

Normal Equation

- Finally, given a least squares problem, $A\mathbf{x} \simeq \mathbf{b}$, we obtain

$$A^T A \hat{\mathbf{x}} = A^T \mathbf{b},$$

which is called a normal equation.

- This can be viewed as a new linear system, $C\mathbf{x} = \mathbf{d}$,
where a square matrix $C = A^T A \in \mathbb{R}^{n \times n}$, and $\mathbf{d} = A^T \mathbf{b} \in \mathbb{R}^n$.
- If $C = A^T A$ is invertible, then the solution is computed as
$$\hat{\mathbf{x}} = (A^T A)^{-1} A^T \mathbf{b}$$