Stochastic Programming Project - Emergency Response Crew Procurement Problem

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1 Introduction

The following report models emergency procurement for external emergency crews (known as trouble crews) to address power events in a city in North America. The model is formulated as a 2 stage stochastic program and multiple formulations are presented, of which 1 is implemented in python using a variety of methods and models (eg solving the extensive form, benders decomposition, level method, applying a risk measure as well as sample average approximation). In section 2, the problem we're trying to solve is presented. In section 3, a brief literature on stochastic programming methods and models along with a review on how similar systems are modelled is considered. Section 4 is broken into 2 distinct sections: basic formulation - a simplistic model representing the problem at hand which was then implemented in Python using a variety of methods and models and an advanced formulation wherein 2 different formulations are presented. Implementation details and results are discussed in Section 5, followed by future work listed in Section 6 and the report is concluded in section 7.

2 Problem Background

Similar to Emergency Response in medical facilities, fire stations, police units etc., electrical utilities are required to respond to emergency electrical events to restore power to customers in a timely manner. Emergency events refer to any abnormal system condition wherein remedial action is required to prevent or limit loss of a distribution system or supply that could adversely affect the reliability of the electrical system [1].

2.1 Event Call Categories

Calls are reviewed and depending on location, impact and severity of events, events are categorized into 1 of 6 possible categories as represented below:

Category A: Urgent Response These events are calls where the assistance of the distributor has been requested by fire, ambulance or police services. The Ontario Distribution System Code requires response of such calls within 60 minutes in urban areas, at least 80 percent of the time on a yearly basis. The arrival of a qualified service person on site is sufficient to constitute as a response [1]

Category B: 3 Phase primary outages These events are calls that require restoration of critical or priority loads. The goal is to restore the greatest number of customers in the shortest period of time prioritized as follows - Transformer Stations, Primary Feeder trunks, municipal substations or feeder laterals

Category C: Crews should be on site within an hour These events are unsubstantiated reports from non - authoritative sources/non-emergency services where there is a reasonable possibility that there is no hazard to the public.

Category D: Crews should be on site within an hour These events are partial power on a 3 phase system. Typically, customers are not out of power but an abnormal situation exists, necessitating response to rectify the situation.

Category E: Crews should be on site within 4 hours Restoration of secondary buses and distribution transformers, restoration on part power situations and restoration of individual secondary service connections

Category F: Crews should be on site within 24 hours These relate to other non critical emergency events.

2.2 Emergency Response Crew Types

Emergency events are responded via 3 resource types which are as follows:

2.2.1 Crew Type A: Trouble Truck Crews

These crews are the primary emergency response crews that typically consistent of 2 line persons and a single bucket truck. The purpose of these crews is to be on site as soon as possible and conduct an initial triage, make the site safe, restore power if possible and then move on to the next emergency. These crews are not responsible for the completion of these events. These are procured externally and can be multiple crews for each type of 12 hour shift (day and night). These crews work 7 days a week, year round. The contracting firm providing these crews is responsible for ensuring the agreed upon crew volume is supplied without exception.

2.2.2 Crew Type B: Reactive Crews

These crews are primarily responsible to help restore emergency events (ie conduct the required repair/replace work to complete the emergency events). These typically comprise of 4 line persons and a single bucket truck. These crews are called upon to respond to emergency calls when there are no trouble truck crews available to respond to events in a timely manner. These crews can again be broken down into 2 subcategories:

Crew Type B: Internal: These crews are procured internally and currently work 7 hour day shifts (ie events between 7am - 3pm).

Crew Type B: External: These crews are procured externally and currently work 10 hour day, evening and night shifts (ie events between 7am - 5pm, 12pm - 10pm, 10pm - 7am).

2.2.3 Crew Type C: Triage Crews

These crews are primarily procured internally (in non storm events) and for the purposes of this analysis can be considered to be of infinite supply. These crews are called upon when neither the trouble truck crews or reactive crews can be called upon to respond to emergency calls. As these are regular crews, these crews typically work 7 hour shifts from 7 am - 3 pm.

2.3 Description of Emergency Process

Typically, emergency events are reported to utilities through a call center which are then reviewed by the Control Centre. Dispatchers review these events and upon review of active crews (availability, proximity) will dispatch events to trouble truck crews to ensure crews are on site within the stipulated time period. If no trouble truck crews are available, reactive crews will be called upon if available. Finally, if

none of these crews are available, triage crews will be called upon to address emergency events. Typically, the latter situation is only applied when response is required for Category A type events.
Day to day operations are dictated by Standard Operating Procedures, wherein the objective of this procedure is to ensure that the utility meets it's commitments as mandated by the regulatory body. Per this Standard Operating Procedure, there should be enough field resources to dispatch to events within prescribed resource targets. If enough resources are not available, additional crews must be procured which takes those crews away from planned work required to enhance, maintain and develop the grid infrastructure of this utility.

These events range from crews being on site to assist with other emergency services (eg: Police and Fire crew involvement), leaking oil from electrical equipment or responding, outages due to animal or vegetation contact. Due to ongoing attrition and a lack of certified electrical professions in north America , utilities face an ongoing challenge to balance crew support to respond to emergencies in a timely manner while ensuring regular planned upgrades, maintenance or reactive replacements or repairs are being conducted as effectively and efficiently as possible. Further, external factors including regulatory requirements, necessitate timely response to respond to emergency events. In a specific utility in Canada, emergency events are categorized into 1 of 6 categories ranging from urgent response requiring response within an hour (ie crew on site within an hour from initiation) to events requiring response within 24 hours. The response time represents the amount of time required for a crew to be on site within a certain time duration. Actual response to events could occur by the original crews dispatched to the event or by follow up crews that have the required skill set and resources to restore normal service. The purpose of this project is to determine the number of emergency response crews to be procured from external vendors (typically a 1-2 contracting firms). These crews are required to work on 12 hour shifts that occur either during the day time (7am - 7pm) or the night time (7pm - 7am).

3 Literature Review

3.1 Scheduling and Staffing Models:

In scheduling problems, two concepts are generally used to denote how events arrive and how they are serviced in the form of arrival rates (or counts) and service rates. The arrival process records the epochs in which the calls or events arrive. Typically these arrivals are random. The randomness of these arrivals are usually justified as follows - there may be many statistically identical callers to a call center, however there is a very small non-negligible probability for these arrivals to come in at any given time interval. Based on previous published research, such events arrive mimic a Poisson process [5]. Similarly, service durations are typically assumed to be exponentially distributed. This is partly due to the lack of empirical evidence that would prove otherwise. The assumption of service rates following an exponential distribution simplifies models and makes them analytically tractable [5]. Further, studies as those performed by Kort confirm that the exponential assumption is typically an acceptable fit in a call center type of problem [6]. For the purposes of this report, we make the same assumptions as in existing literature around the treatment of arrival and service rates.

