目录

D.4 Induction

-Induced emf: the amount of mechanical energy converted into electrical energy per unit charge

$\varepsilon = BLv$ (B: magnetic flux density; L: length of conductor; v: speed of conductor)

-Magnetic Flux: the product of the component of magnetic flux density perpendicular to the loop and the area of the loop when a uniform magnetic field penetrates through a loop of area A

$$\varphi = B_{perpendicular}A = BAcos\theta$$

-Unit: Weber, 1Wb=1Tm^2

-Magnetic Flux Linkage: magnetic flux times the number of turns in the coil

$$N\varphi = NBAcos\theta$$

-Faraday's Law:

The induced emf is equal to the rate of change of magnetic flux linkage

*when a coil in a field is flipped through 180°, the change in magnetic flux linkage is twice the original value

$$\varphi - (-\varphi) = 2 * \varphi$$

-Lenz's Law:

The induced emf will be in such a direction as to oppose the change in the magnetic flux that created the current

-(equivalent to the conservation of energy)

How to use:

1. Determine the change in magnetic flux inside the loop
2. Determine the magnetic field due to the induced current
3. Use the right-hand law to find the direction of emf

Alternating Current (AC) generators

The flow of electric charge periodically reverses direction

*The induced emf is **sinusoidal** if the rotation is at constant speed (same with the current)

$$N\varphi = NBAcos(\omega t)$$
$$\varepsilon = \varepsilon_0 \sin(\omega t)$$
$$\varepsilon_{max} = \varepsilon_0 = NBA\omega$$

$$I = I_0 \sin(\omega t)$$

$$P = P_0 \frac{1 - \cos(2\omega t)}{2}$$

$$P_{average} = \frac{1}{2} P_0$$

*The negative of gradient of flux-time graph shows the induced emf

*The period of P is half of the period of other variables

C: Wave Behavior

Simple Harmonic Motion (SHM)

-Oscillations: periodic vibrations about a central or equilibrium value

-SHM: A motion in which the acceleration is proportional to the displacement from equilibrium position, and always directed towards equilibrium position

$$a \propto -x$$

$$F_{net} \propto -x$$

*正方向取位移方向

-Angular frequency (ω): circular representation of frequency

$$\omega = 2\pi f$$

Fundamental Equations for SHM:

- Equations of motion:
  - $x = x_0 sin(\omega t + \varphi)$
  - $v = x_0 \omega \cos(\omega t + \varphi)$
  - $a = -x_0 \omega^2 \sin(\omega t + \varphi)$
- Defining Equation: $a = -\omega^2 x$
- Restoring Force: $F = ma = -m\omega^2 x$

-Displacement: distance from the equilibrium position (vector)

-Amplitude: maximum magnitude of displacement from the equilibrium position

-Period: the time taken for one complete oscillation

-Frequency: number of complete oscillations per unit time

Mass-Spring system:

$$F = -kx$$

$$a = -\frac{k}{m} x$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Oscillating Pendulum system:

$$F = -mg\frac{x}{l}$$

$$a = -\frac{g}{l}x$$

$$T = 2\pi\sqrt{\frac{l}{g}}$$

-Phase Difference:

Given two oscillations with phases φ_1 and φ_2, the difference $|\varphi_1 - \varphi_2|$ is called the phase difference between the two oscillations:

$$v = \pm\omega\sqrt{{x_0}^2 - x^2}$$

*上式可以用于计算 SHM 中物体在任意一点时的速度

*Total mechanical energy is conserved for simple harmonic motion

$$E_k = \frac{1}{2}m\omega^2({x_0}^2 - x^2)$$

令 equilibrium 处 E_p=0:

$$E_T = E_k + E_p = \frac{1}{2}m\omega^2{x_0}^2 = \frac{1}{2}m{v_0}^2 \text{（Conserved）}$$

$$E_p = \frac{1}{2}m\omega^2x^2$$

*Energy 的 period 是原物体 period 的一半

$$E_p = \frac{1}{2}m\omega^2x^2 = \frac{1}{2}m\omega^2{x_0}^2\frac{1+\cos(2\omega t)}{2}$$

Wave Model

-Wave Pulse: there's no net motion of medium through which the wave travels

-Progressive / Travelling Waves

Waves which move energy from place to place

-Transverse Wave 横波

A transverse wave is one in which the direction of oscillation of particles in the wave are at right angles to the direction of energy propagation

-crest 波峰

-trough 波谷

-e.g. radio wave, string waves

-Longitudinal Wave 纵波

A longitudinal wave is one in which the direction of the vibrations of particles in the wave

are along the direction of energy propagation

-compression 密部

-rarefaction 疏部

-e.g. sound waves, spring waves

-Displacement-Position graph for waves（某一时刻）:

- Displacement: a particle's distance from its mean position on a wave
  - Displacement is a vector quantity; can be + or –
- Amplitude: Maximum displacement of a particle in the wave
- Wavelength(λ): the distance moved by wave during one oscillation of the source of the waves

-Displacement-Position graph shows displacement of **all the particles** along the wave **at one particular time constant**

-Displacement-Time graph shows displacement of **one particle at different time constants**

-Wave Speed:

Distance travelled wave energy per unit time

$$v = \frac{\lambda}{T} = \lambda f$$

-Mechanical waves:

-require a medium such as a fluid or a solid for propagation

-e.g. water waves, sound waves

-Electromagnetic Waves:

-can travel through a vacuum or medium

-are TRANSVERSE waves

-consisting of electric and magnetic fields at right angles to each other(in direction of wave travelling)

-all wave speed = 3*10^8ms-1

-Wavefronts: surfaces connecting points with the phase, normal to the rays

-Rays: lines in the direction of energy transfer of the wave

*Distance between two adjacent wavefronts is wavelength

-Intensity:

The amount of energy passing through unit area per unit time

$$I = \frac{P}{A}$$

*For a wave of amplitude A, intensity I is proportional to A^2

-Inverse Square Law:

$$I \propto x^{-2}$$

-e.g.: For a spherical wave with power P, the intensity at a distance x from the source is

$$I = \frac{P}{4\pi x^2}$$

Reflection and Refraction

All waves can be reflected, refracted and diffracted and can produce interference patterns

-Reflection: when a wave hits a barrier, it is reflected

Reflection of Pulses

- Fixed end
  - The incident pulse exerts an upward force on the fixed end. The wall exerts an equal and opposite force on the rope. The reflected pulse is **inverted** (undergoes a 180° pulse change)
- Free end
  - A pulse reflecting from a free end and is **not inverted** (no phase-change)

Reflection and Transmission:

-travelling into a "denser" medium=类似 fixed end，进入新介质的波 amplitude 变小

-travelling into a less "dense" medium=类似 free end，进入新介质的波 amplitude 变小

The law of reflection:

Angle of reflection = angle of incidence

-Refraction: the change in direction of a wave due to a change of speed

-refractive index(n): the speed of light in vacuum (air) over that of in the material

$$n = \frac{c}{v}$$

-Snell Law:

$$\frac{n_1}{n_2} = \frac{v_2}{v_1} = \frac{sinr}{sini}$$

$$n_1 sini = n_2 sinr$$

$$v_2 sini = v_1 sinr$$

-relative refractive index: A relative to B=

$$n_{ab} = \frac{n_a}{n_b}$$