An Estimated Model of Household Inflation Expectations: Information Frictions and Implications *

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Abstract

This paper proposes and estimates a dynamic model of household inflation expectations. The information flow constraint of the household leads to costly information monitoring. Households use a Bayesian learning model to form and update inflation expectations. The model identifies and corrects for sizable reporting and sampling errors prevalent in household surveys. The estimates show that better-educated households track inflation more closely and report their expectations more accurately. Household inflation expectations are less responsive to changes in the inflation target after the Great Recession. Model-implied household inflation expectations improve the fit of the expectation-augmented Phillips curve. Inattention from households makes it more costly for the Fed to lower inflation than would be the case if everyone is perfectly informed.

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1 Introduction

Household inflation expectations play a central role in monetary macroeconomic models (Woodford, 2003; Galí, 2015). Households make consumption, saving, and labor market decisions based on their perception of future inflation levels. When interest rates are at the zero lower bound, the effectiveness of inflation expectations management as an alternative monetary policy tool is being actively studied (Coibion et al., 2018). While conventional models assume that households form full-information rational expectations, existing work using survey data on inflation expectations provides evidence in support of information frictions (Carroll, 2003; Coibion and Gorodnichenko, 2012, 2015a). I provide and estimate a realistic model of household inflation expectation formation using rational inattention (Sims, 2003). My model allows for different speeds of learning across households and my estimates imply a more sluggish learning process for lower-educated households. When the Fed lowers inflation, household inattention leads to higher unemployment rates, particularly for the lower educated, than what rational expectation models would predict.

I employ this new framework to understand the household expectation updating process and its heterogeneity, and construct new measures of inflation expectations. I first establish a Bayesian learning model of inflation, subject to the household's costly inflation monitoring (Maćkowiak et al., 2018). I solve this household's signal extraction problem and show that its solution is a time-varying inflation updating procedure dependent on the information processing capacity of the household. I then build a novel estimation framework that simultaneously models differences in forecasts across groups as well as survey measurement errors. In my estimation, I treat households' information sets and their beliefs about the inflation process as unobservable. Therefore, my estimation strategy adds on another layer of signal extraction to the problem. By using model estimates, I decompose the discrepancy in inflation forecasts among different groups of forecasters into three sources: information rigidities, reporting errors, and sampling errors. Because I allow for time variation in the inflation process, my estimates show that household inflation expectations are more anchored after the Great Recession.

I begin by modeling households information frictions in the inflation expectation formation process following the rational inattention literature, and extend it to implement a time-varying autoregressive inflation process. Households never fully observe the true underlying state of the inflation process. Instead, they seek to infer about the true state by solving a signal extraction problem each time they receive new information about inflation. I model the signal structure such that the quality of the signal is positively related to the household's information flow capacity. As a result, households with larger information capacity track inflation more closely and update their expectations more rapidly.

Households update their forecasts in a Bayesian manner. Forecasts are a weighted average of the household's prior beliefs and the new signal received. The assignment of weights reflects the degree of information rigidity of the household. Because the inflation process is nonlinear with time-varying parameters, the weight is a time-varying function of the household's belief about the inflation process. Linearizing the model iteratively around the household's posterior, I obtain a closed-form solution to the household problem, which implies a time-varying rule for the household to update inflation expectations.

To estimate the household model, I develop an econometric framework to account for measurement errors in the data from the Michigan Survey of Consumers (MSC). One source of error is the sampling error, which is common in surveys with a limited number of participants. Another source is the reporting error, i.e., the number recorded in the survey does not necessarily represent the respondent's actual expectations. An example of the reporting error is the respondent's inclination to round numbers to multiples of five or ten. Since these measurement errors are unobservable, my econometric framework identifies and estimates them through another signal extraction problem. In this problem, survey-based expectations are noisy measures of households' actual expectations. The unobserved actual expectations are state variables that evolve according to the solution to the household problem. I provide a computationally efficient method to estimate this structural model by linearizing it using the extended Kalman Filter. Based on model estimates, I provide inference about actual household inflation expectations free of measurement errors.

