Numerische Implementierung der linearen FEM

Homework 4

Group: Diamond

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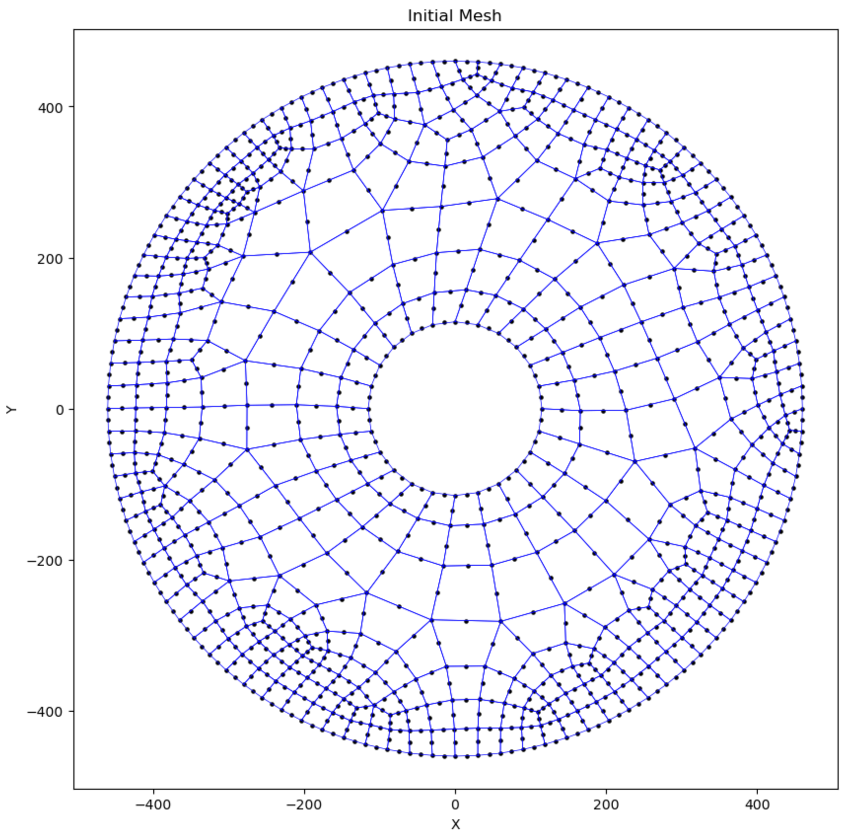
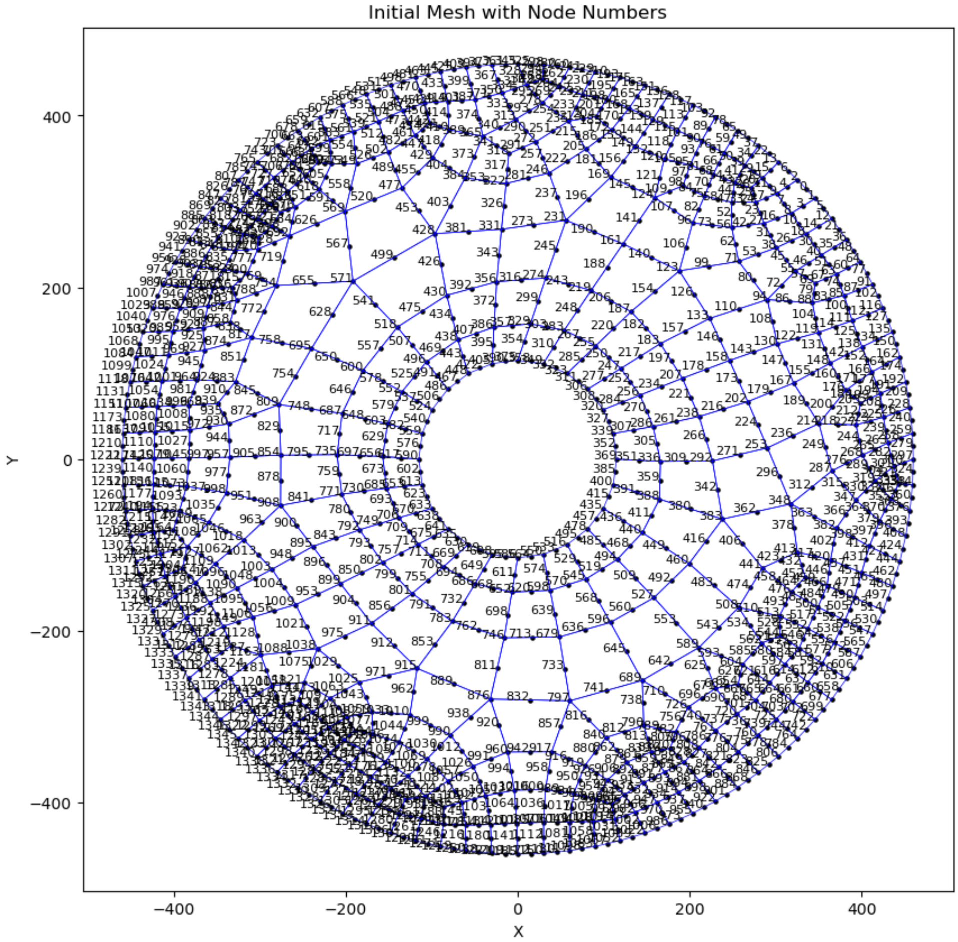
➢ Preprocessor

1. Create 2D Model of train wheel with GiD program (attached in the submission zip file) and generate the mesh, then export the mesh data to python.