There is a wide variety of literature related to multiple aspects of scheduling and staffing resources in a variety of applications including the medical field (nurse scheduling problem), call centres, emergency response etc. When formulating such problems as a Stochastic Integer Program, the first stage consists of integer decision variables where the decision is made to select the number of servers assigned to a possible schedule, whereas the associated arrival rates (or counts) can be modeled as random variables [2][3]. Constraints associated with these first stage decision variables seek to ensure that schedules are chosen to ensure all staffing levels are met. Recourse decisions in such problems typically tend to be the amount of jobs to allocate to available crews and the number of jobs abandoned. Constraints associated with the second stage either seek to track the number of jobs abandoned as well as limit server allocations to available staffing levels. Hence, the overall objective in such problems aims to minimize the scheduling costs and the abandonment costs.

Other models, such as those proposed by Kim and Mehrotra [4] also aim to minimize the scheduling costs while minimizing any potential under or over staffing of resources and corresponding scheduling adjustments as information on demand becomes more accurate. Some models also have a horizon greater than a day (typically in weeks). The first stage decisions in such models is still to determine the number of servers to allocate to a possible schedule subject to similar first stage constraints as above. In the recourse stage, the recourse variables track the amount of over and under staffing at a given time interval as well as the decision to increase or reduce procured resources as the demand for those resources becomes clearer. In such formulations, the goal is to minimize the staffing cost over the horizon as well as the expected recourse cost over that horizon (ie penalties due to over or under staffing, to increase or reduce crews etc.). Typical recourse stage constraints aim to ensure that anticipated demands are satisfied by staffing levels after scheduling adjustments.

3.2 Stochastic Programming Solution Methods and Review:

Many papers, including the ones stated above, typically propose enhancements to regular algorithms that involve speeding up the computation of formulations. Some of these novel approaches involve prioritizing branching strategies or exploiting mixed integer rounding to improve cut generations. For the purposes of this report, traditional algorithms and methods were implemented instead. Models considered and their associated algorithms include:

3.2.1 Stochastic Programming Extensive Form:

For the purposes of our initial implementation, we utilize a solver (GurobiPy) to solve the extensive form of our formulation which typically takes the following format[7]: In addition to this, we will make

$$
\begin{array}{lcccccccccc}
\min\ c^\top x & + & p_1(q^1)^\top y_1 & + & p_2(q^2)^\top y_2 & + & \cdots & + & p_S(q^S)^\top y_S & & \\
\text{s.t. } Ax & & & & & & & & & = & b \\
T^1 x & + & W^1 y^1 & & & & & & & = & h^1 \\
T^2 x & & & + & W^2 y^2 & & & & & = & h^2 \\
\vdots & & & & & & \ddots & & & & \vdots \\
T^S x & & & & & & & + & W^S y^S & = & h^S \\
x \geq 0 & & y^1 \geq 0 & & y^2 \geq 0 & & & & y^S \geq 0 & &
\end{array}
$$

use of the L Shaped Benders Decomposition Method (Single and Multi Cut), Single Cut Level Method and Sample Average Approximation sampling that are further discussed in [7].

3.3 Risk Measures - alpha Conditional Value at Risk

Typical Stochastic Programming implementations are risk neutral wherein expectation is taken as the preference criterion to identify the best decisions. Through the use of Risk Measures (specifically CVaR), we can consider a risk averse two stage Stochastic Program. In past literature, decision making under risk typically uses mean risk models wherein the mean risk function is minimized and represented as:

$$\min_{x \in X} \{\mathbb{E}[f(\mathbf{x}, \omega)] + \lambda \rho [f(\mathbf{x}, \omega)]\} \tag{1a}$$

Where ρ is a specified risk measure. Using the results presented in [8], we get the following: Assuming the following:

$$f(\mathbf{x},\omega) = \{\mathbf{c}^T\mathbf{x}, \xi(\omega))\}$$

And, we know:

$$\min_{x\in X}\{\mathbb{E}[f(\mathbf{x},\omega)] + \lambda \text{CVaR}_\alpha[f(\mathbf{x},\omega)]\} \tag{2a}$$

The above equation can be reformulated as follows with the addition of a constraint to deal with the new introduced variable v_i. The below primarily follows from the Transalation Invariance property of CVaR demonstrated in [7]:

$$\min(1+\lambda)\mathbf{c}^T\mathbf{x} + \sum_{i=1}^{N} p_i q_i^T \mathbf{y}_i + \lambda(\eta + \frac{1}{1-\alpha}\sum_{i=1}^{N} p_i v_i) \tag{3a}$$

4 Formulation

4.1 Basic Formulation - No Risk Measure

For a simplistic formulation of the problem at hand, we modify a model similar to one propsoed in Bodur, Leudtke (Mixed Integer Rounding Enhanced Benders Decomposition for Multiclass Service System Staffing and Scheduling with Arrival Rate Uncertainty). In this problem, let I be the set of event types described in the section above and T be the set of time period. Further, let S denote the set of possible schedules for the trouble truck crews ie (day shift crew from 7am - 7pm and night shift crew from 7pm - 7am). As a result, the duration of these schedules is represented by l_s which is equivalent to 12 hours for the trouble truck crews. Further, we define a binary parameter a_{st} which is equal to the following:

$$a_{st} = \begin{cases} 1 & \text{schedule s includes time t} \\ 0 & \text{otherwise} \end{cases}$$

Let $C_{TT,Reg}$ represent the hourly rate to pay for a trouble truck crew. Further, a non negative random variable E_{it} is defined which models the arrival of event types $i \in I$ arriving in time period $t \in T$.