The econometric framework I build is general enough to incorporate many households with heterogeneous inflation expectation updating mechanisms simultaneously. Since my focus is the heterogeneity in households' information capacity, I estimate the Bayesian learning model using average expectations from households with different levels of education. Given that the individual household's information sets are unobservable, this method recovers estimates of the size of information frictions, by exploiting the difference between households' reported inflation expectations and model-implied rational expectation forecasts.

The estimation reveals heterogeneities in the size of information frictions and measurement errors among households with different education levels. Households with a college degree behave as if they monitor inflation very closely, while households without a college degree observe inflation with an error whose 95% confidence band is about one percent wide. Both groups of households report their expectations with sizable persistent biases, with 1.54 percent for the better educated and 2.28 percent for the less educated. Rounding errors (Binder, 2017) can explain about half of this persistent bias. Sizes of idiosyncratic sampling errors are comparable between two groups at around 0.25 percent, which is in line with the model-free average sampling error observed in the survey data.

Differences in household inflation experiences may explain part of the heterogeneities in inflation expectations from the estimated model. Using scanner data, Kaplan and Schulhofer-Wohl (2017) find that households with lower income experience higher inflation. Since my model does not separately identify systematic differences in experienced inflation from a bias in expectation reporting, part of the large persistent bias estimated could be attributed to the fact that households with lower education tend to have lower income and on average experience a persistently higher level of inflation.[1]

I conduct several policy analysis that highlights the importance of correctly measuring household inflation expectations. After the Great Recession, inflation has been persistently higher than what was predicted by the historical Phillips curve. Coibion and Gorodnichenko (2015b) found that using household inflation expectations makes up for the missing disinflation in the Phillips curve. My analysis demonstrates that using model-implied inflation expectations further improves the fit of the expectation-augmented Phillips curve.

[1]However, the findings in Kaplan and Schulhofer-Wohl (2017) could be limited due to their choice of data and the sample period. Hobijn and Lagakos (2005) does not find such persistent differences when using expenditure data from BLS.

Inattention from households has strong implications to the conduct of monetary policy. First, it is more costly in terms of unemployment rate for the Fed to lower the inflation target. I demonstrate this using my estimates from the Phillips curve. This is a consequence of the household's sluggish behavior in updating inflation expectations. Second, household inflation expectations are more anchored after the Great Recession. This arises from the decline in the household's perception of the autoregressive parameter in the inflation process. Consequently, it takes longer for the Fed to manage inflation expectations than it would in the 1980s.

The current study contributes to the literature on deviations from full-information rational expectations. My household learning model is motivated by the rational inattention literature (Sims, 2003; Mackowiak and Wiederholt, 2009; Matějka and McKay, 2015; Hébert and Woodford, 2019). This class of noisy signal models has also gained popularity in the recent empirical literature (see Coibion and Gorodnichenko (2012), Coibion and Gorodnichenko (2015a)). I extend this class of models to a case with time-varying parameters and provide analytical solutions. I build an econometric framework to estimate this model and provide direct evidence on rational inattention.

This paper also contributes to the use of survey-based expectations to study the formation of household inflation expectations. Existing works in the literature study the heterogeneity in the formation process from different perspectives. Malmendier and Nagel (2016) study how household experiences about inflation affects their expectations. D'Acunto et al. (2019) find that average absolute forecast errors for inflation declines monotonically with IQ. By conducting survey experiments in Argentina and U.S., Cavallo et al. (2017) find that individuals in low inflation contexts have significantly weaker priors about the inflation rate. This study focuses on differences in information capacities by using the education level as a proxy. It is the first paper that constructs measures of household inflation expectations that corrects for measurement errors in the survey data.

