CO 351 : Network Flow Theory A10

Term: Spring 2019

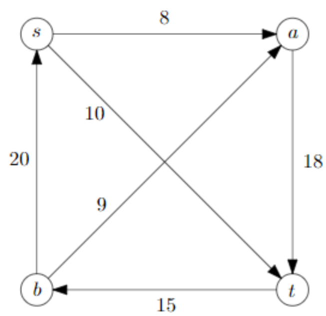
Instructor: M. Pei

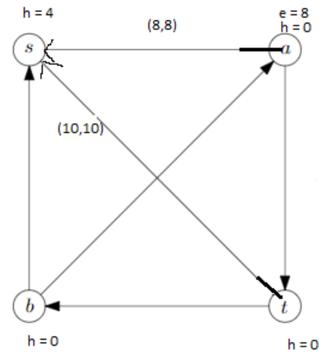
1. Global min cut
   We can run the algorithm twice once on G to figure out the minimum s cut in G and then we reverse all the arcs in G to create $G^{'}$. We then run the algorithm on this new graph and output a min s-cut $S^{'}$. Consider the most smallest s cut in $G^{'}$ we see that since the arcs are reversed' $S^{'} = V - S$ so $c(\delta(S^{'})) = c(\delta(V - S))$ in the original graph G. So what ever min s-cut we produce from Hao Orlin in $G^{'}$ will have an equivalent cut in G that does not include s. Since we are only missing min cuts without s in the original run we just need to take $\min(c(\delta(S)), c(\delta(S^{'}))$

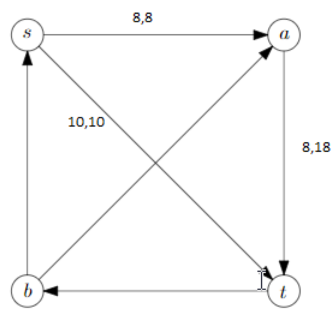
2. Hao Orlin

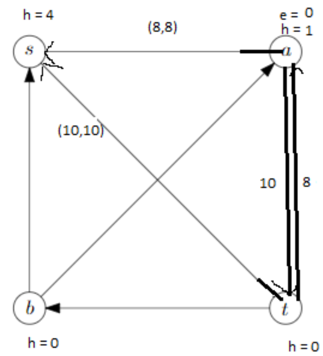
l = 3

x = {s}





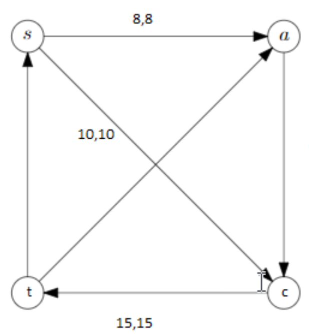


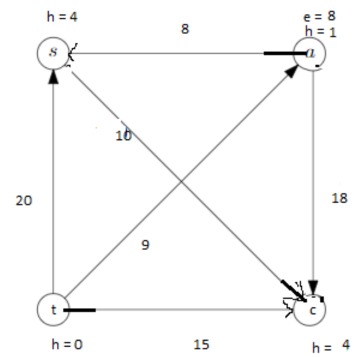


min X, t-cut {s} cap 18

l = 4

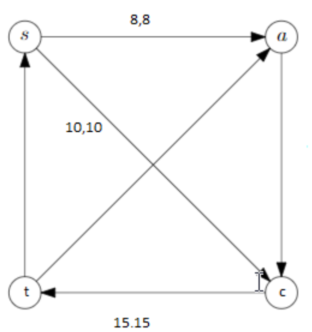
x={s, c}

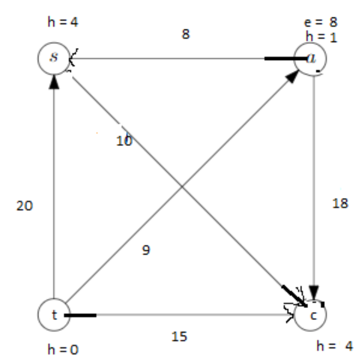




l = 1

x = {s,c,b}
min X,t-cut {s,c,a} cap 15

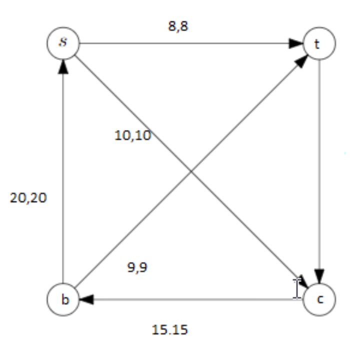


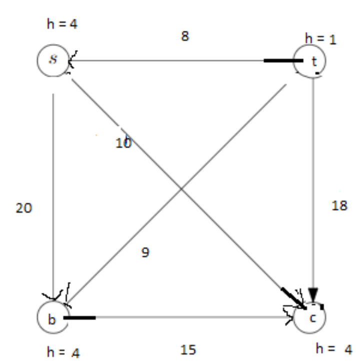


l = 1

x= {s,a,b,c}

min x,t-cut{s,b,c}
cap 17





3. We know that the chances of the global minimum cut surviving all the contractions of the algorithm is $\frac{2}{|v|(|v|-1)}$. We can extend this probability to every global minimum cut that belongs to G. So let p_i be the probability that $\delta(S_i)$ is returned by the algorithm. Where $\delta(S_i)$ is a global minimum cut. In order to return a different cut from the previous we restart the algorithm and contract the edges in different orders. This means that each contraction is a bernoulli trial and are independent, so $\sum_n^i p_i \leq 1$ so that means $n * \frac{2}{|v|(|v|-1)} \leq 1$ so $n \leq \frac{|v|(|v|-1)}{2}$ which is equal to $\binom{|v|}{2}$.

4. Similarly with the proof from class we have the numerator being $c(\delta(S^*))$ this is the probability we pick an edge from the cut. With the demoninator being $\sum_{e \in E} c_e$. Considering the cuts of the form $\delta(v)$ for all $v \in V$, We see that $\sum_{e \in E} c_e = \sum_{v \in V} \delta(v) \geq \sum_{v \in V} \delta(S^*)$. Since $\delta(S^*)$ is a minimum cut. Then $\sum_{v \in V} \delta(S) = |V| c(\delta(S^*))$. So the probability that an edge from $\delta(S^*)$ survives the first contraction is $1 - \frac{c(\delta(S^*))}{|V| c(\delta(S^*))}$ which is just $1 - \frac{1}{|V|}$. Now suppose we have contracted k edges and our global mincut in $\delta(S^*))$ is still intact. Then we have $|v| - k$ vertices left in our graph. The numberator stays the same at $c(\delta(S^*))$ since our cut is still intact and the denominator is $\sum_{e \in E'} c_e$ where $E^{'}$ is the new set of remaining edges. So now $\sum_{e \in E'} c_e = \sum_{v \in V'} \delta(v) \geq \sum_{v \in V'} \delta(S^*)$ which is equal to $((|V| - k) * c(\delta(S^*))$ So the probability of the min cut surviving the $k + 1^{th}$ contraction is $1 - \frac{1}{|v| - k}$. Then we see that the largest number of contractions is $|V| - 3$ so with the proof from class we see that it is still $\frac{2}{\binom{|v|}{2}}$ chance of producing the min cut.