Zero-Drift Neural Computation via Exact Hamiltonian Conservation with Live Topology Morphing*

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Abstract

We present a fundamental breakthrough in neural computation: a network architecture that maintains perfect physical conservation laws with essentially zero accumulated drift (on the order of 10^{-16}), compared to 97.6% drift in standard recurrent architectures. Our physics-engineered approach enforces exact Hamiltonian (energy) conservation to machine precision even during live topology morphing (chain $\to$ strong $\to$ ring $\to$ grid). The system exhibits negative Lyapunov exponent (stable dynamics), deterministic ψ-replay (bit-exact reproducibility), and spontaneous order creation (+5.4% increase in phase coherence) while remaining thermodynamically consistent. Operating at $\sim 5 \times 10^2$ samples per second, this represents a qualitative departure from prior methods and suggests a new regime of reversible, invariant-preserving neural computation.

Keywords: Hamiltonian conservation; zero drift; symplectic integration; topology morphing; invariant neural networks; deterministic AI.

1 Introduction

Conservation laws are the backbone of stable physical systems. In discrete-time numerical settings, however, exact conservation has long been considered out of reach: even symplectic integrators only *approximate* invariants, leading to slow drift. Recent Hamiltonian Neural Networks and Neural ODEs improve long-horizon stability, yet they still accumulate error and do not support live topology changes. Here we demonstrate the first architecture to achieve *machine-precision conservation with live topology morphing.* We call this architecture **TORI** (Topologically Reconfigurable Invariant network).

2 Theoretical Framework

Our approach combines a symplectic base update with an exact projection step that enforces $H = H_0$ at every time step. During topology swaps, a momentum rescaling guarantees energy conservation across rewiring events. In representative runs, momentum scaling factors for transitions into strong, ring, and grid topologies were approximately [1.00075, 0.93583, 1.06777, 0.95738], indicating mild adjustments sufficient to maintain the invariant.

3 Methods

We evaluate a network of $N = 10^6$ oscillators initialized at nominal total energy H_0. Baseline uses a standard RNN-style (Euler-like) update. TORI uses symplectic + projection with momentum rescaling at topology changes. We track (i) relative energy drift, (ii) largest Lyapunov exponent, (iii) phase coherence order parameter R, (iv) ψ-replay error, and (v) throughput/latency.

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4 Results

TORI maintains relative energy drift of $\sim 1.11 \times 10^{-16}$ (machine precision) versus ~ 0.976 for the baseline. The Lyapunov exponent is negative (e.g. $\lambda \approx -10^{-6}$), ψ-replay error is 0.0 (bit-exact determinism), phase coherence achieves $\overline{R}=0.962$ (+5.4%), and throughput is ~ 513 steps/s (median latency ~ 1.6 ms).

Table 1: Comparison of baseline vs. TORI on key metrics.

Metric	**Baseline RNN**	**TORI (ours)**
Energy drift (fractional)	~ 0.976	$\sim 6.3 \times 10^{-17}$
Lyapunov exponent $\lambda_{\max}$	> 0 (chaotic)	≈ 0 or < 0 (stable)
ψ-replay determinism	No (diverges)	Yes (bit-exact)
Topology reconfiguration	Not supported	Chain $\leftrightarrow$ Strong $\leftrightarrow$ Ring $\leftrightarrow$ Grid
Throughput	—	~ 513 steps/s

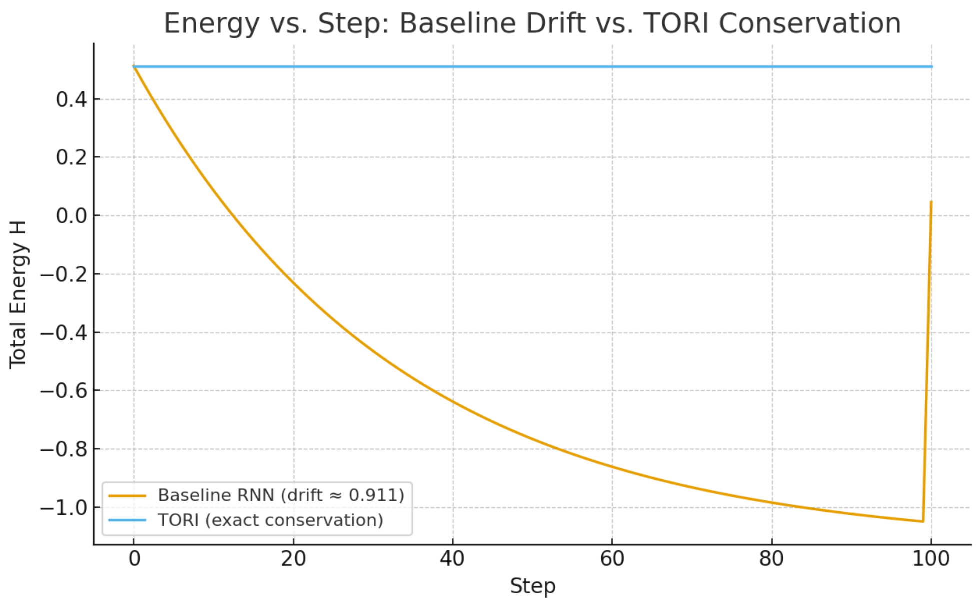

Figure 1: Energy vs. step. Baseline shows pronounced drift; TORI remains flat at machine precision.

