Singular Value Decomposition (SVD)

- Given a rectangular matrix $A \in \mathbb{R}^{m \times n}$,
  its singular value decomposition is written as

$$A = U\Sigma V^T$$

where

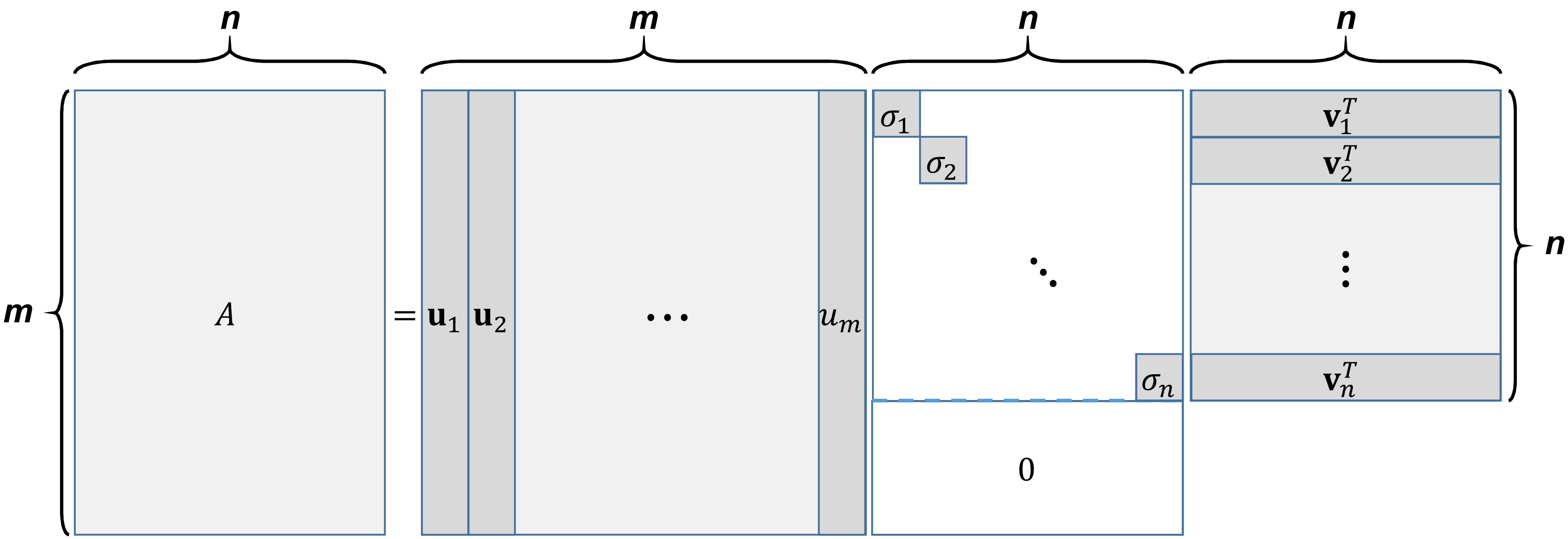
- $U \in \mathbb{R}^{m \times m}$, $V \in \mathbb{R}^{n \times n}$: matrices with orthonormal columns, providing an orthonormal basis of Col A and Row A, respectively

- $\Sigma \in \mathbb{R}^{m \times n}$: a diagonal matrix whose entries are in a decreasing order, i.e., $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{\min(m,n)}$

Basic Form of SVD

- Given a matrix $A \in \mathbb{R}^{m \times n}$ where $m > n$, SVD gives

$$A = U\Sigma V^T$$

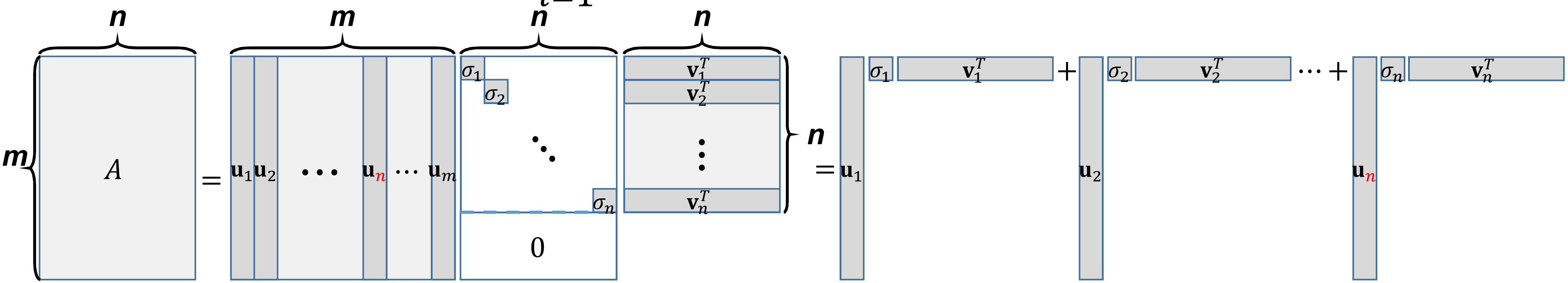


$$A \quad = \quad U \quad \times \quad \Sigma \quad \times \quad V^T$$

SVD as Sum of Outer Products

- A can also be represented as the sum of outer products

$$A = U\Sigma V^T = \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \qquad \text{where } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$$



Reduced Form of SVD

- A can also be represented as the sum of outer products

$$A = U\Sigma V^T = \sum_{i=1}^{n} \sigma_i \mathbf{u}_i \mathbf{v}_i^T, \qquad \text{where } \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n$$