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In this article we discuss a few models developed to explain the general public's inflation expectations formation and provide some relevant estimation results. Furthermore, we suggest a simple Bayesian learning model which could explain the expectations formation process on the individual level. When the model is aggregated to the population level it could explain not only the mean values, but also the variance of the public's inflation expectations. The estimation results of the mean and variance equations seem to be consistent with the results of the questionnaire studies in which the respondents were asked to report their thoughts and opinions about inflation.

I. Introduction

The inflation expectations of the general public are an important determinant of inflation and other macroeconomics fundaments, since they at least influence the process of wage bargaining, price setting and speculative buying. For example, higher inflation expectations may lead employees to demand higher wage settlements, push firms to raise the prices of their products, and encourage agents to purchase more commodities. In addition, public concern about actual inflation has even certainly had an impact on political elections – see Cartwright and Delorme (1985), Parker (1986), Golden and Poterba (1980), Cuzan and Bundrick (1992), Fair (1994) and Shiller (1997). Thus, the inflation expectations of the general public play an essential role in modern market economies.

The assumption of rational expectations, which presumes that the agents know the true structure and probability distribution of the economy, is most commonly used in theoretical and empirical exercises today. However, having observed problems with this assumption,[1] researchers have started to search alternative models for the expectations formation process. For example, in the models of limited information flows, developed by Mankiw and Reis (2002) and Sims (2003), the agents have rational expectations but are not based on complete information, while in the boundedly rational learning models they behave as professional scientists and use methods of scientific inference (see Sargent, 1993; Evans and Honkapohja, 2001 for surveys). The reader should note that when these models are used, it is important to distinguish between the expectations

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[1] See for example Zarnowitz (1985), Bonham and Cohen (1995), Jeong and Maddala (1996) and Lloyd (1999).

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of ordinary people and professionals, because these two groups use different methods and resources to form their expectations (for example, see results of Schiller, 1997).

In recent empirical literature of inflation expectations formation, Branch (2004a) developed a promising model of heterogeneous agents, in which the general public form their inflation expectations using a prediction function from a set of costly alternatives. Specifically, he assumes that consumers use three alternative types of forecast functions in their formation process: vector autoregression (VAR), adaptive and naive type models. His relatively contradictory results have led scientists to think more closely about the process of consumers' inflation expectations formation. We, for example, find the assumption that households have access to VAR estimates to be unrealistic.[2] This is because the ordinary person cannot perceive the causes of inflation. Shiller (1997) in his questionnaire study, asks the respondents to list causes of inflation. The responses to this question were diverse and almost equally represented. Most assumed 'factors' of inflation were of a general type, such as 'greedy' or 'government'. Thus, identification of any more or less complex econometric or economic models seems to be an overwhelming task for ordinary people. In addition, we of course agree with Branch (2004a) and many others in that the agents are heterogeneous. However, we believe that heterogeneity is mainly concerned with the thought process of individuals and is therefore hardly identifiable. Of more importance is that, it is unclear how important this heterogeneity is in the evolution of aggregate consumer expectations.

Carroll (2003) explores the causality of the Michigan households' mean inflation expectations and the survey of professional forecasters' (SPF) mean inflation forecasts. Using the standard Granger causality test he finds that the professional forecast Granger-causes the household forecast, but that there is no Granger causality in the opposite direction. This evidence of Granger causality plays an essential role in his theory of epidemiological expectations formation. In his epidemiology model, households form their expectations when they randomly come into contact with the relevant information set' which Carroll assumes to consist of news articles about professional forecasters' forecasts. This epidemiology model is closely linked to the sticky information model of Mankiw and Reis (2002), for which Khan and Zhu (2006) and Mankiw *et al.* (2003) acquire empirical estimates. All these authors employ different identification schemes, and estimate that individuals update their information sets on average every 12 months. If this is the case, a large proportion of the population always use lagged news-media forecasts as their information set. Consequently, the inflation expectations of the general public should be modelled as a function of lagged professional expectations. Finally, Branch (2004b) compares these 'sticky information' models to the model uncertainty approach of Branch (2004a) and states that model uncertainly is a more robust element of the Michigan data.

In this article, we show that the Michigan inflation expectations data support neither Carroll's epidemiological model nor the sticky information model of Mankiw and Reis (2002). Moreover, we suggest a simple Bayesian learning model for the formation process of inflation expectations and show that by using this type of a boundedly rational model we can describe individuals' opinions and their uncertainty about them. When the model is aggregated from the individual to national level and certain assumptions are made it can explain not only the mean values, but also the variance of the public's inflation expectations. The estimation results of the mean and variance equations seem to be consistent with the results of the questionnaire studies in which the respondents were asked to report their thoughts and opinions about inflation.

In the inflation expectations literature, there has been almost no work testing learning models using actual empirical data. An important exception is Caskey (1985), who in his excellent paper uses a learning model, similar to ours, for professional forecasters' views about future inflation. However, the formation of inflation expectations requires these professionals to assimilate media reports, personal observations, macroeconomic data, and other forms of information that might be generated in obscure ways. Therefore, the use of an econometric model or some simple alternative behaviour model to explain their expectation formation might be problematic (Manski, 2004). However, the expectation formation of the general public is likely to be more straightforward. A typical individual observes inflation through news media reports, which are mainly based on annualized monthly inflation figures, and perceives the process of inflation on a very superficial level.