2. Initialization and read the nodes and elements from the mesh file, then plot the initial mesh.

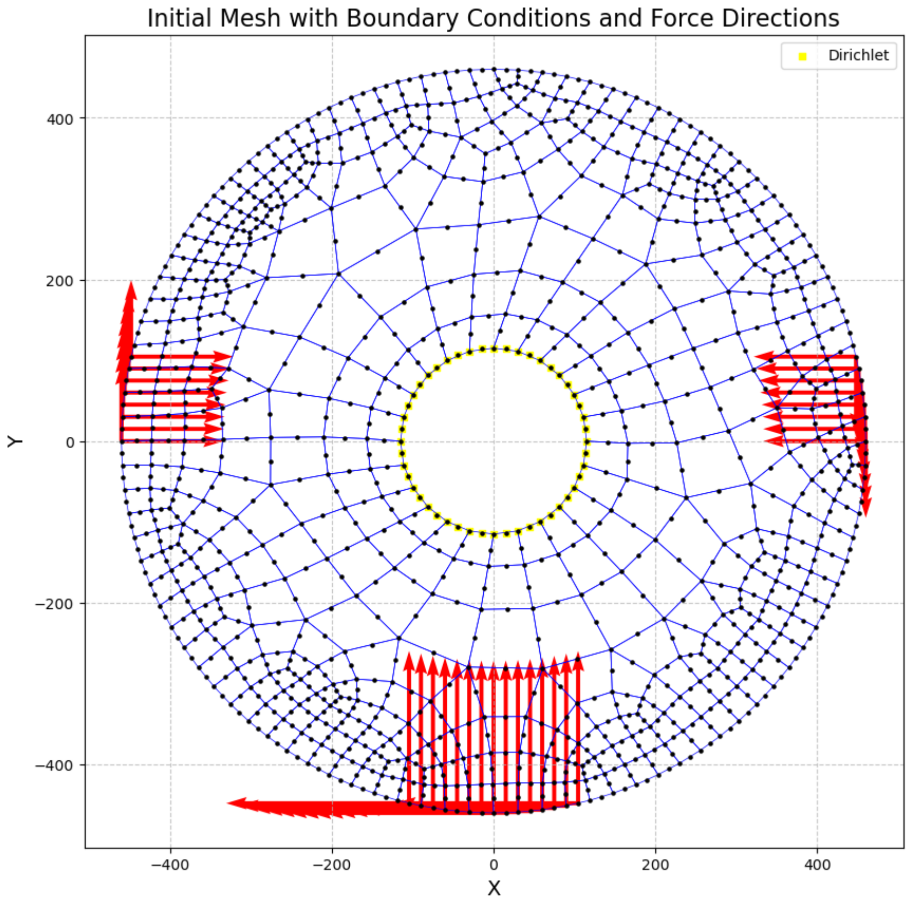
   - The wheel mesh is generated with specified inner (230 mm) and outer diameters (920 mm), and the number of radial and circumferential elements.
   - Nodes and elements are defined using 8-noded quadrilateral elements with mid-side nodes.
   - The initial mesh and specific nodes are plotted.





3. Apply Dirichlet and Neumann boundary conditions to the wheel.

- Dirichlet boundary conditions (fixed nodes) are applied to the inner nodes of the wheel.
- Neumann boundary conditions (forces) are applied to the outer nodes of the wheel. (55835N at the bottom side, 10^4 N on both right and left side)
- The mesh with boundary conditions and forces visualized using red arrows to indicate force directions and magnitudes.

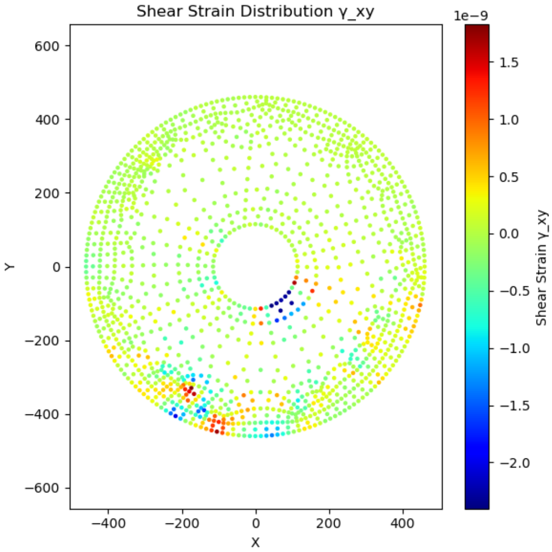


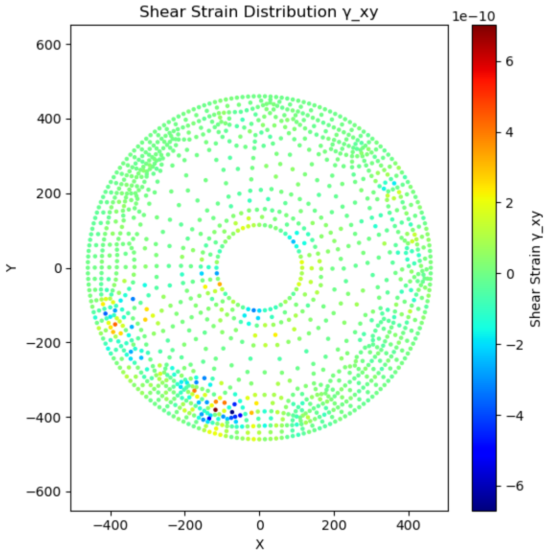
4. Define shape functions and their derivatives for 8-noded quadratic elements.
5. Set up the Gauss quadrature points (9 integration points per element) and weights for numerical integration.
6. Assemble the global stiffness matrix K and force vector F.
   - The local stiffness matrix for each element is computed using numerical integration and assembled into the global stiffness matrix.
7. Apply the boundary conditions and solve the system, in order to get the displacements.
   - Dirichlet boundary conditions are applied by modifying the global stiffness matrix and force vector to account for fixed nodes.

➢ Vergleich zwischen Q8R und Q8

Was passiert, wenn ein einzelnes Element Q8R berechnet werden soll (im Vergleich zu Q8)?

Compared to Q8 (left picture, 9 Gauss quadrature points), Q8R (right picture, 4 Gauss quadrature points) can reduce computational effort because fewer integration points per element need to be calculated. This can improve the efficiency of the algorithm. However, by using fewer integration points, the formulation becomes less sensitive to the shear strain as we can see in these two pictures when comparing with each other, this can mitigate the locking effect. But reduced integration (Q8R) can sometimes lead to hourglassing. And using too few Gauss points (less than required for exact integration) can lead to inaccurate results, which may not capture all aspects of the system response correctly. In area where singularity occurs, it's recommended to avoid using reduced integration.





