Cavity and Feedback Solitons

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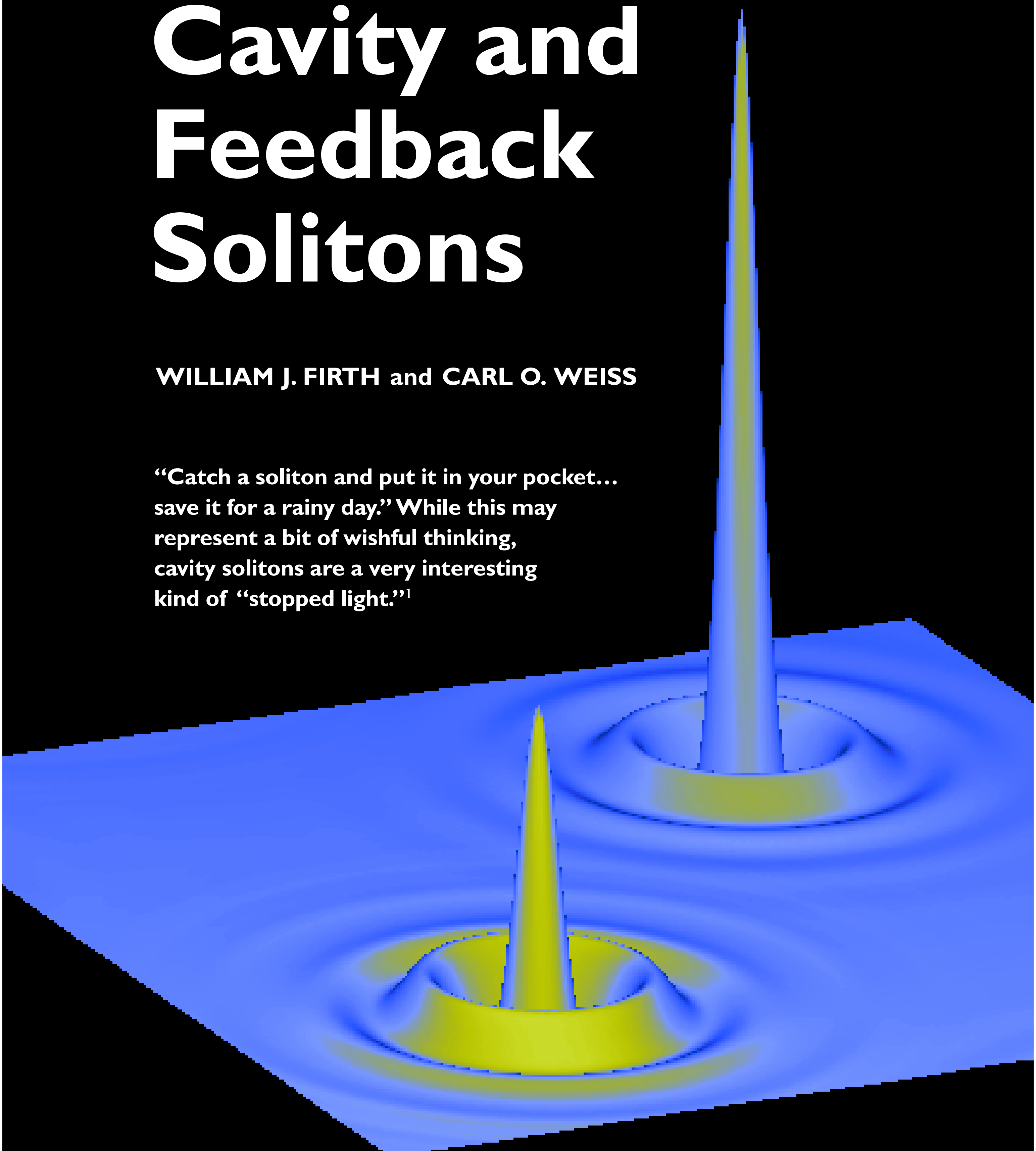
"Catch a soliton and put it in your pocket… save it for a rainy day." While this may represent a bit of wishful thinking, cavity solitons are a very interesting kind of "stopped light."[1]



Created by a pulse of light, cavity solitons can survive indefinitely, stable and stored within an optical cavity. Because they are self-localized through material nonlinearity, they don't diffract. Trapped between mirrors, cavity solitons don't propagate either, except around the cavity. They can be very small: those recently observed in semiconductor microresonators[2] had transverse dimensions on the order of 10 μm in a cavity only a few micrometers thick (Fig. 1).