The third contribution of this study is that I quantify the time-varying cost of changing inflation targeting as a result of household inattention. There is an ongoing debate on what the optimal inflation target is. A few notable studies argued for low rates of inflation even during the zero lower bound (Schmitt-Grohé and Uribe, 2010; Coibion et al., 2012). Adam and Weber (2019) finds that the optimal inflation rate ranges between 1 percent and 3 percent with a downward trend over 1975-2015. Much of the recent policy debate since the Great Recession has been whether the Fed should increase its target (Blanchard et al., 2010; Ball, 2014; Krugman, 2014). Nakamura et al. (2018) show that the welfare cost of inflation implied by the New Keynesian models is not supported by the empirical evidence and its implications need to be reassessed. Using my estimates of the sacrifice ratio (Ball, 1994), I find that household inattention makes it more costly for the Fed to decrease the inflation target in the 1980s. My finding also suggests that it takes longer for the Fed to achieve a new inflation target after the Great Recession, since household inflation expectations are more anchored.

The plan of the paper is as follows. Section 2 discusses survey measures of inflation expectations. Section 3 presents the household Bayesian learning model with costly information monitoring. Section 4 introduces the econometric framework used for estimation, followed by details of structural estimation in Section 5. Applications of estimation results are presented in Section 6. Section 7 concludes this study.

2 Data on inflation expectations

In this section, I discuss measures of household and professional inflation expectations from survey data. I provide evidence on heterogeneity in inflation forecast errors between professionals and various groups of households. Factors that account for such heterogeneity are modeled and discussed in Section 3-5.

2.1 Michigan survey of consumers

I use data from the Michigan survey of consumers (MSC) to construct measures of household inflation expectations. This dataset has been popularly used in the literature to study household expectations (Bhandari et al., 2019; Kamdar, 2018). In this section, I discuss briefly the design of the survey, followed by the construction of measures used in estimation. The MSC questionnaires are designed to track consumer attitudes and expectations. The survey has been conducted by telephone monthly since 1978 and constitutes a sample of over 500 households representative of the US population. It contains demographic information such as each respondent's education level, age, and his/her household income and home value. Two questions in the survey are most relevant for household short-run inflation expectations: 1) "During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?" 2) "By about what percent do you expect prices to go (up/down) on the average, during the next 12 months?" Answers to these questions provide coarse measures of individual household inflation expectations. The MSC has a long time series but does not track individual households over time. Only about 40% of households from the interview six months before are recontacted. In my analysis, I group households based on their education and I use the mean response from each group in every quarter as a measure for a representative household from that group. In this way, I construct a panel of household inflation expectations dating from 1978Q4 to 2018Q4. These aggregate measures suffer from sampling errors due to large variations across individuals within a group as well as changes in individuals sampled from period to period. My econometric framework in Section 4 proposes a method to handle this issue.

2.2 Survey of professional forecasters

I consider measures of professional forecasts as a benchmark for comparison with household inflation expectations. Quarterly forecasts for headline CPI inflation rate are available since 1981Q3 from the Survey of Professional Forecasters (SPF). Forecasts are reported as annualized quarter-over-quarter percent changes of the quarterly average price index level and are available for 6 quarters ahead. I use the four-quarter average as their 1-year inflation forecasts.

2.3 Heterogeneous forecast errors

Individuals differ in reported inflation expectations. Surveys of expectations from consumers and professional forecasters show substantial differences in their inflation forecasts (Mankiw et al., 2003). In particular, consumers disagree about inflation forecasts and exhibit persistent biases in forecast errors. Such heterogeneity could originate from differences in households' updating mechanisms, as well as sampling and measurement methods of the survey. Models built on representative households cannot explain this expectation heterogeneity.

In this section, I provide evidence on heterogeneity in inflation forecast errors between professionals and two groups of households with different education levels. Inflation forecast errors are calculated as the difference between the actual annual inflation rate and the inflation expectations of the respective group reported in the previous quarter. Figure 1 provides a summary of forecast errors for 1) professional forecasters, 2) households with a college degree and 3) households without a college degree.

