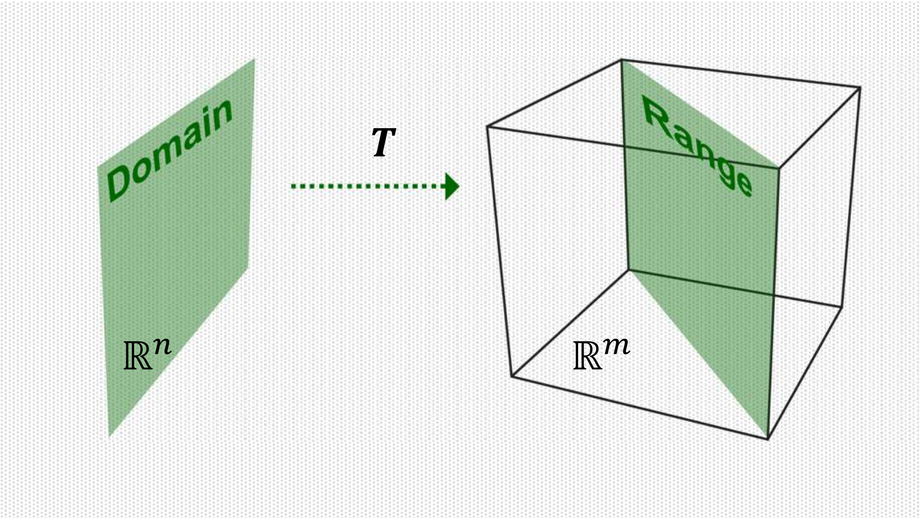
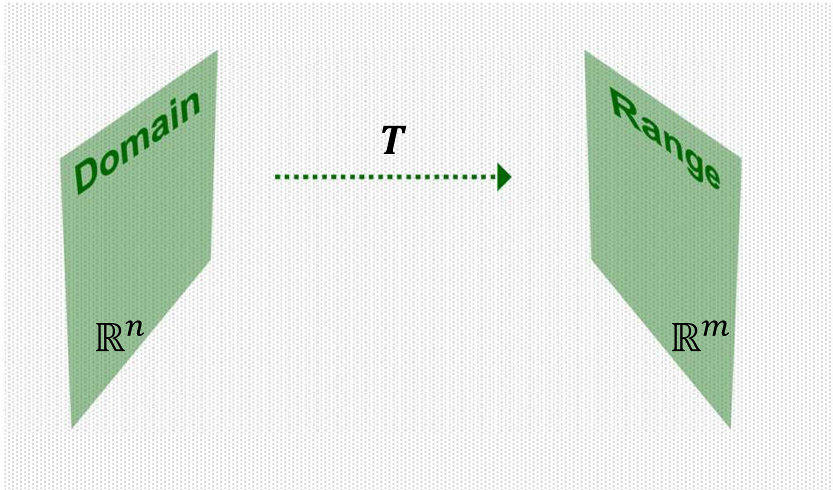
# ONTO and ONE-TO-ONE

- **Definition:** A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be onto $\mathbb{R}^m$ if each $\mathbf{b} \in \mathbb{R}^m$ is the image of at least one $\mathbf{x} \in \mathbb{R}^n$.
  That is, the range is equal to the co-domain.



