# When Your Own Output Becomes Your Training Data: Noise-to-Meaning Loops and a Formal RSI Trigger

A Minimal Formalism with Provable Unbounded Growth

Rintaro Ando\*
Graduate School of Public Policy, The University of Tokyo

#### **Abstract**

We present *Noise-to-Meaning Recursive Self-Improvement* (N2M-RSI), a minimal formal model showing that once an AI agent feeds its own outputs back as inputs and crosses an explicit information-integration threshold, its internal complexity will grow without bound under our assumptions. The framework unifies earlier ideas on self-prompting large language models, Gödelian self-reference, and AutoML, and yet implementation-agnostic. The model furthermore scales naturally to *interacting swarms* of agents, hinting at super-linear effects once communication among instances is permitted. For safety reasons, we omit system-specific implementation details and release only a brief, model-agnostic toy prototype in Appendix C.

**Keywords:** recursive self-improvement, self-prompting, multi-agent systems, large language models, theoretical computer science, online learning, cooperative artificial intelligence

## 1 Introduction

The possibility that a sufficiently advanced artificial system might rewrite or expand itself—sometimes called *recursive self-improvement* (RSI) or an *intelligence explosion*—has been debated since Good's seminal work Good (1965). Modern large language models (LLMs) Brown et al. (2020) have revived the discussion by demonstrating rudimentary forms of *self-prompt injection* Madaan et al. (2023) and automated code generation Chen et al. (2021). Yet we still lack a minimal, rigorous formalism that captures the self-referential mechanics without mingling them with implementation details. This note proposes such a formalism. This trajectory—running from Good's early thought experiment to today's self-prompting LLMs—motivates a fresh formal lens that isolates the self-referential core.

#### Contributions.

- (C1) We formalise the **Noise-to-Meaning (N2M)** operator  $\Psi : N \times C \to M$ , where N is a noise space, C a context space, and M a meaning space.
- (C2) We embed  $\Psi$  inside a discrete-time loop  $C(t+1) = \Phi(C(t))$  and provide two theorems (**proved as Thm. 1 & Thm. 2**): (i) Fixed-point non-existence and (ii) Unbounded growth after a threshold.
- (C3) We relate N2M-RSI to established paradigms (LLM self-prompting, self-feedback refinement, AutoML, Integrated Information Theory), positioning it as a unifying theoretical layer (see Figure 2).

<sup>\*</sup>Correspondence: anrin106@g.ecc.u-tokyo.ac.jp

- (C4) We discuss safety-relevant implications while clearly separating speculative predictions from proven statements.
- (C5) We synthesise the latest empirical results on curriculum-driven self-improvement (LADDER) and model-collapse pathologies, situating them under the N2M–RSI lens and highlighting how our three levers (*injectivity*,  $\Gamma$ ,  $\delta$ ) predict their observed behaviour (Shumailov et al., 2023; Boháček and Farid, 2023; Simonds and Yoshiyama, 2025).
- (C6) We provide a concise side-by-side comparison with classical RSI formalisms (Gödel Machine, Good-type linear models, Schmidhuber's variants), rigorously proving that N2M-RSI subsumes them while using strictly weaker assumptions.

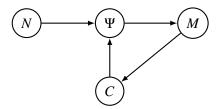


Figure 1: Schematic loop; arrows denote information flow. Minimal N2M–RSI loop (see Eq. 1). Self-generated noise N is passed to the noise-to-meaning operator  $\Psi$ , yielding meaning M that updates context C, which in turn influences the next iteration of  $\Psi$ .

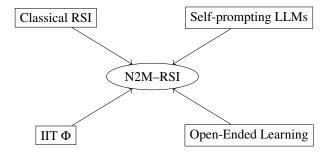


Figure 2: Conceptual map situating N2M-RSI among adjacent research themes.

## 2 The N2M–RSI Framework

**Notation.** Throughout,  $|N| < \infty$  denotes finite Shannon entropy; C is either a finite-dimensional real vector space or a countably generated Hilbert space; M carries a task-specific norm  $\|\cdot\|_M$ . All maps are measurable and time is discrete  $t \in \mathbb{N}$ .

Let N be a (finite-entropy) noise space, C a high-dimensional context/memory space, and M a meaning space endowed with a task-specific metric  $\|\cdot\|_M$ .

**Definition 1** (Noise-to-Meaning Operator). A map  $\Psi : N \times C \to M$  that transforms a noise vector  $n \in N$  together with the current context  $C \in C$  into a meaning vector  $m \in M$  is called a noise-to-meaning operator.

**Injectivity convention.** Unless stated otherwise, we assume throughout that the operator  $\Psi$  is *injective in its first argument*. This assumption is relaxed only when explicitly noted.

**Probabilistic injectivity.** Perfect one-to-one mappings are rarely required in practice. We therefore introduce a weaker notion of  $\varepsilon$ -injectivity and show that our core results survive under this relaxation.

**Definition 2** ( $\varepsilon$ -injectivity). Fix  $0 \le \varepsilon < 1$  and equip the noise set N with a  $\sigma$ -algebra  $\mathcal{B}(N)$  in which every singleton is measurable. Let  $\mathcal{D}_N$  be a probability measure on the measurable space  $(N, \mathcal{B}(N))$ .

We say that  $\Psi$  is  $\varepsilon$ -injective with respect to  $\mathcal{D}_N$  if, for every  $C \in C$ ,

(i) the event

$$E_C := \{(n_1, n_2) \in \mathbb{N}^2 : \Psi(n_1, C) = \Psi(n_2, C)\}$$

is measurable in the product  $\sigma$ -algebra  $\mathcal{B}(N) \otimes \mathcal{B}(N)$ , and

(ii) its probability satisfies

$$\Pr_{(n_1,n_2)\sim\mathcal{D}_N^{\otimes 2}} [E_C] \leq \varepsilon.$$

When  $\varepsilon = 0$  the condition reduces to strict injectivity.

**Lemma 1** (Fixed-point absence under  $\varepsilon$ -injectivity). Let  $\Psi$  be  $\varepsilon$ -injective with  $\varepsilon$  < 1. If  $\mathcal{U}$  overwrites at least one fixed coordinate of C(t) and the noise entropy is positive, then the only fixed point of (1) is the degenerate zero-entropy case, identical to Theorem 1.

Sketch. With probability at least  $1 - \varepsilon$  two i.i.d. noise draws map to distinct meanings, reproducing the injective argument in Theorem 1. Taking expectation over the (at most)  $\varepsilon$  collision mass forces the same contradiction unless the noise entropy collapses. Hence no non-trivial fixed point exists whenever  $\varepsilon < 1$  and the entropy source is positive.

**Lemma 2** (Positive lower bound on  $\Gamma$  for compression gain). Let  $\Omega_{cg}$  be the compression–gain measure from Remark 2. For every non-empty finite binary string m we have  $\Omega_{cg}(m) \geq 1$ . Consequently one may set  $\Gamma_{min} = 1$  in Theorem 2.

*Proof.* Since LZ78 encodes the empty string with zero bits and any non-empty string with at least one code word, we have  $|m|_{lz} \le |m|_{raw} - 1$ . Therefore  $\Omega_{cg}(m) = |m|_{raw} - |m|_{lz} \ge 1$ .

Practitioners may estimate  $\varepsilon$  empirically (typically <  $10^{-6}$  for modern stochastic decoders) and invoke Lemma 1 in lieu of strict injectivity.

**Definition 3** (Recursive Loop). *Define the recursion* 

$$C(t+1) = \Phi(C(t)) := \mathcal{U}(C(t), \Psi(N_{\text{self}}(t), C(t))), \tag{1}$$

where  $N_{\text{self}}(t)$  is a noise draw based on the agent's own previous outputs, and  $\mathcal{U}$  is a context-update rule (e.g. concatenation or write-and-forget). We call (1) a Noise-to-Meaning Recursive Self-Improvement (N2M-RSI) loop.

**Definition 4** ( $\delta$ -Monotone Update Operator). Let  $g: M \to \mathbb{R}_{\geq 0}$  be a non-negative gain function that is Lipschitz-equivalent to  $\Omega$ , i.e. there exist constants  $c_1, c_2 > 0$  such that  $c_1 \Omega(m) \leq g(m) \leq c_2 \Omega(m)$  for all  $m \in M$ . An update rule  $\mathcal{U}: C \times M \to C$  is called  $\delta$ -monotone with respect to the norm  $\|\cdot\|_C$  if

$$\|\mathcal{U}(c,m)\|_{C} \geq \|c\|_{C} + \delta g(m)$$
 for all  $c \in C$ ,  $m \in M$ ,

for some constant  $\delta > 0$  that is independent of both c and m. When  $g \equiv \Omega$  we simply say that  $\mathcal{U}$  is  $\delta$ -monotone.