➢ Plot stress distributions σ_x, σ_y, τ_{xy} and von Mises stress

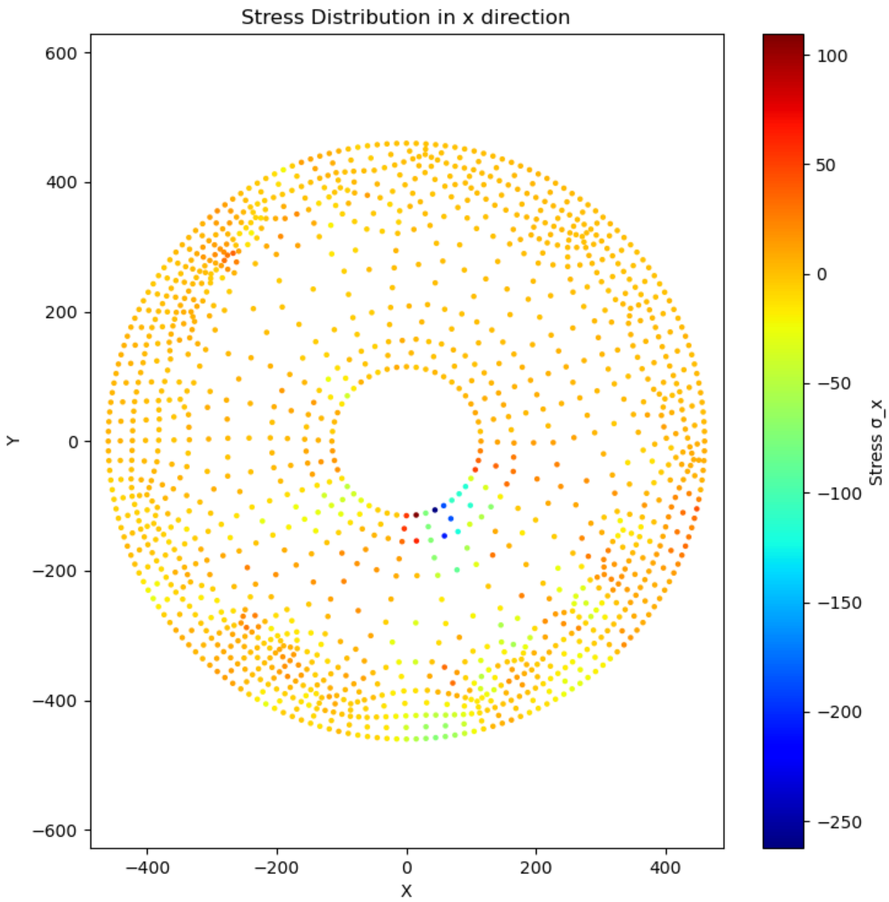
The following 4 pictures show the normal stress distribution in x-, y-direction, shear stress on xy-area and the equivalent Von Mises stress.

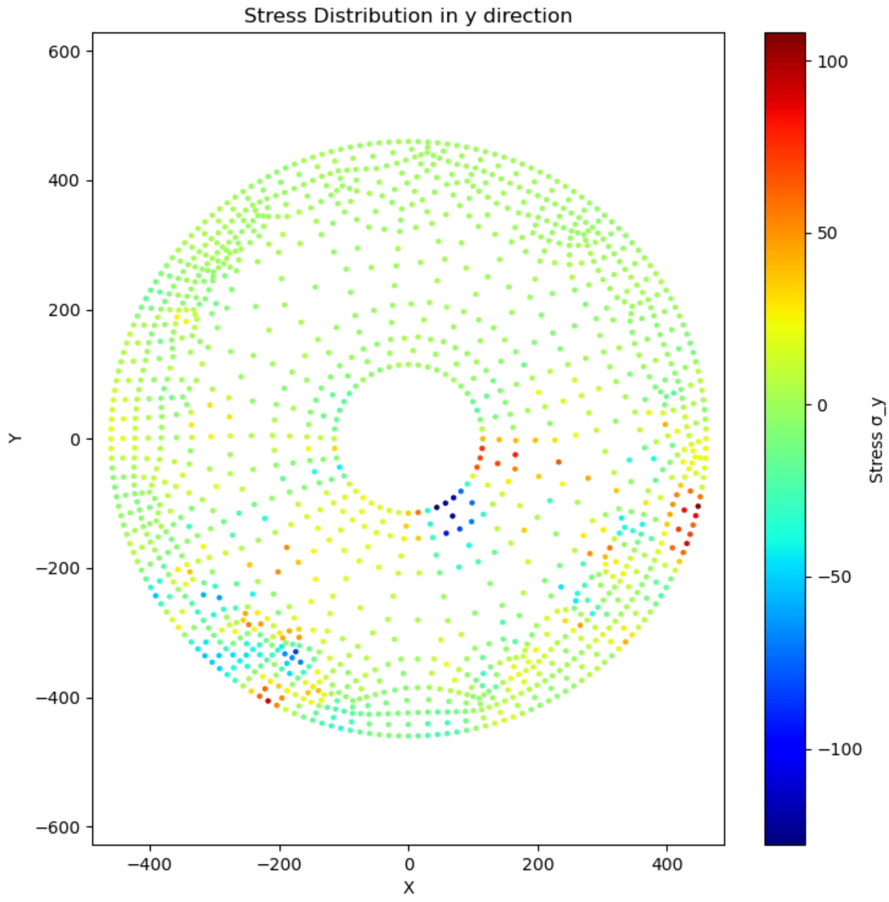
Normal stress in x direction has a uniform stress distribution across the structure (mostly yellow). Small areas of high compressive stress (blue) are near the inner circle at the 4 o'clock position. Most areas of the wheel are covered in low stress (orange).

