Non-Invertible Matrix A for $A\mathbf{x} = \mathbf{b}$

- Note that **if** A is invertible, the solution is uniquely obtained as
$$\mathbf{x} = A^{-1}\mathbf{b}.$$
- **What if** A is non-invertible, i.e., the inverse does not exist?
  - E.g., For $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$, in $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, the denominator $ad - bc$ $= 0$, so A is not invertible.

- For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $ad - bc$ is called the determinant of A, or $\det A$.

Does a Matrix Have an Inverse Matrix?

- $\det A$ determines whether A is invertible (when $\det A \neq 0$) or n
  ot
  (when $\det A = 0$).

- For more details on how to compute the determinant of a ma
  trix $A \in \mathbb{R}^{n \times n}$ where $n \geq 3$, you can study the following:
  - https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/lecture-18-properties-of-determinants/
  - https://ocw.mit.edu/courses/mathematics/18-06-linear-algebra-spring-2010/video-lectures/lecture-19-determinant-formulas-and-cofactors/

Inverse Matrix Larger than 2×2

- If invertible, is there any formula for computing an inverse matrix of a matrix $A \in \mathbb{R}^{n \times n}$ where $n \geq 3$?

- No, but one can compute it.
- We skip details, but you can study Gaussian elimination in Lay Ch1.2 and then study Lay Ch2.2.

Non-Invertible Matrix A for $A\mathbf{x} = \mathbf{b}$

- Back to the linear system, if A is non-invertible, $A\mathbf{x} = \mathbf{b}$ will hav e either no solution or infinitely many solutions.