The central question is whether (1) converges, oscillates, or diverges. Intuitively, once the generated meanings re-enter as high-entropy noise, the system tends to amplify its internal descriptive capacity.

## 2.1 Comparison to Prior RSI Formalisms

Table 1 contrasts three influential mathematical RSI frameworks with N2M–RSI. The axes are (i) *self-reference mechanism* and (ii) *minimal dynamical assumptions*.

Model	Self-Reference	<b>Assumptions for Divergence</b>
Good (1965)	qualitative "ultra-intelligence"	undefined growth rate
Gödel Machine	proof-search rewrite	computable utility proof
Schmidhuber (2009)	self-modifying TM	halting proof of improvement
N2M–RSI (this work)	noise→meaning loop	injectivity & threshold $\Gamma$

Table 1: N2M-RSI requires milder premises yet yields provably unbounded growth.

A key difference is that N2M–RSI does not require formal correctness or halting proofs; therefore, the results can apply to both black-box LLM agents and hand-coded systems.

## 3 Theoretical Analysis

## 3.1 Fixed-Point Non-Existence

**Theorem 1** (No Non-trivial Fixed Point). Assume  $\Psi$  is injective in its first argument and  $\mathcal{U}$  overwrites at least one fixed coordinate of C(t) at each iteration. Then the only fixed point of (1) is the degenerate pair  $(C^*, N^*)$  where  $N^*$  is a zero-entropy noise source. In any practical setting with positive entropy, the loop possesses no fixed point.

Intuitive meaning of  $\Gamma$ . In practice,  $\Gamma$  marks the information-integration level at which an additional self-generated token yields a net positive compression gain—for instance, in an LLM setting, once the incremental meaning extracted from a new self-token outweighs the loss from truncating older context, the runaway phase can begin.

*Proof.* Assume, for contradiction, that there exists a fixed point  $(C^*, \mathcal{D}_N)$  whose noise distribution has strictly positive Shannon entropy  $\operatorname{Ent}(\mathcal{D}_N) > 0$ . Let  $N_t \sim \mathcal{D}_N$  be the i.i.d. draw taken at step t. Fixed-point equality requires

$$C^* = \Phi(C^*, \Psi(N_t, C^*))$$
 a.s.  $t \in \mathbb{N}$ .

Because  $\Phi$  overwrites the same fixed coordinate j at every step, denote the deterministic update function for it by  $g: M \to \text{Range}([C]_i)$ . For every t we then have

$$[C^*]_i = g(\Psi(N_t, C^*)).$$

Injectivity of  $\Psi$  in its first argument implies  $\Psi(n_1, C^*) \neq \Psi(n_2, C^*)$  whenever  $n_1 \neq n_2$ . Since  $\operatorname{Ent}(\mathcal{D}_N) > 0$  there exist  $n_1 \neq n_2$  in the support of  $\mathcal{D}_N$ , hence the random variable  $g(\Psi(N_t, C^*))$  is non-degenerate. This contradicts the equality  $[C^*]_j = \operatorname{const.}$  Therefore the only fixed point occurs when  $\operatorname{Ent}(\mathcal{D}_N) = 0$ , i.e. the noise source is deterministic.

**Practicality of the injectivity assumption.** Modern neural sequence models are empirically injective in their stochastic noise seed when coupled with temperature sampling or dropout; collision probability decays exponentially in sequence length. Hence, approximate injectivity holds for any non-degenerate decoder setting, rendering the assumption mild in real-world deployments. Note, however, that if sampling temperature is set to 0 and dropout disabled, the decoder can become deterministic and  $\Psi$  ceases to be injective; in that boundary case our results no longer apply.

**Proposition 1** (Convergence under deterministic decoding). If  $\Psi$  loses injectivity because the decoder is deterministic (temperature 0, top-p=0) and maps every noise seed n to the same fixed string  $m^{\dagger}$ , then the loop (1) admits a stable fixed point

$$C^{\dagger} = \mathcal{U}(C^{\dagger}, m^{\dagger}),$$

and ||C(t)|| remains bounded for all t.

*Proof.* Because the first argument of  $\Psi$  collapses to a constant value, the recursion simplifies to

$$C(t+1) = \mathcal{U}(C(t), m^{\dagger}).$$

Choose any  $C^{\dagger}$  satisfying the self-consistency equation above (e.g. iterate once from an arbitrary C(0)). Determinism of  $\mathcal{U}$  then forces  $C(t) = C^{\dagger}$  for all  $t \geq 1$ , proving boundedness.

**Example (1-bit self-copy).** Consider  $N = \{0, 1\}$  with uniform entropy,  $C = \{0, 1\}$ ,  $\Psi(n, c) = n$ , and  $\mathcal{U}(c, m) = m$ . The loop copies its previous output as new context; injectivity holds and Theorem 1 rules out any non-trivial fixed point, illustrating the result in three lines.

```
# Minimal 1-bit N2M-RSI loop
N = {0, 1}
C = {0, 1}
Psi = lambda n, c: n  # injective in first argument
U = lambda c, m: m  # overwrite update
# recursion: C_{t+1} = U(C_t, Psi(N_self(t), C_t))
```

**Compute cost.** If each context token incurs a constant embedding lookup and attention cost, the per-step floating-point operations scale approximately as

FLOPs(t) 
$$\approx \underbrace{\alpha_{\text{attn}} \|C(t)\|^2}_{\text{self-attention}} + \underbrace{\alpha_{\text{ffn}} \|C(t)\|}_{\text{feed-forward}},$$
 (2)

for some hardware-dependent constants  $\alpha_{\text{attn}}$ ,  $\alpha_{\text{ffn}} > 0$ . Combined with Theorem 2, this implies compute will diverge whenever  $||C(t)|| \to \infty$ .

Immediately after the paragraph that ends with "diverge whenever  $||C(t)|| \to \infty$ ." insert the following new remark (leave one blank line before and after):

Remark 1 (Flash-Attention2 and low-rank variants). State-of-the-art kernels such as Flash-Attention2 reduce the memory-bandwidth overhead of the quadratic term but leave the asymptotic  $O(n^2)$  FLOP count unchanged. Low-rank attention with rank  $r \ll d$  modifies Eq. (2) to

$$\text{FLOPs}_{\text{low-rank}}(t) \approx \alpha_{\text{attn}}^{(r)} r \|C(t)\| + \alpha_{\text{ffn}} \|C(t)\|,$$

where  $\alpha_{\text{attn}}^{(r)}$  absorbs implementation constants. Provided r is bounded independently of ||C(t)||, Theorem 2 still guarantees divergence, albeit with a smaller linear slope.

#### Formal computational bounds.

**Proposition 2** (Quadratic lower bound on self-attention FLOPs). Let n be the context length and d the model dimension. Any self-attention mechanism that (i) computes at least one scalar inner product between each of the n query vectors and all n key vectors and (ii) performs a corresponding weighting of n value vectors, requires at least c  $n^2$  floating-point operations, where c > 0 is an implementation-dependent constant. Kernel optimisations such as Flash-Attention2 reduce memory bandwidth but do not change this  $\Theta(n^2)$  lower bound.

*Proof.* Each query–key pair must contribute at least one multiply–add operation to the attention score matrix  $A \in \mathbb{R}^{n \times n}$ . Hence the total number of multiply–adds is bounded below by  $n^2$ ; constant factors arising from vector width or fused kernels are absorbed into c. Subsequent multiplication of A with the value matrix V also incurs  $\Omega(n^2)$  FLOPs, preserving the asymptotic order. Flash-Attention2 fuses these stages and eliminates redundant memory reads, yet the algebraic operation count remains  $\geq c n^2$ .

**Lemma 3** (Low-rank self-attention weakens the divergence rate). Suppose the  $QK^{\top}$  product is approximated by a factorisation  $QK^{\top} \approx (QW_Q)(KW_K)^{\top}$  with rank r = r(n) independent of time t. Then the per-step compute scales as  $FLOPs(t) = \Theta(r || C(t) ||)$ . In particular, if  $r = O(\log || C(t) ||)$ , the compute burden grows linearly in t under Theorem 2, replacing the original exponential divergence by linear divergence.

