Inverse Matrix

- **Definition**: For a square matrix $A \in \mathbb{R}^{n\times n}$, its inverse matrix A^{-1} is defined such that

$$A^{-1}A = AA^{-1} = I_n.$$

- For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, its inverse matrix A^{-1} is defined as

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Solving Linear System via Inverse Matrix

- We can now solve $A\mathbf{x} = \mathbf{b}$ as follows:

$$A\mathbf{x} = \mathbf{b}$$
$$A^{-1}A\mathbf{x} = A^{-1}\mathbf{b}$$
$$I_n\mathbf{x} = A^{-1}\mathbf{b}$$
$$\mathbf{x} = A^{-1}\mathbf{b}$$

Solving Linear System via Inverse Matrix

- **Example:**

$$\underbrace{\begin{bmatrix} 60 & 5.5 & 1 \\ 65 & 5.0 & 0 \\ 55 & 6.0 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{\mathbf{x}} = \underbrace{\begin{bmatrix} 66 \\ 74 \\ 78 \end{bmatrix}}_{\mathbf{b}} \Longrightarrow A^{-1} = \begin{bmatrix} 0.0870 & 0.0087 & -0.0870 \\ -1.1304 & 0.0870 & 1.1314 \\ 2.0000 & -1.0000 & -1.0000 \end{bmatrix}$$

- One can verify

$$A^{-1}A = AA^{-1} = I_n.$$

- $\mathbf{x} = A^{-1}\mathbf{b} = \begin{bmatrix} 0.0870 & 0.0087 & -0.0870 \\ -1.1304 & 0.0870 & 1.1314 \\ 2.0000 & -1.0000 & -1.0000 \end{bmatrix} \begin{bmatrix} 66 \\ 74 \\ 78 \end{bmatrix} = \begin{bmatrix} -0.4 \\ 20 \\ -20 \end{bmatrix}$

Solving Linear System via Inverse Matrix

- Now, the life-span can be written as

$$\text{(life-span)} = -0.4 \times \text{(weight)} + 20 \times \text{(height)} \\ -20 \times \text{(is\_smoking)}.$$