These "bit-in-a-box" features pave the way for the possible use of cavity solitons to buffer serial optical data. Longitudinally confined, these solitons are essentially two-dimensional, as the figures illustrate. Many cavity solitons could be accommodated in a single resonator of high Fresnel number. Image and data capture and parallel processing are thus both possible. Although whether such devices would be cost effective or not remains to be seen, cavity-soliton arrays possess a unique advantage over conventional electronic or optoelectronic arrays: plasticity. It is possible in fact to directly manipulate and maneuver cavity solitons in the transverse plane.[1,2] This plasticity could give rise to novel forms of image or array processing.

Conventional spatial solitons mostly involve propagation in an essentially lossless medium. Optical cavities, in contrast, are naturally dissipative due to mirror and other losses. They can support stable solitonic waves only if the losses are balanced by some kind of gain or external driving. Sustained in such a way, stable solitons can exist even in the presence of large dissipation. Indeed, even if one cavity mirror is removed completely (Fig. 2), solitons can still be found in the resulting single-mirror feedback system.[3] The latter systems are related to hybrid feedback systems based on liquid-crystal light valves (LCLV), which also support solitons.[4]

Although cavity and feedback solitons are self-organized spatial structures, in other respects they differ considerably from most solitons familiar in optics. While cavity solitons are sometimes considered "just solitons in a cavity," this is generally not true. The cavity is more often an essential feature; in other words, "no cavity = no soliton." For example, a saturable absorber can support stable, robust cavity solitons[5] which have no propagating counterpart (Fig. 3).

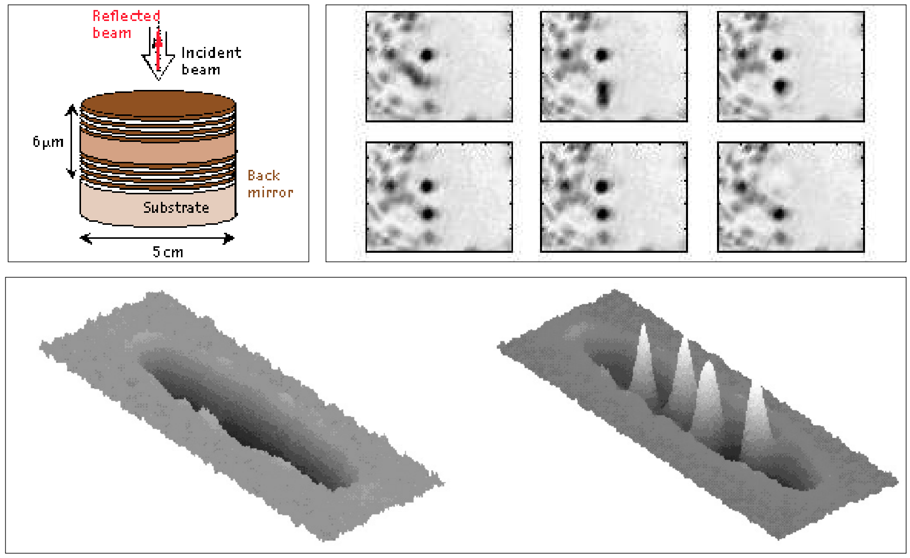


Figure 1. (*Top Left*): A typical vertical-cavity semiconductor microresonator consisting of an active layer sandwiched between Bragg mirrors. (*Right*): Writing and erasing cavity solitons in a similar structure (VCSEL in amplifier mode). In these negative images, the dark spot just above left of center in the top left frame is a previously written stable cavity soliton. In the next frames, a second soliton is written just below it. Then the address beam is moved and a phase-reversed pulse erases the first soliton. (*Courtesy J. Tredicce, INLN, Nice, France*). (*Bottom Right*): An array of bright (as viewed in reflection) solitonic structures in an uninverted MQW microresonator, formed following bistable switching (*left*). [Weiss *et al.*, (1b)].

