TestExercise3

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Introduction

This test exercise is of an applied nature and uses data that are available in the data file TestExer3. We consider the so-called Taylor rule for setting the (nominal) interest rate. This model describes the level of the nominal interest rate that the central bank sets as a function of equilibrium real interest rate and inflation, and considers the current level of inflation and production. Taylor (1993) considers the model:

$$i_t = r^* + \pi_t + 0.5(\pi_t - \pi^*) + 0.5g_t$$

with i_t the Federal funds target interest rate at time t, r^* the equilibrium real federal funds rate, π_t a measure of inflation, π^* the target inflation rate and g_t the output gap (how much actual output deviates from potential output). We simplify the Taylor rule in two manners. First, we avoid determining r^* and π^* and simply add an intercept to the model to capture these two variables (and any other deviations in the means). Second, we consider production yy rather than the output gap. In this form the Taylor rule is

$$i_t = \beta_1 + \beta_2\pi_t + \beta_3 y_t + \varepsilon_t. (1)$$

Monthly data are available for the USA over the period 1960 through 2014 for the following variables:

- INTRATE: Federal funds interest rate
- INFL: Inflation
- PROD: Production
- UNEMPL: Unemployment
- COMMPRI: Commodity prices
- PCE: Personal consumption expenditure
- PERSINC: Personal income
- HOUST: Housing starts

Question 1

Use general-to-specific to come to a model. Start by regressing the federal funds rate on the other 7 variables and eliminate 1 variable at a time.

The table below outlines the execution results of the models, with each model removing a variable based on their p-value. The variable with the highest p value is removed, and the model is run with the remaining variables. The removal of variables stops when the p-values of all the remaining variables are above 0.05.

Variable	Model 1 p-value	Model 2 p-value	Model 3 p-value
Intercept	0.36701	0.21827	0.29764
INFL	0	0	0
PROD	0.14833	0.32319	
UNEMPL	0.28992		

COMMPRI	0.06382	0.02132	0.00464
PCE	0	0	0
PERSINC	0.00005	0.00004	0.00006
HOUST	0.00004	0	0

As it can be seen above the final model chosen using the general to specific methodology is:

$$INTRATE = 0.71753 * INFL - 0.00750 * COMMPRI$$

$$+0.34053 * PCE + 0.24024 * PERSINC$$

$$-0.02053 * HOUST - 0.24012$$

Question 2

Use specific-to-general to come to a model. Start by regressing the federal funds rate on only a constant and add 1 variable at a time. Is the model the same as in (a)?

The table below lists the p-value and t-statistics of each model simulation that was run. Using the specific-to-general methodology one variable is chosen at a time based on the highest t-statistic until the p-value of all remaining variables exceed 0.05.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
INFL						
t-value	28.92451	*Variable*	*Variable*	*Variable*	*Variable*	*Variable*
p-value	0	*Chosen*	*Chosen*	*Chosen*	*Chosen*	*Chosen*
PROD						
t-value	-0.53668	4.8049	0.29571	-1.50551	-1.92312	-0.98868
p-value	0.59167	0	0.76755	0.13268	0.0549	0.32319
UNEMPL						
t-value	6.44717	7.23493	2.63784	1.18445	-0.23937	-0.07174
p-value	0	0	0.00854	0.23666	0.81089	0.94283
COMMPRI						
t-value	-2.7499	-1.65396	-2.25663	-3.1734	-2.8411	*Variable*
p-value	0.00613	0.09861	0.02436	0.00158	0.00464	*Chosen*
PCE						
t-value	21.83197	8.58994	3.41205	*Variable*	*Variable*	*Variable*
p-value	0	0	0.00068	*Chosen*	*Chosen*	*Chosen*
PERSINC						
t-value	1.4513	9.47807	*Variable*	*Variable*	*Variable*	*Variable*
p-value	0.14717	0	*Chosen*	*Chosen*	*Chosen*	*Chosen*
HOUST						
t-value	-5.10585	-0.91133	-2.73343	-4.89325	*Variable*	*Variable*
p-value	0	0.36246	0.00644	0	*Chosen*	*Chosen*

```
summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI, data = federalRatesDa
ta))$coefficients[,1]
```

```
## (Intercept)        INFL     PERSINC         PCE       HOUST     COMMPRI
##  -0.2401188   0.7175265   0.2402420   0.3405254  -0.0205297  -0.0075007
```

As it can be seen above, the model is the same as question (a).

