目录

D.4 Induction

-Induced emf: the amount of mechanical energy converted into electrical energy per unit charge

$\varepsilon = BLv$ (B: magnetic flux density; L: length of conductor; v: speed of conductor)

-Magnetic Flux: the product of the component of magnetic flux density perpendicular to the loop and the area of the loop when a uniform magnetic field penetrates through a loop of area A

$$\varphi = B_{perpendicular}A = BAcos\theta$$

-Unit: Weber, $1Wb=1Tm^2$

-Magnetic Flux Linkage: magnetic flux times the number of turns in the coil

$$N\varphi = NBAcos\theta$$

-Faraday's Law:

The induced emf is equal to the rate of change of magnetic flux linkage

*when a coil in a field is flipped through 180°, the change in magnetic flux linkage is twice the original value

$$\varphi - (-\varphi) = 2 * \varphi$$

-Lenz's Law:

The induced emf will be in such a direction as to oppose the change in the magnetic flux that created the current

-(equivalent to the conservation of energy)

How to use:

1. Determine the change in magnetic flux inside the loop
2. Determine the magnetic field due to the induced current
3. Use the right-hand law to find the direction of emf

Alternating Current (AC) generators

The flow of electric charge periodically reverses direction

*The induced emf is **sinusoidal** if the rotation is at constant speed (same with the current)

$$N\varphi = NBAcos(\omega t)$$
$$\varepsilon = \varepsilon_0 \sin(\omega t)$$
$$\varepsilon_{max} = \varepsilon_0 = NBA\omega$$
$$I = I_0 \sin(\omega t)$$
$$P = P_0 \frac{1 - \cos(2\omega t)}{2}$$
$$P_{average} = \frac{1}{2} P_0$$

*The negative of gradient of flux-time graph shows the induced emf

*The period of P is half of the period of other variables

C: Wave Behavior

Simple Harmonic Motion (SHM)

-Oscillations: periodic vibrations about a central or equilibrium value

-SHM: A motion in which the acceleration is proportional to the displacement from equilibrium position, and always directed towards equilibrium position

$$a \propto -x$$
$$F_{net} \propto -x$$

*正方向取位移方向

-Angular frequency (ω): circular representation of frequency

$$\omega = 2\pi f$$

Fundamental Equations for SHM:

- Equations of motion:
  - $x = x_0 sin(\omega t + \varphi)$
  - $v = x_0 \omega \cos(\omega t + \varphi)$
  - $a = -x_0 \omega^2 \sin(\omega t + \varphi)$
- Defining Equation: $a = -\omega^2 x$
- Restoring Force: $F = ma = -m\omega^2 x$

-Displacement: distance from the equilibrium position (vector)

-Amplitude: maximum magnitude of displacement from the equilibrium position

-Period: the time taken for one complete oscillation

-Frequency: number of complete oscillations per unit time

Mass-Spring system:

$$F = -kx$$
$$a = -\frac{k}{m} x$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Oscillating Pendulum system:

$$F = -mg\frac{x}{l}$$

$$a = -\frac{g}{l}x$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

-Phase Difference:

Given two oscillations with phases φ_1 and φ_2, the difference $|\varphi_1 - \varphi_2|$ is called the phase difference between the two oscillations:

$$v = \pm\omega\sqrt{{x_0}^2 - x^2}$$

*上式可以用于计算 SHM 中物体在任意一点时的速度

*Total mechanical energy is conserved for simple harmonic motion

$$E_k = \frac{1}{2}m\omega^2({x_0}^2 - x^2)$$

令 equilibrium 处 E_p=0:

$$E_T = E_k + E_p = \frac{1}{2}m\omega^2{x_0}^2 = \frac{1}{2}m{v_0}^2 \text{ (Conserved)}$$

$$E_p = \frac{1}{2}m\omega^2x^2$$

*Energy 的 period 是原物体 period 的一半

$$E_p = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2{x_0}^2\frac{1+\cos(2\omega t)}{2}$$

Wave Model

-Wave Pulse: there's no net motion of medium through which the wave travels

-Progressive / Travelling Waves

Waves which move energy from place to place

-Transverse Wave 横波

A transverse wave is one in which the direction of oscillation of particles in the wave are at right angles to the direction of energy propagation

-crest 波峰

-trough 波谷

-e.g. radio wave, string waves

-Longitudinal Wave 纵波

A longitudinal wave is one in which the direction of the vibrations of particles in the wave are along the direction of energy propagation

-compression 密部

-rarefaction 疏部

-e.g. sound waves, spring waves

-Displacement-Position graph for waves（某一时刻）:

- Displacement: a particle's distance from its mean position on a wave
  - Displacement is a vector quantity; can be + or –
- Amplitude: Maximum displacement of a particle in the wave
- Wavelength(λ): the distance moved by wave during one oscillation of the source of the waves

-Displacement-Position graph shows displacement of **<u>all the particles</u>** along the wave **<u>at one particular time constant</u>**

-Displacement-Time graph shows displacement of **<u>one particle at different time constants</u>**

-Wave Speed:

Distance travelled wave energy per unit time

$$v = \frac{\lambda}{T} = \lambda f$$

-Mechanical waves:

-require a medium such as a fluid or a solid for propagation

-e.g. water waves, sound waves

-Electromagnetic Waves:

-can travel through a vacuum or medium

-are TRANSVERSE waves

-consisting of electric and magnetic fields at right angles to each other(in direction of wave travelling)

-all wave speed = 3*10^8ms-1

-Wavefronts: surfaces that move with the wave and are perpendicular to the direction of the wave motion. Consecutive wavefronts are one wavelength apart