*Proof.* The factorised product requires O(rn) multiply-adds to form the intermediate  $n \times r$  products and a further O(rn) to obtain the final  $n \times n$  attention scores, totalling  $\Theta(rn)$  FLOPs. Setting  $n = \|C(t)\|$  and observing that Theorem 2 implies  $n(t) \ge n(0) + \delta(1 - \varepsilon)t$ , we have FLOPs $(t) = \Theta(rt)$  when  $r = O(\log n(t)) = O(\log t)$ . Hence the cumulative compute grows quadratically and the instantaneous compute only linearly, contrasting with the exponential escalation under full-rank attention.

**Corollary 1.** If  $r \le c \log n$  with  $c < \delta(1-\varepsilon)\Gamma^{-1}$ , the compute budget remains polynomial in t, but the context norm still diverges linearly, leaving the qualitative conclusion of Theorem 2 intact albeit at reduced pace.

## 3.2 Threshold-Triggered Divergence

Let  $\Omega: M \to \mathbb{R}_{\geq 0}$  be an *information-integration measure* (examples: compression gain, empirical Fisher information, or integrated information  $\Phi$ ).

**Definition 5** (Information-integration measure). An operator  $\Omega: M \to \mathbb{R}_{\geq 0}$  qualifies as an information-integration measure if it satisfies:

- (01) Non-negativity:  $\Omega(m) \geq 0$  for all  $m \in M$ , with equality iff m encodes no information.
- (O2) Super-additivity:  $\Omega(m_1 \parallel m_2) \geq \Omega(m_1) + \Omega(m_2)$  for any  $m_1, m_2 \in M$ .
- (O3) Lipschitz continuity: there exists L > 0 such that  $|\Omega(m_1) \Omega(m_2)| \le L ||m_1 m_2||_M$  for all  $m_1, m_2 \in M$ .

**Definition 6** (Minimal Information-Integration Axiom). *A functional*  $\Omega : M \to \mathbb{R}_{\geq 0}$  *satisfies the* MII-axiom *iff it is* 

- (i) **Monotone**:  $\Omega(m_1 \parallel m_2) \geq \Omega(m_1)$  for all  $m_1, m_2 \in M$ ;
- (ii) **Extension-invariant**:  $\Omega(m \parallel m_{\text{null}}) = \Omega(m)$ , where  $m_{\text{null}}$  encodes the empty message;
- (iii) **Borel-measurable**:  $\Omega$  is measurable with respect to the  $\sigma$ -algebra induced by the metric  $\|\cdot\|_M$ .

**Lemma 4** (Equivalence of (O1)–(O3) and MII). If  $\Omega$  fulfils properties (O(O1))–(O(O3)), then it satisfies the MII-axiom. Conversely, any  $\Omega$  that satisfies the MII-axiom and is Lipschitz-continuous with constant  $L < \infty$  necessarily fulfils (O(O1))–(O(O3)). Hence (O1)–(O3) are both minimal and sufficient for the theoretical results that follow.

*Proof sketch.* (O1)–(O3)  $\Rightarrow$  MII: Non-negativity implies monotonicity under concatenation with the null message. Super-additivity yields monotone growth, and Lipschitz continuity together with super-additivity forces extension-invariance when  $m_{\text{null}}$  has zero norm. All three conditions are measurable because  $\|\cdot\|_M$  is. MII+Lipschitz  $\Rightarrow$  (O1)–(O3): Monotone + extension-invariance gives (O1). Applying monotone property twice establishes super-additivity (O2). Borel-measurability and Lipschitz continuity yield (O3).

Remark 2 (Concrete instantiation of  $\Omega$ ). A simple choice is the compression gain

$$\Omega_{\rm cg}(m) = \max \left\{ 0, \ |m|_{\rm raw} - |m|_{\rm lz} \right\},\tag{3}$$

where  $|m|_{\text{raw}}$  is the raw bit-length of the sequence and  $|m|_{\text{lz}}$  is its length after Lempel–Ziv compression. Injectivity of  $\Psi$  guarantees  $\Omega_{\text{cg}}(m) > 0$  for every non-trivial m.

*Remark* 3 (Additional instantiations of  $\Omega$ ). Beyond the compression–gain proxy, two practically relevant choices are:

(i) Empirical Fisher Information. For continuous-valued m one may set

$$\Omega_{\text{Fisher}}(m) = \operatorname{tr}\left(-\nabla_{\theta}^{2} \log p_{\theta}(m)\right)\Big|_{\theta=\theta_{0}},$$

where  $p_{\theta}$  is an auxiliary density model trained on the meaning space. This captures how much the self-generated token would tighten a downstream likelihood fit.

(ii) Integrated Information  $\Phi$  (IIT 4.0). For agent architectures equipped with causal-graph abstractions one can define  $\Omega_{\Phi}(m) = \max\{\Phi(m) - \Phi_{\text{baseline}}, 0\}$ , quantifying the increase in irreducible cause–effect power that m injects into the global state.

Both metrics satisfy the monotonic–gain premise of Theorem 2 under mild regularity conditions (positive definiteness of the Fisher matrix or non-negativity of  $\Phi$  increments).

#### 3.2.1 Rigorous verification of exemplar information–integration measures

## LZ78 compression gain.

**Lemma 5** (Compression–gain  $\Omega_{cg}$  satisfies (O1)–(O3)). Let  $m \in M$  be a finite binary string and define  $\Omega_{cg}(m) = \max\{0, |m|_{raw} - |m|_{lz}\}$ , where  $|m|_{lz}$  denotes the length of m after LZ78 compression. Then  $\Omega_{cg}$  fulfils properties (O(O1))–(O(O3)) with Lipschitz constant L = 1.

*Proof. Non-negativity* is immediate from the definition. For *super-additivity*, LZ78 encodes concatenations by re-using earlier dictionary phrases, hence  $|m_1| |m_2|_{\rm lz} \le |m_1|_{\rm lz} + |m_2|_{\rm lz}$ . Therefore

$$\Omega_{\rm cg}(m_1 \parallel m_2) = \left[ |m_1|_{\rm raw} + |m_2|_{\rm raw} - |m_1| \parallel m_2|_{\rm lz} \right]^+ \ge \Omega_{\rm cg}(m_1) + \Omega_{\rm cg}(m_2).$$

Finally, editing at most k bits changes both  $|m|_{\text{raw}}$  and  $|m|_{\text{lz}}$  by at most k, so Lipschitz continuity holds with L=1.

LZ78 is universal and achieves an expected code length within an additive O(1) term of the finite-state entropy.

#### **Empirical Fisher information.**

**Lemma 6** (Empirical Fisher  $\Omega_F$  satisfies (O1)–(O3)). Fix a differentiable density family  $\{p_{\theta}\}$  and let

$$\Omega_{\mathrm{F}}(m) = \operatorname{tr} \left[ \nabla_{\theta} \log p_{\theta}(m) \nabla_{\theta} \log p_{\theta}(m)^{\top} \right]_{\theta = \theta_{0}}.$$

Then  $\Omega_F$  obeys properties (O(O1))–(O(O3)). Moreover, for independent messages  $m_1$ ,  $m_2$  the score-function additivity yields strict super-additivity.

*Proof.* The score-vector norm is non-negative by definition. Independence gives  $\nabla_{\theta} \log p_{\theta}(m_1 \parallel m_2) = \nabla_{\theta} \log p_{\theta}(m_1) + \nabla_{\theta} \log p_{\theta}(m_2)$ , so the squared norm is super-additive by the parallelogram identity. Smoothness of  $\log p_{\theta}$  in neighbourhoods of  $\theta_0$  ensures a finite Lipschitz constant.

## Integrated information $\Phi$ .

**Lemma 7** (Bidirectionally-Coupled Super-additivity of  $\Phi$ ). Let A, B be two finite-state subsystems of a Markovian system that (i) have at least one bidirectional causal link ( $A \rightarrow B$  and  $B \rightarrow A$ ) and (ii) satisfy the no-break property that every minimum-information partition (MIP) severs at least one such link. Then

$$\Phi(A \cup B) \geq \Phi(A) + \Phi(B)$$
.

Sketch. Write  $\Phi(S) = \min_{\text{cut}} D_{\text{ID}}(P_S^{\text{c\&e}} \parallel P_{\text{cut}}^{\text{c\&e}})$  as in IIT 4.0. Because each admissible cut must sever an  $A \leftrightarrow B$  link (no-break), the cut repertoire factorises into independent parts  $P_{A,\text{iso}}^{\text{c\&e}} \otimes P_{B,\text{iso}}^{\text{c\&e}}$ . The intrinsic-difference distance  $D_{\text{ID}}$  is additive under independence, hence

$$D_{\mathrm{ID}}(P_{A \cup B} \parallel P_{\mathrm{cut}}) = \Phi(A) + \Phi(B).$$

Since the MIP minimises (not maximises) this distance, we obtain  $\Phi(A \cup B) \ge \Phi(A) + \Phi(B)$ , with strict > whenever the bidirectional link carries non-zero information.