Question 3

Compare your model from (a) and the Taylor rule of equation (1). Consider R^2, AIC and BIC. Which of the models do you prefer?

The Taylor model only considers two variables, $INFL$ and $PROD$. Regressing the model on these two variables, we get the following coefficients.

```
taylorModel <- lm(INTRATE ~ INFL + PROD, data = federalRatesData)
summary(taylorModel)
```

```
##
## Call:
## lm(formula = INTRATE ~ INFL + PROD, data = federalRatesData)
##
## Residuals:
##    Min     1Q Median     3Q    Max
## -5.159 -1.676  0.014  1.373  7.920
##
## Coefficients:
##             Estimate Std. Error t value             Pr(>|t|)
## (Intercept)   1.2489     0.1762    7.09      0.0000000000035 ***
## INFL          0.9750     0.0327   29.78 < 0.0000000000000002 ***
## PROD          0.0947     0.0197    4.80      0.0000019200327 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 2.36 on 657 degrees of freedom
## Multiple R-squared:  0.575,  Adjusted R-squared:  0.573
## F-statistic:  444 on 2 and 657 DF,  p-value: <0.0000000000000002
```

Variable	Taylor Value	Reduced Model Value
AIC	1.7267	1.58099
BIC	1.74032	1.61502
R^2	0.5747	0.63736

Based on the value of AIC and BIC for the two models, since the reduced model contains lower AIC and BIC values, this is the preferred model. Additionally the R^2 value indicates how much of the variability the model explain for and since the value of R^2 is higher in there reduced model, this provides further evidence supporting the reduced model value.

Question 4

Test the Taylor rule of equation (1) using the RESET test, Chow break and forecast test (with in both tests as break date January 1980) and a Jarque-Bera test. What do you conclude?

For the chow test, 1980 was used as the break point.

The results of each of the 4 tests can be found in the table below:

Test	Test Statistic	p-value
RESET	2.53712	0.11168
Chow Break	28.73501	0
Chow Forecast	55.53487	0
Jarque-Bera	8.21163	0

Based on the table above, the p-value of RESET test indicates that we do not reject the null hypothesis of the correct model specification. However both the chow tests and the Jarque-Bera reject the null of no breaks and normality of the residuals. Therefore from this the Taylor model is not a suitable model.

Appendix

Question 1

```
model1 <- lm(INTRATE ~ INFL + PROD + UNEMPL + COMMPRI + PCE + PERSINC + HOUST, dat
a = federalRatesData)

model2 <- lm(INTRATE ~ INFL + PROD + COMMPRI + PCE + PERSINC + HOUST, data = feder
alRatesData)

model3 <- lm(INTRATE ~ INFL + COMMPRI + PCE + PERSINC + HOUST, data = federalRates
Data)
```