-Rays: lines in the direction of energy transfer of the wave

*Distance between two adjacent wavefronts is wavelength

-Intensity:

The amount of energy passing through unit area per unit time

$$I = \frac{P}{A}$$

*For a wave of amplitude A, intensity I is proportional to A^2

-Inverse Square Law:

$$I \propto x^{-2}$$

-e.g.: For a spherical wave with power P, the intensity at a distance x from the source is

$$I = \frac{P}{4\pi x^2}$$

Reflection and Refraction

All waves can be reflected, refracted and diffracted and can produce interference patterns

-Reflection: when a wave hits a barrier, it is reflected

Reflection of Pulses

- Fixed end
  - The incident pulse exerts an upward force on the fixed end. The wall exerts an equal and opposite force on the rope. The reflected pulse is **inverted** (undergoes a 180° pulse change)
- Free end
  - A pulse reflecting from a free end and is **not inverted** (no phase-change)

Reflection and Transmission:
-travelling into a “denser” medium=类似 fixed end，进入新介质的波 amplitude 变小
-travelling into a less “dense” medium=类似 free end，进入新介质的波 amplitude 变小

The law of reflection:

Angle of reflection = angle of incidence

-Refraction: the change in direction of a wave due to a change of speed
-refractive index(n): the speed of light in vacuum (air) over that of in the material

$$n = \frac{c}{v}$$

-Snell Law:

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{sinr}{sini}$$

$$n_1 sini = n_2 sinr$$

$$v_2 sini = v_1 sinr$$

-relative refractive index: A relative to B=

$$n_{ab} = \frac{n_a}{n_b}$$

Total Internal Reflection:

For an angle of incidence greater than the critical angle, no refraction takes place, all of the light is reflected

Critical Angle (c):

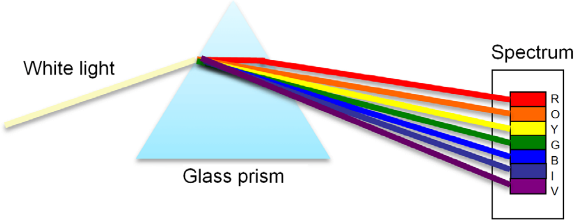
The angle of incidence for which the angle of refraction is 90°

$$\frac{\sin c}{\sin 90°} = \frac{n_2}{n_1}$$

$$\sin c = \frac{n_2}{n_1} \ (n_2 < n_1)$$

Dispersion of Light:

White light passes through a triangular prism and split into a spectrum of different colors



*red light is bent the least, violet light is bent the most; this is because that violet light has shorter wavelength and thus greater refractive index (don't ask me why this is just how it works according to *Sellmeier Equation/ Cauchy's Equation*)

Interference and Diffraction

Interference: when two waves meet at a point, there is a change in overall displacement

Principle of Superposition of Waves:

When two or more waves meet at a point, the resultant displacement at that point is equal to the vector sum of the displacements of the individual waves at that point.

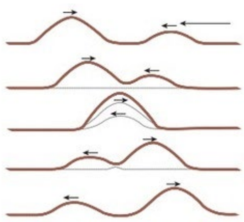


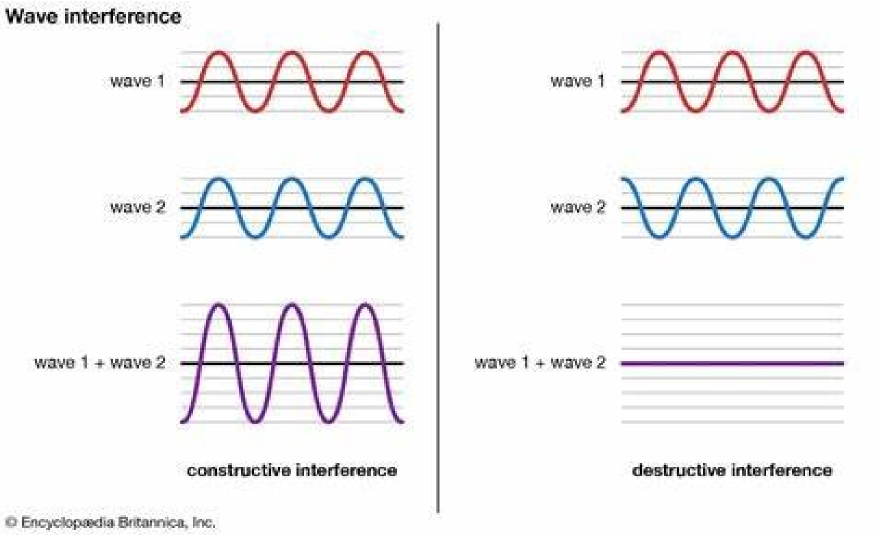
Figure 11.26 Superposition of two waves

Constructive Interference:

- If two waves arriving at a point are in phase, the waves will interfere constructively
- The resultant wave will have an amplitude two times that of the original amplitude

Destructive Interference:

- If two waves arriving at a point are out of phase (phase difference = π), the waves will interfere destructively
- The resultant wave will have an amplitude of zero



Producing an Interference Pattern

Two waves meet at a point P, path difference is the difference in distance of the point from the two sources

-if path difference = nλ

The waves would arrive in phase and constructive interference occurs

-if path difference = (n+0.5)λ

The waves would arrive out of phase and destructive interference occurs

*phase difference = $\frac{path\ difference}{\lambda} \times \pi$

	Constructive	Destructive
Path difference	nλ	(n+0.5)λ
Phase difference	2π	π

Coherence

Conditions to produce an observable interference pattern:

1. Two wave sources have the same frequency
2. Two wave sources have a constant phase relationship

(其实 2 里面就包括了 1)

*if the question says "two coherent waves", it is implying that they have the same frequency

conventional light sources (独立光源) are **incoherent sources

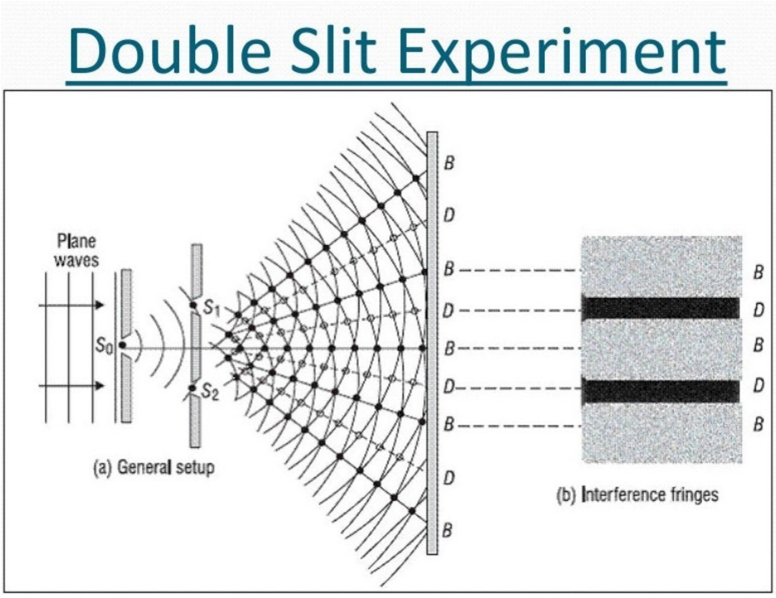
-they emit light with frequent and random changes of phase between the photons

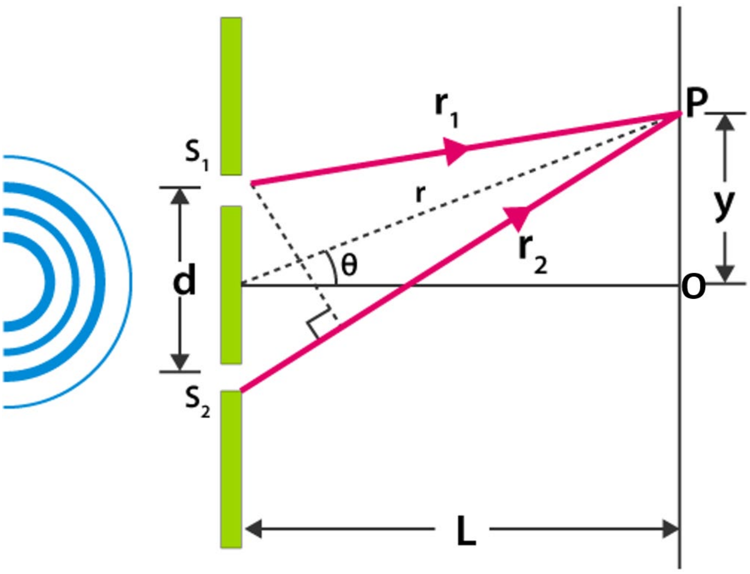
**separate light sources, even of the same frequency, produce incoherent waves

***the light that comes from a laser is **coherent**

Young's Double-Slit Experiment

1. A monochromatic light source is positioned behind a single slit; diffraction occurs at the slit, producing a point light source
2. Diffraction occurs at the double slit to produce two coherent light sources
3. Bright-Dark fringes are produced on the screen





O: the Central Maximum (zero-order bright fringe)
P: N^{th} order bright fringe (e.g. first-order bright fringe)
y: fringe spacing

For bright fringes:

$$d \sin\theta = n\lambda$$

For dark fringes:

$$d \sin\theta = (n + 0.5)\lambda$$

From Equations:

$$\tan\theta = \frac{x}{L}$$

$$\sin\theta = \frac{\lambda}{d}$$

We can derive:

$$x = \frac{\lambda L}{d}$$

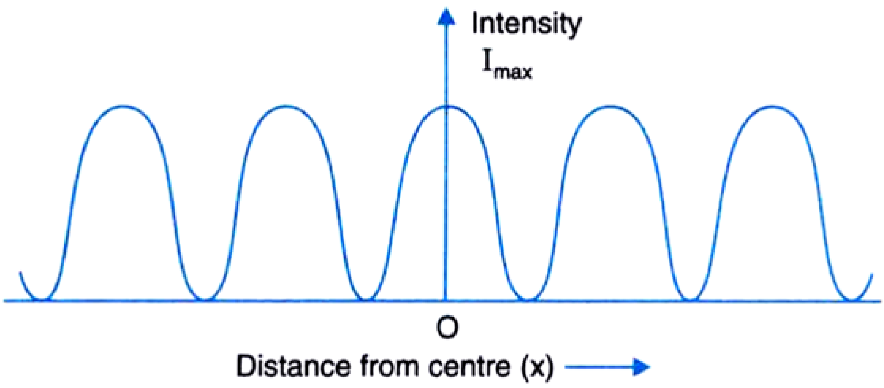
Note: this equation can only be applied to first-order bright fringe (where θ is small)

Re-distribution of Energy:

Suppose that the (coherent) wave sources have intensity I and amplitude A

- At the dark fringes intensity = 0; amplitude = 0
- At bright fringes intensity = 4I; amplitude = 2A (intensity is proportional to amplitude squared)

