CO 351 : Network Flow Theory A9

Term: Spring 2019

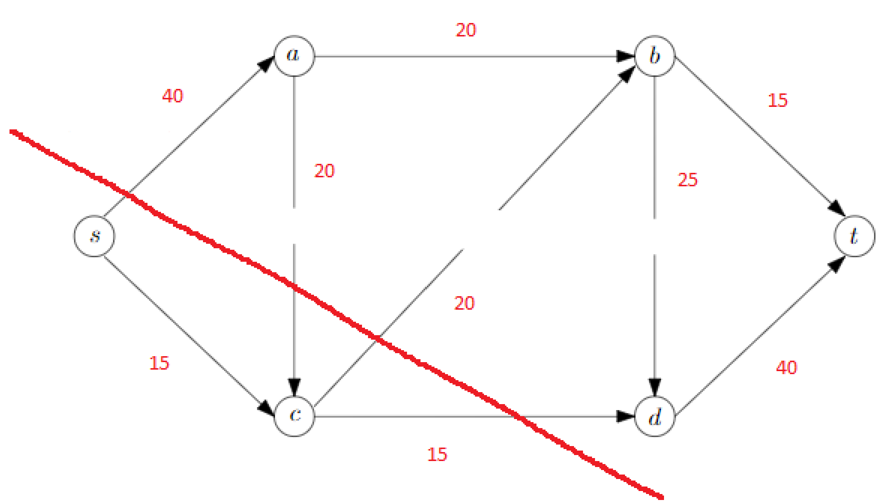
Instructor: M. Pei

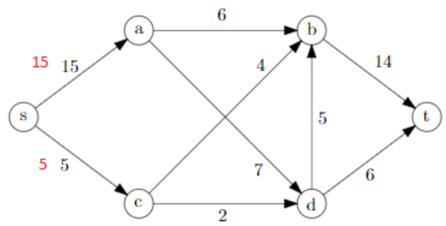
1. min flow max cut

   (a) FF
   Modified Ford Fulkerson Algorithm:
   Start with any x that is a feasible flow for a directed graph G. And form a residual digraph $D^{'}$ where the value for residual is $x_e - l_e$ for forward arcs and $c_e - x_e$ for backward arcs.
   While $D^{'}$ has an s-t dipath:
   Find s,t-dipath P in $D^{'}$
   Push flow r along p ($-r$ for forward arcs and $+r$ for backwards arcs) where r is the minimum residual in P
   Update $D^{'}$

   (b) Theorem
   The value of a minimum flow is equal to the maximum value of $l(\delta(S)) - c(\delta(\bar{S}))$ among all s, t cuts $\delta(S)$
   Proposition: Given a digraph G with sink and source t, s, capacities c and lower bounds l the value of any s,t flow is at least $l(\delta(S)) - c(\delta(\bar{S}))$
   Proof: Let x be any s,t flow and $\delta(S)$ be any s,t cut. We know that the s,t flow in an s,t cut is equal to the flow at s so $x(\delta(s)) - x(\delta(\bar{s})) = x(\delta(S)) - x(\delta(\bar{S})) \geq l(\delta(S)) - c(\delta(\bar{S}))$
   Proof of theorem above: Let x be a minimum flow for G. This means that in $D^{'}$ there is no s,t dipath remaining otherwise modified FF Alg can find a smaller flow. Then that means there exists an empty s,t cut $\delta_{D'}(S)$. If uv is an arc in $\delta_D(S)$ then it is not an arc in $D^{'}(S)$ since $\delta_{D'}(S)$ is empty. So then by the algorithm $x_{uv} = l_{uv}$. If uv is an arc in $\delta_D(\bar{S})$ then vu is not an arc in $D^{'}$ and $x_{uv} = c_{uv}$ So then the value of x is $x(\delta(S)) - x(\delta(\bar{S})) = l(\delta(S)) - c(\delta(\bar{S}))$. Then by our proposition we see that x is a minimu flow and $\delta(S)$ is a cut with the maximum value of $l(\delta(S)) - c(\delta(\bar{S}))$

   (c) Sol
   red numbers indicate flow on each arc. Optimal value is 55





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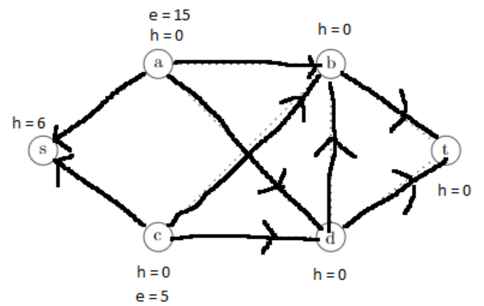