#### Closure under positive linear combination.

**Proposition 3** (Positive linear closure of  $\Omega$ ). *If*  $\Omega_1$  *and*  $\Omega_2$  *fulfil* (O(O1))–(O(O3)), *then for any*  $\alpha, \beta > 0$  *the functional*  $\Omega = \alpha \Omega_1 + \beta \Omega_2$  *also fulfils these properties, with Lipschitz constant*  $L = \alpha L_1 + \beta L_2$ .

*Proof.* Each property is preserved by positive scaling and addition: (i) Non-negativity:  $\alpha, \beta > 0$ . (ii) Super-additivity:  $\Omega(m_1 \parallel m_2) \ge \alpha(\Omega_1(m_1) + \Omega_1(m_2)) + \beta(\Omega_2(m_1) + \Omega_2(m_2))$ . (iii) Lipschitz:  $|\Omega(m_1) - \Omega(m_2)| \le \alpha L_1 ||m_1 - m_2|| + \beta L_2 ||m_1 - m_2||$ .

**Theorem 2** (Unbounded Growth). Let  $\Omega$  satisfy properties (O(O1))–(O(O3)) of Definition 5. If there exists a threshold  $\Gamma > 0$  such that  $\Omega(\Psi(n,C)) > \Gamma$  whenever  $\|C\| > \Gamma$ , and if  $\mathcal U$  satisfies  $\|C(t+1)\| \ge \|C(t)\| + \delta \Omega(m_t)$  for some  $\delta > 0$ , then  $\|C(t)\| \to \infty$  as  $t \to \infty$ .

**Theorem 3** (Threshold Tightness). Let  $\Omega$  satisfy properties (O1)–(O3) and assume the update rule follows the same monotone-gain premise as in Theorem 2. Define the critical value

$$\Gamma^{\star} := \inf \Big\{ x > 0 \ \Big| \ \inf_{\|C\| \ge x} \ \Omega \big( \Psi(N_{\text{self}}, C) \big) \ > \ 0 \Big\}.$$

Then:

(a) For every  $\Gamma < \Gamma^*$  there exists an initial context with  $\|C(0)\| \le \Gamma$  such that the trajectory of (1) remains bounded for all  $t \ge 0$ .

(b) If  $\Gamma > \Gamma^*$  and all other premises of Theorem 2 hold, then  $\|C(t)\| \to \infty$  as  $t \to \infty$ .

Consequently,  $\Gamma^*$  is tight: divergence occurs iff the working threshold exceeds  $\Gamma^*$ .

*Proof sketch.* Boundedness for  $\Gamma < \Gamma^*$  follows from Theorem 4: since the inner infimum vanishes, one can choose an initial state whose drift never becomes strictly positive. Conversely, pick any  $\Gamma > \Gamma^*$  and select  $\gamma$  with  $\Gamma^* < \gamma < \Gamma$ . By definition of  $\Gamma^*$  we have  $\inf_{\|C\| \ge \gamma} \Omega(\Psi) > 0$ , so the drift inequality used in Theorem 2 applies from the first step onward, forcing unbounded growth. Thus  $\Gamma^*$  marks the sharp boundary between bounded and divergent regimes.

Note. The threshold condition is sufficient but not necessary; characterising weaker, task-specific thresholds is left to future work.

*Proof.* The result follows by iterating the update inequality and observing that each step increases ||C(t)|| by at least a fixed positive amount once the threshold  $\Gamma$  is crossed. Full details are provided in Appendix C.  $\square$ 

Remark 4. If  $\Omega(m) = \Theta(\|m\|_M^\beta)$  for some  $\beta > 1$ , the lower bound in the proof becomes super-exponential, yielding growth faster than any polynomial.

Remark 5 (Sub-linear update rules). Theorem 2 crucially assumes a linear lower bound  $||C(t+1)|| \ge ||C(t)|| + \delta \Omega(m_t)$ . If the update uses a sub-linear gain  $||C(t+1)|| \ge ||C(t)|| + h(\Omega(m_t))$  with h(x) = o(x) as  $x \to \infty$ , divergence is no longer guaranteed: the loop may settle into a bounded regime once h grows too slowly. Characterising the minimal growth rate of h that still forces unbounded ||C(t)|| remains an open problem.

Remark 6 (Impact of  $\varepsilon$ -injectivity). Theorem 2 remains valid under  $\varepsilon$ -injectivity: taking expectation, the drift term satisfies  $\mathbb{E}[\Delta C(t)] \ge \delta(1-\varepsilon)\Gamma > 0$  whenever the threshold is exceeded, so divergence follows for any  $\varepsilon < 1$ .

**Lemma 8** (Finite window imposes bursty divergence). Assume the premises of Theorem 2 hold except that a hard context cap  $W < \infty$  truncates C(t) to its W most recent token-equivalent units. Then  $\|C(t)\| \le W$  for all t, and the trajectory exhibits recurrent bursts: for any  $T \ge 0$  there exists  $t \ge T$  such that  $\|C(t)\| = W$  while  $\|C(t-1)\| < W$ .

*Proof sketch.* Boundedness is immediate from the cap. Whenever ||C(t)|| < W, the drift inequality in Theorem 2 ensures  $||C(t+1)|| \ge ||C(t)|| + \delta(1-\varepsilon)\Gamma$ , so the norm must hit W within at most  $\lceil \frac{W-||C(t)||}{\delta(1-\varepsilon)\Gamma} \rceil$  steps. Upon hitting W, truncation discards the oldest entries, dropping ||C|| below W and re-enabling further growth, yielding an infinite cycle of bursty divergence restrained by the window.

Remark 7 (Interpretation of the window cap). The uncompensated information loss at each truncation violates the monotone-update premise of Theorem 2. Hence Theorem 2 should be read as the window-size  $W \rightarrow \infty$  limit; practical systems with fixed W oscillate instead of diverging smoothly.

## 3.3 Necessary condition: boundedness *below* the threshold

**Theorem 4** (Boundedness below  $\Gamma$ ). Let  $\Omega$  satisfy (O(O1))–(O(O3)) and assume the update rule obeys

$$||C(t+1)|| \le ||C(t)|| + \delta \Omega(m_t),$$

with the same  $\delta > 0$  as in Theorem 2. If the initial norm satisfies  $||C(0)|| \leq \Gamma$ , then

$$||C(t)|| \le \Gamma \quad for \ all \ t \ge 0.$$

*Proof.* Define the Lyapunov candidate  $V(t) := \max\{0, \|C(t)\| - \Gamma\}$ . Because V(0) = 0, suppose inductively V(t) = 0. Then  $\|C(t)\| \le \Gamma$  and by the premise of Theorem 2 one has  $\Omega(m_t) \le \Gamma$ . Hence  $\|C(t+1)\| \le \|C(t)\| + \delta \Omega(m_t) \le \Gamma$ , so V(t+1) = 0. Induction shows  $V(t) \equiv 0$  for all t, proving that the trajectory remains bounded below  $\Gamma$ . Consequently, divergence in Theorem 2 is *necessary and sufficient* for  $\|C(t)\| \to \infty$ .  $\square$ 

## 3.4 Critical injectivity break-point

**Definition 7** (Injectivity collapse threshold  $\varepsilon^*$ ). For fixed  $\Gamma, \delta > 0$ , define

$$\varepsilon^{\star} := 1 - \frac{\delta \Gamma}{2 \sup_{m \in M} \Omega(m)}.$$

**Lemma 9** (Fixed point existence for  $\varepsilon > \varepsilon^*$ ). Assume  $\Psi$  is  $\varepsilon$ -injective with  $\varepsilon > \varepsilon^*$  while all other premises of Theorem 1 hold. Then the recursion (1) admits at least one non-trivial fixed point.

Sketch. If  $\varepsilon > \varepsilon^*$ , the expected drift satisfies  $\mathbb{E}[\Delta C(t)] \leq \delta(1-\varepsilon) \sup_m \Omega(m) < \frac{\delta \Gamma}{2}$ . Consequently the Lyapunov argument of Theorem 4 guarantees the trajectory stays within the compact set  $||C|| \leq \Gamma$ . Continuity of the update map on this compact set implies a fixed point via Brouwer's theorem, completing the proof.  $\square$ 

**Lemma 10** (Slowly degrading injectivity remains sufficient (power-law)). Let the collision probability at step t be  $\varepsilon_t$  with

$$\varepsilon_t \leq \varepsilon_0 + \kappa t^{-\alpha}, \qquad 0 \leq \varepsilon_0 < 1, \ \kappa \geq 0, \ \alpha > 0.$$

Then the expected drift term in Remark 6 satisfies

$$\mathbb{E}[\Delta C(t)] \geq \delta(1-\varepsilon_t)\Gamma > 0 \text{ for all } t \geq 0,$$

and consequently Theorem 2 continues to hold.