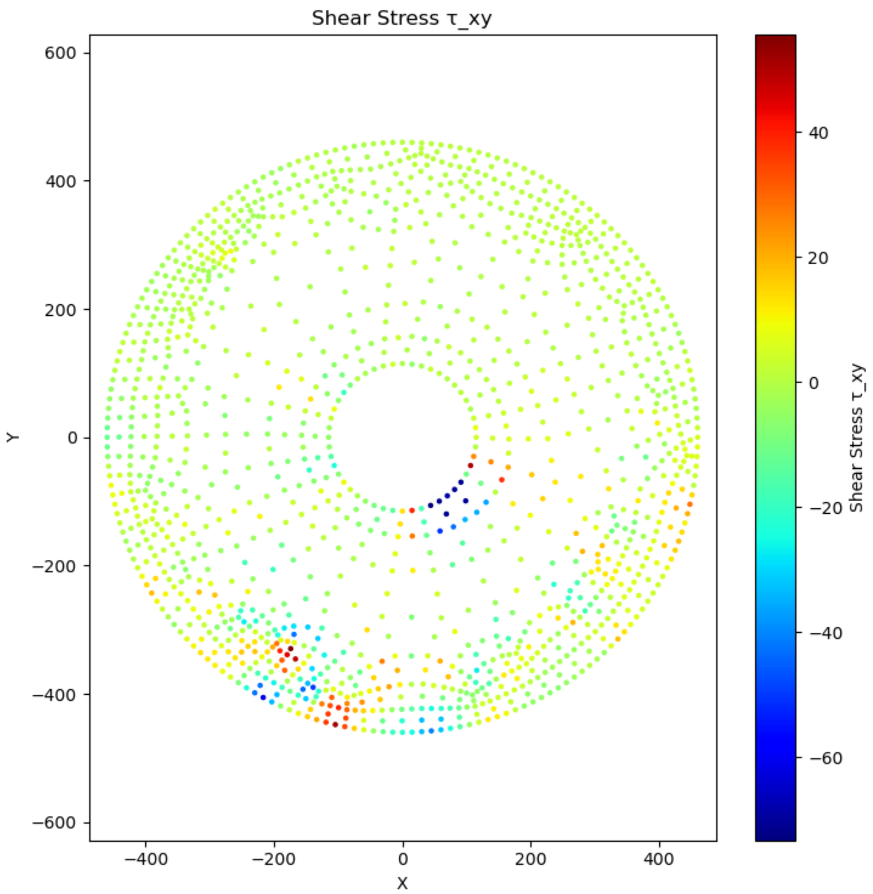
Stress Distribution in y-direction has a more varied stress distribution compared to the one in x-direction. High tensile stress (red) occurs at 4 o’clock position of inner circle as well as 4 and 7 o’clock of the outer edge. Areas of high compressive stress (blue) at the 5 o'clock positions near the inner circle and 7 o'clock positions near the outer edge.

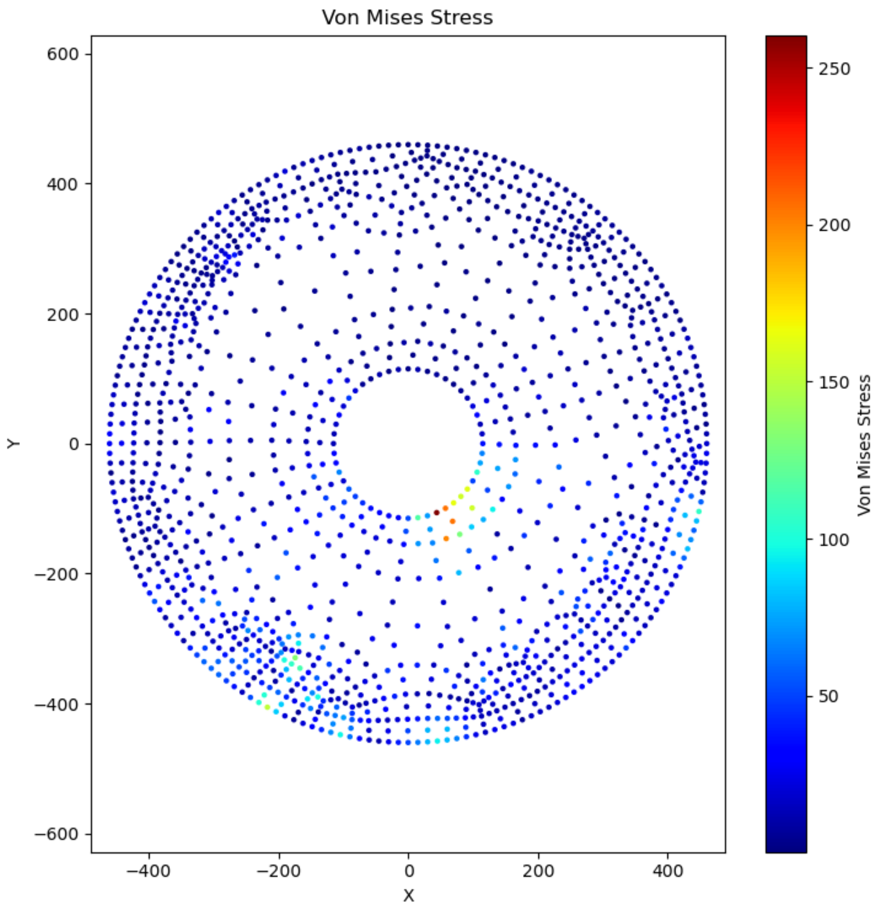
Shear tress has a complex stress distribution throughout the structure. The highest positive shear stresses (red) occur in the lower right quadrant, especially near the inner and outer circle area. The highest negative shear stresses (blue) appear at around 5 o'clock position near inner circle and 7 to 8 o'clock position near outer circle. Most areas of the wheel are covered in low shear stress (green).

Von Mises stress shows the overall stress intensity, combining effects of normal and shear stresses. Therefore, Von Mises stress diagram is particularly useful for identifying potential failure points in this wheel structure. In this diagram, we can notice that the highest stress concentration (red/orange) near the inner circle, particularly around the 3 to 5 o'clock position. And stress generally decreases towards the outer edge (blue), which provides us a good indication of where material failure is most likely to occur. The inner circle area seems to be a critical region with high stress concentrations. As a result, we will further investigate whether there is any convergence behavior if more elements are applied in the inner circle of the wheel structure.