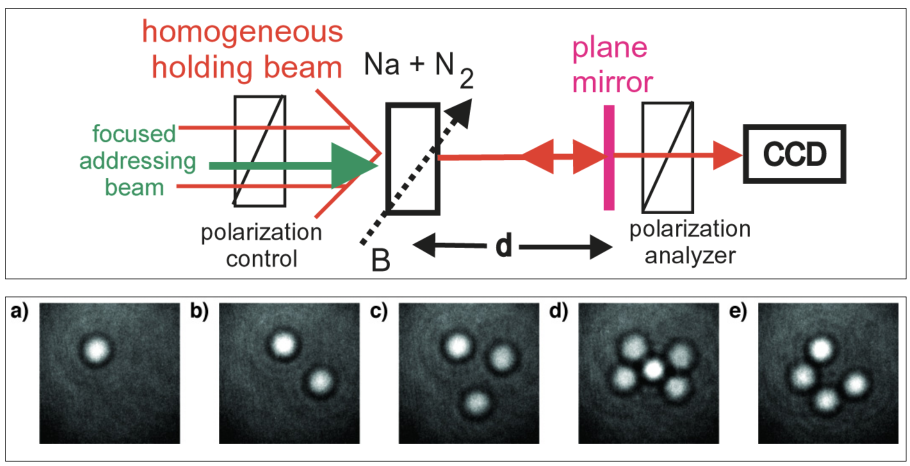


Figure 2. (*Top*): Single-mirror feedback system which sustains solitons in Na vapor. (Courtesy T. Ackemann, Muenster, Germany). (*Bottom*): Clusters of solitons observed in this system.[3]

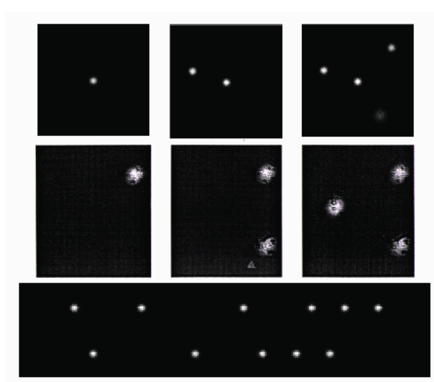


Figure 3. Cavity solitons in a saturable absorber resonator. (*Top*): Simulation in the model of [5] with C=5.4, θ=-1.2. Successive localized address pulses write stable 2D cavity solitons in the transverse cross section of the resonator (panel shows the transmitted intensity). (*Middle*): Experimental realization using nonlinear absorber "Aberchrome" in a cavity driven by a frequency-doubled Nd:YAG laser, addressed by pulses from an independent A^+ laser. (*Bottom*): Two frames from a soliton buffer memory simulation. Solitons are written at the left by "1" pulses of an optical bit-stream and drift rightwards at constant speed on a uniform phase gradient.

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One remarkable feature of dissipative solitons is that they exist in materials like saturable absorbers with no analogous "bulk" solitons. Another is that they are all substantially identical: the need to balance gain and loss, as well as diffraction and nonlinearity, makes for a unique solution. As attracting states, they literally create themselves from any suitably similar starting condition, such as an address pulse. They can thus capture and store bits of spatial information (Fig. 3). In the transverse plane, they possess particlelike properties: they can be steered and controlled optically, for example, by means of a spatial structure imposed on the phase of the driving field[2,5] as illustrated in Fig. 3. It is this property which gives rise to the plasticity of cavity-soliton arrays.

The transverse dynamics of these solitons is intimately linked to symmetry. Transverse translational symmetry implies the existence of a neutral eigenmode (Goldstone mode) of the soliton. Excitation of this mode leads to transverse motion. Often all other eigenmodes are strongly damped, and the soliton behaves like a particle with no internal degrees of freedom. In such a case, its dynamics are non-Newtonian: its velocity, rather than its acceleration, is proportional to the applied force. Spatial variation of any parameter will normally exert a force on the soliton and make it move. Since it lacks inertia, if it reaches a point at which all gradient forces vanish or are cancelled, it simply stops.

Laser-cavity solitons

In lasers the phase of the field is free, which makes topological solitons possible. The simplest soliton structure of this kind is the optical vortex,[6] a point at which the phase of the field has a singularity. In a large Fresnel number laser with high gain, the time needed to synchronize phase across the entire cross section by diffraction may be longer then the build-up time of the field in the resonator. In this case, the field in different regions of the cross section can develop with different phases seeded from uncorrelated noise. Where three such phase domains intersect, a vortex forms: a point around which the phase increases (or decreases) by 2≠. The spatial intensity distribution for left- and right-handed vortices is indistinguishable.