[2] One may assume that the VAR forecasts are almost same as the forecasts of professionals made available to the public through news articles, but they cannot be directly compared since there is no cost to read those news articles.

Thus, modelling inflation expectations using a simple, well-defined random process might be ideal in this case.

Our report is organized as follows. In Section II, we explore the empirical relationship between the professionals' and consumers' forecasts. In Section III, we discuss the formation process of the general public's inflation expectations and present a Bayesian learning model. In Section IV, we test how well the outcomes of this model can explain the mean and variance of inflation expectations. Finally, in Section V, we conclude the article.

II. Exploring the Relationship between the Professionals' and Households' Forecasts and Monthly Inflation

The most commonly published economic news articles for the general public likely concern the annualized monthly inflation figures

$$\Pi_t^m = 1200 \times \ln\left(\frac{\mathrm{CPI}_t}{\mathrm{CPI}_{t-1}}\right) \tag{1}$$

where CPI is the seasonally adjusted consumer price index for all urban consumers. However, every month, the Survey Research Center at the University of Michigan asks a random sample of at least 500 households the following question: 'During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?'. If a respondent expects that the prices will change during the next 12 months then he is simply asked to supply a 12 months ahead forecast for annual inflation

$$\Pi_t = 100 \times \ln\left(\frac{\mathrm{CPI}_t}{\mathrm{CPI}_{t-12}}\right) \tag{2}$$

This provides us with a well-defined absolute numerical scale for responses; hence, the respondents understand what the survey questions mean and interpret them similarly. Thus, modelling the Michigan households' responses is sensible; see Manski (2004) for further discussion on the topic.

We use the quarterly means of the above series, since the only relevant candidate series for the views of professional forecasters which has the same forecasting horizon as the Michigan series is the four-quarter inflation forecast from the SPF conducted by the federal reserve bank of Philadelphia.[3] Moreover, here we use only the so-called real-time series, i.e., series which were available to the public when they formed their beliefs about future inflation. Our main source of data is the federal reserve bank of Philadelphia[4] (Croushore and Stark, 2001). The missing values of the CPI data were acquired from Norman R. Swanson's home pages.[5]

To explore the relationship between the professionals' and households' forecasts and monthly inflation, we start our analysis by estimating Carroll's (2003) Equation 12 with constant term and recently published annualized monthly inflation Π_t^m (not annual inflation as in Carroll),

$$\begin{aligned} M_t[\Pi_{t+4}] = \gamma_0 + \gamma_1 S_t[\Pi_{t+4}] + \gamma_2 M_{t-1}[\Pi_{t+3}] \\ + \gamma_3 \Pi_t^m + \nu_t \end{aligned} \tag{3}$$

where M_t and S_t are operators that yield the population means (or medians) of the Michigan and SPF inflation expectations at time t, respectively, and ν_t is an error term. The estimates obtained using the mean and median series are presented in Table 1.

The estimates of γ_3 in the mean and median cases are positive and significant at the 5 and 1% levels, respectively. This suggests that annualized monthly inflation, which is the most commonly reported inflation figure in the news media, is an important factor to explain the Michigan mean and median series. Our estimates are in disagreement with Carroll's (2003) finding that inflation has no influence on an individual's expectation formation process. In our opinion, his finding arises, first, from the high correlation (0.865) between recent annual inflation and the lagged value of the Michigan series, and second, from using the annual inflation series instead of the monthly annualized inflation series.

Surprisingly, the estimate of γ_2 is statistically significant in the median case. Therefore, we could possibly exclude the lagged value of the Michigan series from the regression Equation 3. This empirical finding causes a real problem to Carroll's epidemiology model. Moreover, the reader should note that as Curtin (1996) and Mankiw *et al.* (2003) argue, the long tails of the Michigan expectation series are not particularly informative and, therefore, the results acquired using median values might be more sensible. Therefore, in the following section we will focus on the median values of the Michigan series.

Carroll (2003) argues that the constant term in Equation 3 is spuriously significant because it implies, for example, that if both actual inflation and the

[3] Data are available at http://www.phil.frb.org/econ
[4] Data are available at http://www.phil.frb.org/econ/forecast/readow.html
[5] Data are available at http://econweb.rutgers.edu/nswanson/realtime.htm