4.1.1 First Stage Model

We design the number of Trouble Truck crews to be assigned to a shift schedule $s \in S$ which then determines the number of trouble truck crews available to respond to emergencies in each period $t \in T$. To do this, we introduce 2 decision variables x_s which represents the number of trouble crews assigned to schedule s. Further, the decision y_t is introduced to represent the number of trouble truck crews at time $t \in T$. After emergency events materialize, a second stage problem allocates the available procured crews to the events in order to minimize the number of crews that won't be served. If crews are not served, the other tiers of crews (ie the reactive crews or the triage crews) will be assigned to serve these events. For the purposes of the simple formulation these will be grouped together. The problem is formulated as follows:

$$\min_{x,y} \sum_{s\in S} C_{TT,Reg} * l_s * x_s + \sum_{t\in T} \mathbb{E}_{E_{it}}[Q_t(y, E_{it})] \tag{4a}$$

$$\text{s.t} \quad \sum_{s\in S} a_{st} x_s \geq y_t \quad \forall t \in T \tag{4b}$$

$$\sum_{s\in S} x_s \leq 6 \tag{4c}$$

$$x_s \in \mathbb{Z}_+^S \quad \forall s \in S \tag{4d}$$

$$y_t \in \mathbb{Z}_+^T \quad \forall t \in T \tag{4e}$$

The objective function aims to minimize the costs of procuring trouble truck crews and the expected costs to pick up events that procured crews cannot meet (which can also be though of as abandonment costs). The constraints listed above can be explained as follows:

Schedule Enforcement: To ensure that the schedules chosen ensure sufficient staffing levels are met. Represented by the following representation:

$$\sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T$$

Procurement Limitation: As per current policy at the utility, no more than 6 trouble truck crews can be procured for the purposes of responding to emergency response. This limitation is in part set by the multi year budget approved by the regulatory body.

$$\sum_{s \in S} x_s \leq 6$$

Decision Variables must be non negative integers: The decision variables must be non negative integers ie the number of crews procured must be positive integers. This is represented as:

$$x_s \in \mathbb{Z}_+^S, y_t \in \mathbb{Z}_+^T \quad \forall s \in S, \forall t \in T$$

4.1.2 Second Stage Model

Further, $Q_t(.,.)$ is the function used to measure the Quality of Service, measured as the weighted sum of the events that are not fulfilled by the trouble truck crews. For a given time period $t \in T$, $Q_t.,.$ can be represented as:

$$Q_t(y, E_{it}) := \min_{v,w} \quad C_{p,i} * w_i \tag{5a}$$

$$\text{s.t} \quad \mu_{it} * v_i + w_i = \lambda_{it} \quad \forall i \in I, \forall t \in T \tag{5b}$$

$$\sum_{i \in I} v_i \leq y_t \tag{5c}$$

$$v_i \in \mathbb{R}_+^I, w_i \in \mathbb{R}_+^I \quad \forall i \in I \tag{5d}$$

The above Objective function is the second stage model which calculates the cost of events not being fulfilled by the trouble truck crews, $Q_t(y, E_{it})$ for the period $t \in T$ with fixed staffing levels y and the incoming event arrival rate of λ. In this stage, we introduce the following additional variables v_i represents the number of emergency events $i \in I$ that are assigned to the procured trouble truck crews and w_i represents the number of emergency events $i \in I$ that are not allocated to the trouble trucks. These events as a result need to be assigned to either the reactive or the triage crews to be addressed. The objective function listed above aims to reduce the cost of allocating unassigned work to other crews and can be thought of as a penalty cost for not meeting demand. The parameter $\mu_{it} \leq 0$ represents the service rate of the trouble trucks crews when serving event type $i \in I$ during time $t \in T$.

Incoming arrivals must be met: The number of events allocated to trouble truck crews and unmet demand is equal to the the number of events of type $i \in I$ raised in time $t \in T$. This is represented as follows:

$$\mu_{it} * v_i + w_i = \lambda_{it} \quad \forall i \in I, \forall t \in T$$

Crew Allocations constrained by Staffing levels: The following constraint limits the number of server allocations to the available staffing levels represented as follows:

$$\sum_{i \in I} v_i \leq y_t$$

Number of events allocated or unmet can be a positive real number: As per existing literature review (eg: Bassamboo, Harrison and Zevi, Bodur, Leudtke), crews can be fractionally allocated to meet demand and demand can be split among different crews.

4.1.3 Scenario Enumeration:

We note that the proposed objective function,consists of an expectation term which aims to determine the expectation over a random vector (defined as E_{it}). We can utilize Sample Average Approximation to evaluate the expected value. Using Monte Carlo Sampling, we can obtain a set of independent and identically distributed samples from the random vector. These realizations represent scenarios. Now we define a set K which will represent the scenario indices . Further, $\lambda^1, \lambda^2, ..., \lambda^k$ will be realizations of $E_i t$. The expected value $\mathbb{E}$ will be replaced by the sample average over scenarios which can be determined via a probabilistic distribution. Hence:

$$\sum_{t \in T} \mathbb{E}_{E_{it}}[Q_t(y, E_{it}] = \sum_{t \in T} \sum_{k \in K} p_k * Q_t(y, \lambda^k)$$

where p_k represents the probability of scenario k. Hence the above model can be reformulated as follows:

$$\min_{x,y} \sum_{s \in S} C_{TT,Reg} * l_s * x_s + \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} p_k * C_{p,i} * w_{it}^k \tag{6a}$$

$$\text{s.t} \quad \sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T \tag{6b}$$

$$\sum_{s \in S} x_s \leq 6 \tag{6c}$$

$$\mu_{it} * v_{it}^k + w_{it}^k = \lambda_{it}^k \quad \forall i \in I, \forall t \in T, \forall k \in K \tag{6d}$$

$$\sum_{i \in I} v_{it}^k \leq y_t \quad \forall t \in T, \forall k \in K \tag{6e}$$

$$x_s \in \mathbb{Z}_+^S \quad \forall s \in S \tag{6f}$$

$$y_t \in \mathbb{Z}_+^T \quad \forall t \in T \tag{6g}$$

$$v_{it}^k \in \mathbb{R}_+^{ITK}, w_{it}^k \in \mathbb{R}_+^{ITK} \quad \forall i \in I, \forall t \in T, \forall k \in K \tag{6h}$$

4.1.4 Remarks for Simplistic Model:

Model Recourse: The above model represents a 2 stage stochastic integer program wherein we have integer variables in the first stage and continuous variables in the second stage. As a result, the second stage will always be feasible (ie the model has complete recourse). Further, as the second stage is feasible and bounded linear program, the second stage value function must be piecewise linear and convex.