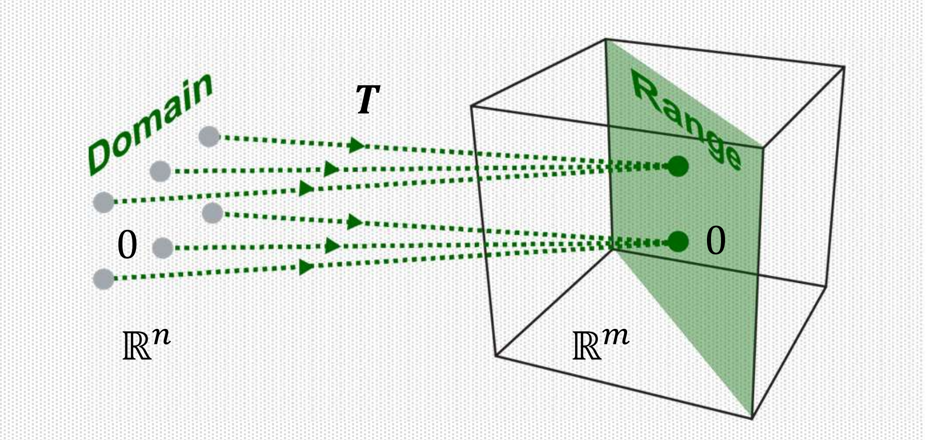

$T$ is NOT onto $\mathbb{R}^m$



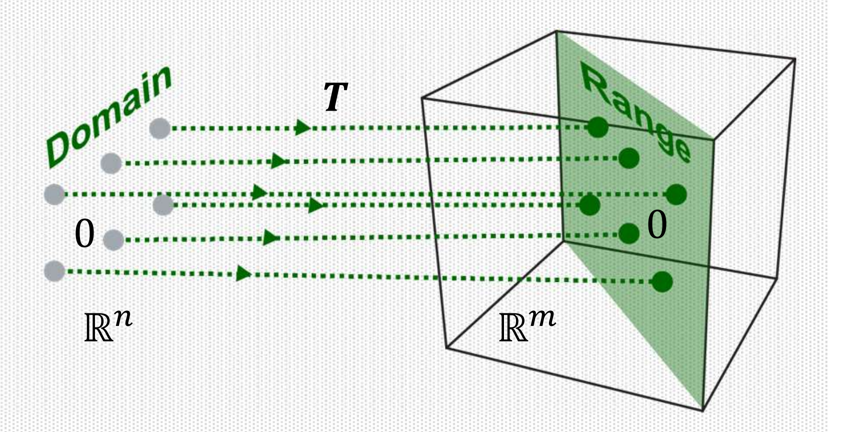

$T$ is onto $\mathbb{R}^m$

# ONTO and ONE-TO-ONE

- **Definition:** A mapping $T : \mathbb{R}^n \to \mathbb{R}^m$ is said to be one-to-one if each $\mathbf{b} \in \mathbb{R}^m$ is the image of at most one $\mathbf{x} \in \mathbb{R}^n$.
  That is, each output vector in the range is mapped by only one input vector, no more than that.