Table 1. Estimation results for Equations 3, 5 and 6

Equation	γ_0		γ_1		γ_2		γ_3		R^2
Equation 3									
with mean series	1.176**	(0.181)	0.510**	(0.080)	0.233*	(0.090)	0.042*	(0.019)	0.849
with median series	1.324**	(0.145)	0.386**	(0.059)	0.074	(0.112)	0.079**	(0.020)	0.794
Model $M_t[\Pi_{t+4}] = \gamma_0 + \gamma_1 S_t[\Pi_{t+4}] + \gamma_2 M_{t-1}[\Pi_{t+3}] + \gamma_3 \Pi_t^m + v_t$									
Equation 5	β_1		β_2						
	0.149	95.2%	0.349	100%					
Model(VECM) $\Delta y_t = \psi + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \alpha\beta' y_{t-1} + \varepsilon_t$ where $y_t = (M_t[\Pi_{t+4}]\ S_t[\Pi_{t+4}]\ \Pi_t{}^m)$ and the cointegrating relation is $M_t[\Pi_{t+4}] = \beta_1 S_t[\Pi_{t+4}] + \beta_2 \Pi_t^m + z_t$									
Equation 6	$\Sigma\gamma_{21}{}^{(i)}$		β_1		α_1		α_2		
	−0.500	84.0%	0.418	96.9%	0.487	98.7%	0.223	95.5%	–
Model(VECM) $\Delta y_t = \psi + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \alpha\beta' y_{t-1} + \varepsilon_t$ where $y_t = (M_t[\Pi_{t+4}]\ S_t[\Pi_{t+4}])$ and the cointegrating relation is $M_t[\Pi_{t+4}] = \beta_1 S_t[\Pi_{t+4}] + z_t$									

Notes: Newey–West SEs in parentheses. The results are not sensitive to the choice of lags (4 lags are used).
[a]The signs ** and * denote statistical significance at the 1 and 5% levels, respectively.
[b]In the Bayesian analysis part we report the posterior median and the share of posterior mass which lies above (below) zero when the median is positive (negative).
Sample, 1981/$Q$3–2004/$Q$1

professionals' expectations were to go to zero forever, people would continue to expect a positive inflation rate forever, i.e. they wouldn't eventually learn. However, this is only true when we expect that individuals form their expectations as Carroll assumes. For example, if individuals form their expectations according learning models, the presence of a positive constant in regression poses no problem.

Unfortunately, the results presented above are not very reliable, because using nonstationary time series in regression analysis may yield spurious regression and inconsistent parameter estimates (Hamilton, 1994). On the other hand, if the series are cointegrated, the ordinary least squares (OLS) estimates might be biased due to endogeneity and serial autocorrelation (Banerjee *et al.*, 1993, Chapter 7). Therefore, we studied whether the series are unit root processes. The results of the augmented Dickey–Fuller test are shown in Table 2. Since the null hypothesis of a unit root is not rejected in either case of an expectation series, and only slightly rejected in the case of the annualized inflation series, we model them as $I(1)$ processes.

If the expectations series move together in the long run, which seem to be a reasonable assumption, they can be modelled using cointegrated vector autoregression (CVAR). Let therefore, $y_t = (M_t[\Pi_{t+4}]\ S_t[\Pi_{t+4}]\ \Pi_t{}^m)$ be the vector of the SPF median inflation forecast, the Michigan household median inflation expectation and annualized monthly inflation, respectively. Then the CVAR model can be parameterized in the error-correction form

$$\Delta y_t = \psi + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \alpha\beta' y_{t-1} + \varepsilon_t \qquad (4)$$

where ψ is a vector of parameters, α, a matrix/vector of adjustment coefficients, β a cointegrating matrix/vector and Γ_i: s parameter matrices. The error vectors ε_t are assumed to be independent over time and normally distributed with zero mean and covariance matrix Ω.[6]

We will use the Bayesian approach and posterior density simulations to make exact inference on the parameters. To see whether the data confirm the existence of a cointegrating relation between the Michigan, professional and actual inflation series and to find the proper lag length for the model (4) we follow Corander and Villani (2004), and compute approximate fractional marginal likelihoods (FML). The FML results (not reported here) indicate that the proper lag length is 1 and the cointegration rank is 2.

[6] Note that we do not assume the SPF nor inflation series to be exogenous. For example, even if the lagged values of the Michigan series do not help forecast the future SPF values, one should not take this as a sign of noncausality, since professional forecasters should use the information offered by consumer expectations when they form their forecasts. For example, they might expect that the high inflation expectations of the general public cause consumer inflation to rise.

Table 2. Augmented Dickey–Fuller tests

Augmented Dickey–Fuller test for Michigan median inflation expectations series, professional forecasters' inflation forecast series and annualized monthly inflation series

Variable	*t*-adf
Michigan mean series	−1.993
Michigan median series	−1.529
SPF mean series	−0.5551
SPF median series	−0.7668
Annualized monthly inflation (Real time)	−3.201*

Sample, 1981/$Q$3–2004/$Q$1
* indicates statistical significance at the level of 5%.

However, we restrict our model to include only one cointegrating vector and write the long-run relationship in an informative form

$$M_t[\Pi_{t+4}] = \beta_1 S_t[\Pi_{t+4}] + \beta_2 \Pi_t^m + z_t \qquad (5)$$

where z_t is a stationary term; i.e. we use the parameterization $\beta = (-1 \quad \beta_1 \quad \beta_2)$. From the estimates of β_1 and β_2 we can see which of the series $S_t[\Pi_{t+4}]$ or Π_t^m is more closely related to the Michigan series.