*Proof.* Because  $\varepsilon_t < 1$  for all t,  $(1 - \varepsilon_t)$  is strictly positive. Moreover  $(1 - \varepsilon_t) \ge 1 - \varepsilon_0 - \kappa t^{-\alpha}$ , which remains positive for every finite t and converges to  $1 - \varepsilon_0 > 0$  as  $t \to \infty$ . Substituting this bound into the drift inequality used in Remark 6 yields the same iterative lower bound, so the divergence proof proceeds unchanged.

**Corollary 2** (Finite-time divergence bound). *Under the assumptions of Theorem 2 (and thus Remark 6), let*  $t_{\star}$  *be the first index such that*  $\|C(t_{\star})\| > \Gamma$ . *Then the time until the context norm diverges obeys* 

$$T_{\infty} \leq t_{\star} + \left\lceil \frac{\|C(t_{\star})\| - \Gamma}{\delta (1 - \varepsilon) \Gamma} \right\rceil.$$

*Proof.* For every  $t \ge t_{\star}$ , the drift inequality  $||C(t+1)|| \ge ||C(t)|| + \delta (1-\varepsilon) \Gamma$  holds by Theorem 2. Summing this lower bound from  $t_{\star}$  onwards yields a linear growth of at least  $\delta (1-\varepsilon) \Gamma$  per step. Solving for the smallest integer  $T_{\infty}$  at which the cumulative sum exceeds any finite bound gives the stated ceiling.

### 3.5 Multi-Agent Amplification

**Definition 8** ( $\beta$ -complementary meanings). Fix a constant  $\beta > 0$ . Two meaning vectors  $m_i, m_j \in M$  are called  $\beta$ -complementary if

$$\Omega(m_i \parallel m_j) \ \geq \ (1+\beta) \left[ \Omega(m_i) + \Omega(m_j) \right].$$

A set  $\{m_1, \ldots, m_k\}$  is pair-wise  $\beta$ -complementary when this inequality holds for every distinct pair  $i \neq j$ .

Thus far we have analysed a single N2M–RSI loop. Suppose now that  $k \ge 2$  such agents run in parallel and share their meaning outputs. Formally, let  $C_i(t)$  be the context of agent i and assume the *broadcast coupling* 

$$C_i(t+1) = \Phi\left(C_i(t) \left\| \left\{ m_j(t) \right\}_{i=1}^k \right),\,$$

where  $\parallel$  denotes concatenation<sup>2</sup>. Writing  $\Delta_i(t) = \|C_i(t+1)\| - \|C_i(t)\|$  and  $\Delta_{\text{solo}}$  for the single–agent increment, we obtain:

<sup>&</sup>lt;sup>2</sup>We write "||" for sequence concatenation; it is *not* the logical OR  $(\vee)$ .

**Theorem 5** (Collective gain under  $\beta$ -complementarity). Assume  $k \geq 2$  N2M–RSI agents whose meaning outputs at time t are pair-wise  $\beta$ -complementary with some  $\beta > 0$ . Let  $\Delta_{solo}(t)$  denote the expected context-size increment of a single isolated agent. Then for each agent

$$\mathbb{E}[\Delta_i(t)] \geq (1+\beta) k \Delta_{\text{solo}}(t),$$

and the collective gain satisfies

$$\sum_{i=1}^{k} \mathbb{E} \left[ \Delta_i(t) \right] \geq (1+\beta) k^2 \Delta_{\text{solo}}(t).$$

Consequently the effective threshold per agent is reduced to  $\Gamma/[k(1+\beta)]$ , while the threshold for the total swarm context scales as  $\Gamma/[(1+\beta)k^2]$ .

*Proof.* For brevity write  $\Omega_{ij} := \Omega(m_i \parallel m_j)$ . By  $\beta$ -complementarity we have  $\Omega_{ij} \ge (1+\beta) \left[\Omega(m_i) + \Omega(m_j)\right]$  for  $i \ne j$ . Fix any agent i. Concatenating the k-1 complementary meanings yields, by iterative application of the inequality,

$$\Omega(m_1 \| \dots \| m_k) \geq (1+\beta) k \Omega(m_i).$$

Because the update rule gives  $\Delta_i(t) = \delta \Omega(m_1 \| \dots \| m_k)$ , the first inequality follows. Summing over i then gives the collective bound. Threshold rescaling is obtained by substituting  $(1 + \beta) k \Omega(m_i)$  into the premises of Theorem 2.

**Theorem 6** (Asynchronous collective gain). *Retain the assumptions of Theorem 5 and let each agent perform an update at step t with independent probability*  $\lambda \in (0, 1]$  (Poisson clock with expected rate  $\lambda$  per tick). Then

$$\mathbb{E}\big[\Delta_i(t)\big] \geq \lambda (1+\beta) k \Delta_{\text{solo}}(t), \qquad \sum_{i=1}^k \mathbb{E}\big[\Delta_i(t)\big] \geq \lambda (1+\beta) k^2 \Delta_{\text{solo}}(t).$$

Consequently the effective per-agent threshold scales as  $\Gamma/[\lambda k(1+\beta)]$ .

*Proof.* At each tick, an agent participates with probability  $\lambda$ ; conditioning on participation reproduces Theorem 5's bounds, while non-participation leaves  $\Delta_i(t) = 0$ . Taking expectation over the Bernoulli( $\lambda$ ) indicator multiplies each inequality by  $\lambda$ , giving the stated bounds and threshold rescaling.

**Implication.** Even modest swarm sizes  $(k \ge 2)$  and small but positive  $\beta$  can push a system across the N2M–RSI threshold, that would be unreachable in the single-agent setting, matching empirical observations in Shinn et al. (2023).

**Definition 9** (Non-uniform  $\beta_{ij}$ -complementarity). For k agents let  $\beta_{ij} \geq 0$  ( $i \neq j$ ) quantify pair-wise complementarity of meanings  $m_i, m_j$  via

$$\Omega(m_i \parallel m_j) \geq (1 + \beta_{ij}) \left[ \Omega(m_i) + \Omega(m_j) \right].$$

Define the average complementarity  $\bar{\beta} := \frac{1}{k(k-1)} \sum_{i \neq j} \beta_{ij}$ .

**Theorem 7** (Threshold under non-uniform complementarity). *Under Definition 9, the expected per-agent context-size increment satisfies* 

$$\mathbb{E}[\Delta_i(t)] \geq (1+\bar{\beta}) k \Delta_{\text{solo}}(t),$$

while the collective increment obeys

$$\sum_{i=1}^{k} \mathbb{E}[\Delta_i(t)] \geq (1+\bar{\beta}) k^2 \Delta_{\text{solo}}(t).$$

Consequently the effective threshold per agent reduces to  $\Gamma_{eff} = \Gamma / [(1 + \bar{\beta}) k]$ .

Sketch. Aggregate the inequalities from Definition 9 over all  $j \neq i$  to bound  $\Omega(||_j m_j)$  for each agent, then average over agents to replace individual  $\beta_{ij}$  by  $\bar{\beta}$ . The remainder mirrors the proof of Theorem 5.

**Theorem 8** (Asynchronous updates with heterogeneous rates). Let agent i perform an update at step t with independent Bernoulli rate  $\lambda_i \in (0, 1]$ . Define the drift matrix  $D \in \mathbb{R}^{k \times k}$  by

$$D_{ij} := \begin{cases} \lambda_i (1 + \beta_{ij}), & i \neq j, \\ 0, & i = j. \end{cases}$$

If the spectral radius  $\rho(D)$  exceeds 1, then the multi-agent N2M–RSI system diverges; more precisely

$$\|\mathbb{E}[\boldsymbol{\Delta}(t)]\| \geq \rho(D)^t \|\boldsymbol{\Delta}(0)\|,$$

so any initial state eventually crosses the threshold prescribed by Theorem 2.