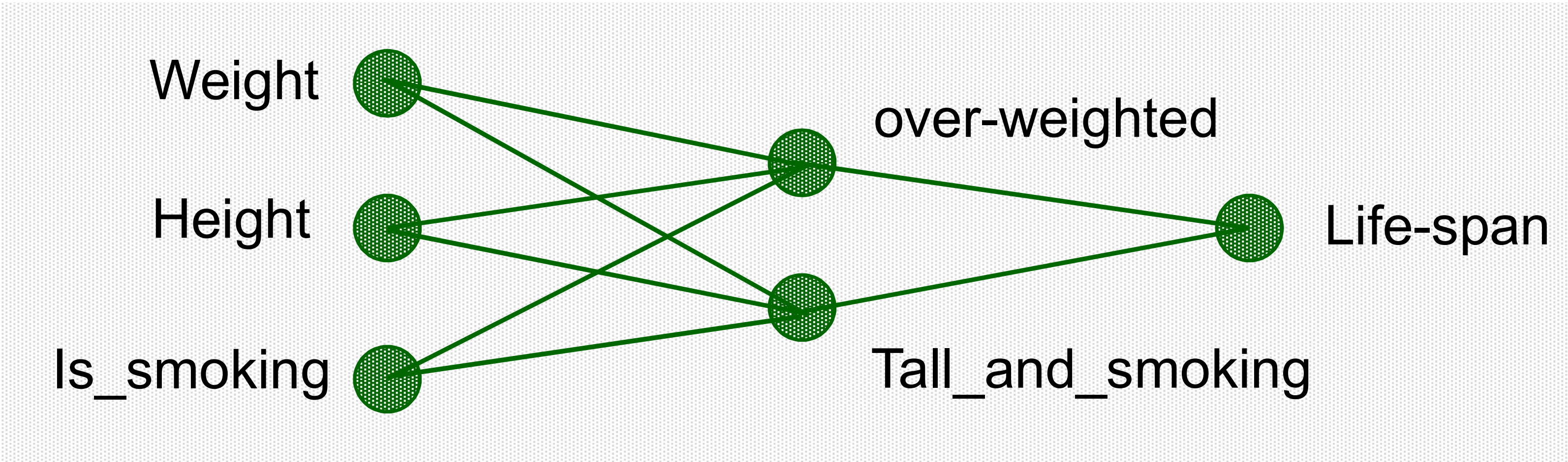

$T$ is NOT one-to-one




$T$ is one-to-one

# Neural Network Example

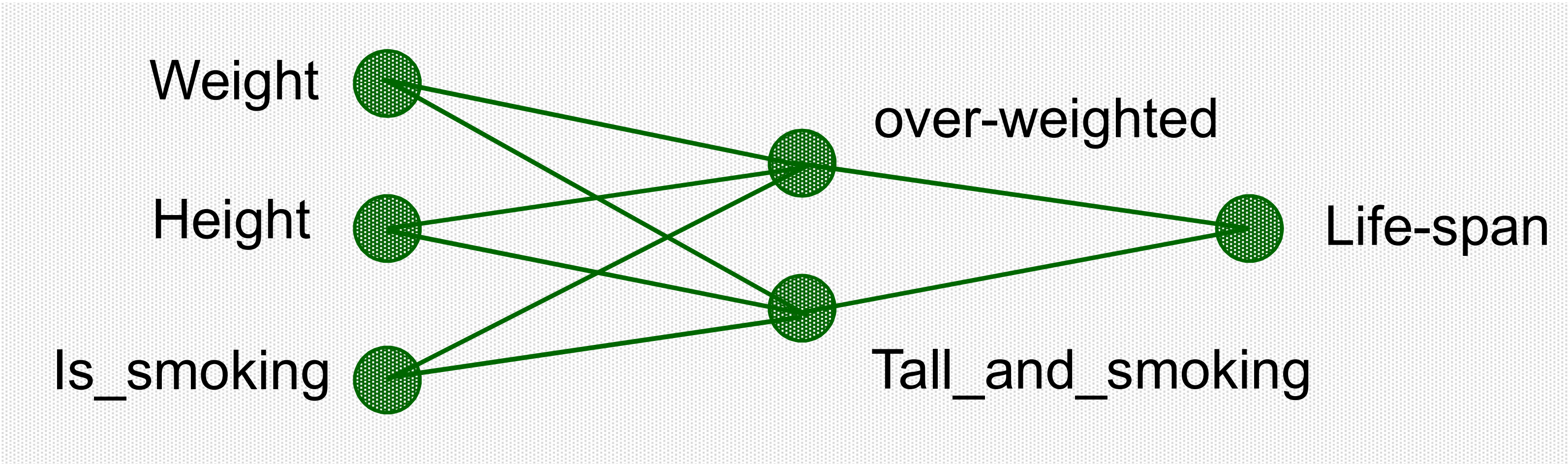
- Fully-connected layers




$$\mathbf{x} \xrightarrow[T_1]{} \mathbf{y} \xrightarrow[T_2]{} \mathbf{z}$$

# Neural Network Example: ONE-TO-ONE

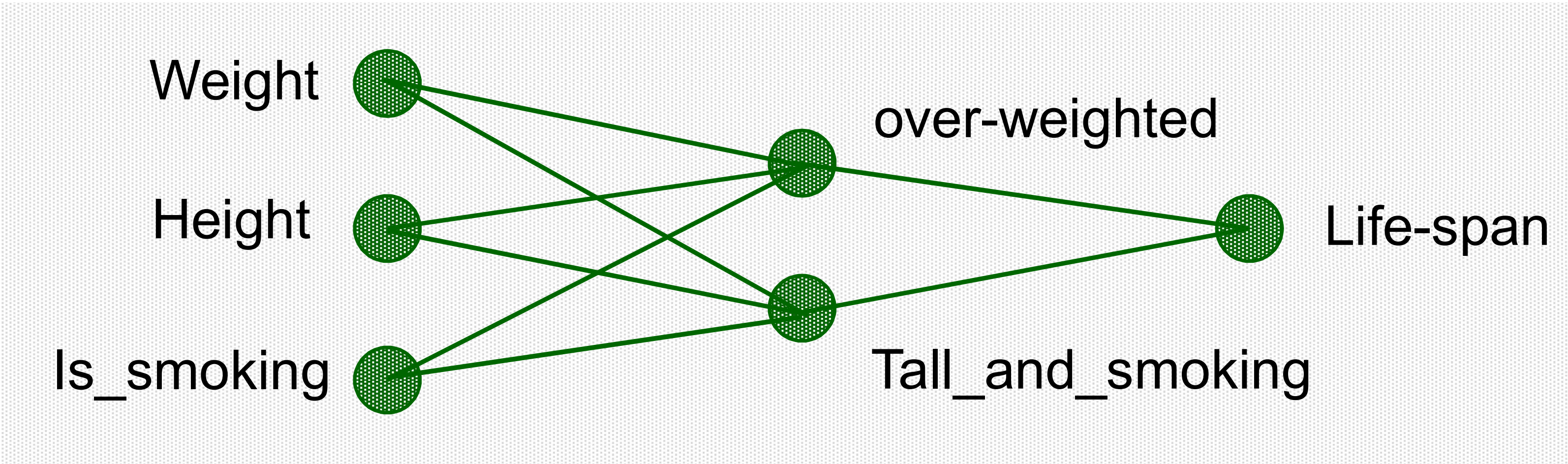
- Will there be many (or unique) people mapped to the same (over_weighted, tall_and_smoking)?




$$\mathbf{x} \underset{T_1}{\longrightarrow} \mathbf{y} \underset{T_2}{\longrightarrow} \mathbf{z}$$

# Neural Network Example: ONTO

- Is there any (over_weighted, tall_and_smoking) that does not exist at all?




$$\mathbf{x} \xrightarrow[T_1]{} \mathbf{y} \xrightarrow[T_2]{} \mathbf{z}$$

# ONTO and ONE-TO-ONE

- Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation, i.e.,

$$T(\mathbf{x}) = A\mathbf{x} \text{ for all } \mathbf{x} \in \mathbb{R}^n.$$

- $T$ is one-to-one if and only if the columns of $A$ are linearly independent.

- $T$ maps $\mathbb{R}^n$ onto $\mathbb{R}^m$ if and only if the columns of $A$ span $\mathbb{R}^m$.

# ONTO and ONE-TO-ONE

- **Example:**

  Let $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 2 & 0 \\ -1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

- Is $T$ one-to-one?
- Does $T$ map $\mathbb{R}^2$ onto $\mathbb{R}^3$?

# ONTO and ONE-TO-ONE

- **Example:**

  Let $T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

- Is $T$ one-to-one?
- Does $T$ map $\mathbb{R}^3$ onto $\mathbb{R}^2$?