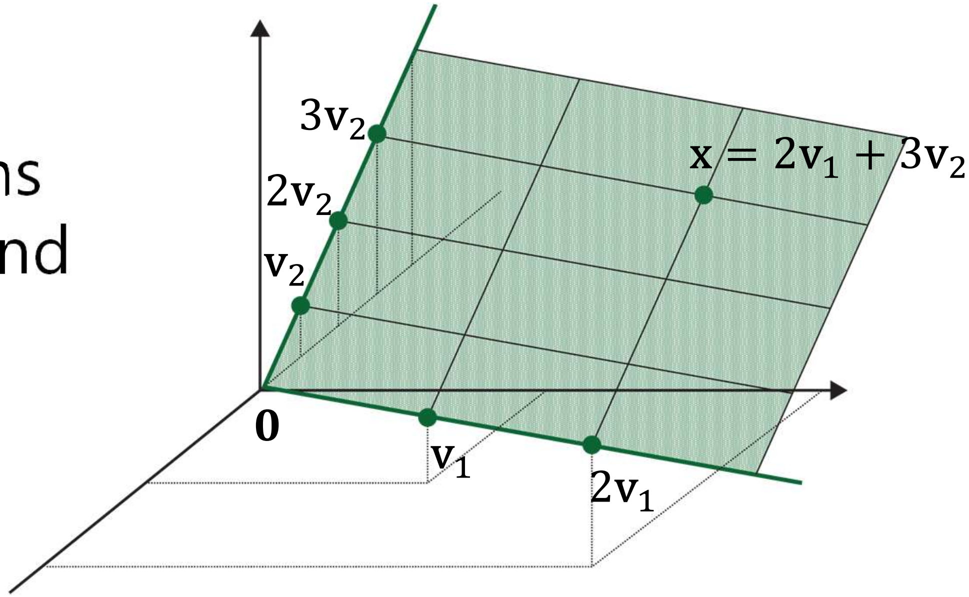
Geometric Description of Span

- If $\mathbf{v}_1$ are $\mathbf{v}_2$ nonzero vectors in $\mathbb{R}^3$, with $\mathbf{v}_2$ not a multiple of $\mathbf{v}_1$, then Span $\{\mathbf{v}_1, \mathbf{v}_2\}$ is the plane in $\mathbb{R}^3$ that contains $\mathbf{v}_1, \mathbf{v}_2$ and $\mathbf{0}$.

- In particular, Span $\{\mathbf{v}_1, \mathbf{v}_2\}$ contains the line in $\mathbb{R}^3$ through $\mathbf{v}_1$ and $\mathbf{0}$ and the line through $\mathbf{v}_2$ and $\mathbf{0}$.



Geometric Interpretation of Vector Equation

- Finding a linear combination of given vectors $\mathbf{a}_1, \mathbf{a}_2$, and $\mathbf{a}_3$ to be equal to $\mathbf{b}$:

$$\begin{bmatrix} 60 \\ 65 \\ 55 \end{bmatrix} x_1 + \begin{bmatrix} 5.5 \\ 5.0 \\ 6.0 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} x_3 = \begin{bmatrix} 66 \\ 74 \\ 78 \end{bmatrix}$$

$$\mathbf{a}_1 x_1 + \quad \mathbf{a}_2 x_2 \quad + \quad \mathbf{a}_3 x_3 = \quad \mathbf{b}$$

- The solution exists only when $\mathbf{b} \in \text{Span}\ \{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.