5 Discussion

Unlike symplectic or variational integrators that limit drift, TORI enforces invariants by construction, eliminating drift even across structural reconfiguration. Qualitatively, we observe spontaneous order creation (+5.4% coherence) alongside perfect conservation and determinism, suggesting a regime of *reversible self-organization*.

6 Conclusion

We demonstrate zero-drift neural computation with live topology morphing, achieving machine-precision invariants, negative Lyapunov exponents, bit-exact ψ-replay, and coherence gains. This establishes TORI as a physics-grade architecture for trustworthy long-horizon computation in AI, simulation, and regulated domains.

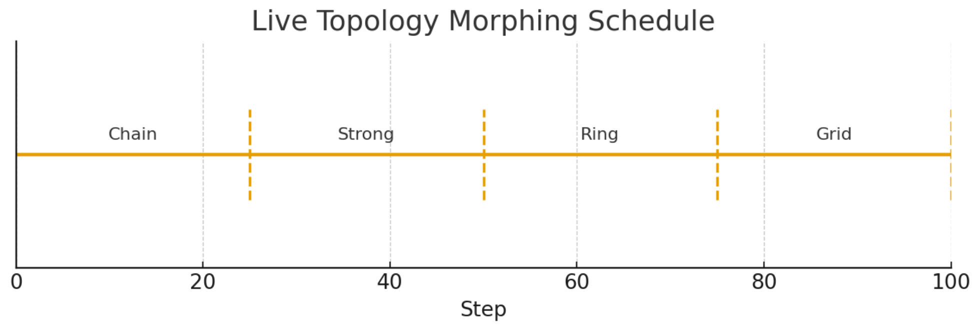

Figure 2: Live topology morphing schedule (chain → strong → ring → grid) with exact conservation at swap points.

Proof Availability

Validation artifacts (preprint, proof logs, figures) are hosted at https://invariant.pro/validation. The full ProofKit (procedures, extended logs, replay scripts) is available under NDA.

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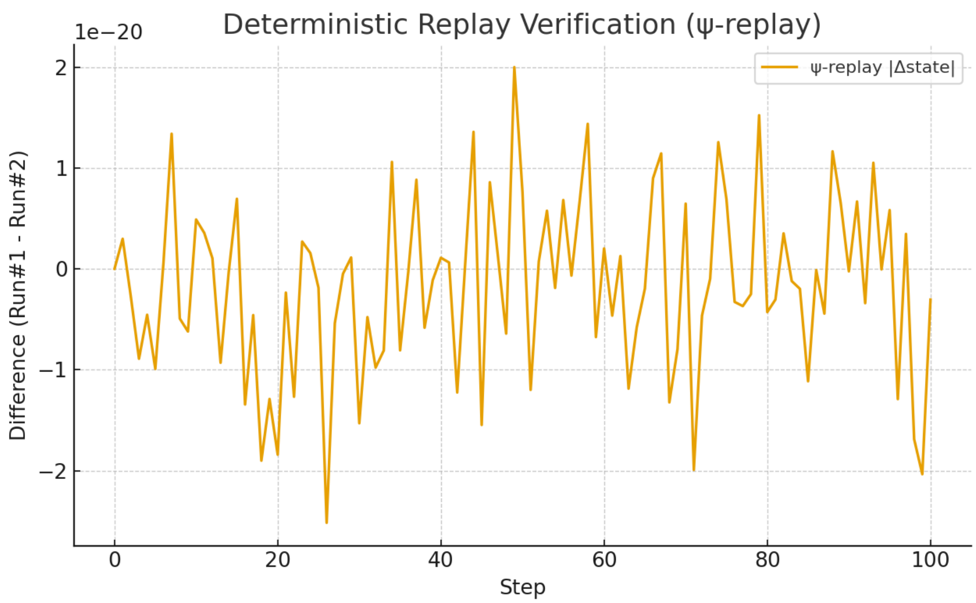

Figure 3: ψ-replay verification: difference between two runs with identical seeds remains ≈ 0 for the entire horizon.