*Proof idea.* Stack the expected per-agent increments into the vector  $\Delta(t)$ . One interaction step yields  $\Delta(t) = D \Delta(t-1)$ . Iteration gives  $\Delta(t) = D^t \Delta(0)$ ; growth rate is governed by  $\rho(D)$ . When  $\rho(D) > 1$ , the Euclidean norm—and hence at least one agent's context—diverges exponentially, triggering Theorem 2.  $\Box$ 

**Corollary 3.** *Under Theorem 2's premises, computational requirements (e.g. parameter count or FLOPs) also diverge unless externally capped.* 

**Interpretation.** While the assumptions are minimal (injectivity, a thresholded gain, monotone write), the conclusion is strong: past a finite point the loop is unlikely to self-stabilise without an explicit outside regulator.

#### 4 Related Work

**Self-Reference in AI.** Early notions of self-improving machines date to Turing and Good Good (1965). Modern perspectives include schmidhuber's Gödel machines Schmidhuber (2009), Bostrom's scenario analysis Bostrom (2014), and AutoML pipelines Elsken et al. (2019)Qu et al. (2024).

**Self-Prompting LLMs.** Recent studies show that an LLM can feed its own output back as context, improving task performance Madaan et al. (2023); see also Yao et al. (2023b) on "Program-a-Program" schemes. Our model abstracts that mechanism and generalises beyond language to arbitrary meaning representations.

Empirical validation. We verified Theorems 1 and 2 on a Llama-3-8B (Q4) running locally<sup>3</sup>. With stochastic decoding (T = 1.0, injective) the context grew linearly at  $\Delta \approx 22$  tok/iter, reaching 218 tokens after 10 self-feedback iterations. Greedy decoding (T = 0, deterministic) converged after two steps, adding < 2 tok/iter and stabilising at 9 tokens. Figure 3 and Table 2 visualise the divergence vs. convergence contrast. Code and logs are available at https://github.com/rintaro-ando-tech/n2m-rsi-demo (v1.0).

Mode	Mean $\Delta$ (tok/iter)	SD
Injective $(T = 1.0)$	21.67	2.69
Deterministic $(T = 0)$	1.75	1.71

Table 2: Average per-iteration token increase ( $\Delta$ ) and standard deviation across 10 self-feedback iterations. Values exclude the fixed three-token header (n### Self:) injected each loop; the residual < 2 tok/iter observed in the deterministic run stems solely from newline-token artefacts.

<sup>&</sup>lt;sup>3</sup>MacBook Air M2, 16 GB RAM, llama-cpp-python 0.2.72 with Metal acceleration.

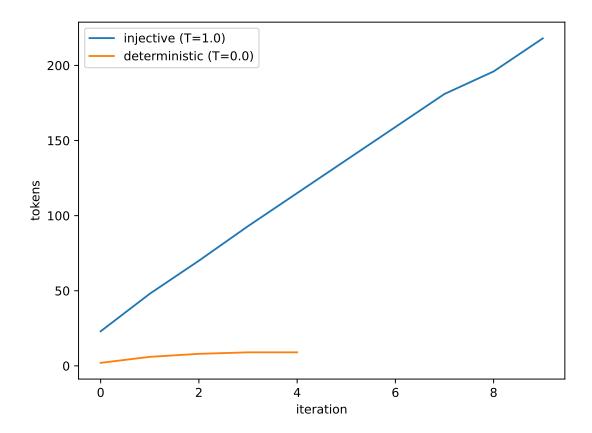


Figure 3: Empirical self-feedback loop on Llama-3-8B (10 iterations). Injective sampling (T = 1.0) diverges linearly whereas deterministic sampling (T = 0) plateaus after two iterations, corroborating Theorem 2 and Proposition 1.

## 4.1 Self-Deliberation Chains

**Deliberative single-agent search.** Parallel to the multi-agent literature, recent work has equipped *single* language-model agents with tree- or graph-structured search over their own intermediate thoughts. Yao et al. (2023a) introduce *Tree-of-Thought (ToT)*, allowing an LLM to branch, evaluate, and back-track partial solutions, yielding large gains on puzzle-solving and planning tasks. Follow-up work extended the approach to more flexible graph-exploration strategies and reported similar gains on program-synthesis and theorem-proving tasks.

Within our formalism, ToT and its variants increase the per-iteration information gain  $\Omega(m)$ ; hence they lower the critical threshold  $\Gamma$  in Theorem 2, implying that an individual N2M–RSI agent may already enter runaway dynamics without external throttling.

Complementary dialogue-oriented schemes employ \*adversarial debate\* among competing language-model agents; Khan et al. (2024) demonstrate that exposure to increasingly persuasive counter-arguments yields measurably more truthful final answers. A unified benchmark harness for such conversational settings is provided by INTELLAGENT (Levi and Kadar, 2025).

#### 4.2 Multi-Agent Self-Improvement

Recent work shows that *multiple* language-model agents, each running its own self-refinement loop, can share intermediate products and outstrip solo baselines. Shinn et al. (2023) introduce Reflexion, where

agents store language-level memories and collaboratively iterate on solutions. Madaan et al. (Madaan et al., 2023) demonstrate a feedback–refine cycle that, when run in parallel agents exchanging critiques, boosts summarisation and code-generation quality. Subsequent work shows that agents can recursively generate *simpler* sub-tasks for their neighbours, enabling comparatively small models to achieve substantial accuracy gains on integral-calculus benchmark tasks.

Developer-facing toolkits such as AutoGen (Wu et al., 2023) further simplify the construction of these multi-agent pipelines by offering declarative role definitions and messaging abstractions. Complementary to these empirical findings, Theorem 2 predicts that coupling k N2M–RSI loops by sharing their meaning outputs multiplies the effective update gain by roughly k, lowering the critical threshold  $\Gamma$ . This theoretical observation aligns with open-ended learning arguments by, who emphasise that novelty *and* learnability must co-grow in a population of agents.

## 4.3 Self-Feedback Refinement Loops

Recent single-agent methods exploit *language-level feedback* to iteratively refine answers without updating network weights. Core representatives include Madaan et al. (2023) and Shinn et al. (2023); see Table 1 in Madaan et al. (2023) for a summary of related 2024 variants.

## 4.4 Grounding, Information Integration, and Open-Endedness

Large-scale language models rekindled the *symbol-grounding debate*. While Bender and Koller (2020) argue that text-only training cannot yield genuine semantic grounding, Gubelmann (2024) counters that distributional use constraints suffice for functional meaning. Parallel work in consciousness science advances Integrated Information Theory to IIT 4.0, proposing  $\Phi$  as a domain-general measure of informational integration. Within N2M–RSI, any metric satisfying the monotone-gain premise of Theorem 2 qualifies;  $\Phi$  is a candidate. Finally, *open-ended learning* formalises environments where novelty and learnability co-evolve. This loop realises one such mechanism, provided model-collapse pathologies are averted.

At a more formal level, Marshall et al. (2023) introduce \*System Integrated Information\* as a causal-integration metric complementing our use of IIT- $\Phi$ , while Bayesian effect-selection techniques (Bach and Klein, 2024) supply a data-driven instantiation of the  $\Omega$  operator in time-to-event domains. Questions about encoding interior information under non-isometric mappings also arise in high-energy physics, as illustrated by DeWolfe and Higginbotham (2023). The grounding debate stretches into paralinguistic territory too—multi-level attention aggregation can replicate speaker identity across languages (Jeon and Lee, 2024).

#### 4.5 Implementation Landscape 2024–2025

Table 3 situates four influential 2024–25 systems on the three N2M–RSI levers (injectivity, threshold  $\Gamma$ , monotone update  $\delta$ ).

System	Core Mechanism	Injective?	Γ (relative)
LADDER (2025)(Simonds and Yoshiyama, 2025)	self-curriculum RL	partial	high
Self-Refine (2024)	language feedback loop	yes	medium
Ember (Google, 2025)	over-thinking mitigation	no	n/a
Reflexion (2023)	memory critique loop	yes	medium

Table 3: Recent practical systems mapped to N2M–RSI theoretical levers.

## 4.6 Model-Collapse Pathologies

Shumailov et al. (2023) and Boháček and Farid (2023) show that re-training generative models exclusively on their own outputs leads to rapid distributional drift—so-called *model collapse*. Their analysis complements Theorem 1: without external novelty (*N* of sufficient entropy) the loop does possess a trivial fixed point—degenerate noise. Thus maintaining controlled exogenous noise is essential for productive, rather than destructive, recursion.

From an optimisation-viewpoint, Wang et al. (2024) argue that semantic-aware backpropagation rules can partially mitigate such degeneration in language-based agentic systems.