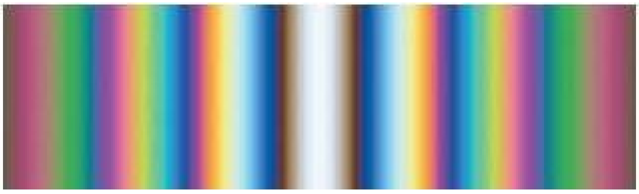
Intensity Graph:



*其实 x 轴好像一般用弧度😳

In case of **white light** as light source:

- Central white fringe
- Fringes on either side show a range of colors
- The red fringe is always at the furthest; the blue/violet fringe is always at the closest to the center

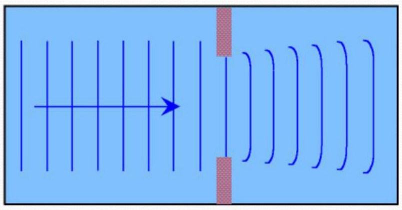


Diffraction

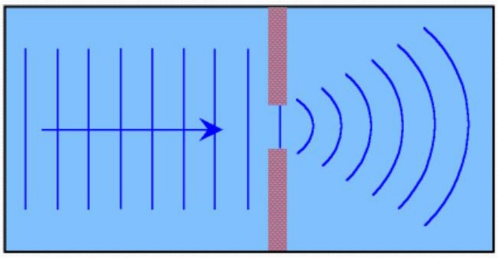
=waves spreads out into geometric shadow region

-Diffraction is strong when the width of the gap is similar in size to the wavelength of the waves

-Diffraction is negligible if the wavelength is much smaller than the opening size



If the wavelength does not match the size of the gap, then only a little diffraction will occur (at the edge of the wave).



Strong diffraction as gap width ≈ λ

Huygens Principle:

At any instant, all points on a wavefront would be regarded as secondary wavelet sources, producing their own outward-spreading circular wavelets. The interference of these wavelets produce the new wavefront.

Young's Single Slit Experiment (Diffraction of light through a single slit)

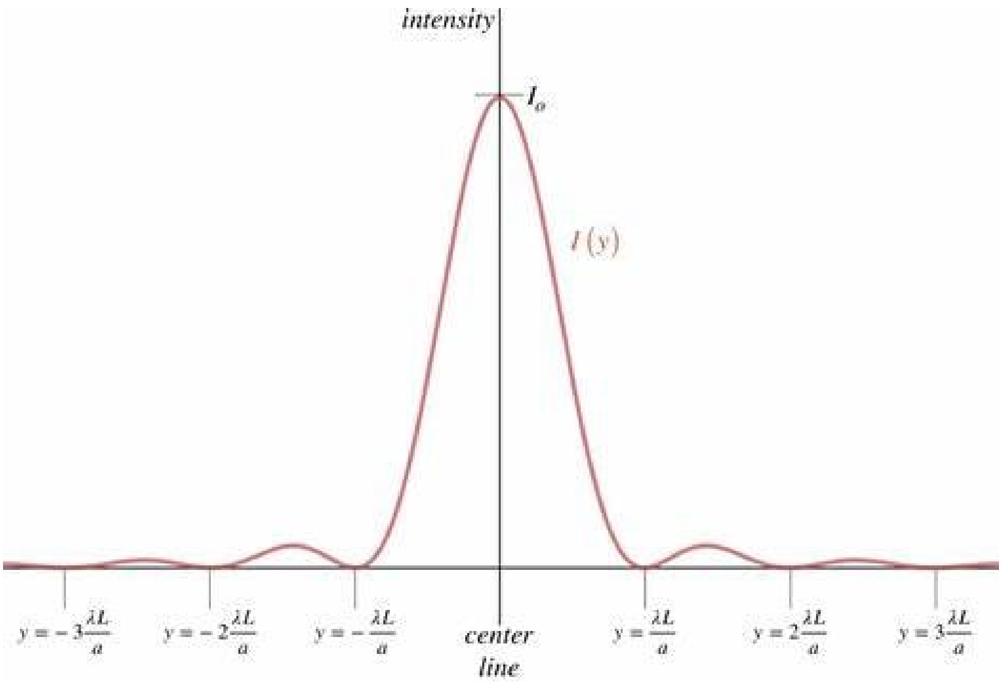
- The diffraction pattern is a series of fringes of light and dark
- The central maximum is brighter and about twice the width of the secondary maximum
- The bright area gets fainter and fainter as it is further away from the center

$b \sin\theta = n\lambda$ (b is the width of the slit)

*as the slit width decreases, the pattern becomes wider—diffraction gets more significant

Intensity of Maxima:

- The maxima of the pattern are approximately half-way between the minima
- The intensity of the first secondary maximum is approximately 5% of that of the principal (center) maximum; the second secondary 2%; the third secondary 1%



