Characteristic Equation

- How can we find the eigenvalues such as 8 and -3?
- If $(A - \lambda I)\mathbf{x} = \mathbf{0}$ has a nontrivial solution, then the columns of $(A - \lambda I)$ should be noninvertible.
- If it is invertible, $\mathbf{x}$ cannot be a nonzero vector since

$$(A - \lambda I)^{-1}(A - \lambda I)\mathbf{x} = (A - \lambda I)^{-1}\mathbf{0} \Longrightarrow \mathbf{x} = \mathbf{0}$$

- Thus, we can obtain eigenvalues by solving

$$\det(A - \lambda I) = 0$$

called a **characteristic equation**.

- Also, the solution is not unique, and thus $A - \lambda I$ has linearly dependent columns.

Example: Characteristic Equation

- In the previous example, $A = \begin{bmatrix} 2 & 6 \\ 5 & 3 \end{bmatrix}$ is originally invertible since

$$\det(A) = \det\begin{bmatrix} 2 & 6 \\ 5 & 3 \end{bmatrix} = 6 - 30 = -24 \neq 0.$$

- By solving the characteristic equation, we want to find λ that makes $A - \lambda I$ non-invertible:

$$\det(A - \lambda I) = \det\begin{bmatrix} 2-\lambda & 6 \\ 5 & 3-\lambda \end{bmatrix}$$

$$= \ (2-\lambda)(3-\lambda) - 30$$

$$= -\lambda^2 - 5\lambda - 25 = (8-\lambda)(-3-\lambda) = 0$$

$\lambda = -3$ or 8