Since the stabilizing force in a laser comes from the population inversion and acts on the intensity of the field only, there is bistability between vortices of opposite handedness. This bistability is related to the "structural stability" of the vortices. If one wants to destroy or transform a vortex, one has to "cut" or "break" wave fronts. Such a change of the wave front is associated with changes of the spatial intensity distribution on which the restoring force can act. Thus, to convert a vortex from left handed to right handed, one has to cross the barrier of a double-well potential. The barrier height is twice the creation energy of a vortex, proportional to the total energy of the field in which the vortex is embedded. It follows that single vortices can be created spontaneously only in the dark. Yet the spontaneous creation of pairs of vortices is possible in bright areas. Higher-order vortices, with a phase circulation of 2N≠, also exist. Although they split easily into simple vortices, they are observable: Figure 4 shows the phase structure of an N=3 vortex, visualized experimentally in an interferogram.

Figure 4. Interferogram of a charge-3 vortex in a laser resonator (Weiss *et al.* [1b,10]).

Can one create a stable structure in which an "island" of one stable state is embedded in a "sea" of instability?

If, on the other hand, a laser shows *intensity* bistability, it can support bright solitons on a dark background. This is possible if there is an intracavity nonlinear absorber in the laser. These solitons may exhibit spontaneous motion in the resonator cross section. They move at fixed speed, but without fixed direction of motion. While stationary solitons are round, moving ones are elliptically shaped because by their motion they have broken the transverse spatial symmetry of the system.

Phase-locked dissipative solitons

Suppose a planar optical cavity is irradiated by a constant plane wave input [see, for example, Fig. 1(a)]. In the linear-response regime, the field transmitted into the cavity sustains an intracavity field proportional to, and thus phase locked with, the input field. The magnitude of the intracavity field naturally depends on the reflectivity of the mirrors and on intracavity absorption, or in other words, on the cavity's finesse. The intracavity field is also sensitive to the relationship between the frequency of the driving field and the cavity's resonances, i.e., to the mistuning of the input field. For any parameters, the intracavity field is stable and unique, and therefore cannot support any kind of soliton. Nonlinearity is a prerequisite for soliton formation.

If any material within the cavity has a nonlinear optical response, then the finesse, the mistuning, or both, become amplitude dependent. There is no longer any guarantee that the intracavity field is either unique or stable. The phenomenon of optical bistability (OB) has been studied for decades.[7] Typically in OB, the cavity exhibits three stationary plane-wave responses to a single input field, of which two are stable and the intermediate one unstable. Can one create a stable structure in which an "island" of one stable state is embedded in a "sea" of instability? If so, is a cavity soliton such a structure? The answer to both questions is "Sometimes."

Let's consider a simple model of a cavity exhibiting a dispersive (Kerr) nonlinearity. If the finesse is high enough, a single longitudinal mode dominates the cavity field E, and the response to a plane-wave input field E_{in} obeys the following equation[8]:

$$i\frac{\partial E}{\partial t}+\frac{1}{2}\nabla^2 E+|E|^2 E=i\gamma(-E-i\theta E+E_{in}) \quad (1)$$

Here γ is the cavity loss parameter. If γ is zero, Eq. 1 reduces to the nonlinear Schrödinger equation (here in two transverse dimensions), for which solitons exist. For finite loss, the terms on the right side of Eq. 1 come into play. They represent loss, mistuning (θ), and driving, respectively. It is easy to show that OB requires $\theta > \sqrt{3}$. However, cavity solitons exist to much smaller mistunings, well outside the bistable domain (Fig. 5). If a cavity soliton is not "just a soliton in a box," nor, it seems, is it "just a switched island in OB." In the absence of OB, the plane-wave cavity field E_s is unique, and therefore so is the normalized intensity $I = |E_s|^2/\gamma$ used as the second parameter in Fig. 5.

Two-dimensional spatial solitons propagating in a bulk Kerr medium are always unstable, suffering self-focusing collapse. In a Kerr cavity, however, they can be stable over a considerable parameter range (Fig. 5). Even when unstable, they do not collapse. Figure 5 shows a "still" from a movie in which two separate Kerr cavity solitons are created by address pulses, settle down to steady self-oscillation, and are then erased by phase-reversed address pulses.[2,9] These solitons "sit" on a pedestal, which must be stable—if the soliton is. This pedestal is the plane-wave solution E_s, known to be unstable if $I > 1$ due to spontaneous pattern formation.[8]

The Kerr-cavity model is a fundamental one in the theory of cavity solitons. It might even be termed its "hydrogen atom" but for the inconvenient fact that 2D Kerr-cavity solitons have yet to be observed. Fortunately, it is easy to generalize Eq. 1 to more practical nonlinear media while still retaining soliton solutions. The simplest models just replace the Kerr term $|E|^2$ on the left side with some other function N(E), describing, for example, saturable absorption. Figure 3 shows a compilation of experimental and numerical results on solitons in a saturable absorber cavity [the experiment used Aberchrome (PTB)]. In the upper images, three stable solitons are written at different locations by address pulses. In the lower, solitons are written at a single point by the "1"s of a bit-stream of optical data but are drifted out of the address beam by a constant phase gradient imposed on the holding field. This simulates a simple buffer memory, which is an interesting potential application of cavity solitons.