4.2 Basic Formulation: Protecting against Uncertainty: Conditional Value at Risk

As the problem at hand is to plan for Emergency events where arrivals are stochastic and we would like to limit the risk of large values ie we would like to have just enough crews to limit the risk of a large rate of arrivals while minimizing the cost it takes to procure these costs. We can consider the following reformulation:

$$\min \mathbb{E}[Z] + \lambda \text{CVaR}_\alpha(Z)$$

Where Z represents the random cost or the total of the first and second stage costs in our example. As a results, our reformulation becomes:

$$\min_{x,y} (1 + \lambda) \sum_{s \in S} C_{TT,Reg} * l_s * x_s + \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} p_k * C_{p,i} * w_{it}^k + \lambda(\text{CVaR}_\alpha \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} p_k * C_{p,i} * w_{it}^k)$$

$$
\begin{aligned}
\text{s.t} \quad & \sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T && \text{(7a)} \\
& \sum_{s \in S} x_s \leq 6 && \text{(7b)} \\
& \mu_{it} * v_{it}^k + w_{it}^k = \lambda_{it}^k \quad \forall i \in I, \forall t \in T, \forall k \in K && \text{(7c)} \\
& \sum_{i \in I} v_{it}^k \leq y_t \quad \forall t \in T, \forall k \in K && \text{(7d)} \\
& x_s \in \mathbb{Z}_+^S \quad \forall s \in S && \text{(7e)} \\
& y_t \in \mathbb{Z}_+^T \quad \forall t \in T && \text{(7f)} \\
& v_{it}^k \in \mathbb{R}_+^{ITK}, w_{it}^k \in \mathbb{R}_+^{ITK} \quad \forall i \in I, \forall t \in T, \forall k \in K && \text{(7g)}
\end{aligned}
$$

As we know, the $alpha$ Conditional Value at Risk is equivalent to:

$$
\text{CVaR}_\alpha(Z) = \min_{y \in R} \{y + \frac{1}{1-\alpha} \mathbb{E}[Z - y]_+\}
$$

As mentioned in the literature review, CVaR can be re-written and new variables r and u_k will be introduced to deal with the CVaR specific variables. The above model can be reformulated as:

$$
\begin{aligned}
\min_{x,y} \quad & (1+\lambda) \sum_{s \in S} C_{TT,Reg} * l_s * x_s + \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} p_k * C_{p,i} * w_{it}^k + \lambda(r + \frac{1}{1-\alpha} \sum_{k \in K} p_k * u_k) && \text{(8a)} \\
\text{s.t} \quad & \sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T && \text{(8b)} \\
& \sum_{s \in S} x_s \leq 6 && \text{(8c)} \\
& \mu_{it} * v_{it}^k + w_{it}^k = \lambda_{it}^k \quad \forall i \in I, \forall t \in T, \forall k \in K && \text{(8d)} \\
& \sum_{i \in I} v_{it}^k \leq y_t \quad \forall t \in T, \forall k \in K && \text{(8e)} \\
& u_k \geq \sum_{t \in T} \sum_{i \in I} C_{p,i} * w_{it}^k - r && \text{(8f)} \\
& x_s \in \mathbb{Z}_+^S \quad \forall s \in S && \text{(8g)} \\
& y_t \in \mathbb{Z}_+^T \quad \forall t \in T && \text{(8h)} \\
& v_{it}^k \in \mathbb{R}_+^{ITK}, w_{it}^k \in \mathbb{R}_+^{ITK} \quad \forall i \in I, \forall t \in T, \forall k \in K && \text{(8i)} \\
& u_k \geq 0 \quad \forall k \in K && \text{(8j)} \\
& r \in \mathbb{R} && \text{(8k)}
\end{aligned}
$$

The last three constraints are added due to the expansion of the CVaR expression, as explained in the literature review [7][8]. In our implementation, we will consider multiple values of λ and α to test how results change depending on the values selected.

4.3 Protecting against Uncertainty: Robust Optimization UPDATING UNCERTAINTY SET ATM

Another approach to dealing with uncertainty is to reformulate the Stochastic Program as a Robust Optimization problem wherein the aim is to protect against all possible realizations in an uncertainty set, where the size of the set would be controlled through the Budget of Uncertainty.

We introduce a variable θ and reformulate our base formulation as follows:

$$\min_{x,y} \sum_{s \in S} C_{TT,Reg} * l_s * x_s + \theta \tag{9a}$$

$$\text{s.t} \quad \sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T \tag{9b}$$

$$\sum_{s \in S} x_s \leq 6 \tag{9c}$$

$$\mu_{it} * v_{it}^k + w_{it}^k = \lambda_{it}^k \quad \forall i \in I, \forall t \in T, \forall k \in K \tag{9d}$$

$$\sum_{i \in I} v_{it}^k \leq y_t \quad \forall t \in T, \forall k \in K \tag{9e}$$

$$\theta \geq \sum_{t \in T} \sum_{k \in K} \sum_{i \in I} p_k * C_{p,i} * w_{it}^k \tag{9f}$$

$$x_s \in \mathbb{Z}_+^S, y_t \in \mathbb{Z}_+^T \quad \forall s \in S, \forall t \in T \tag{9g}$$

$$v_{it}^k \in \mathbb{R}_+^{ITK}, w_{it}^k \in \mathbb{R}_+^{ITK} \quad \forall i \in I, \forall t \in T, \forall k \in K \tag{9h}$$

$$\theta \in \mathbb{R} \tag{9i}$$

Not implemented in Python

4.4 Advanced Formulations:

The following section considers more advanced formulations which were not implemented in python but could be followed up on as part of future work for this project. Two formulations are presented: The first considers an integrated staffing and scheduling problem similar to models presented in Kim and Mehrortra[]. The second seeks to formulate the original problem that was presented that seeks to minimize the costs required to incur trouble truck crews while ensuring over staffing or under staffing, the overall unmet demand and costs due to procuring additional crews are minimized.

4.4.1 Advanced Formulation 1:

As crews need to be procured over multiple days, and often times over multiple weeks. The formulation above is simplistic in that there are temporal impacts between consecutive days in terms of demand for resources required to address emergency events. As a result, we now consider procuring crews over a multiple time horizon, wherein the decision maker can adjust the procured crew structure. The goal would also be to ensure crews are staffed adequately ie over staffing and under staffing of crews is minimized while there is uncertainty in the demand, which itself will As in the previous formulation, we let S represent the set of schedules possible in the duration of a week. The set T will represent the hourly time periods expanded to a week (ie set of comprising of each hour in the week). Further, let the set X represent potential staffing and scheduling rules. As before, $C_{TT,Reg}$ represents the hourly rate of a trouble crew. The length of each shift in set S, is represented by the parameter l_s which is currently assumed to be 12 hours. As before, the binary parameter a_{st} represents:

$$a_{st} = \begin{cases} 1 & \text{schedule s includes time t} \\ 0 & \text{otherwise} \end{cases}$$

Now, we introduce the following first stage decision variables: let x_s represent the number of crews working in schedule shift s. Further, y_t is the variable that represents the number of crews working at time t $\in$ T. For the second stage, we introduce the set S' which is the set of potential schedule adjustments that can be made once the demand is realized. We introduce the cost parameters $C_{S',add}$ and $C_{S',canc}$ which represents the cost of adding and cancelling a shift respectively. Further, we introduce the following costs that represent the penalty of over staffing and under staffing, $C_{p,over}$ and $C_{p,under}$. Further, let $u_{s',t}$ represent the binary parameter:

$$u_{s't} = \begin{cases} 1 & \text{schedule s' includes hour t} \\ 0 & \text{otherwise} \end{cases}$$