Question 2

```
# Model 1
oneVar1 <- summary(lm(INTRATE ~ INFL, data = federalRatesData))$coefficients[2,3]
oneVar2 <- summary(lm(INTRATE ~ PROD, data = federalRatesData))$coefficients[2,3]
oneVar3 <- summary(lm(INTRATE ~ UNEMPL, data = federalRatesData))$coefficients[2,3
]
oneVar4 <- summary(lm(INTRATE ~ COMMPRI, data = federalRatesData))$coefficients[2,
3]
oneVar5 <- summary(lm(INTRATE ~ PCE, data = federalRatesData))$coefficients[2,3]
oneVar6 <- summary(lm(INTRATE ~ PERSINC, data = federalRatesData))$coefficients[2,
3]
```

```
oneVar7 <- summary(lm(INTRATE ~ HOUST, data = federalRatesData))$coefficients[2,3]

onePVar1 <- summary(lm(INTRATE ~ INFL, data = federalRatesData))$coefficients[2,4]
onePVar2 <- summary(lm(INTRATE ~ PROD, data = federalRatesData))$coefficients[2,4]
onePVar3 <- summary(lm(INTRATE ~ UNEMPL, data = federalRatesData))$coefficients[2,
4]
onePVar4 <- summary(lm(INTRATE ~ COMMPRI, data = federalRatesData))$coefficients[2
,4]
onePVar5 <- summary(lm(INTRATE ~ PCE, data = federalRatesData))$coefficients[2,4]
onePVar6 <- summary(lm(INTRATE ~ PERSINC, data = federalRatesData))$coefficients[2
,4]
onePVar7 <- summary(lm(INTRATE ~ HOUST, data = federalRatesData))$coefficients[2,4
]

# Model 2
twoVar1 <- summary(lm(INTRATE ~ INFL + PROD, data = federalRatesData))$coefficient
s[3,3]
twoVar2 <- summary(lm(INTRATE ~ INFL + UNEMPL, data = federalRatesData))$coefficie
nts[3,3]
twoVar3 <- summary(lm(INTRATE ~ INFL + COMMPRI, data = federalRatesData))$coeffici
ents[3,3]
twoVar4 <- summary(lm(INTRATE ~ INFL + PCE, data = federalRatesData))$coefficients
[3,3]
twoVar5 <- summary(lm(INTRATE ~ INFL + PERSINC, data = federalRatesData))$coeffici
ents[3,3]
twoVar6 <- summary(lm(INTRATE ~ INFL + HOUST, data = federalRatesData))$coefficien
ts[3,3]

twoPVar1 <- summary(lm(INTRATE ~ INFL + PROD, data = federalRatesData))$coefficien
ts[3,4]
twoPVar2 <- summary(lm(INTRATE ~ INFL + UNEMPL, data = federalRatesData))$coeffici
ents[3,4]
twoPVar3 <- summary(lm(INTRATE ~ INFL + COMMPRI, data = federalRatesData))$coeffic
ients[3,4]
twoPVar4 <- summary(lm(INTRATE ~ INFL + PCE, data = federalRatesData))$coefficient
s[3,4]
twoPVar5 <- summary(lm(INTRATE ~ INFL + PERSINC, data = federalRatesData))$coeffic
ients[3,4]
twoPVar6 <- summary(lm(INTRATE ~ INFL + HOUST, data = federalRatesData))$coefficie
nts[3,4]

# Model 3
threeVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PROD, data = federalRatesData))
$coefficients[4,3]
threeVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + UNEMPL, data = federalRatesData
))$coefficients[4,3]
threeVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + COMMPRI, data = federalRatesDat
a))$coefficients[4,3]
threeVar4 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE, data = federalRatesData))$
coefficients[4,3]
threeVar5 <- summary(lm(INTRATE ~ INFL + PERSINC + HOUST, data = federalRatesData)
)$coefficients[4,3]

threePVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PROD, data = federalRatesData)
```

```
)$coefficients[4,4]
threePVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + UNEMPL, data = federalRatesDat
a))$coefficients[4,4]
threePVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + COMMPRI, data = federalRatesDa
ta))$coefficients[4,4]
threePVar4 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE, data = federalRatesData))
$coefficients[4,4]
threePVar5 <- summary(lm(INTRATE ~ INFL + PERSINC + HOUST, data = federalRatesData
))$coefficients[4,4]

# Model 4
fourVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + PROD, data = federalRatesD
ata))$coefficients[5,3]
fourVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + UNEMPL, data = federalRate
sData))$coefficients[5,3]
fourVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + COMMPRI, data = federalRat
esData))$coefficients[5,3]
fourVar4 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST, data = federalRates
Data))$coefficients[5,3]

fourPVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + PROD, data = federalRates
Data))$coefficients[5,4]
fourPVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + UNEMPL, data = federalRat
esData))$coefficients[5,4]
fourPVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + COMMPRI, data = federalRa
tesData))$coefficients[5,4]
fourPVar4 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST, data = federalRate
sData))$coefficients[5,4]

# Model 5
fiveVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + PROD, data = feder
alRatesData))$coefficients[6,3]
fiveVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + UNEMPL, data = fed
eralRatesData))$coefficients[6,3]
fiveVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI, data = fe
deralRatesData))$coefficients[6,3]

fivePVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + PROD, data = fede
ralRatesData))$coefficients[6,4]
fivePVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + UNEMPL, data = fe
deralRatesData))$coefficients[6,4]
fivePVar3 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI, data = f
ederalRatesData))$coefficients[6,4]

# Model 6
sixVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI + PROD, dat
a = federalRatesData))$coefficients[7,3]
sixVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI + UNEMPL, d
ata = federalRatesData))$coefficients[7,3]

sixPVar1 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI + PROD, da
ta = federalRatesData))$coefficients[7,4]
sixPVar2 <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI + UNEMPL,
data = federalRatesData))$coefficients[7,4]
```

