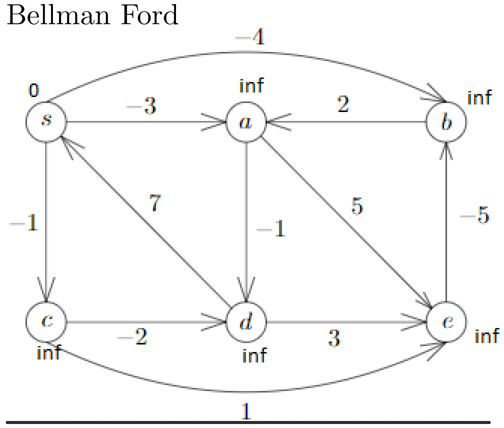
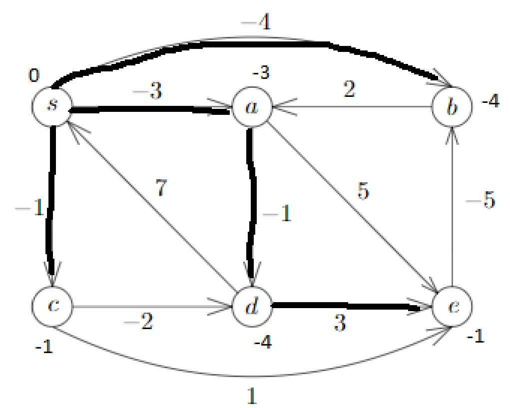
CO 351 : Network Flow Theory A7

Term: Spring 2019

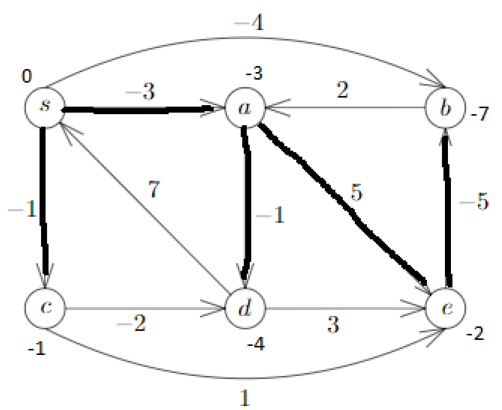
Instructor: M. Pei

1. Bellman Ford

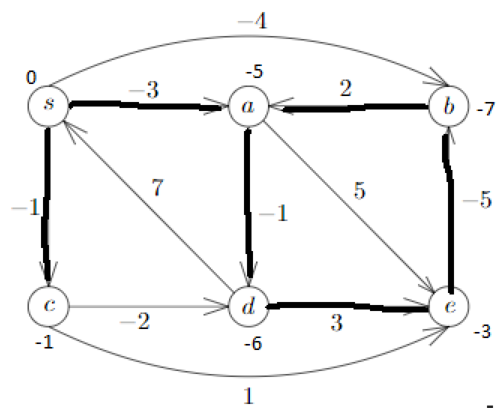




$sb, sa, \quad sc, \quad ad, ae, \quad de,$



$ae, eb,$



$ba, \quad ad, \quad de,$

negative dicycle at b->a->
d->e

2. alpha and beta

   (a) Suppose that uv is not in any dicycles and not in T. Since it is not in a dicycle, we cannot produce a negative dicycle and since its not in T we can increase w_{uv} by any amount and T will still be the tree of shortest path, since there is no incentive to add uv into T. There is no incentive because we persume adding w_{uv} increases the cost of T, so $\alpha_{uw} = \infty$. No matter how we increase it it doesn't affect T. Now we claim that $\beta_{uv} = y_u + w_{uv} - y_v$. which is just $\bar{w}_{uv}$ we show this using a algorithm from Bellman Ford. Since T is a tree rooted at s we know that in order for the algorithm to terminate all arcs in T are equality arcs and all y is feasible. So in order for T to change and add uv and replace some arcs in T, $y_u + w_{uv} - y_v = 0$ must be true, so $\bar{w}_{uv}$ must equal to 0. So adding uw into T lets us set $y_v = y_u + w_{uv}$ but since we are adding uv into T, we want y_v to stay the minimum, so we attempt to keep the arc tight by reducing the arc cost w_{uv}. So we must change w_{uv} by the difference between $y_u + w_{uw}$ and y_v at least if we want to add uv. The difference is just $y_u + w_{uv} - y_v = \bar{w}_{uv}$ Therefore β_{uv} at most is $\bar{w}_{uv}$ before T is no longer the shortest path tree.

   (b) Two scenarios, if uv was the only possible path to v and v is a sink then uv can increase by any amount and it would still be in T in order to form a spanning tree. If uv was not the only path to v or v was not a sink then it be α_{uv} would be $\min\{\bar{w}_{xy} | xy \notin T$ and xy is part of a s-v dipath$\}$

3. TP

Let $D = (N, A)$ be a digraph with demands b_v for $v \in N$. Let $\delta(S)$ be any cut on D. The netflow of the cut is the sum of b_v within the cut, in order for the flow to be feasible at least one arc in a cut with non-zero net flow needs to have flow on it. So let there be a dipath P that consists of arc of non-zero flow. We define chracterictic vectors of the dipath to be 0 if $e \notin A(P)$ and x_e if $e \in A(P)$. Let $x^{'} = x - x^p$ then that means $x^{'}(\delta(\bar{v})) - x^{'}(\delta(v)) = b_v - 1$ As for $x^{'}$ to be feasible the demands and supply values are all now $b_v - 1$ since we essentially remove 1 flow from every single arc. We continue to remove x^p from subsequent $x^{'}$ for b_v times, until $b_v = 0$ So by theorm in class $x^{'}$ is a sum of characteristic vectors of dicycles.