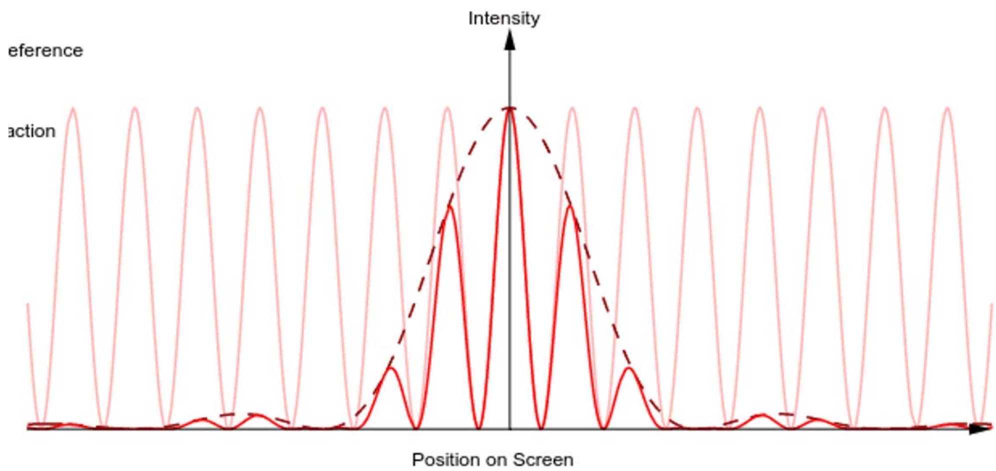
Intensity in Two/Multiple slit interference

Two-slit interference

-If the slit width is negligible, fringes are equally bright (no single-slit diffraction)

-If the slit width is not ignored, the intensity pattern will be modulated by the diffraction effects of the slits

-The positions of the maxima and minima remains the same, but the intensity is modulated by the single-slit pattern (基本上就是把 double-slit 的效果塞到 single-slit 的 pattern 里面)



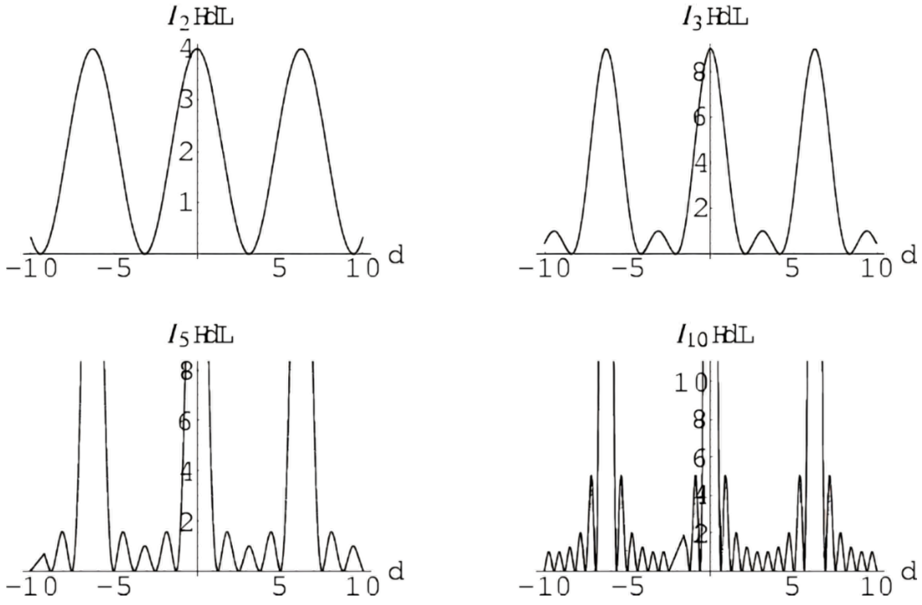
Multiple-slit diffraction

-The primary maxima maintains the same separation

-The primary maxima becomse much sharper

-The pattern increases in density

-As the number of slits increases, the width of the maxima decreases but their position stays the same



Diffraction Grating

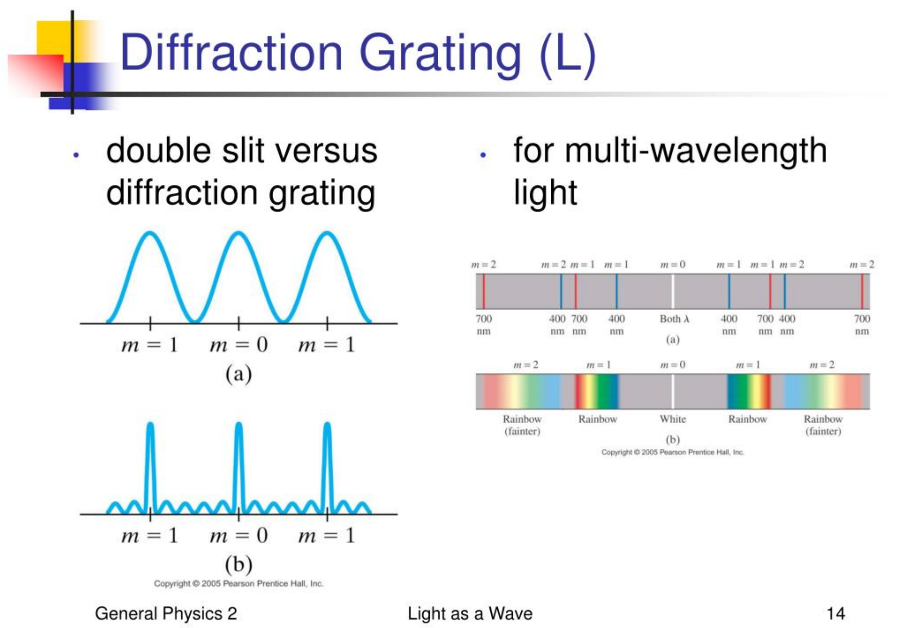
A plate with very large number of parallel, identical, very closely spaced slits

Condition for a maximum of intensity:

$$d\sin\theta = n\lambda$$

Calculating the maximum numbers of fringes observed:

$$n_{max} = \frac{d\sin\theta}{\lambda} = \frac{d}{\lambda}$$



Standing Waves and Resonance

Standing Wave

When an oscillator is vibrated at some certain frequencies, or the string is plucked at some certain points, the whole string will vibrate, and the amplitude of the vibration is large