Stress Distribution in x direction
Y
X
Stress σ_x
600
400
200
0
−200
−400
−600
−400
−200
0
200
400
100
50
0
−50
−100
−150
−200
−250


Stress Distribution in y direction
Y
X
Stress σ_y
600
400
200
0
−200
−400
−600
−400
−200
0
200
400
100
50
0
−50
−100


Shear Stress τ_xy
600
400
200
0
−200
−400
−600
Y
−400
−200
0
200
400
X
40
20
0
−20
−40
−60
Shear Stress τ_xy


Von Mises Stress
600
400
200
0
−200
−400
−600
Y
−400
−200
0
200
400
X
250
200
150
100
50
Von Mises Stress

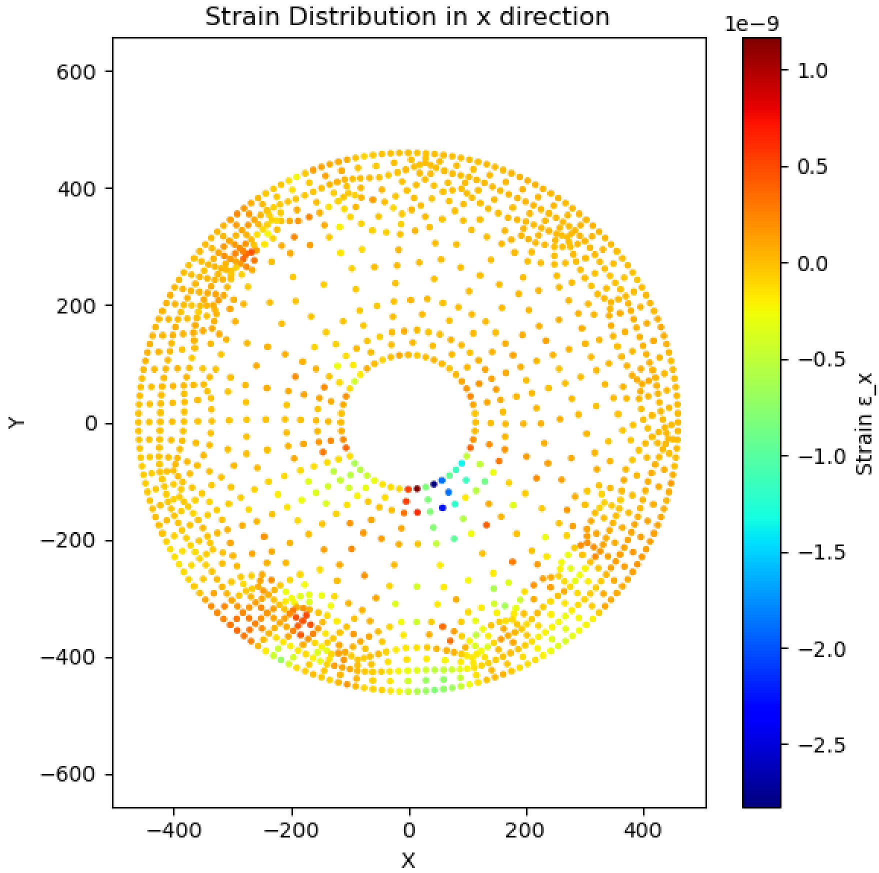
- **Plot strain distributions ε_x, ε_y, γ_{xy}**

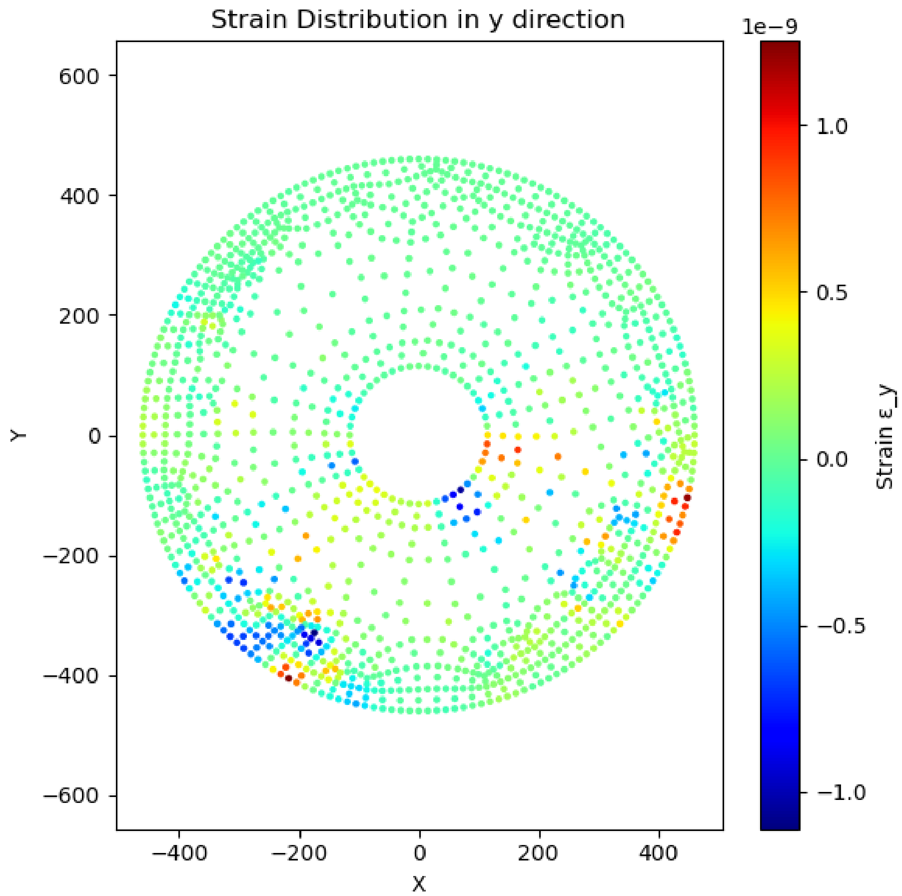
Most of the strain distribution in x direction are covered in orange (positive strain), which indicates the expansion in the x direction. Areas of high negative strain (blue) appear near the inner circle at the 5 o'clock position, showing localized compression. Areas of higher positive strain (orange to red) occur at the top-left and bottom-left quadrants of the outer edge.

