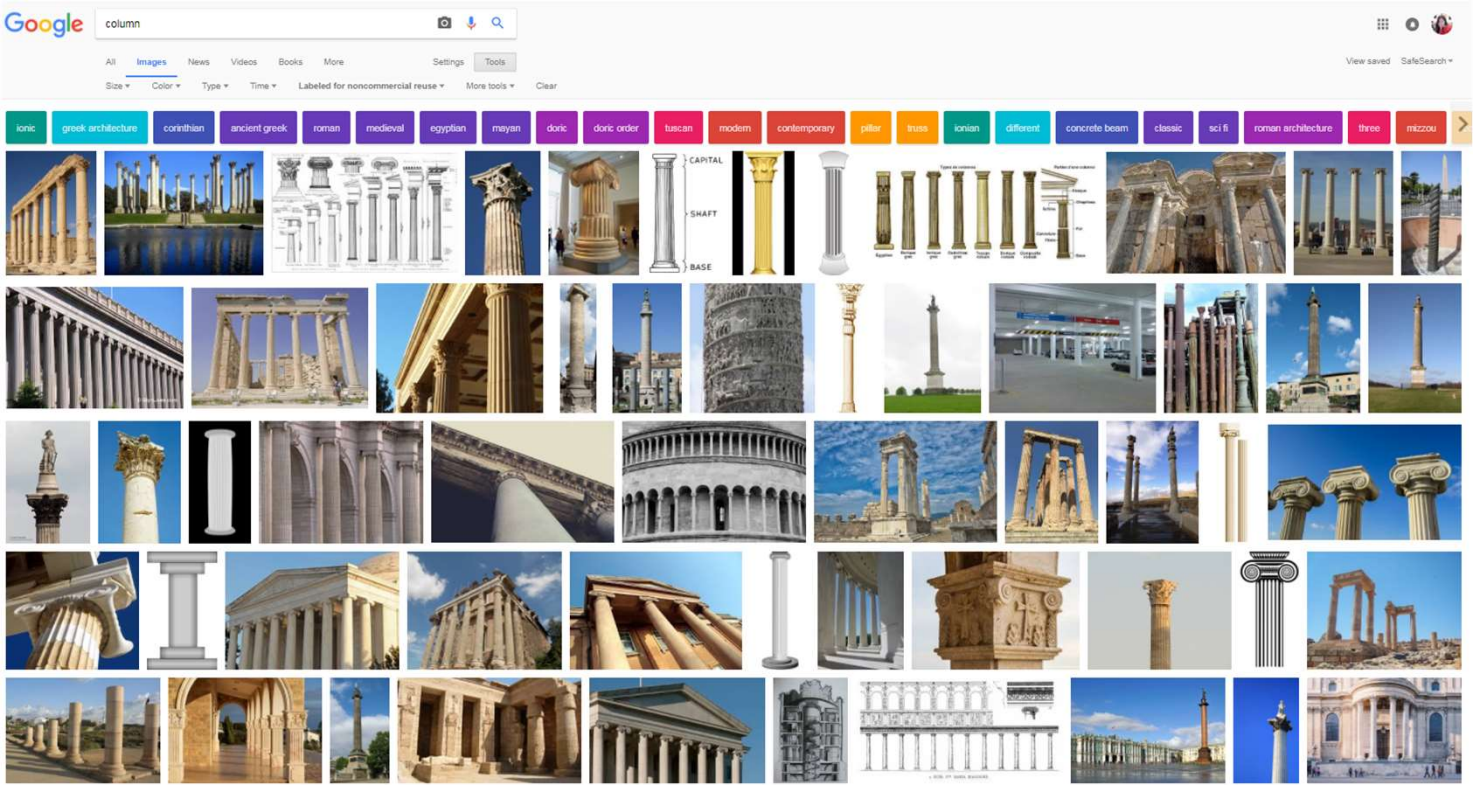
Scalar, Vector, and Matrix

- Scalar: a single number $s \in \mathbb{R}$ (lower case), e.g., 3.8
- Vector: an ordered list of numbers, e.g. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$ (boldface, lower-case), e.g., $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \in \mathbb{R}^3$
- Matrix: a two-dimensional array of numbers, e.g. $A = \begin{bmatrix} 1 & 6 \\ 3 & 4 \\ 5 & 2 \end{bmatrix} \in \mathbb{R}^{3\times 2}$ (capital letter)
  - Matrix size: 3×2 means 3 rows and 2 columns
  - Row vector: a horizontal vector
  - Column vector: a vertical vector

Column is Vertical Vector
(Don’t be Confused!)



Column Vector and Row Vector

- A vector of n-dimension is usually a column vector, i.e., a mat rix
  of the size $n \times 1$
  - $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n = \mathbb{R}^{n \times 1}$
- Thus, a row vector is usually written as its transpose, i.e.,
  - $\mathbf{x}^T = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}^T = [x_1 \quad x_2 \quad \cdots \quad x_n] \in \mathbb{R}^{1 \times n}$

Matrix Notations

- $A \in \mathbb{R}^{n \times n}$: Square matrix (#rows = #columns)
  - e.g., $B = \begin{bmatrix} 1 & 6 \\ 3 & 4 \end{bmatrix}$
- $A \in \mathbb{R}^{m \times n}$: Rectangular matrix (possible: #rows ≠ #columns)
  - e.g., $A = \begin{bmatrix} 1 & 6 \\ 3 & 4 \\ 5 & 2 \end{bmatrix}$
- A^T: Transpose of matrix (mirroring across the main diagonal)
  - e.g., $A^T = \begin{bmatrix} 1 & 3 & 5 \\ 6 & 4 & 2 \end{bmatrix}$
- A_{ij} : (i, j)-th component of A, e.g., $A_{2,1} = 3$
- $A_{i,:}$: i-th row vector of A, e.g., $A_{2,:} = [3 \quad 4]$
- $A_{:,j}$: j-th column vector of A, e.g., $A_{:,2} = \begin{bmatrix} 6 \\ 4 \\ 2 \end{bmatrix}$