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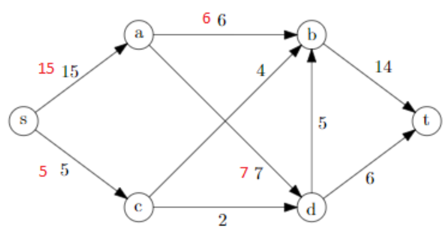

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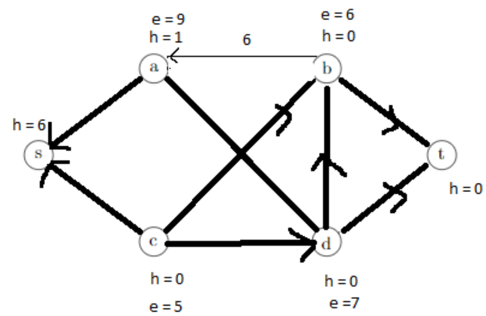

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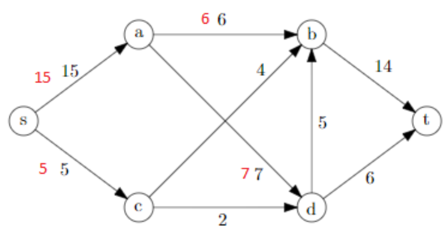

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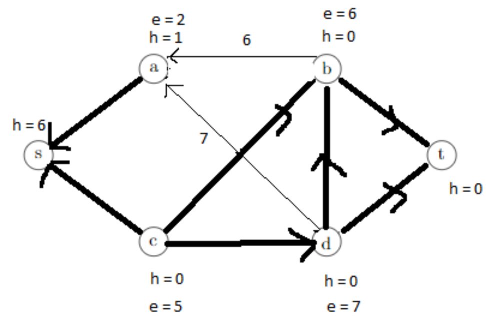

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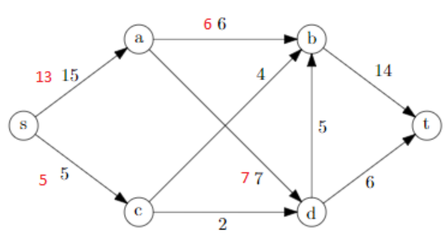

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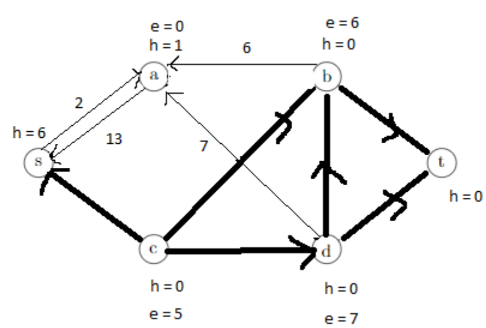

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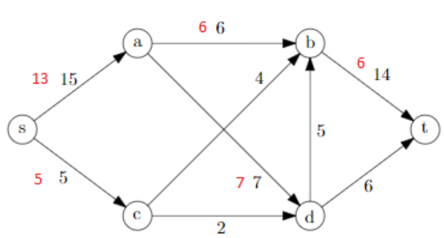

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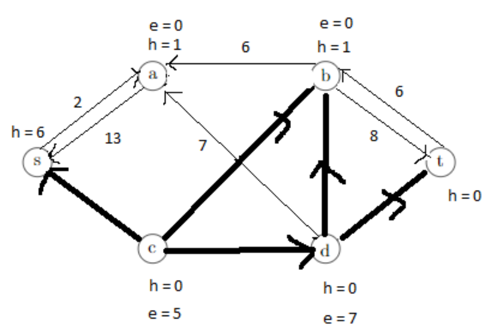

