

Another Derivation of Normal Equation

- $\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{b} - A\mathbf{x}\| = \arg \min_{\mathbf{x}} \|\mathbf{b} - A\mathbf{x}\|^2$
 $= \arg \min_{\mathbf{x}} (\mathbf{b} - A\mathbf{x})^T (\mathbf{b} - A\mathbf{x}) = \mathbf{b}^T \mathbf{b} - \mathbf{x}^T A^T \mathbf{b} - \mathbf{b}^T A\mathbf{x} + \mathbf{x}^T A^T A\mathbf{x}$
- Computing derivatives w.r.t. \mathbf{x} , we obtain
$$-A^T \mathbf{b} - A^T \mathbf{b} + 2A^T A\mathbf{x} = \mathbf{0} \quad \Leftrightarrow \quad A^T A\mathbf{x} = A^T \mathbf{b}$$
- Thus, if $C = A^T A$ is invertible, then the solution is computed as
$$\mathbf{x} = (A^T A)^{-1} A^T \mathbf{b}$$