In order to generate conditional and marginal posteriors, we use Normal likelihood and an improper prior $p(\Psi, \alpha, \beta, \Gamma_1, \ldots, \Gamma_{p-1}, \Omega) \propto |\Omega|^{-0.5(m+1)}$ in our Bayesian analysis. With this choice of prior, the joint posterior distribution of β_1 and β_2 has a 1-1 poly-*t* density (see Corollary 3.1 in Bauwens and Lubrano, 1996) and we can use the algorithms of Richard and Tompa (1980) to generate random numbers from it.

The estimates of the CVAR model are presented in Table 1. Since more than 4% of the probability mass of β_1 lies below zero, one must be careful when stating that the general public uses professional forecasters' forecasts when they form their expectations. On the other hand, the high CVAR estimate $\beta_2 = 0.349$ confirms our earlier results that annualized monthly inflation is an important factor in the public's inflation expectations formation process. One can also interpret the estimated cointegrating relation, to mean the expectations of the public are more closely connected to annualized monthly inflation figures than the professionals' forecasts, since professionals also use other information in their forecasts such as unemployment rates and lagged inflation estimates. See also the estimates of Equation 6 in Table 1, where annualized inflation is excluded from regression. The link between the Michigan and SPF expectations does not seem strong here either, since the 95% posterior interval of β_1 includes zero.

Khan and Zhu (2002), Carroll (2003) and Mankiw *et al.* (2003) acquire empirical estimates which indicate that in sticky information models, individuals update their information sets on average every 12 months. If this is the case, the resulting median forecast of the Michigan survey should be closely related to the median of geometrically weighted averages of past professional inflation forecasts and the cross coefficients of the lagged SPF series and their sum $\Sigma \gamma_{21}^{(i)}$ should be different from zero. Therefore, we estimated the CVAR model of Equation 4 with data $y_t = (M_t[\Pi_{t+4}] \; S_t[\Pi_{t+4}])$ and cointegrating relationship

$$M_t[\Pi_{t+4}] = \beta_1 S_t[\Pi_{t+4}] + z_t \qquad (6)$$

The FML based estimates for rank support this restriction (rank = 1) and the estimated lag length is 1. However, because we are interested in the coefficients of the lagged SPF series and their sum ($\Sigma \ \gamma_{21}^{(i)}$) we estimate the CVAR model using four lags.

The estimates of this model are shown in Table 1 [Equation 6]. As the lag length estimate indicates, the posterior density of $\Sigma\gamma_{21}^{(i)}$ has a lot of its probability mass close to zero. Thus, the cumulative effect of the lagged SPF series on the Michigan series is not economically or statistically significant.[7] It seems that the data do not support Carroll's (2003) epidemiology model nor Mankiw's and Reis's (2002) sticky information model.

Finally, Table 3 shows the medians and SDs of the cross coefficients $\gamma_{21}^{(i)}$ $(i = 1, \ldots, p)$ and their sums when the standard Bayesian vector autoregressive (BVAR) model[8] with different lag lengths is used. We can see from the table that the parameters $\gamma_{21}^{(i)}$ are not significantly different from zero except in the model with one lag. However, as Carroll's Granger causality test indicates, the cumulative effect of the SPF forecast on the Michigan series is positive with high probability. Thus, there seems to be a relation between the professionals' and the general public's forecasts, but this relation is probably a kind of

[7] Also, note that none of the parameters $\gamma_{21}^{(i)}$ had a posterior distribution deviating significantly from zero (the results were similar when we use mean series). We also estimated the error-correction model with the assumption that the SPF series is exogenous and the results looked similar.
[8] The model is of the form $y_t = \psi \sum_{i=1}^{p} \Gamma_i y_{t-i} + \varepsilon_t$, where y_t is a vector of m variables, ψ is a vector of parameters Γ_i:s are parameter matrices and ε_t is the Normally distributed error vector with zero mean and Σ covariance. We use standard noninformative prior distribution $p_t(\psi, \Gamma_1, \ldots, \Gamma_p, \Sigma) \infty |\Sigma|^{-0.5(m+1)}$

Table 3. Point estimates of BVAR models

Number of lags	Parameter (lag)	median	
Model $y_t = \psi \sum_{i=1}^{p} \Gamma_i y_{t-i} + \varepsilon_t$ with different lag lengths ($p = 1, \ldots, 4$) where $y_t = (M_t[\pi_{t+4}]\ S_t[\pi_{t+4}])$			
1	${\gamma_{21}}^{(1)}$	0.31	(0.07)
2	${\gamma_{21}}^{(1)}$	0.29	(0.23)
	${\gamma_{21}}^{(2)}$	−0.03	(0.21)
	sum $\gamma_{21}(i)$	0.25	(0.09)
3	${\gamma_{21}}^{(1)}$	0.31	(0.24)
	${\gamma_{21}}^{(2)}$	0.06	(0.33)
	${\gamma_{21}}^{(3)}$	−0.11	(0.21)
	sum $\gamma_{21}(i)$	0.27	(0.10)
4	${\gamma_{21}}^{(1)}$	0.27	(0.25)
	${\gamma_{21}}^{(2)}$	0.12	(0.35)
	${\gamma_{21}}^{(3)}$	−0.11	(0.34)
	${\gamma_{21}}^{(4)}$	−0.03	(0.23)
	sum $\gamma_{21}(i)$	0.25	(0.10)

SDs in parentheses.

long-run comovement in which the general public adjust their expectations as the significant estimate for the adjustment parameter $\alpha_1 = 0.487$ indicates (Equation 6 in Table 1). However, households do not necessarily adjust their expectations toward the forecasts of professionals. Rather, both professionals and the general public adjust their expectations toward actual annual inflation, i.e. both groups correct their expectations toward the fully rational outcome.

