Vector/Matrix Additions and Multiplications

- $C = A + B$: Element-wise addition, i.e., $C_{ij} = A_{ij} + B_{ij}$
  - A, B, C should have the same size, i.e., $A, B, C \in \mathbb{R}^{m \times n}$
- $c\mathbf{a}$, cA : Scalar multiple of vector/matrix
  - e.g., $2\begin{bmatrix}3\\2\\1\end{bmatrix} = \begin{bmatrix}6\\4\\2\end{bmatrix}$, $2\begin{bmatrix}1 & 6\\3 & 4\\5 & 2\end{bmatrix} = \begin{bmatrix}2 & 12\\6 & 8\\10 & 4\end{bmatrix}$
- $C = AB$: Matrix-matrix multiplication, i.e., $C_{ij} = \sum_k A_{i,k} B_{k,j}$
  - e.g., $\begin{bmatrix}1 & 6\\3 & 4\\5 & 2\end{bmatrix}\begin{bmatrix}1 & -1\\2 & 1\end{bmatrix} = \begin{bmatrix}13 & 5\\11 & 1\\9 & -3\end{bmatrix}$, $\begin{bmatrix}3 & 2 & 1\end{bmatrix}\begin{bmatrix}1\\3\\5\end{bmatrix} = [14]$, $\begin{bmatrix}1\\3\\5\end{bmatrix}\begin{bmatrix}1 & 2\end{bmatrix} = \begin{bmatrix}1 & 2\\3 & 6\\5 & 10\end{bmatrix}$

Size: $(3 \times 2)(2 \times 2) = 3 \times 2$, $(1 \times 3)(3 \times 1) = 1 \times 1$, $(3 \times 1)(1 \times 2) = 3 \times 2$

Matrix multiplication is NOT commutative

- $AB \neq BA$: Matrix multiplication is NOT commutative.
- e.g., Given $A \in \mathbb{R}^{2\times 3}$ and $B \in \mathbb{R}^{3\times 5}$, AB is defined, but BA is n ot even defined.
- What if BA is defined, e.g., $A \in \mathbb{R}^{2\times 3}$ and $B \in \mathbb{R}^{3\times 2}$? Still, the sizes of $AB \in \mathbb{R}^{2\times 2}$and $BA \in \mathbb{R}^{3\times 3}$ does not match, so $AB \neq BA$.
- What if the sizes of AB and BA match, e.g., $A \in \mathbb{R}^{2\times 2}$ and $B \in \mathbb{R}^{2\times 2}$? Still in this case, generally, $AB \neq BA$.
- E.g., $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 19 & 22 \\ 43 & 50 \end{bmatrix}$, $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 23 & 34 \\ 31 & 46 \end{bmatrix}$

Other Properties

- $A(B+C) = AB + AC$: Distributive
- $A(BC) = (AB)C$: Associative
- $(AB)^T = B^T A^T$: Property of transpose