We introduce recourse variables $z_{s',add}(\xi)$ and $z_{s',canc}(\xi)$ represents the variable that represents the number of shifts added to the adjusted pattern s'. Further, $v_{over,t}(\xi)$ and $v_{under,t}(\xi)$ represents the amount of over and under staffing at hour t. Hence, we formulate our stochastic program as one that seeks to minimize the costs to procure crews as well as to minimize the expected penalties incurred due to over and under staffing at a time t as well as the cost to procure additional shifts or cancel existing shifts. This is represented by:

$$\min_{x_s} \quad \sum_{s \in S} C_{TT,Reg} * l_s * x_s + \sum_{k \in K} \{ \sum_{s' \in S} [C_{S',add} * l_{s'} * z^k_{s',add}(\xi) + C_{S',canc} * l_{s'} * z^k_{s',canc}(\xi)] \tag{10a}$$

$$+ \sum_{t \in T} [C_{P,over} * v^k_{over,t}(\xi) + C_{P,under} * v^k_{under,t}(\xi)] \} \tag{10b}$$

$$\text{s.t} \quad \sum_{s \in S} a_{st} x_s \geq y_t \quad \forall t \in T \tag{10c}$$

$$\sum_{s \in S} x_s \leq 6 \tag{10d}$$

$$\sum_{s' \in S'} u_{s',t} * (z^k_{s',add}(\xi) - z^k_{s',canc}(\xi)) - v^k_{over,t}(\xi) + v^k_{under,t}(\xi) = d^k_t(\xi) - y_t \quad \forall t \in T \tag{10e}$$

$$z^k_{s',add}(\xi) \in \mathbb{Z}_+ \quad \forall s' \in S', \forall k \in K \tag{10f}$$

$$z^k_{s',canc}(\xi) \in \mathbb{Z}_+ \quad \forall s' \in S', \forall k \in K \tag{10g}$$

$$v^k_{under,t}(\xi) \quad \forall t \in T, \forall k \in K \tag{10h}$$

$$v^k_{over,t}(\xi) \quad \forall t \in T, \forall k \in K \tag{10i}$$

4.4.2 Advanced Formulation 2:

In the following section, we seek to formulate the originally proposed model (as described in the Section 2. As mentioned, we have 3 distinct crew categories represented by crew r in resource set R. We have 6 types of emergency events denoted by event i in event type set I. The crews will be procured to work on shift s in Shift set S of duration l_s, whereas the hourly time periods will be represented by period t in Time period shift T. We also define binary parameters a_st which is represented as:

$$a_{st} = \begin{cases} 1 & \text{schedule s includes time t} \\ 0 & \text{otherwise} \end{cases}$$

Crews have an hourly rate denoted by the parameter $C_{r,RT}$ and $C_{r,OT}$ for regular and over time respectively.

First Stage Decision Variables : Similar to our previous formulation and the original problem, we have the decision variables x_s which represents the number of external trouble truck crews working at shift s. Further, the decision variable y_t represents the number of trouble truck crews working at time t.

Second Stage Decision Variables : As mentioned in the original problem statement, after demand of events materializes in scenario k of set K, we introduce the following recourse decision variables:
$w^k_{i,t}$:= The volume of events of type i is unmet at time t under scenario k
$v^k_{i,t,r}$:= The volume of events of type i allocated to resource r at time t under scenario k

$z^k_{t,r,RT}$:= Binary variable to obtain non trouble truck crews for 1 hour at time t under scenario k
$z^k_{t,r,OT}$:= Binary variable to extend the shift of crew r for 1 hour at time t under scenario k
$u^k_{t,\alpha,A}$:= The variable to track over or under staffing of trouble truck crews at time t under scenario k
These decision variables have a penalty cost of $C_{P,\alpha}$ which represents the penalty cost of under or over staffing trouble truck crew at time t. The penalty cost of $C_{P,i}$ represents the cost of unmet demand of events i at time t. Finally, an additional hourly rate of $C_{r,RT}$ or $C_{r,OT}$ is incurred to procure additional crews (whether through shift extensions.

Objective Function : Hence the objective function for this formulation is to minimize the cost incurred to procure external trouble truck crews for shift s while minimizing the costs of under/over staffing these trouble truck crews, the costs to procure additional crews as well as the penalty cost of not meeting demand via any crew. Hence:
Minimize:
Cost of procuring trouble truck crews for shift S +
$\mathbb{E}$[Cost of Over Staffing or Under Staffing trouble truck crews at time t +
Cost of not meeting demand + Cost of procuring additional crews to meet demand]

$$\min \sum_{s\in S} C_{A,RT} l_s x_s + \sum_{k\in K} p_k * [\sum_{t\in T}\sum_{\alpha} C_{P,\alpha} u^k_{t,\alpha,A} + \sum_{t\in T}\sum_{r\in R'} C_{r,RT} z^k_{t,r,RT} + \sum_{t\in T}\sum_{r\in R} C_{r,OT} z^k_{t,r,OT} + \sum_{t\in T}\sum_{i\in I} C_{P.i} w^k_{it}]$$

where p_k represents the probability of scenario k.

First Stage Constraints:

Schedule Enforcement: To ensure that the schedules chosen for the trouble truck crews ensure sufficient staffing levels are met, represented as:

$$\sum_{s\in S} a_{st} x_s \geq y_t \quad \forall t \in T$$

Procurement Limitation: As mentioned in the previous formulation, no more than 6 trouble truck crews can be procured , hence:

$$\sum_{s\in S} x_s \leq 6$$

First Stage Decision Variables must be non negative integers: The decision variables must be non negative integers ie the number of crews procured must be positive integers. This is represented as:

$$x_s \in \mathbb{Z}^S_+, y_t \in \mathbb{Z}^T_+ \quad \forall s \in S, \forall t \in T$$

Second Stage Constraints:

Employment Standards Act Constraint: Based on the Ontario Employments Standard Acts, crews can work upto 16 hours on a single day (8 hours of downtime is mandated). Hence, we introduce a parameter H_r which represents the remaining hours constrained by the act. Hence:
For Non Trouble truck crews:

$$\sum_{t\in T} [z^k_{t,r,RT} + z^k_{t,r,OT}] \leq H_r \quad \forall r \in R', \forall k \in K$$