In summary, we do not find that the data support Carroll's (2003) epidemiology model or Mankiw's and Reis's (2002) sticky information model. Therefore, there is a need for some alternative theory for the general public's inflation expectations.

III. Modelling the Consumers' Formation of Inflation Expectations

In our candidate theory, which offers an alternative explanation for the general public's inflation expectations formation process, we assume that each individual has his personal beliefs about the process of inflation. He updates this prior knowledge regularly using annualized monthly inflation figures and possibly some other variables offered by the news media and uses the updated information to form his expectations. We find several reasons why inflation expectations of the general public should be modelled using a well-defined random process with personal probabilities about the parameter values of the process. First, individuals cannot observe inflation directly but through the news media, since in everyday life they observe the prices not the inflation. Second, it is well known that typical individuals' views about the inflation process are relatively poor. They may not have any clear idea about the causes of inflation. It is very descriptive that the most common answer was 'greed' or 'greedy', when Shiller (1997), in his questionnaire study, asked people to list the causes of inflation. Third, we, unlike Mankiw and Reis (2002) or Carroll (2003), expect that most people follow economic news daily or at least regularly. We agree with Shiller (1997) who argues: 'Because the word "inflation" is so much a part of everyday lives, it has many associations and connotations to ordinary people. Moreover, because shopping, and thereby noticing prices, is an everyday activity for ordinary people, thinking about prices is also a major part of people's thinking, and the subject "inflation" is one of great personal interest for most people'. Furthermore, when Shiller (1997) asked people if they find news stories about inflation interesting, 89% reported that inflation news reports are very or somewhat interesting (see more discussion on individuals' expectation behaviour: Kahneman and Tversky, 1979; Nisbett and Ross, 1980; El-Gamal and Grether, 1995; Gleitman, 1996; Shiller, 1997; Akerlof *et al.*, 2000; Manski, 2004). Therefore, we assume that a typical individual believes that the process of inflation is captured by the model

$$\Pi_t = X_{t-4}B + \varepsilon_t \tag{7}$$

where ε_t is a Normally distributed error term with zero mean and σ^2 variance and X_{t-4} is a row vector which includes annualized monthly inflation and other possible explanatory variables. We believe that a simple univariate process describes the public's views about the process of inflation better than a multivariate process, since ordinary people seem to fail to understand the concept of general equilibrium (Shiller, 1997).

However, the opinions about the parameter values of the above process may vary strongly among individuals. To allow for this kind of disagreement, we further assume that every individual has his personal beliefs about the parameters of the model (7). Moreover, we assume that the personal prior distribution of the ith individual for the parameter vector B^i is multivariate normal:

$$B^i \sim N\left(B_0^i, \sigma^2 \left(V_0^{-1}\right)^i\right) \tag{8}$$

where B_0 is the individual's prior estimate of B and matrix $(V_0^{-1})^i$ is a measure of his uncertainty, relative to σ^2. If, for example, the ith individual believed that inflation follows a random walk process, the parameter of recent inflation in B_0 would be $= 1$ and the corresponding diagonal entry in V_0^{-1} very small. The parameters of other variables in B_0 would be zero and the corresponding variances in V_0^{-1} very small. Thus we have a very flexible behavioural model which covers the observed disagreement about the causes of inflation and explains the variance of inflation expectations among households (Shiller, 1997; Mankiw *et al*., 2003).

We further assume that the ith individual forecasts inflation on the basis of his evolving beliefs about B (Caskey, 1985). Each period he obtains new information on annualized monthly inflation and other possible explanatory variables from news articles he uses it to update his beliefs about the parameter vector B. This updating process can be best described using the following recursive equations of the Kalman filter:

$$\hat{B}_t^i = \hat{B}_{t-1}^i + K_t^i\left(\Pi_t - X_{t-4}\hat{B}_{t-1}^i\right) \tag{9}$$

$$\left(\hat{V}_t^{-1}\right)^i = \left(\hat{V}_{t-1}^{-1}\right)^i - K_t^i X_{t-4}\left(\hat{V}_{t-1}^{-1}\right)^i \tag{10}$$

$$K_t^i = \left(\hat{V}_{t-1}^{-1}\right)^i X'_{t-4}\left(X_{t-4}\left(\hat{V}_{t-1}^{-1}\right)^i X'_{t-4} + 1\right)^{-1} \tag{11}$$

where vector K_t is the so-called Kalman gain. We do not expect that individuals fully remember the observations of possible explanatory variables, presented in past news articles; however, our point is that this kind of a recursive system might be the best description of their learning process. The model also covers the possibility that some people have such strong prior beliefs that they do not change them.

