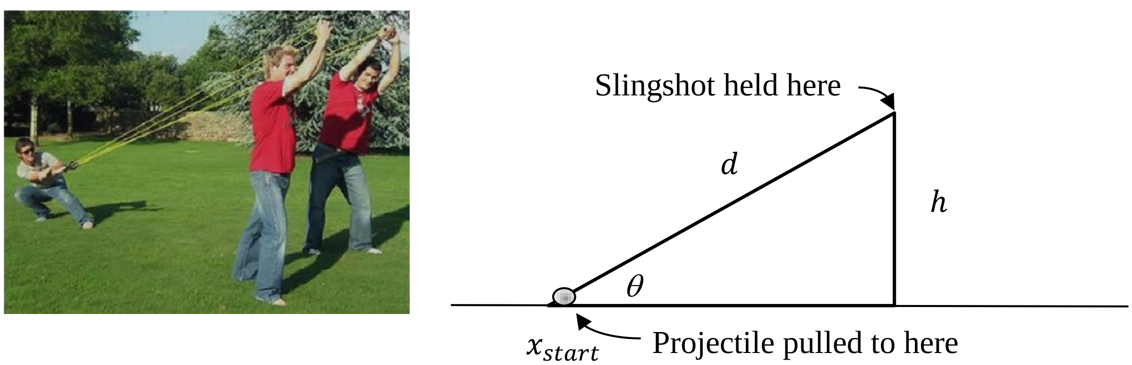

Figure 2: Image and Model of Stretched Slingshot.

Exercise 2: You are now asked to determine the angle associated with maximum distances traveled when considering the stretch allowed is limited by ground. You are encouraged to do the following:

1) Based on Figure 2, determine the distance *d* as a function of θ.

2) Based on Equation 5, write a function "*intital_v.m*" that takes θ, *k*, *m*, and *h* and returns the initial velocity of the body as it leaves the slingshot. Let the weight of the body be 0.5 lbs, k = 10 lbs/ft, and the height at which the slingshot is held be h = 3 ft.

3) Use this function and those from Exercise 1 to generate a new plot indicating the angle associated with both the maximum vertical height and the maximum horizontal distance.

Checkpoint 2: Show your instructor your plot from Exercise 1. If the stretch *d* is set to 6 ft, what is the corresponding vertical and horizontal distance traveled?

Exercise 3: Now let's take to the quad to validate our plots from Exercise 2. Collect enough data relating θ to total horizontal distance traveled.

Checkpoint 3: Generate the experimental plot relating θ to total horizontal distance traveled. Talk about how the simulated results compare to the experimental results both qualitatively and quantitatively. Identify primary sources for discrepancy. Share any input about how this process could be made better.