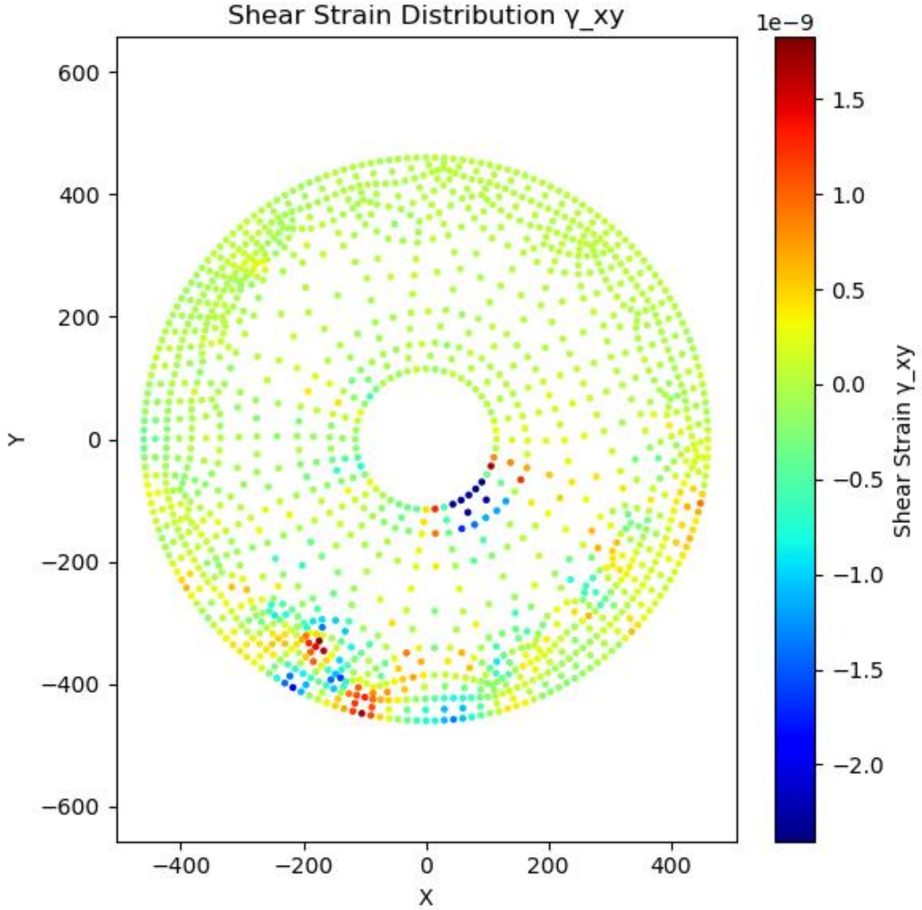
Strain in y direction has a more varied strain distribution compared to the one in x direction. Localized areas of high positive strain (red) concentrate at around 4 o'clock position of inner and outer circle. We can see the significant area of high negative strain (dark blue) appear in the bottom-left quadrant and 5 o'clock position of inner circle.

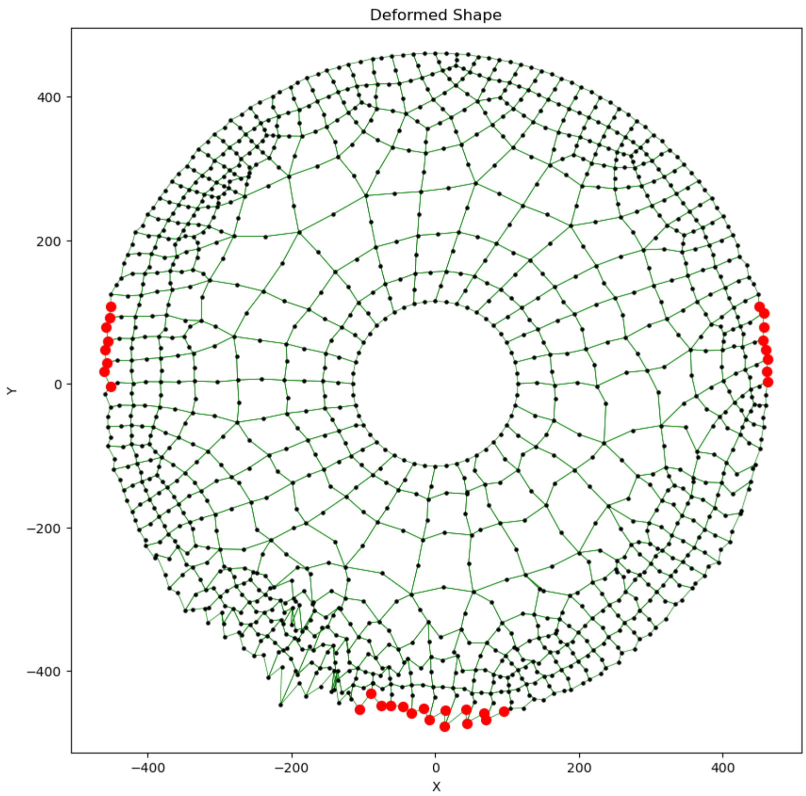
Shear Strain Distribution has a similar distribution like strain in y direction. Highest positive shear strains (red) in the lower right quadrant near the edge. Highest negative shear strains (dark blue) occur in small areas near the inner circle, particularly around the 4 o'clock position and around 7 o'clock position of outer edge.

The x-direction strain is predominantly positive, while the y-direction strain is mostly negative, showing that the structure may be elongating in x direction and compressing in y direction. The inner circle areas, especially around the 3 to 5 o'clock positions, show significant strain concentrations in all three distributions, which may require further analysis, e.g. mesh refinement. This phenomenon correlates with the results of Von Mises stress distribution.