--as opposed to progressive waves, the waveforms do not move

--it is formed by the superposition of two progressive waves of equal amplitude and frequency, travelling in opposite directions

Standing Wave on a String

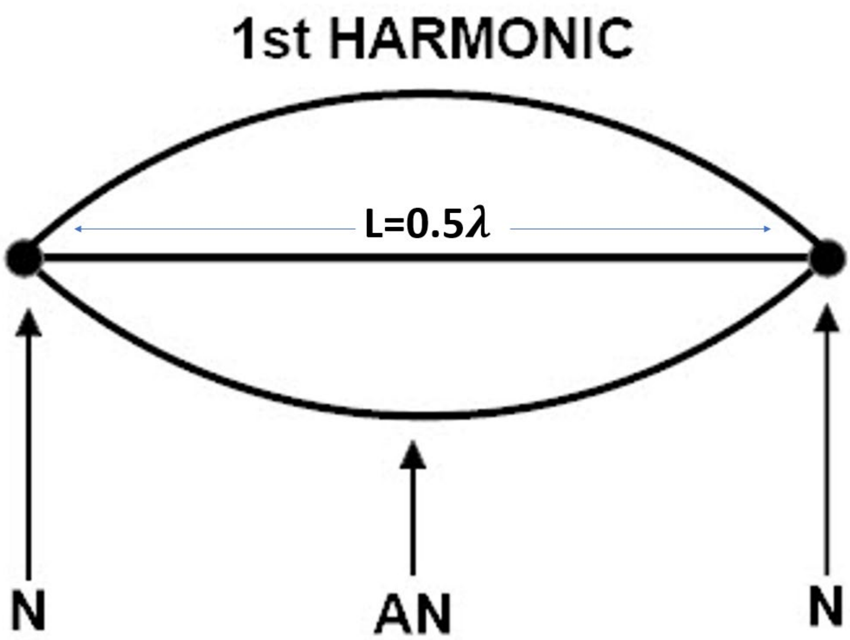
Node: point of no vibration

Antinode: points of maximum amplitude

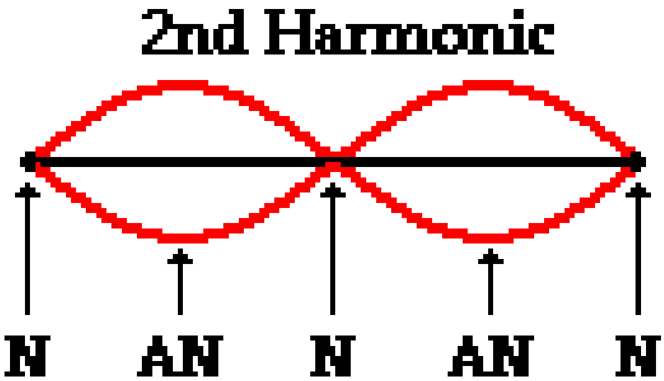
*the distance between two neighboring bodes is always 0.5λ

**相邻两个 node 之间的 wave 中的点全部都 in phase；隔着一个 node，则一定全部 out of phase

The wave pattern with a single loop is called the fundamental mode of vibration, or the first harmonic



$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$



$$f_2 = \frac{v}{\lambda_2} = \frac{v}{L} = 2f_1$$

…………

$$\lambda_n = \frac{2L}{n} \qquad f_n = \frac{nv}{2L}$$

Boundary Conditions:

It is possible that none, one, or both boundaries to free

Difference between Progressive & Standing Waves

	Standing Wave	Progressive Wave
1	Nodes and antinodes do not change	Crest and troughs move along the string
2	Energy is not transferred	Energy is transferred
3	Amplitude varies with position	All points have the same amplitude
4	Points are either in phase or out of phase	Phase varies continuously

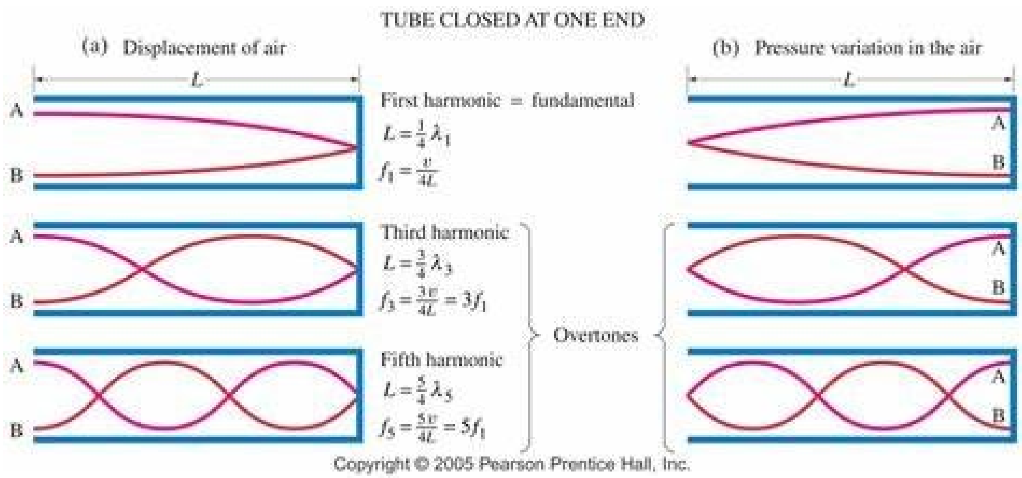
Standing Wave in a Pipe (closed)