For Trouble Truck crews:

$$\sum_{t\in T} [z^k_{t,r,OT}] \leq H_r \quad \forall r \in R, \forall k \in K$$

Incoming arrivals must be met: The number of events allocated to crews r as well as overall unmet demand must equal the number of events of type $i \in I$ raised in time $t \in T$, represented as:

$$\sum_{r \in R} \mu_{t,i,r} v^k_{i,t,r} + w^k_{it} = \lambda^k_{it} \quad \forall k \in K, \forall t \in T, \forall i \in I$$

As defined above, $\mu_{t,i,r}$ represents the service rate to address event type i by resource r during time t. Further, λ^k_{it} represents the arrival rate of event type i during time t

Performance Constraints: As stated in section 2, there are specific performance constraints ie unmet demand over the entire time set T must be less than some percentage amount of the arrival rate λ^k_{it}, represented by the known parameter β_i which is specific for an event type i. This is represented as:

$$\sum_{t \in T} [w^k_{it} - \beta_i \lambda^k_{it}] \leq 0 \quad \forall i \in I, \forall k \in K$$

Trouble Truck Additional Crew Procurement Constraint: As the problem has the first stage decision of procuring these crews during shift s, we assume that we will not procure these crews during regular time. Hence for the trouble truck crews (r = A):

$$z^k_{t,A,RT} = 0 \quad \forall t \in T, \forall k \in K$$

Non Trouble Truck Crew Procurement Restriction: For non trouble truck crews, either the shift of the crew can be extended (ie paid at OT) or crew is procured (ie paid at RT):

$$z^k_{t,r,RT} + z^k_{t,r,OT} \leq 1 \quad \forall t \in T, \forall r \in R'$$

Additional crew Constraint: If additional crew are procured, work must be allocated to the procured crew:

$$v^k_{i,t,r} - z^k_{t,r,RT} - z^k_{t,r,OT} \geq 0 \quad \forall i \in I, \forall t \in T, \forall r \in R, \forall k \in K$$

Crew Allocations constrained by Staffing Levels: The following constraint limits the number of server allocations to available staffing levels represented as follows:

$$\sum_{i \in I} v^k_{i,t,r} \leq y_t \quad t \in T, \forall r \in R, \forall k \in K$$

Resource Demand should be met: The number of crews procured as well as the potential over or under staffing of the procured trouble truck crews must equal the demand of resources at time t D^k_t, represented as: To add

Second Stage Decision Variables: Binary Constraint The variables $z^k_{t,r,RT}$ and $z^k_{t,r,OT}$ are binary variables by definition:

$$z^k_{t,r,*} = \begin{cases} 1 & \text{if additional crew r is procured for an additional hour at time t under scenario k} \\ 0 & \text{otherwise} \end{cases}$$

Second Stage Decision Variables: Continuous

$$w^k_{i,t} \in \mathbb{R}^{ITK}_{+} \quad \forall i \in I, \forall t \in T, \forall k \in K$$

$$v^k_{i,t} \in \mathbb{R}^{ITK}_{+} \quad \forall i \in I, \forall t \in T, \forall k \in K$$

$$u^k_{t,\alpha,A} \in \mathbb{R}_{+} \quad \forall t \in T, \forall k \in K$$

5 Implementation and Results

5.1 Data Acquisition and Prep:

Service Rates and Arrival Rates: Data was obtained from a North American utility. The emergency event data was cleansed and coded to identify the distinct type of events that were identified using data for 6 years (2012-2018). Once the data was obtained for each hour t in set T and for each event type i in set I. As a result, the mean of the arrivals and the service rates was obtained. Arrivals were defined to be the number of events of type i created at time t. Similarly, the service rate defined to be the amount of time taken to complete events of type i. Based on existing research, arrival rates tend to follow a Poisson distribution and service rates follow a Exponential Distribution[]. Upon building the respective distributions, different methods and models were applied.

Input Parameters: Hourly rates for procuring a trouble crew as well as penalty costs for not meeting demand were assumed keeping with market standards but different from actual rates utilized. Further, the length of shift s in set S was assumed to be 12 hours.

Additional Constraint: During implementation an additional constraint was added that is not listed in the formulation listed in section 4.1.3. This constraint was added to each of the implementations and is a first stage constraint as follows: For shift s in set S, where set S comprises of a Day and night shift as explained in section 2, we introduce a constraint to ensure the day crews procured are more than the night crews procured. Specifically, in keeping with current practice at the utility, we introduce the constraint:

$$2 * x_d \geq 3 * x_e$$

Where $x_s = x_d$ if shift is day shift and $x_s = x_e$ if shift is night shift

5.2 Extensive Form:

The model detailed in Section 4.1.3 was formulated under the function $extensive\_formulation$. This function takes in service rates, arrival rates and the number of scenarios. The model is then solved using GurobiPy to obtain the results shown in Fig. 1. The solution suggests procuring 4 crews during the day shift (ie from 7am - 7pm) and 2 crews during the night shift (ie from 7pm - 7am).

```
    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0 94638.3151    0   26 142525.600 94638.3151  33.6%     -    1s
H    0     0                    95575.845183 94638.3151  0.98%     -    1s
     0     0     cutoff    0      95575.8452 95575.8452  0.00%     -    1s

Cutting planes:
  MIR: 1195

Explored 1 nodes (6724 simplex iterations) in 1.72 seconds
Thread count was 4 (of 4 available processors)

Solution count 2: 95575.8 142526

Optimal solution found (tolerance 1.00e-04)
Best objective 9.557584518269e+04, best bound 9.557584518259e+04, gap 0.0000%
The time used for extensive formulation is:  1.7310459613800049
```

Figure 1: Results of Extensive Formulation

5.3 L Shaped Bender's Decomposition: Multi-Cut

The algorithm detailed in [7]was implemented in Python under the functions $regular\_multicut\_benders$ and $ModifyAndSolveSP\_MultiCut$. The results were obtained and are shown in Fig. 2. Similar to the extensive form, we obtain the same procurement strategy with an objective value of $95, 576, requiring 3 iterations and 2000 cuts.

```
Iteration:  1
UB: 151525.6000000002
LB: 9000.0
Iteration:  2
UB: 95575.84518268508
LB: 18495.12682974354
Iteration:  3
UB: 95575.84518268508
LB: 95575.84518268525

The optimal solution has been found with an objective value:  95575.84518268508
Time used is:  2.4755475521087646
Total number of cuts added is:  2000
Number of Iterations is : 3
```

Figure 2: Results of Multi-cut Benders Decomposition

5.4 L Shaped Bender's Decomposition: Single-Cut

The algorithm detailed in [7] was implemented in Python under the functions $SingleCutBenders$ and $ModifyAndSolveSP\_SingleCut$. The results were obtained and are shown in Fig. 3. Similar to the extensive form, we obtain a similar procurement strategy of 4 day crews and 2 night crews with an objective cost of $95, 576. The model took longer than the Extensive form to run, requiring 3 iterations with 2 added cuts.

```
Iteration:  1
UB: 151525.6000000002
LB: 9000.0
Iteration:  2
UB: 95575.84518268508
LB: 18000.0
Iteration:  3
UB: 95575.84518268508
LB: 95575.84518268531

The optimal solution has been found with an objective value:  95575.84518268508
Time used is:  2.7940268516540527
Total number of cuts added is:  2
Number of Iterations is : 3
```

Figure 3: Results of Single-cut Benders Decomposition

5.5 Single Cut Level Method

The algorithm detailed in [7] was implemented in Python under the functions funcnamehere and fncnamehere. The results were obtained and are shown in Fig. 4. The results obtained is similar to previous methods but the model took longer to solve (2X Benders and 4X Extensive form, approx.).