Strain Distribution in x direction
1e−9
Strain ε_x
Y
X


Strain Distribution in y direction
1e−9
Strain ε_y
Y
X

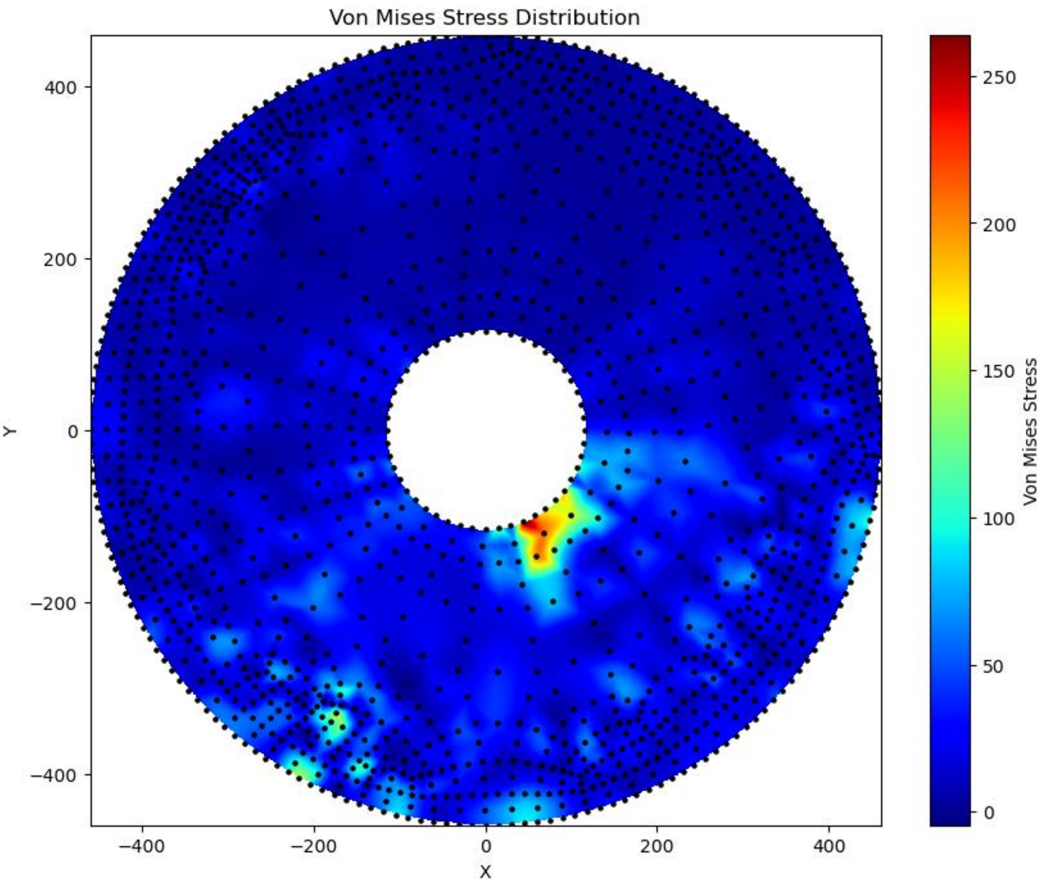

Shear Strain Distribution γ_xy
1e−9
600
400
200
0
−200
−400
−600
Y
−400
−200
0
200
400
X
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Shear Strain γ_xy


Deformed Shape
400
200
0
−200
−400
Y
−400
−200
0
200
400
X

➢ Plot the global interpolation

Global interpolation method can utilize more data points across the entire region to produce a smoother and more continuous stress distribution, compared to the result with nodal interpolation, because nodal interpolation displays the calculated stress values at each node, accurately reflecting the stress state at each node.

The picture below shows the Von Mises stress distribution with global interpolation of the entire structure. This aids in identifying weak spots and potential failure locations within the structure. Although the results may not be as precise as the one with nodal interpolation, it offers better visual effects and stress distribution trends.



➢ Convergence analysis

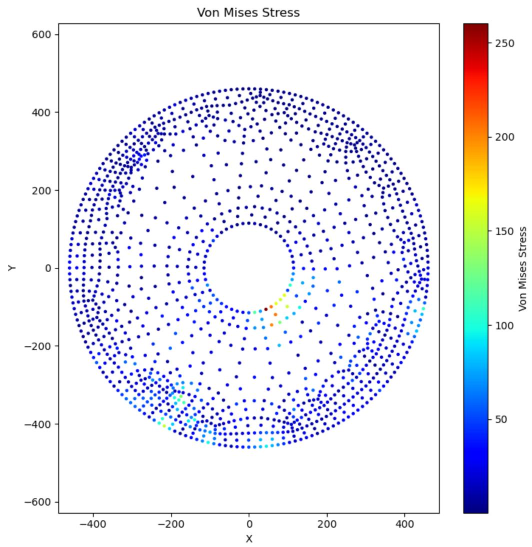
To analyze the convergence behavior of the stress distributions between the three different mesh densities 409 (up-left picture), 674 (up-right picture) and 868 elements (bottom-left picture).

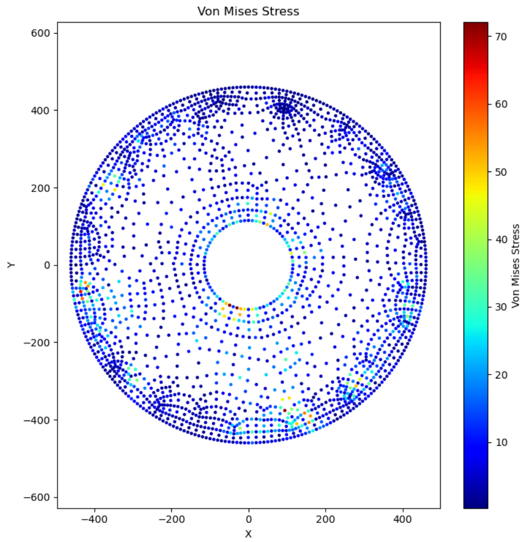
As the number of elements increases, the stress range decreases for all components and stress patterns become more uniform and less concentrated, indicating convergence towards a more stable solution and better resolution of the stress field.