Finally, assuming that the ith individual uses a quadratic loss function in his forecasts and taking into consideration that his thought processes cannot be fully replicated, we model his inflation expectations as

$$E_t^i(\Pi_{t+4}) = X_t\hat{B}_t^i + u_t^i \tag{12}$$

where the operator E_t^j denotes the ith individual's expectation at time t and u_t a Normally distributed error with zero mean and σ_u^2 variance. Unfortunately, because of the nature of consumer surveys, such as the Michigan survey data, we cannot test the Bayesian learning model directly. The reason is that consumer surveys use random samples of respondents and the respondents are different every month. Therefore, it is not possible to track individual learning processes using the data. However, the Bayesian learning model has a couple of consequences which we can test empirically. First, the model implies that the mean of the expectations can be calculated as

$$m\left\{E_t^i[\Pi_{t+4}]\right\} = X_t m\left\{\hat{B}_t^i\right\} + m\left\{u_t^i\right\} \tag{13}$$

where $m\{\}$ denotes the mean over all individuals in the population. Second, the variance of expectations is given by

$$\operatorname{var}\left(E_t^i[\Pi_{t+4}]\right) = X_t\operatorname{cov}\left(\hat{B}_t^i\right)X'_t + \operatorname{var}\left(u_t^i\right) \tag{14}$$

where $\operatorname{cov}(\hat{B}_t^i)$ is the covariance matrix measuring the dispersion of opinions among the general public.

Equation 13 implies that the population variance of inflation expectations is a function of annualized monthly inflation and possibly some other variables reported in the news media and of the variances and covariances of the individuals' parameter estimates.

IV. Empirical Test of the Bayesian Learning Model

The respondents of the monthly Michigan surveys have learning paths with different lengths and starting points. Therefore, it seems reasonable to assume that the mean vector $m\{\hat{B}_t^i\}$ and dispersion matrix $\operatorname{cov}(\hat{B}_t^i)$ in Equations 13 and 14 stay constant over time. Then we can estimate them using OLS. However, the reader should note that we can not trust the standard t-statistics related to these parameters, since the time series involved in these models are unit root or nearly unit-root processes. However, our main objective is to investigate how well the outcomes of the Bayesian learning model can predict the mean and variance series of the actual Michigan inflation expectations. We suggest that the key things here are the correct signs of the variance estimates and the prediction power of the estimated models. To proceed further, we set[9]

$$\operatorname{var}(M_t[\Pi_{t+4}]) = \operatorname{var}\left(E_t^i[\Pi_{t+4}]\right) + error = X'_t\operatorname{cov}\left(\hat{B}_t^i\right)X_t + \operatorname{var}\left(u_t^i\right) + error \tag{15}$$

[9] To analyse the predicted variance of the learning model we need a longer sample period than the period, 1981/3 to 2004/1, for which the professionals' CPI inflation forecast series is available. To obtain a longer forecast period, 1970/1 to 2004/1, we used SPF's GDP deflator forecast series. We regressed the CPI inflation forecast series on the GDP deflator inflation forecast series and a constant and predicted the CPI inflation forecast series for the period 1970/1 to 1981/2 using the estimated regression model. The parameter estimates were 0.68 and 0.89 for the constant and the GDP deflator forecast series, respectively).

$$M_t[\Pi_{t+4}] = m\{E_t^i[\Pi_{t+4}]\} + error = X_t m\{\hat{B}_t^i\} + m\{u_t^i\} + error \quad (16)$$

Moreover, we assume that the most probable components of X_t are annualized monthly inflation $\Pi_t^{\prime m}$ and the 3-month T-bill rate, since most commonly published economic news articles on the subject concern these figures and since the public seem to perceive the relation between inflation and the interest rate (Shiller, 1997). Our alternative candidate series, which may compensate for the T-bill series, are those which are most probably regularly reported by all major media in the United States:

(a) SPF forecast
(b) Unemployment rate
(c) Annualized quarterly growth of gross domestic product (GDP)
(d) Annualized monthly growth of M1

The results for the variance and mean equations are summarized in Table 4. We can see that the variance model based on inflation and the 3-month T-bill rate and the variance model based on inflation alone give similar correlations between the predicted variance series and the actual Michigan variance. However, the actual mean series has a remarkably higher correlation with the predicted mean series, based on inflation and the T-bill rate than with the predicted series based on inflation alone. These results suggest that a part of the population always believes the process of inflation to be random walk, which causes the variance of the Michigan survey to be high, while another part of them is aware of the relationship between inflation and the interest rate (Shiller, 1997).