5.6 Risk Measure Application: CVaR

The model detailed in section 4.2 was implemented in Python under the functions fncnamehere and fncnamehere, and the following results were obtained:

```
Iteration:  1
UB: 142525.6000000001
LB: 18000.0
Iteration:  2
UB: 111493.55419759324
LB: 88456.47759246262
Iteration:  3
UB: 98452.34174962889
LB: 94775.68705132011
Iteration:  4
UB: 96376.81931062779
LB: 95511.05874255141
Iteration:  5
UB: 95575.84518268508
LB: 95575.84518268531
The optimal solution has been found with an objective value:  95575.84518268508
Time used is:  4.47679877281189
Total number of cuts added is:  5
Number of Iterations is : 5
```

Figure 4: Results of Single-cut Level Method

```
Iteration    Objective       Primal Inf.    Dual Inf.      Time
   30519    1.0188142e+06   4.222989e+04   0.000000e+00      5s
   37452    1.0534003e+06   0.000000e+00   0.000000e+00      7s

Root relaxation: objective 1.053400e+06, 37452 iterations, 4.33 seconds

    Nodes    |    Current Node    |     Objective Bounds      |     Work
 Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time

     0     0     cutoff    0      1067007.20 1067007.20  0.00%     -    8s

Explored 0 nodes (40795 simplex iterations) in 8.12 seconds
Thread count was 4 (of 4 available processors)

Solution count 1: 1.06701e+06
No other solutions better than 1.06701e+06

Optimal solution found (tolerance 1.00e-04)
Best objective 1.067007204151e+06, best bound 1.067007204151e+06, gap 0.0000%
The time used for extensive formulation is:  8.135231733322144
```

Figure 5: Results of CVaR

5.7 SAA Sampling:

Using methods detailed in [7] and applying SAA sampling, we obtained the following results: As with

```
mean SAA objval of LB part =  96013.87923032543
unbiased stdev of SAA objval of LB part =  891.7633087457701
objective values of sample of LB part:  [96200.3506389  96600.33528163 96665.55741019 94335.34922149
 94953.5365929  96474.12326311 95690.78261546 96439.26303653
 97293.15158121 95486.34266184]
solutions of sample of LB part - Day Shift Crews:  {0: 4.0, 1: 4.0, 2: 4.0, 3: 4.0, 4: 4.0, 5: 4.0, 6: 4.0, 7: 4.0, 8: 4.0,
9: 4.0}
solutions of sample of LB part - Night Shift Crews:  {0: 2.0, 1: 2.0, 2: 2.0, 3: 2.0, 4: 2.0, 5: 2.0, 6: 2.0, 7: 2.0, 8: 2.
0, 9: 2.0}
```

Figure 6: Lower Bound - SAA

the other methods, we obtain an objective value close to $96,000$. The optimality gap for the worst case bounds is found to be between $95,376$ and $96,037$ where we have 95% confidence that the lower bounds are between $95,376$ and $96,652$ and the upper bounds are between $94,800$ and $96,037$

5.8 Remarks:

The performance of all models was compared for a variety of scenarios to test how the performances of the models and the objective value changes (if it does).

We have tested four number of scenarios: 1000, 3000, 5000, and 10000. For each number of scenarios, we have run three problem instances with all solution methods. The results are shown in Fig. 8, 9, 10, 11. We also generated a graph to visualize the average performances of different methods as shown in Fig. 12.

With regards to comparing the impact on the runtime to the optimal solution/second vs the number of scenarios, as the scenarios increase the runtime increases. The runtime is the largest for a single-cut

```
The index of the best sample average objective value:  3
candidate solution =  4.0 2.0 {0: 2.0, 1: 2.0, 2: 2.0, 3: 2.0, 4: 2.0, 5: 2.0, 6: 2.0, 7: 4.0, 8: 4.0, 9: 4.0, 10: 4.0, 11:
4.0, 12: 4.0, 13: 4.0, 14: 4.0, 15: 4.0, 16: 4.0, 17: 4.0, 18: 4.0, 19: 2.0, 20: 2.0, 21: 2.0, 22: 2.0, 23: 2.0}
mean solution objval of UB part =  95418.58203750722
unbiased stdev of solution objval of UB part =  8411.251432273344

95% ci on lower bound = [ 95375.96630885021 , 96651.79215180065 ]
95% ci on upper bound = [ 94799.79456121767 , 96037.36951379677 ]
The worst optimality gap = [ 95375.96630885021 , 96037.36951379677 ]
```

Figure 7: Upper Bound - SAA

Runtime to optimal solutions				
Method	Instance 1	Instance 2	Instance 3	Mean
Extensive Form	2.197279	2.0684662	1.3773153	**1.88102**
Single-cut Benders	3.7630327	3.2904668	3.2827485	**3.44542**
Multi-cut Benders	3.5295548	3.2772679	3.2505214	**3.35245**
Single-cut Level Method	4.4573259	4.994122	5.0002863	**4.81724**
CVaR	10.557514	6.8243144	6.9005089	**8.09411**
Objective value and LB & UB				
Method	Instance 1	Instance 2	Instance 3	
Extensive Form	88746.229	84423.346	93558.824	
Single-cut Benders	88746.229	84423.346	93558.824	
Multi-cut Benders	88746.229	84423.346	93558.824	
Single-cut Level Method	88746.229	84423.346	93558.824	
CVAR	991677.32	943275.05	1044092.6	
95% CI_LB_left	88264.767	83710.191	93217.761	
95% CI_LB_right	89314.188	84478.525	94260.096	
95% CI_UB_left	88089.375	83354.237	92644.386	
95% CI_UB_left	89266.137	84451.638	93889.034	

Figure 8: Performances of Different Methods for Three Instances with 1000 Scenarios