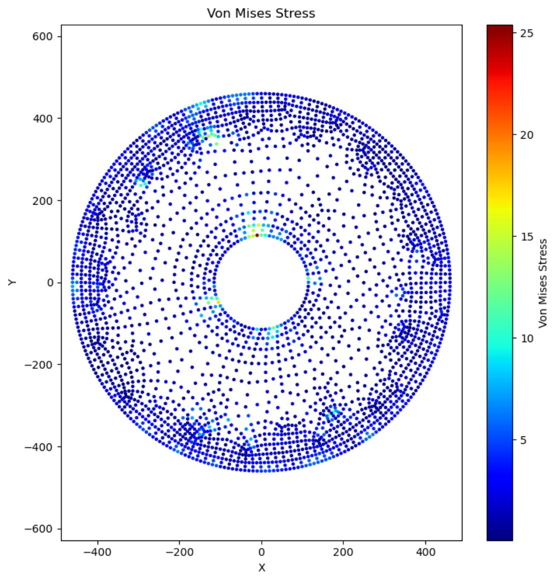
Furthermore, the value of maximum stress values decline as mesh density increases, which shows that coarser meshes may overestimate maximum stresses. The 868-element mesh has the most stable and uniform stress distributions across all components, and is therefore, closest to a converged solution.

Elements	Max Stress X	Max Stress Y	Max Shear Stress	Max Von Mises Stress
409	100	100	50	250
674	50	40	30	100
868	15	5	12.5	25

Elements	Stress Range X	Stress Range Y	Shear Stress Range	Von Mises Stress
409	350	200	110	250
674	110	60	70	100
868	25	15	17.5	25



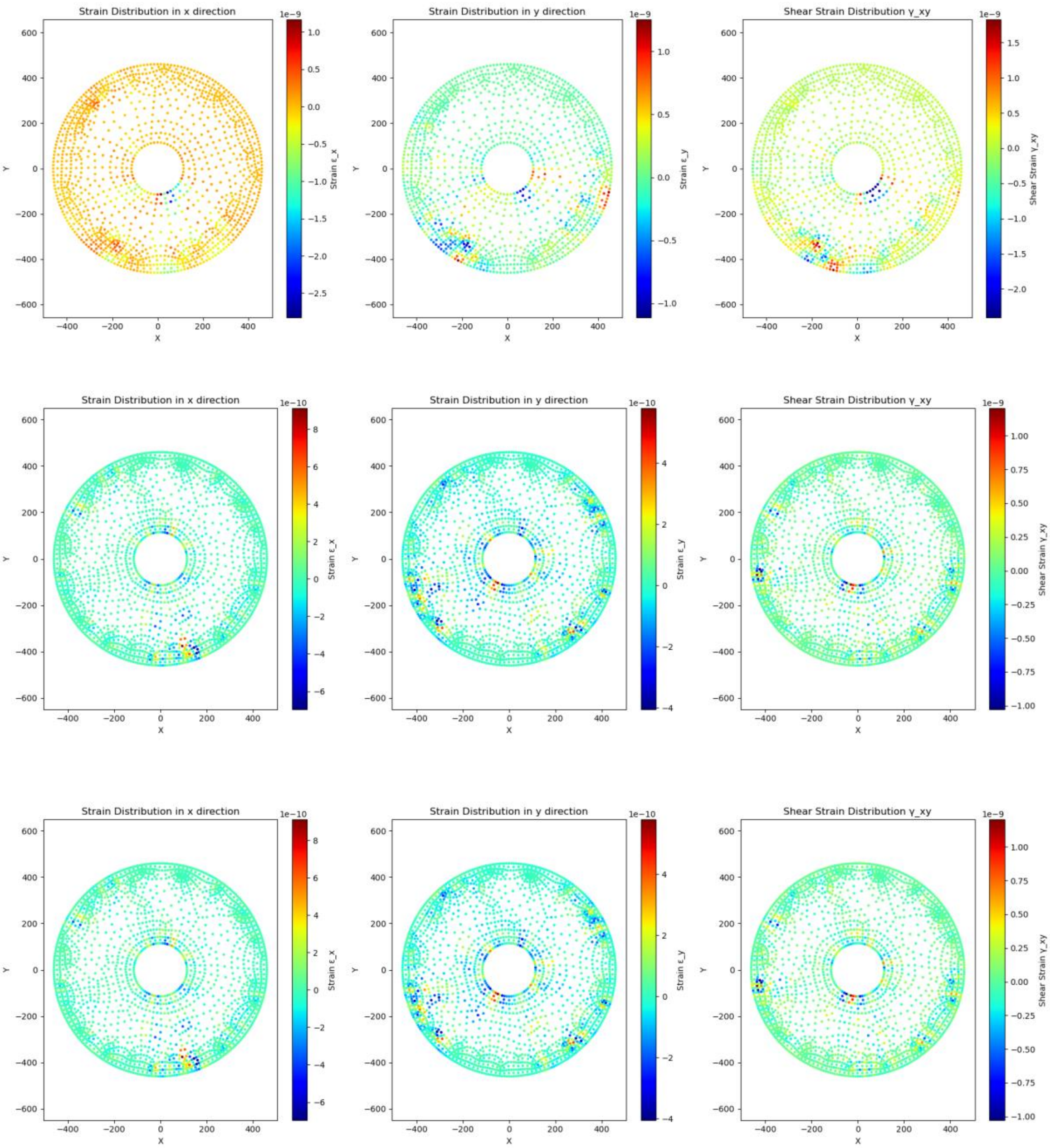




As the number of elements increases, the overall strain patterns become more refined and show more local detail. The table below shows that the maximum and minimum value of strain in both x, y direction as well as shear stress decrease as number of elements increases, suggesting that the coarser mesh may be overestimating strain.

Moreover, the changes between 674-element mesh (middle 3 pictures) and 868-element mesh (bottom 3 pictures) are less dramatic than the difference between 409-element mesh (top 3 pictures) and 674-element mesh, which shows that the solution is converging towards a more accurate representation of the actual strain field.

Elements	Max X	Min X	Max Y	Min Y	Max Shear	Min Shear
409	1.0e-9	-2.5e-9	1.1e-9	-1.0e-9	1.7e-9	-2.2e-9
674	6.5e-10	-8.0e-10	4.5e-10	-3.0e-10	1.1e-9	-1.5e-9
868	1.7e-10	-1.0e-10	7.5e-11	-1.0e-10	4.8e-10	-1.5e-10



Conclusion: The results of both strain and stress distribution from our calculation are trustworthy as the values are generally approaching convergence with increased number of elements. However, the more refined the meshes are, the more accurate results we can get.