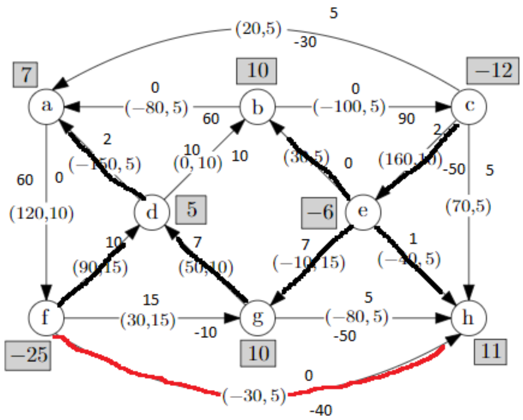
CO 351 : Network Flow Theory A5

Term: Spring 2019

Instructor: M. Pei

1. MCFP

A3 Q1 template. Suggestion: Record the flow value above the arc label, record the reduced cost below the arc label, write explanations on the right.



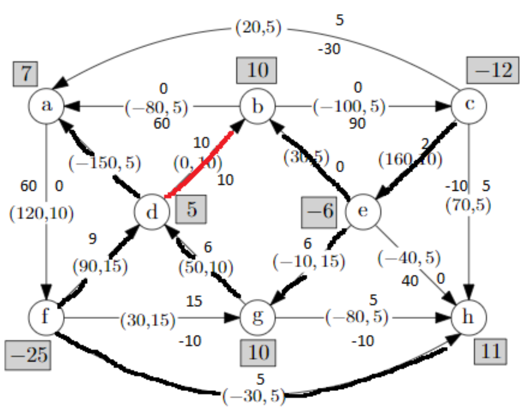
potentials:
y_a = 50
y_f = 110
y_d = 200
y_b = 190
y_e = 160
y_c = 0
y_g = 150
y_h = 120

fh violates optimality condition since it has reduce cost below 0 and has 0 flow

we orient fh as the forward arc in the cycle

min{5, 1, 8, 3, 10}

leaving arc is eh



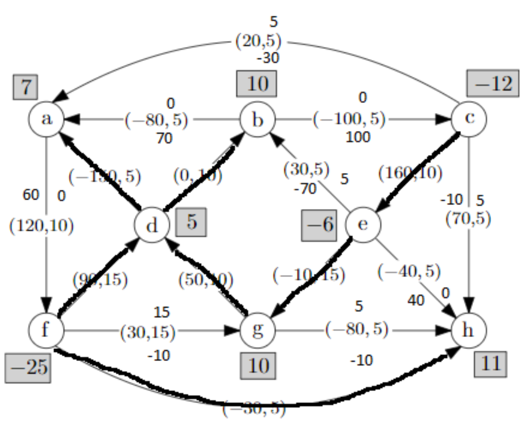
potentials:
y_a = 50
y_f = 110
y_d = 200
y_b = 190
y_e = 160
y_c = 0
y_g = 150
y_h = 80

db violates optimality condition since it has reduced cost > 0 and has flow at capacity

we orient db as the backwards arc in the cycle

min{10, 6, 6, 5}

leaving arc is eb

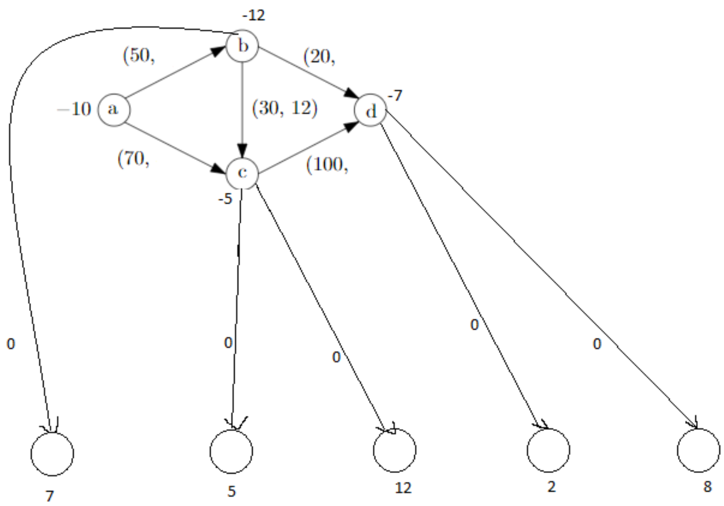


potentials:
y_a = 50
y_f = 110
y_d = 200
y_b = 200
y_e = 160
y_c = 0
y_g = 150
y_h = 80

This is our optimal solution as we see that there are no available entering arcs since each fulfills their optimality condition

2. MCFP as TP

(a) we see that using the example from the question $c_{av} = x_{av} + s_{av}$ and we see that $x_{av} - xvc = b_v$ so $b_v - c_{av} = x_{av} - x_{vc} - x_{av} - s_{av} = -x_{vc} - s_{av}$. We are essentially removing the capacity constraints by subtracting them from our original demands so the slack variables. Since the capacities c_{uv} are made up with the positive flow and the slack variable. So as we remove the lower bound from our supply we are really just subtracting the positive flow and the positive slack variables. Because of this we have positive x_{uv} and positive s_{uv} representing capacities and negative x_{uv} and negative s_{uv} representing the new supply.



(b)

3. Hall's Theorem

(<=) Let every $S \subseteq A$, $|N(S)| \geq |S|$ We model our graph after a MCFP. Where every $v \in A$ is a supply node and every $u \in B$ is a demand node. We set some capacity to each of the arcs going from A to B such that that $c_{uv} > 0$ Since $|N(S)| \geq |S|$ we see that $|\delta(S)| \geq |S|$ so each node in A has a least one out going arc that supplies a node in B. Because of this we set the supply of all $u \in A$ to be -1 and an equal number of nodes in B to have demand 1 any extra nodes will have demand 0 and each arc $uv \in E$ will have a capacity of 1 so MCFP is feasible i.e $b(S) \leq c(\delta(S))$ for all $S \subseteq N$. Since MCFP is feasible each $u \in A$ has an arc with an outflow > 0 and each node in B with a demand of -1 requires an incoming arc to satisfy the demand. In order to have a feasible flow, each A must only send out flow through one arc and each B must only receive flow from one arc so we know that $\delta(u)$ for all $u \in A$ only has one arc with a non-zero flow value. Each $v \in B$ must also only recieve flow from one arc. This is so the $x(\delta(\bar{v}))) - x(\delta(v))) = b_v$ holds for all nodes. So we pick M to only contain arcs with a flow greater than 0. Since each A has only one positive out flow arc and each B only has one positive in flow arc. We can say that M is a matching of G and has arcs adjacent to every $u \in A$.

(=>) Suppose there exists a matching M of G such that every node in A is adjacent to an arc in M. Let $S \subseteq A$ We see that $|N(S)|$ is greater than the cardinality of the set where $u \in B$ such that $uv \in M$ which is equal to $v \in S$ and v is adjancent to an edge in M. Which by our hypothese is just $|S|$ so $|N(S)| \geq |S|$