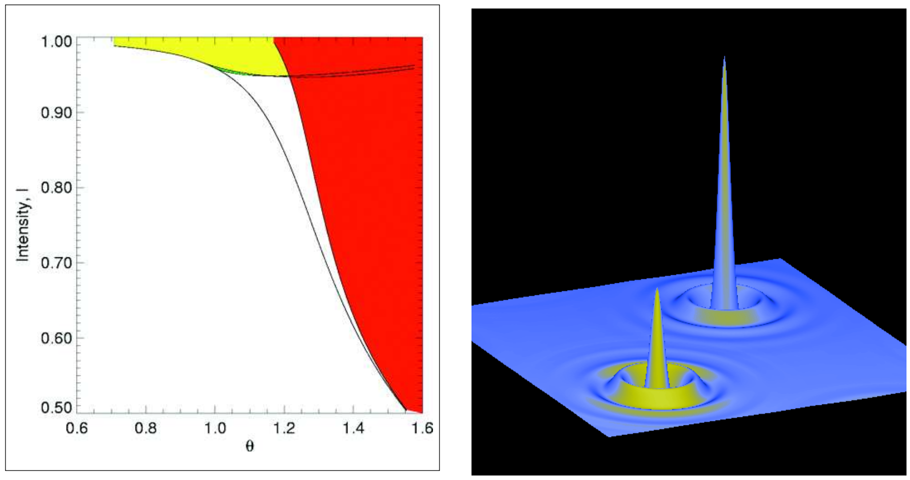

Figure 5. (*Left*): Existence and stability of 2D Kerr-cavity solitons. They exist above the lowest curve but are unstable to pattern formation in the yellow region and to self-oscillation in the red region. (*Right*): Frame from a simulation [9] of the dynamics of 2D Kerr cavity solitons showing stable self-oscillation. (θ=1.3, I=0.9).

We have the right conditions for solitons if nonlinearity induces local lensing or bleaching, acting in opposition to diffraction.

An experimental implementation of such a buffer memory has been realized using the sodium vapor feedback mirror system (see Fig. 2).[3] Although a detailed description of feedback systems is beyond the scope of this article, in essence a bright spot in the feedback field induces a lens (or gain lens) in the vapor. If the phase and/or amplitude perturbation imposed on the input field by this lens evolve in the feedback loop so as to reinforce the lens, a stable soliton can form or be formed.

In feedback systems, the roles of nonlinearity and diffraction are separated, while in a cavity they are intertwined. The similarity of the behaviors of cavity and feedback systems suggests that these differences are really only minor. What is important is that diffraction, nonlinearity, and dissipation are all present. We have the right conditions for solitons if nonlinearity induces local lensing or bleaching, acting in opposition to diffraction.

Given the generality of this prescription, it should be no surprise that cavity solitons have been predicted in a wide variety of optical systems (and observed in quite a few).[1] Among these, semiconductor microresonators (Fig. 1) attract interest because they promise to combine small size with high speed.[2] We discuss these below, but first mention briefly some other cavity soliton recipes.

The nonlinear function N(E) to be inserted into Eq. 1 can be generated by other optical fields, e.g., through cross-phase modulation, polarization coupling or $\chi^{(2)}$ interactions.[1] In degenerate parametric or wave-mixing systems, the phase of the generated field is also tied to that of an external "pump" field, but has two possible values with opposite phase. Thus such systems exhibit phase bistability. In this case, one can have solitons consisting of an "island" of one phase in a "sea" of opposite phase. Fields of opposite phase have the same intensity, but at the boundary between them the intensity goes through zero, and so these solitons appear in intensity as a small dark ring. Figure 6 shows collections of such solitons in a degenerate four-wave-mixing resonator, both calculated and observed.[1,10]

Solitons require nonlinearity, the more the better. Unfortunately, strength of nonlinearity tends to be proportional to response time. The earliest demonstrations of dissipative solitons have used somewhat sluggish media, such as the LCLV, Aberchrome and Na vapor already mentioned. Since cavity solitons are time-independent