Runtime to optimal solutions				
Method	Instance 1	Instance 2	Instance 3	Mean
Extensive Form	5.13326502	7.2444291	4.8709373	**5.74954**
Single-cut Benders	9.5540719	9.6404245	9.5841355	**9.59288**
Multi-cut Benders	9.76827407	9.7799804	9.8049209	**9.78439**
Single-cut Level Method	12.2882309	12.17188	14.628857	**13.0297**
CVaR	55.7946541	50.306978	47.301774	**51.1345**
Objective value and LB & UB				
Method	Instance 1	Instance 2	Instance 3	
Extensive Form	91100.5295	88589.599	88748.468	
Single-cut Benders	91100.5295	88589.599	88748.468	
Multi-cut Benders	91100.5295	88589.599	88748.468	
Single-cut Level Method	91100.5295	88589.599	88748.468	
CVAR	1017137.52	989737.36	991104.94	
95% CI_LB_left	91010.8182	88130.488	88077.37	
95% CI_LB_right	91855.5692	88808.899	89281.124	
95% CI_UB_left	90335.2505	87922.365	88391.049	
95% CI_UB_left	91528.2929	89087.521	89541.727	

Figure 9: Performances of Different Methods for Three Instances with 3000 Scenarios

Runtime to optimal solutions				
Method	Instance 1	Instance 2	Instance 3	Mean
Extensive Form	10.8150635	9.2322717	9.4791474	**9.84216**
Single-cut Benders	16.0485778	15.979492	16.803355	**16.2771**
Multi-cut Benders	16.4538543	16.590645	18.45677	**17.1671**
Single-cut Level Method	20.2687662	20.469409	23.140351	**21.2928**
CVaR	144.015532	143.76651	139.88107	**142.554**
Objective value and LB & UB				
Method	Instance 1	Instance 2	Instance 3	
Extensive Form	90686.7811	89556.203	93730.581	
Single-cut Benders	90686.7811	89556.203	93730.581	
Multi-cut Benders	90686.7811	89556.203	93730.581	
Single-cut Level Method	90686.7811	89556.203	93730.581	
CVAR	1012471.11	1000011.5	1046399.3	
95% CI_LB_left	90061.6024	89072.281	93964.721	
95% CI_LB_right	90821.3655	90113.991	94617.904	
95% CI_UB_left	89983.491	89274.242	93088.085	
95% CI_UB_left	91169.1149	90424.816	94318.116	

Figure 10: Performances of Different Methods for Three Instances with 5000 Scenarios

Runtime to optimal solutions				
Method	Instance 1	Instance 2	Instance 3	Mean
Extensive Form	26.826448	18.268265	33.070396	**26.055**
Single-cut Benders	34.1901455	35.822234	33.04388	**34.3521**
Multi-cut Benders	37.8537452	34.855505	35.060556	**35.9233**
Single-cut Level Method	54.1285956	43.428431	47.177784	**48.2449**
CVaR	579.460775	634.44536	609.7164	**607.874**
Objective value and LB & UB				
Method	Instance 1	Instance 2	Instance 3	
Extensive Form	92313.6777	93154.893	86687.17	
Single-cut Benders	92313.6777	93154.893	86687.17	
Multi-cut Benders	92313.6777	93154.893	86687.17	
Single-cut Level Method	92313.6777	93154.893	86687.17	
CVAR	1030802.29	1040037.1	1032489.2	
95% CI_LB_left	91783.5629	92612.877	86319.253	
95% CI_LB_right	93101.6354	93944.408	87381.86	
95% CI_UB_left	91614.5065	92689.695	85623.677	
95% CI_UB_left	92838.1146	93860.114	86854.683	

Figure 11: Performances of Different Methods for Three Instances with 10000 Scenarios

level method and the smallest for the extensive form solution. As iterations increase, both the extensive form and the single cut level methods there is a dramatic increase in runtime. We also observed that both benders methods are fairly close and the runtime increases linearly as iterations increase.

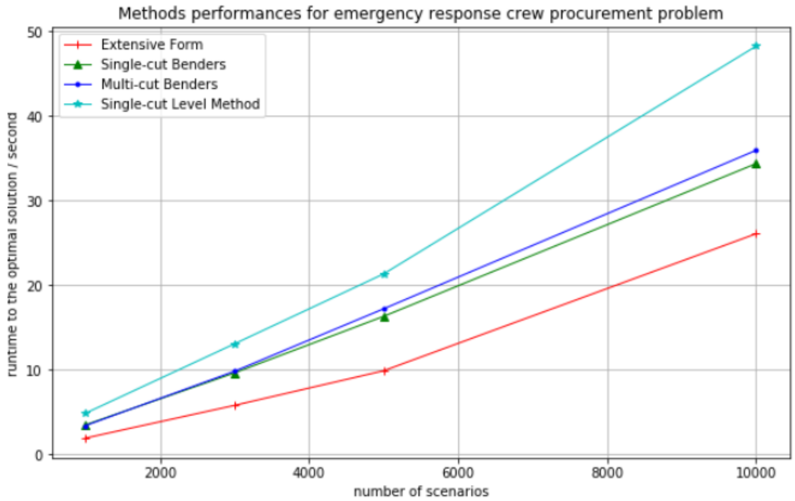

Figure 12: Performances of Different Methods

These results violate our knowledge that level method and benders decomposition often perform better than extensive form when the number of scenarios is large. The reason might be that the Bender's Decomposition and Level Method are not designed to specialize in Stochastic Programming Problems with Integer Variables in the first stage [2][3]. Better results should be obtained by using Integer Benders Decomposition algorithm or prioritizing the branching strategies or integer cuts as suggested by some of the papers mentioned in section 3.

6 Conclusion and Future Work

In this report, we presented 3 distinct formulations to address the problem of procuring emergency response crews for day and night shifts with the goal of minimizing scheduling costs and recourse stage abandonment and/or utilization costs. The basic formulation of the three models was implemented using a variety of standard modelling algorithms (eg Benders, Level, SAA, Extensive Form) yielding similar decisions with similar objective values. Differences in methods were observed wrt the run time of these algorithms. As presented in Section 5, the Single cur Level method takes the longest to run whereas the extensive form solved via GurobiPy takes the least time to run. We posit that this might be due the nature of the problem we've set up ie a Stochastic Integer Program, for which existing literature indicates utilizing novel methods to enhance the speed of convergence, as the extensive form tends to scale poorly as data and scenarios increase.

In terms of future work, an implementation of the third formulation presented should be considered along with utilizing Integer Bender's Decomposition or Mixed Integer Rounding to determine how the implementations perform on this complex formulation.

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