# Symmetric Matrix (2 of 9)

- **Theorem 1:** If $A$ is symmetric, then any two eigenvectors from different eigenspaces are orthogonal.
- **Proof:** Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be eigenvectors that correspond to distinct eigenvalues, say, $\lambda_1$ and $\lambda_2$.
- To show that $\mathbf{v}_1 \cdot \mathbf{v}_2 = 0$, compute

$$\lambda_1 \mathbf{v}_1 \cdot \mathbf{v}_2 = \left(\lambda_1 \mathbf{v}_1\right)^T \mathbf{v}_2 = \left(A\mathbf{v}_1\right)^T \mathbf{v}_2 \quad \text{Since } \mathbf{v}_1 \text{ is an eigenvector}$$

$$= \left(\mathbf{v}_1^T A^T\right)\mathbf{v}_2 = \mathbf{v}_1^T\left(A\mathbf{v}_2\right) \quad \text{Since } A^T = A$$

$$= \mathbf{v}_1^T\left(\lambda_2 \mathbf{v}_2\right) \quad \text{Since } \mathbf{v}_2 \text{ is an eigenvector}$$

$$= \lambda_2 \mathbf{v}_1^T \mathbf{v}_2 = \lambda_2 \mathbf{v}_1 \cdot \mathbf{v}_2$$