The model based on monthly inflation and the 3-month T-bill rate gives relatively high correlation coefficients between the predicted and actual Michigan mean series, compared to the models based on inflation and unemployment, inflation and the quarterly growth rate of GDP or inflation and the monthly growth rate of M1. The drop of the correlation coefficient from the mean model based on inflation and the T-bill rate to the mean model based on inflation and the unemployment rate is quite large, about 11%. Moreover, the correlation between the predicted mean series based on annualized monthly inflation and the actual Michigan mean series is 0.645, which suggests that the growth of money, growth of GDP or unemployment series add practically nothing to the learning model expectations based on monthly inflation (compare correlations reported in Table 4). These findings are in line with Shiller's (1997) questionnaire study, in which he observes that most people fail to think of economic models; people do not tend to see any connection between inflation and unemployment, i.e. the Phillips curve, or between inflation and money growth, i.e. the quantity theory of money.

The variance model based on inflation and unemployment, and the model based on inflation and the SPF series give us the highest correlations between the predicted and actual variance series (0.88 and 0.9). However, if we take a closer look at the parameter estimates of these models, we find that in both these models the estimates of $\mathrm{var}(u) + \mathrm{var}(b_0)$ are negative, which of course does not make sense. One plausible explanation for the high correlation between the professionals' forecast series and the Michigan variance series ($\mathrm{cov}(b_2, b_0) = 14.7$) is that there is a higher probability for individuals who believe inflation to be random walk to give lousy forecasts when inflation is rationally expected to increase rapidly than during a stable low inflation period.

In summary, based on the weak relation between the SPF and Michigan median series, as found when estimating Equations 5 and 6 in Section II, and the results presented in the current section, we conclude that a Bayesian learning model based on annualized monthly inflation and the 3-month T-bill rate offers us a more plausible explanation for the general public's inflation expectations than the limited information flows models based on rational or nearly rational forecast series.

V. Conclusion

In this article, we have empirically showed that a simple Bayesian learning model is a feasible explanation for the general public's inflation expectations formation. We have also showed that the Michigan Survey data do not support models of limited information flows, discussed the theoretical basis of the Bayesian model and come to the conclusion that it gives a more realistic picture of individuals' expectation behaviour in the case of inflation than some new models presented in the literature, such as models of limited information flows or models with uncertainty approach. Learning models have, of course, many variants, but the advantage of Bayesian models is that, by using

Table 4. Estimation results for variance and mean equations

Sample, 1970/Q1–2004/Q1
Variance Model $\mathrm{var}(M_t[\Pi_{t+4}]) = X_t' \mathrm{cov}(\hat{B}_t^i) X_t + \mathrm{var}(u_t^i) + \mathrm{error}$

Variables	var(u) + var(b_0)	var(b_1)	cov(b_1,b$_0$)	var(b_2)	cov(b_2,b$_0$)	cov(b_1, b$_2$)	Correlation between predicted and actual variance series
Monthly inflation	21.617 (1.698)	0.591 (0.082)	−0.272 (0.110)	– –	– 0.842		
Monthly inflation and T-bill	3.178 (5.034)	0.292 (0.110)	3.836 (1.318)	0.116 (0.136)	3.278 (1.473)	−0.406 (0.214)	0.842
Monthly inflation and SPF forecasts	−20.636 (6.530)	0.573 (0.148)	1.994 (1.380)	0.216 (0.534)	14.679 (3.504)	−1.351 (0.493)	0.908
Monthly inflation and unemployment	−69.320 (19.495)	0.131 (0.072)	12.172 (1.999)	−0.498 (0.382)	17.485 (5.497)	−1.302 (0.267)	0.875
Monthly inflation and growth of GDP	24.828 (3.956)	0.162 (0.091)	3.281 (1.290)	0.028 (0.065)	−2.669 (0.930)	0.318 (0.123)	0.809
Monthly inflation and growth of M1	14.348 (3.638)	0.143 (0.087)	4.036 (1.235)	−0.161 (0.403)	3.625 (1.875)	0.053 (0.306)	0.818

Sample, 1981/Q3–2004/Q1
Mean model $M_t[\Pi_{t+4}] = X_t m\{\hat{B}_t^i\} + m\{u_t^i\} + \mathrm{error}$

Variables	b_0	b_1	b_2	correlation between predicted and actual mean series
Monthly inflation (b_1)	3.176 (0.142)	0.295 (0.037)	– 0.645	
Monthly inflation (b_1) and T-bill (b_2)	2.341 (0.129)	0.149 (0.023)	0.228 (0.023)	0.852
Monthly inflation and SPF forecasts	1.516 (0.130)	0.054 (0.025)	0.678 (0.044)	0.918
Monthly inflation and unemployment	1.658 (0.293)	0.270 (0.032)	0.257 (0.045)	0.757
Monthly inflation and growth of GDP	3.369 (0.165)	0.291 (0.036)	−0.070 (0.032)	0.668
Monthly inflation and growth of M1	3.101 (0.158)	0.298 (0.037)	0.053 (0.048)	0.651

personal probabilities, one can describe individuals' opinions and their willingness to update them. From our point of view, Bayesian learning models are worth further development in the context of the *public's* inflation expectations.

References

Akerlof, G. A., Dickens, W. T. and Perry, G. L. (2000) Near-rational wage and price setting and the Long-run Phillips curve, Brookings Papers on Economic Activity, **1**, 1–60.