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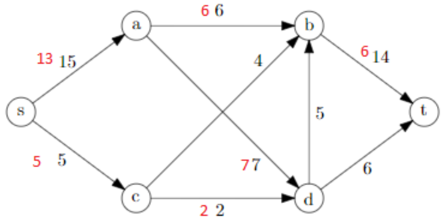
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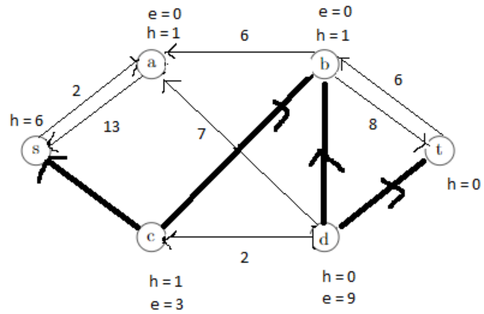
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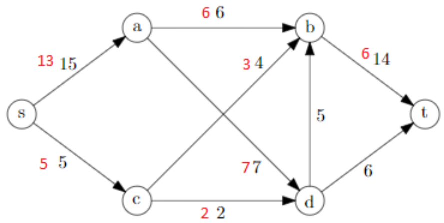
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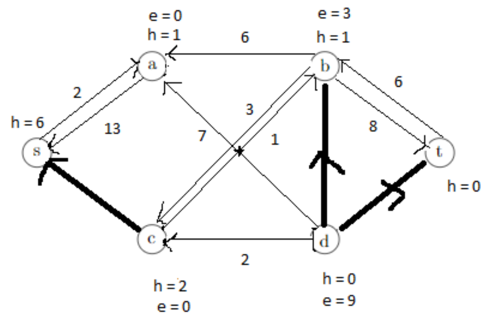
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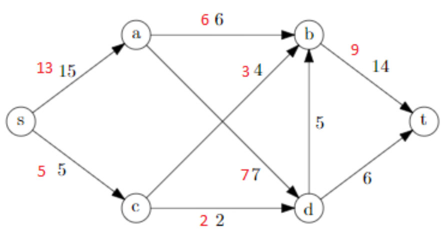
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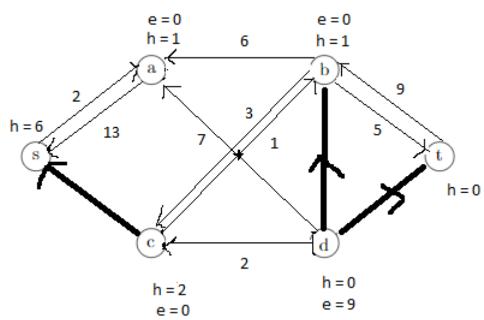
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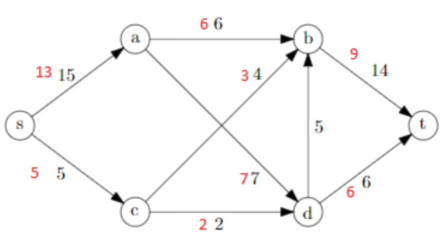
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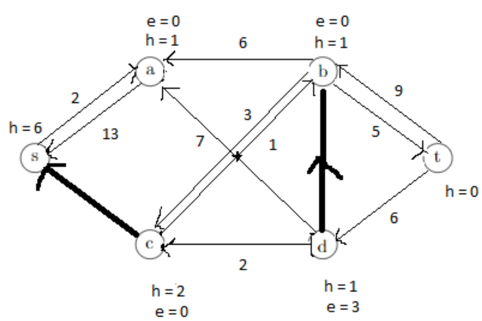
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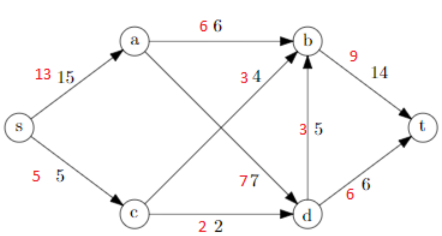
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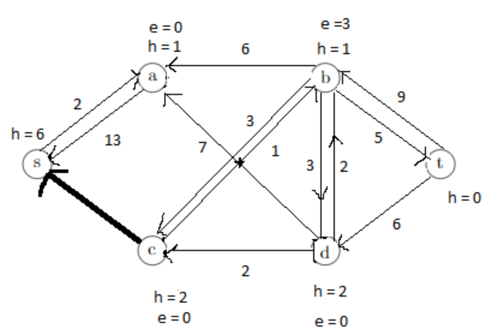
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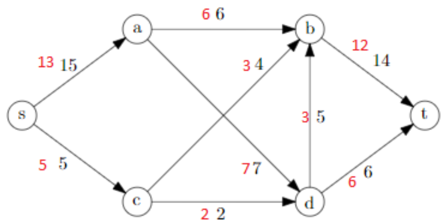

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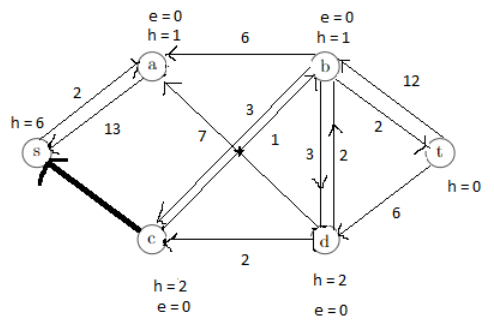

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