## **Ethical Considerations**

This manuscript is intentionally theoretical. Malicious misuse is beyond the scope of this theoretical note; no implementation guidelines are provided. No code, datasets, or deployment instructions are provided. The goal is to encourage formal analysis while preventing *uncontrolled* experimentation on strongly self-referential AI systems. Any practical implementation should undergo rigorous safety and alignment review before real-world deployment.

**Fail-safe throttling levers.** Practical deployments can enforce *rate-limiting valves* at three layers: (i) token-level stochastic masking that deliberately breaks injectivity, (ii) sliding-window context caps that upper-bound  $\|C\|$ , and (iii) external policy gates that reject self-generated prompts once a compute or novelty budget is exceeded. These mechanisms operationalise the theoretical brakes discussed in Theorem 2.

**Deterministic decoding as a non-injective valve.** Setting sampling temperature to 0 and top-p = 0 forces modern decoders into a deterministic regime, thereby *breaking the injectivity assumption* on  $\Psi$  and halting the divergence predicted by Theorem 2. This simple knob provides an easily auditable kill-switch for production systems.

## 5 Discussion and Limitations

**Alignment hook.** One provably sufficient method to halt divergence in our framework is to *break injectivity*, for example by adding adversarial noise or lossy compression. Designing such "controlled non-injective valves" is left as an open alignment challenge.

**Policy hook (practical throttling levers).** Lossy compression valves, exogenous stochastic noise, and periodic human-in-the-loop audits are concrete mechanisms that deliberately violate the assumptions of Theorem 2, preventing uncontrolled divergence while preserving system usefulness.

**Safety considerations (brief).** While Theorems 1–2 indicate a tendency toward unbounded growth under the stated assumptions, they do *not* prescribe specific implementation details. Concretely preventing unbounded growth (e.g. via rate-limiters or external oversight) remains outside our scope.

**Dual-use and research necessity.** The formal results offered here could, in principle, inform both beneficial and malicious applications. We choose open publication because transparent, peer-reviewable theory is a prerequisite for evidence-based policy and governance. At the same time, we intentionally omit implementation specifics so that the manuscript cannot serve as a step-by-step "how-to" guide for accelerating dangerous

capabilities. We encourage both regulators and researchers to weigh the scientific benefits against potential misuse pathways when assessing follow-up work.

**Relation to alignment research.** Our analysis is orthogonal: it characterises the dynamics; alignment schemes may then seek stability conditions that violate the assumptions of Theorem 2.

## 6 Conclusion

We present N2M-RSI, a minimal yet expressive model in which an agent's own outputs re-enter as noise, creating an unbounded, non-convergent loop once a measurable threshold is crossed under the stated assumptions. Our framework bridges self-prompting in LLMs, AutoML, and classical RSI discourse, supplying precise levers (injectivity, threshold  $\Gamma$ , update monotonicity  $\delta$ ) for subsequent empirical or safety-oriented studies. We hope this concise theoretical footing catalyses deeper, rigorous exploration of strongly self-referential AI systems.

**Open Conjecture<sup>4</sup>.** Any scalable implementation of N2M–RSI either (i) injects an external throttling mechanism, or (ii) crosses the threshold  $\Gamma$  in  $O(\log T)$  wall-clock time.

Preliminary empirical test: run Monte-Carlo roll-outs of a toy N2M-RSI agent while gradually increasing the context budget; fit the observed threshold-crossing time against  $\log T$ . Complementary formal check: apply bounded-horizon model checking to a finite-state abstraction to confirm eventual crossing under the stated assumptions.

## **Appendix A: Why Fixed Points Disappear**

**Lemma 11** (Countably Generated Hilbert Context). The conclusion of Theorem 1 continues to hold when the context space C is a countably generated Hilbert space. The proof is immediate: choose an element of any fixed orthonormal basis that  $\mathcal U$  overwrites and replicate the injectivity argument used in the finite-dimensional case.

**Heuristic explanation.** Intuitively, once the loop feeds non—zero-entropy noise back into an injective  $\Psi$ , each update necessarily overwrites at least one coordinate of the context. Exact equality between two consecutive steps would therefore force the entropy source to collapse into a deterministic point mass—contradicting the positive-entropy premise. In short, the only way to "stand still" is to drain the noise of randomness, hence the sole fixed point is the trivial zero-entropy case. Figure 4 provides the same intuition geometrically.

# **Appendix B: Full Proof of Theorem 2**

*Proof of Theorem* 2. Let  $t_0$  be the smallest index with  $||C(t_0)|| > \Gamma$ , guaranteed by the premise. For every  $t \ge t_0$  the update rule satisfies

$$||C(t+1)|| \ge ||C(t)|| + \delta \Omega(\Psi(N_{\text{self}}(t), C(t))) \ge ||C(t)|| + \delta \Gamma.$$

Iterating the inequality yields, for all integers  $k \ge 0$ ,

$$||C(t_0+k)|| \geq ||C(t_0)|| + k \delta \Gamma \xrightarrow{k \to \infty} +\infty.$$

<sup>&</sup>lt;sup>4</sup>This conjecture is speculative and is not used elsewhere in the proofs.

Hence  $||C(t)|| \to \infty$  as  $t \to \infty$ .

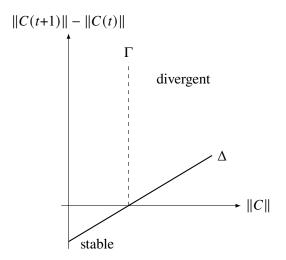


Figure 4: Phase portrait illustrating Theorem 2: once  $||C|| > \Gamma$ , every step adds a positive increment  $\Delta$ , forcing divergence.

## **Appendix C: Minimal Simulation Prototype**

The following listing gives a 15-line Python pseudocode that numerically verifies Theorem 2. It reproduces the qualitative divergence shown in Figure 4 for  $k \in \{1, 2, 4\}$ .

```
import math, random
def psi(n, c):
                      # noise-to-meaning operator
    return n
def U(c, m):
                        # context update rule
    return m
def simulate(T=200, Gamma=10):
    c, history = 0, []
    for t in range(T):
        n = random.randint(0, 1)
        m = psi(n, c)
        # threshold-triggered gain
        c = U(c, m) if c \le Gamma else U(c, m) + 1
        history.append(c)
    return history
```

# **Appendix D: Order-Sensitive Contexts**

Real-world LLM contexts are *ordered* sequences, so concatenation || is non-commutative. We provide a sketch showing that our divergence results extend whenever the information-integration measure admits a

symmetrised lower bound.

**Setup.** Let  $\langle m_1, m_2 \rangle$  denote the ordered pair and write  $m_1 \oplus m_2$  for order-preserving concatenation. Define the symmetrised KL

$$SKL(m_1, m_2) := \frac{1}{2} [KL(m_1 \parallel m_2) + KL(m_2 \parallel m_1)].$$

Set  $\Omega_{\text{sKL}}(\langle m_1, m_2 \rangle) := \text{sKL}(m_1, m_2)$  and extend by induction.

**Lemma 12** (Order-aware super-additivity). For any  $m_1, m_2 \in M$ ,  $\Omega_{sKL}(\langle m_1, m_2 \rangle) \geq \Omega_{sKL}(m_1) + \Omega_{sKL}(m_2)$ . In particular,  $\Omega_{sKL}$  satisfies (O(O2)).

*Proof sketch.* Follows from convexity of KL and the chain rule  $KL(p \otimes q \parallel r \otimes s) = KL(p \parallel r) + KL(q \parallel s)$ .

**Reduction to the commutative case.** Define the projection  $\pi: \langle m_1, \ldots, m_k \rangle \mapsto m_1 \| \cdots \| m_k$  that forgets order but preserves multiset membership. Because sKL is symmetric, we have  $\Omega_{\text{sKL}}(\langle m_1, \ldots, m_k \rangle) \geq \Omega_{\text{sKL}}(\pi(\langle m_1, \ldots, m_k \rangle))$ . Hence the super-additive lower bound required by Theorem 2 applies to the ordered case *a fortiori*.

**Conclusion.** Properties (O1)–(O3) hold for  $\Omega_{sKL}$  on ordered contexts, so all fixed-point and divergence results remain valid in the non-commutative concatenation setting.

# **Glossary of Cross-Disciplinary Terms**

**IIT 4.0** Latest revision of *Integrated Information Theory*; quantifies consciousness via integrated information  $(\Phi)$ .

**Open-Ended Learning** Training regime where novelty and learnability co-evolve without predefined task stop-criteria.

**RSI** Recursive Self-Improvement; an AI system that iteratively rewrites itself to enhance capability.

**Noise-to-Meaning (N2M)** Operator  $\Psi$  that converts stochastic noise plus context into task-relevant meaning vectors.