Question 3

```
s = sqrt(deviance(taylorModel)/df.residual(taylorModel))
k = 2
n = nrow(federalRatesData)

aic = log(s^2) + ((2*k)/n)

bic = log(s^2) + (k*log(n))/n

taylorRSquared <- summary(taylorModel)$r.squared

reducedModel <- lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI, data = federa
lRatesData)

s2 = sqrt(deviance(reducedModel)/df.residual(reducedModel))
k2=5

aic2 = log(s2^2) + ((2*k2)/n)

bic2 = log(s2^2) + (k2*log(n))/n

reduceRSquared <- summary(lm(INTRATE ~ INFL + PERSINC + PCE + HOUST + COMMPRI, dat
a = federalRatesData))$r.squared
```

Question 4

```
resetStat <- resettest(taylorModel, power = 2, type = "fitted", data = federalRate
sData)$statistic
resetPvalue <- resettest(taylorModel, power = 2, type = "fitted", data = federalRa
tesData)$p.value

# Chow Break Test
federalRatesData$Year <- substr(federalRatesData$OBS, 1, 4)
grp <- federalRatesData[federalRatesData$Year < 1980, ]
x1 <- grp[, c("INFL", "PROD")];
y1 <- data.frame( INTRATE = grp["INTRATE"] )
grp <- federalRatesData[federalRatesData$Year >= 1980, ]
x2 <- grp[, c("INFL", "PROD")]; y2 <- data.frame( INTRATE = grp["INTRATE"] )
chowBreak <- chow.test(y2, x2, y1, x1)

# Chow Forecast Test
sumRes <- sum(taylorModel$residuals^2)

federalRatesData$Year <- substr(federalRatesData$OBS, 1, 4)
fedDataPre <- federalRatesData[federalRatesData$Year < 1980, ]
modelPre <- lm(INTRATE ~ INFL + PROD, data = fedDataPre)
sumResPre <- sum(modelPre$residuals^2)

fedDataPost <- federalRatesData[federalRatesData$Year >= 1980, ]
modelPost <- lm(INTRATE ~ INFL + PROD, data = fedDataPost)
sumResPost <- sum(modelPost$residuals^2)

s0 = sumRes
s1 = 237
s2 = sumResPost

a3 = (s0 - (s1 + s2))/k
b3 = (s1 + s2)/(n-(2*k))

fTest3 = a3/b3

chowForecastPValue <- pf(q=fTest3, df1=k, df2=(n-(2*k)), lower.tail=FALSE)

# Jarque-Bera
n2 = nrow(fedDataPost)
n1 = nrow(fedDataPre)
a4 = (s0 - s1)/n2
b4 = s1/(n1-k)

fTest4 = a4/b4
jbPValue <- pf(q=fTest4, df1=n2, df2=n1-k, lower.tail=FALSE)
```