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

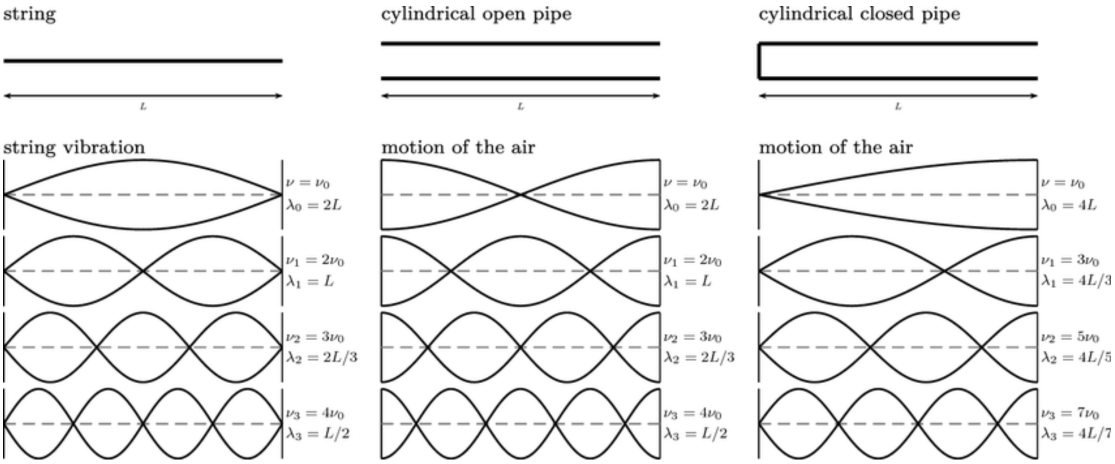
$$f_3 = \frac{v}{\lambda_3} = \frac{v}{\frac{4}{3}L} = 3f_1$$

$$f_5 = \frac{v}{\lambda_5} = \frac{v}{\frac{4}{5}L} = 5f_1$$

*only odd harmonics are formed, for one end of the pipe is closed



Standing Waves in Open Pipes



Forced Vibrations and Resonance

Free Vibrations:

Oscillations without energy loss and without externally applied forces

The amplitude stays constant

Forced Vibrations (Oscillations):

Oscillations driven by an external periodic force

Damping:

Reduction of amplitude of an oscillating system due to loss of energy caused by dissipative forces, such as friction and viscosity

-light/under damping

-critical damping

-heavy/over damping

LIGHT DAMPING (mass on string):

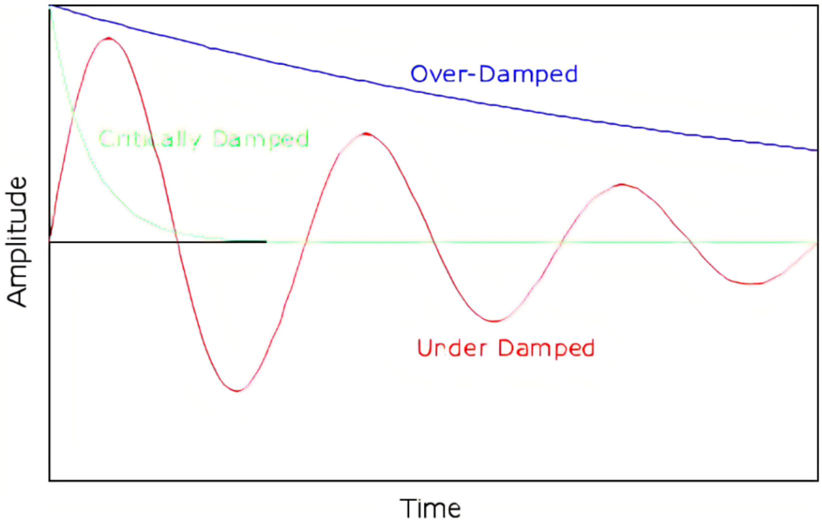
1. Amplitude gradually decreases until it approaches zero
2. Period increases slightly
3. Amplitude/energy decreases exponentially

CRITICAL DAMPING (car suspension):

The system returns to its equilibrium state in the shortest possible time without oscillating

OVER DAMPING (door damper):

The system returns to its equilibrium without oscillations but much slower than in the case of critical damping



*for critical damping, the curve reaches zero displacement in no more than one cycle

Free oscillations===>oscillating at natural frequency

Forced oscillations===>oscillating at driving frequency

Dissipated energy = input energy

Resonance:

Maximum amplitude occurs when driving frequency equals the natural frequency

Resonant frequency:

The frequency at which the amplitude is maximum

- As the degree of damping increases, maximum amplitude decreases, the peak becomes flatter and shifts to lower frequencies
- At very low frequencies, the amplitude is essentially constant

Examples of resonance being useful:

-resonance of quartz oscillators are used in timing circuits

-microwave oven uses resonance to warm food

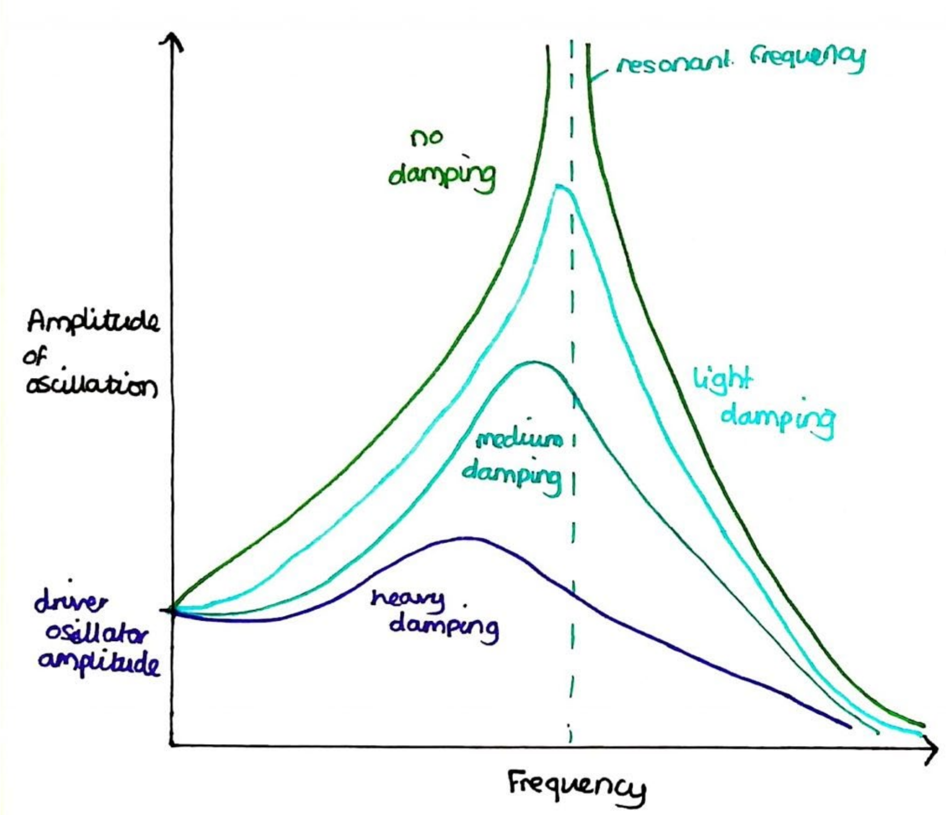
-radio uses resonance to tune into one specific station

Examples of resonance being destructive:

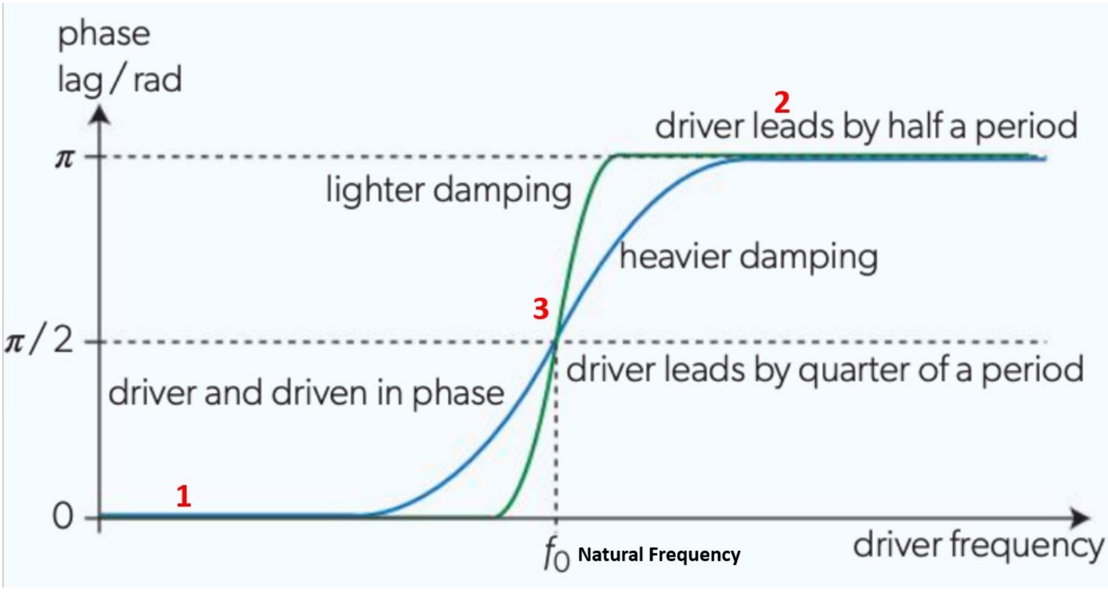
-resonance of buildings in an earthquake

-resonance of a plane's wing

-resonance of bridges



Phase Difference as a Function of External Driving Frequency



1. The driver is dragging the mass along with it, so they stay in phase
2. The driven system tries to respond more slowly than is allowed by the driver and there is a phase shift of 180°
3. The driving frequency and the system's velocity are in phase

**the amount of damping determines the sharpness of the transition

The Doppler Effect

When there is relative motion between the observer and the source, the observer will receive the wave at a frequency that is different from the emitted frequency

1. The source is moving

$$f' = f(\frac{v}{v \pm u_s})$$

*u_s is the velocity of the source

**the wave speed remains unchanged, the wavelength is changing

2. The observer is moving

$$f' = f(\frac{v \pm u_o}{v})$$

*u_o is the velocity of the observer

**the wavelength remains unchanged, the (relative) wave speed is changing

Doppler Effect and EM radiation

If the relative velocities are much smaller than the speed of light, we can use the following approximation:

$$\Delta f = \frac{v}{c} f_0$$

$$\Delta\lambda = \frac{v}{c} \lambda_0$$

Red Shift:

If a source of light is moving away from an observer, the light received by the observer will have a longer wavelength than when it was emitted—the light shifts towards the red end of the spectrum, called "red shift".

Applications of the Doppler Effect:

Ultrasound—measuring blood flow rate

Radars—speed measurement

***Double doppler effect (apply it twice!)