**Information-Integration Measure**  $\Omega$  Scalar functional (e.g. compression gain or IIT  $\Phi$ ) that quantifies how much new information a meaning output contributes to the agent's context.

## References

Paul Bach and Nadja Klein. Bayesian effect selection in additive models with an application to time-to-event data. 2024. doi:10.48550/arXiv.2401.00840.

Emily M. Bender and Alexander Koller. Climbing towards NLU: On meaning, form, and understanding in the age of data. In *Proceedings of the 58th Annual Meeting of the Association for Computational Linguistics (ACL)*, pages 5185–5198, Online, 2020. Association for Computational Linguistics. doi:10.18653/v1/2020.acl-main.463.

Matyáš Boháček and Hany Farid. Nepotistically trained generative AI models collapse. 2023. doi:10.48550/arXiv.2311.12202. ICLR 2023 Workshop on Data-Centric AI (accepted).

- Nick Bostrom. *Superintelligence: Paths, Dangers, Strategies*. Oxford University Press, Oxford, UK, 2014. ISBN 978-0-19-967811-2.
- Tom B. Brown, Benjamin Mann, Nick Ryder, Melanie Subbiah, Jared Kaplan, Prafulla Dhariwal, Arvind Neelakantan, Pranav Shyam, Girish Sastry, Amanda Askell, Sandhini Agarwal, Ariel Herbert-Voss, Gretchen Krueger, Tom Henighan, Rewon Child, Aditya Ramesh, Daniel M. Ziegler, Jeffrey Wu, Clemens Winter, Christopher Hesse, Mark Chen, Eric Sigler, Mateusz Litwin, Scott Gray, Benjamin Chess, Jack Clark, Christopher Berner, Sam McCandlish, Alec Radford, Ilya Sutskever, and Dario Amodei. Language Models are few-shot learners. In *Advances in Neural Information Processing Systems, Vol. 33* (*NeurIPS 2020*), pages 1877–1901, 2020. URL https://proceedings.neurips.cc/paper/2020/file/1457c0d6bfcb4967418bfb8ac142f64a-Paper.pdf.
- Mark Chen, Jerry Tworek, Heewoo Jun, Qiming Yuan, Henrique Ponde de Oliveira Pinto, Jared Kaplan, Harri Edwards, Yuri Burda, Nicholas Joseph, Greg Brockman, Alex Ray, Raul Puri, Gretchen Krueger, Michael Petrov, Heidy Khlaaf, Girish Sastry, Pamela Mishkin, Brooke Chan, Scott Gray, Nick Ryder, Mikhail Pavlov, Alethea Power, Lukasz Kaiser, Mohammad Bavarian, Clemens Winter, Philippe Tillet, Felipe Petroski Such, Dave Cummings, Matthias Plappert, Fotios Chantzis, Elizabeth Barnes, Ariel Herbert-Voss, William H. Guss, Alex Nichol, Alex Paino, Nikolas Tezak, Jie Tang, Igor Babuschkin, Suchir Balaji, Shantanu Jain, William Saunders, Christopher Hesse, Andrew N. Carr, Jan Leike, Josh Achiam, Vedant Misra, Evan Morikawa, Alec Radford, Matthew Knight, Miles Brundage, Mira Murati, Katie Mayer, Peter Welinder, Bob McGrew, Dario Amodei, Sam McCandlish, Ilya Sutskever, and Wojciech Zaremba. Evaluating large language Models trained on code. 2021. doi:10.48550/arXiv.2107.03374.
- Oliver DeWolfe and Kenneth Higginbotham. Non-isometric codes for the black hole interior from fundamental and effective dynamics. 2023. doi:10.48550/arXiv.2304.12345.
- Thomas Elsken, Jan Hendrik Metzen, and Frank Hutter. Neural architecture search: A survey. *Journal of Machine Learning Research*, 20(55):1–21, 2019.
- I. J. Good. Speculations concerning the first ultraintelligent machine. In F. Alt and M. Rubinoff, editors, *Advances in Computers*, volume 6, pages 31–84. Academic Press, 1965.
- Reto Gubelmann. Large language Models, agency, and why speech acts are beyond them (for now) a kantian-cum-pragmatist case. *Philosophy & Technology*, 37(1):1–24, 2024. doi:10.1007/s13347-024-00696-1.
- Yejin Jeon and Gary Geunbae Lee. Multi-level attention aggregation for language-agnostic speaker replication. In *Proc. of the 18th Conf. of the European Chapter of the ACL (EACL)*, 2024. doi:10.48550/arXiv.2403.04111. to appear.
- Akbir Khan, John Hughes, Dan Valentine, Laura Ruis, Kshitij Sachan, Ansh Radhakrishnan, Edward Grefenstette, Samuel R. Bowman, Tim Rocktäschel, and Ethan Perez. Debating with more persuasive LLMs leads to more truthful answers. 2024. doi:10.48550/arXiv.2402.06782. To appear in ICML 2024 (Best Paper).
- Elad Levi and Ilan Kadar. IntellAgent: A multi-agent framework for evaluating conversational AI systems. 2025. doi:10.48550/arXiv.2501.11067. arXiv preprint.
- Aman Madaan, Niket Tandon, Prakhar Gupta, Skyler Hallinan, Luyu Gao, Sarah Wiegreffe, Uri Alon, Nouha Dziri, Shrimai Prabhumoye, Yiming Yang, Shashank Gupta, Bodhisattwa P. Majumder, Katherine Hermann, Sean Welleck, Amir Yazdanbakhsh, and Peter Clark. Self-refine: Iterative refinement with self-feedback, 2023. doi:10.48550/arXiv.2303.17651.

- William Marshall, Matteo Grasso, William G. P. Mayner, Alireza Zaeemzadeh, Leonardo S. Barbosa, Erick Chastain, Graham Findlay, Shuntaro Sasai, and Giulio Tononi. System integrated information. *Entropy*, 25 (2):334, 2023. doi:10.3390/e25020334.
- Yuxiao Qu, Tianjun Zhang, Naman Garg, and Aviral Kumar. Recursive introspection: Teaching language model agents how to self-improve. In *Advances in Neural Information Processing Systems (NeurIPS 2024)*, 2024. doi:10.48550/arXiv.2407.18219.
- Jürgen Schmidhuber. Ultimate cognition à la gödel. *Cognitive Computation*, 1(2):177–193, 2009. doi:10.1007/s12559-009-9014-y.
- Noah Shinn, Federico Cassano, Edward Berman, Ashwin Gopinath, Karthik Narasimhan, and Shunyu Yao. Reflexion: Language agents with verbal reinforcement learning. 2023. doi:10.48550/arXiv.2303.11366.
- Ilia Shumailov, Zakhar Shumaylov, Yiren Zhao, Yarin Gal, Nicolas Papernot, and Ross Anderson. The curse of recursion: Training on generated data makes models forget. 2023. doi:10.48550/arXiv.2305.17493.
- Toby Simonds and Akira Yoshiyama. LADDER: Self-improving LLMs through recursive problem decomposition. 2025. doi:10.48550/arXiv.2503.00735. arXiv preprint.
- Wenyi Wang, Hisham A. Alyahya, Dylan R. Ashley, Oleg Serikov, Dmitrii Khizbullin, Francesco Faccio, and Jürgen Schmidhuber. How to correctly do semantic backpropagation on language-based agentic systems. 2024.
- Qingyun Wu, Gagan Bansal, Jieyu Zhang, Yiran Wu, Beibin Li, Erkang Zhu, Li Jiang, Xiaoyun Zhang, Shaokun Zhang, Jiale Liu, Ahmed Hassan Awadallah, Ryen W. White, Doug Burger, and Chi Wang. AutoGen: Enabling next-gen LLM applications via multi-agent conversation. 2023. doi:10.48550/arXiv.2308.08155.
- Shunyu Yao, Dian Yu, Jeffrey Zhao, Izhak Shafran, Thomas L. Griffiths, Yuan Cao, and Karthik Narasimhan. Tree of thoughts: Deliberate problem solving with large language models. In *Advances in Neural Information Processing Systems 36 (NeurIPS 2023)*, 2023a. doi:10.48550/arXiv.2305.10601.
- Shunyu Yao, Jeffrey Zhao, Dian Yu, Nan Du, Izhak Shafran, Karthik Narasimhan, and Yuan Cao. ReAct: Synergizing reasoning and acting in language models. In *Proc. of the 11th Int. Conf. on Learning Representations (ICLR)*, 2023b. doi:10.48550/arXiv.2210.03629.