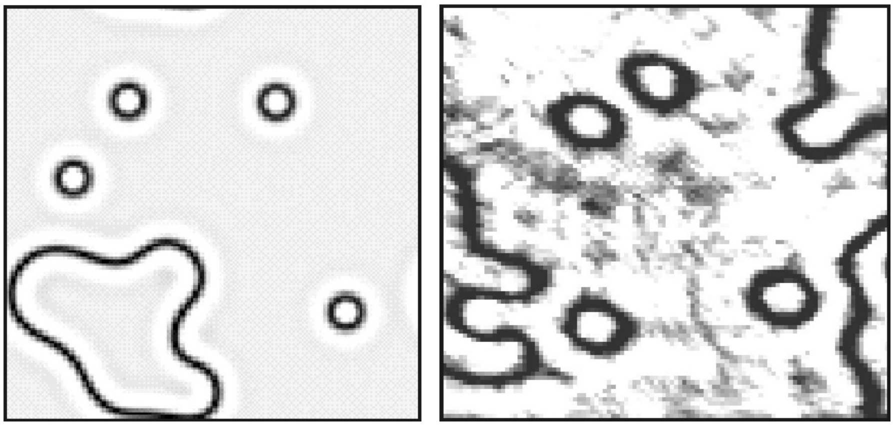

Figure 6. Dark-ring solitons and phase domains in a degenerate 4-wave-mixing resonator: simulation on the left and experiment on the right. [Weiss *et al.,* (1a, 10)].

structures, their existence is not sensitive to the response time of the medium, though of course their dynamics are. Sluggish nonlinear media usually obey their own dynamical equations, so that N(E) becomes a susceptibility change arising from excitation of the medium.

In semiconductor microresonators (such as that illustrated in Fig. 1), the intracavity field, which could respond at THz rates, couples to the electron-hole population, with response times usually one or more orders of magnitude longer. In a passive configuration, the material is uninverted, and the driving field has to do all the work. This requires moderately high intensities. In the experimental observations, the input beam was only a few times broader than the solitons, which ideally should ride on a very broad background field. High intensity also means high thermal load, and so temperature becomes an additional (and much slower) dynamical variable. Despite these complications, groups of up to four solitonlike structures [Fig. 1(c)] have been found in experiments.[1b]

In an alternative active configuration, the resonator is actually a VCSEL (vertical-cavity surface-emitting laser), operated just below its laser threshold. An amplifying medium offers several advantages. The nonlinear index changes sign, becoming self-focusing, which is helpful for soliton formation. Input optical intensities are reduced, which allows larger Fresnel numbers. A practical advantage is that much of the power needed to form and sustain solitons comes cheap, from the wall plug. Figure 1 (b) shows some very recent VCSEL-amplifier results, sequentially writing and erasing two solitons with address pulses. Here erasure depends on the locking of the phases of soliton and driving field. The address pulse is split off from the driver and fed in, localized on the target soliton, with reversed phase. As the address locations were manually selected, the switching sequence shown occupied several seconds, demonstrating this system's long-term stability and true cw operation.

As self-organized dissipative structures, cavity and feedback solitons have useful practical properties like stability and uniqueness.

Cavity-soliton phenomena make stringent demands on semiconductor resonator structures, particularly with regard to uniformity. As mentioned, cavity solitons will move on material or other gradients. Typical speeds in a semiconductor microresonator are in the km/s range, much too fast for CCD imaging detectors to follow. The growth techniques used in microresonator fabrication do not usually control long-range flatness very well. In Fig. 1 (b), there is evidently variation of the optical response across the beam (which is 200 μm in diameter, vast in comparison to typical VCSEL apertures). Solitons are only stationary, and thus easily observable, if some local "trap" exists to stop this intrinsic mistuning gradient from sweeping them out of the beam. Recently achieved improvements in long-range uniformity[2] should be sufficient to allow optical compensation of residual gradients as well as optical manipulation of individual solitons and solitonic arrays.

Conclusion

Stable solitonlike structures exist in driven optical cavities containing any of a wide variety of nonlinear materials. This class includes semiconductor microresonators, a fact which holds promise in terms of future applications of cavity solitons. The first experimental verifications of these solitons have been made.[1,2] Qualitatively similar structures have been predicted, and observed, in feedback systems.[3,4] As self-organized dissipative structures, cavity and feedback solitons have useful practical properties like stability and uniqueness. They are mirror confined in the longitudinal direction, but self confined, with particlelike properties, in the transverse plane. They thus provide a means of trapping and manipulating optical pulses. We believe that cavity and feedback solitons have much to offer, both in optics and in opto-electronic technology, in the coming years.

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