Banerjee, A., Dolado, J. J., Galbraith, J. W. and Hendry, D. F. (1993) *Cointegration, Error Correction, and the Econometric Analysis of Non-Stationary Data*, Oxford University Press.

Bauwens, L. and Lubrano, M. (1996) Identification restrictions and postierior densities in cointegrated Gaussian VAR system, *Advances in Economics*, **11**, Part B, 3–28.

Berk, J. M. (1999) Measuring inflation expectations: a survey data approach, *Applied Economics*, **31**, 1467–80.

Bonham, C. and Cohen, R. H. (1995) Testing the rationality of price forecasts: comment, *American Economic Review*, **85**, 284–9.

Branch, W. A. (2004a) The theory of rationally heterogeneous expectations: evidence from survey data on inflation expectations, *Economic Journal*, **114**, 592–621.

Branch, W. A. (2004b) Sticky Information and Model Uncertainty in Survey Data on Inflation Expectations, Working paper, College of William and Mary.

Carroll, C. D. (2003) Macroeconomic expectations of households and professional forecasters, *Quarterly Journal of Economics*, 269–98.

Cartwright, P. A. and DeLorme Jr, C. D. (1985) The unemployment-inflation voter utility relationship in the business cycle: some evidence, *Southern Economic Journal*, **51**, 898–905.

Caskey, J. (1985) Modeling the formation of price expectations: a bayesian approach, *The American Economic Review*, **75**, 768–76.

Corander, J. and Villani, M. (2004) Bayesian assessment of dimensionality in reduced rank regression, *Statistica Neerlandica*, **58**, 255–70.

Croushore, D. and Stark, T. (2001) A real-time data set for macroeconomists, *Journal of Econometrics*, **105**, 111–30.

Curtin, R. (1996) Procedure to Estimate Price Expectations, mineo, University of Michigan Survey Research Center.

Cuzan, A. G. and Bundrick, C. M. (1992) Selected fiscal and economic effects on presidental elections, *Presidential Studied Quarterly*, **22**, 127–34.

El-Gamal, M. A. and Grether, D. M. (1995) Are people bayesian? Uncovering behavioral strategies, *Journal of the American Statistical Association*, **90**, 1137–45.

Evans, G. W. and Honkapohja, S. (2001) *Learning and Expectations in Macroeconomics*, Princeton University Press, Princeton, New Jersey.

Fair, R. C. (1994) The Effect of Economic Events on Votes for President: 1992 Update, Cowles Foundation Discussion Paper No. 1084.

Gleitman, H. (1996) *Basic Psychology*, Norton, New York.

Golden, D. G. and Poterba, J. M. (1980) The price of popularity: the political business cycle reconsidered, *American Journal of Political Science*, **24**, 696–714.

Hamilton, D. J. (1994) *Time series analysis*, Princeton University Press, Princeton, New Jersey.

Jeong, J. and Maddala, G. S. (1996) Testing the rationality of survey data using the weighted double-bootstrapped method of moments, *Review of Economics and Statistics*, **78**, 296–302.

Kahneman, D. and Tversky, A. (1979) , Prospect theory: an analysis of decision under risk, *Econometrica*, **47**, 263–92.

Khan, H. and Zhu, Z. (2006) Estimates of the Sticky-Information Phillips Curve for the United States, *Journal of Money, Credit and Banking*, **38**, 195–207.

Lloyd Jr., B. T. (1999) Survey measures of expected U.S. inflation, *Journal of Economic Perspective*, **13**, 125–44.

Mankiw, N. G. and Reis, R. (2002) Sticky information versus sticky prices: a proposal to replace the new Keynesian Phillips curve, *Quarterly Journal of Economics*, **117**, 1295–328.

Mankiw, G. N., Reis, R and Wolfers, J. (2003) Disagreement about inflation expectations, *NBER Macroeconomics Annual 2003*.

Manski, C. F. (2004) Measuring expectations, *Econometrica*, **72**, 1329–0000.

Nisbett, R. and Ross, L. (1980) *Human Inference: Strategies and Shortcomings of Social Judgment*, Prentice-Hall, Englewood Cliffs, NJ.

Parker, G. R. (1978) Economic Partisan Advantages in Congressional Contests: 1938-1978, *Public Opinion Quarterly*, **50**, 387–401.

Richard, J. F. and Tompa, H (1980) On the evaluation of poly-t density functions, *Journal of Econometrics*, **12**, 335–51.

Sargent, T. J. (1993) *Bounded Rationality in Macroeconomics*, Oxford University Press, Oxford.

Shiller, R. J. (1997) Why Do People Dislike Inflation, in *Reducing Inflation: Motivation and Strategy* (Eds) C. D. Romer and D. H. Romer, University of Chicago press, Chicago, pp. 13–51.

Sims, C. A. (2003) Implications of rational inattention, *Journal of Monetary Economics*, **50**, 665–90.

Stock, J. H. and Watson, M. W. (1999) Forecasting inflation, *Journal of Monetary Economics*, **44**, 293–335.

Zarnowitz, V. (1985) Rational expectations and macroeconomic forecasts, *Journal of Business & Economic Statistics*, **3**, 293–311.