Professional forecasters on average make much more accurate predictions than households. Households persistently make positive forecast errors over the entire period. Households without a college degree consistently make larger forecast errors compared to households with a college degree. A few potential sources lead to such patterns observed in forecast errors. First, the professional forecasts are reported and recorded more precisely compared to households. Household survey responses suffer from reporting errors that might contribute to the persistent bias. Second, survey sampling errors as discussed in Section 2.1 add noise to measures of household expectations. Third, professional forecasters may have better information on inflation compared to households. Better educated households might be more capable of processing or monitoring information about inflation compared to the less educated. In the following sections, I propose a framework that incorporates these factors and provides estimates to disentangle the contributions from each source of error.

3 A Bayesian learning model of inflation expectations

Households differ in capabilities of monitoring and processing information on inflation. I propose a Bayesian learning model of household inflation expectations that features costly inflation monitoring. In this model, households keep track of some noisy signals of inflation. The quality of signals depends on the household's information capacity. I show that this information structure is a direct result of the rational inattention theory. I extend the baseline model in which agents tracks an AR(1) process to a case with time-varying parameters. Households update their inferences not only about inflation but also about model parameters once they receive new signals. I develop an analytical solution to the household model that describes the households inflation expectation updating procedure.

3.1 Imperfect inflation monitoring

I adapt the dynamic rational-inattention model to allow for a cost of households' inflation monitoring. This section briefly discusses the rational inattention model and its implications on households' information structure. Sims (2003) first proposed to model household's limited attention as a constraint on the household information flow capacity. In this class of models, agents track an optimal action by choosing the information structure subject to an information flow constraint.

In the context of household inflation expectations, the household has to form expectations about future inflation every period given his / her information set, which is denoted as $E(\pi_{t+1}|\mathcal{I}_t)$. I follow the setup in Maćkowiak et al. (2018) to describe the household's dynamic decision making process. The inflation follows a Gaussian process $a(L)\pi_t = \alpha + b(L)\eta_t$. The information set $\mathcal{I}_t$ at any period t includes any initial information and signals received up to the current period

$$\mathcal{I}_t = \mathcal{I}_0 \cup \{S_1, \ldots, S_t\}$$

where $\mathcal{I}_0$ denotes the initial information set and S_t denotes the signal vector received in period t. Information received during this period will also be useful in future periods. The household chooses the composition of signals by choosing the matrices A, B and covariance matrix Σ_ω

$$S_t = A\pi_t + B\eta_t + \omega_t, \quad \omega_t \sim N(0, \Sigma_\omega)$$

The household seeks to minimize the mean squared forecast error:

$$\min_{A,B,\Sigma_\omega} \quad E[(\pi_{t+1} - E(\pi_{t+1}|\mathcal{I}_t))^2]$$

subject to an information flow constraint

$$\lim_{T\to\infty} \frac{1}{T} I(\pi_0, \pi_1, \ldots, \pi_T; S_1, \ldots, S_T) \leq \kappa$$

The information flow to the household is quantified by the reduction in uncertainty, which in turn is measured by reduction in entropy, as defined in the Shannon Mutual Information. It measures the difference in the prior uncertainty about inflation and the posterior after observing the signals.

Dynamic rational inattention problems are difficult to solve. Sims (2003) solved the problem using numerical methods. Maćkowiak et al. (2018) provide analytical solutions to the case where the optimal action is Gaussian. In particular, if inflation follows an AR(1) process

$$\pi_t = \alpha + \phi \pi_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2)$$

The optimal signal is simply inflation plus some noise

$$S_t = \pi_t + \omega_t, \quad \omega_t \sim N(0, \sigma_\omega^2)$$

and the size of the noise σ_ω is inversely related to the size of the information capacity κ. Intuitively, the larger the information capacity, the closer the household tracks inflation.

Maćkowiak et al. (2018) established the inverse relationship between the size of noise in signals and the household's information capacity for this example. In the following sections, I take the signal structure as given in a more general setting and focus on the formation of a household's inflation expectation.

3.2 The household problem

This section presents the Bayesian learning model of households' inflation expectations. This model is based on the signal extraction problem in the rational inattention literature. I extend this class of models by allowing time-varying parameters in the inflation process. This extension enables households to learn about the inflation parameters while tracking the inflation process.

I first characterize the law of motion for inflation. Inflation follows an AR(1) process with time-varying parameters:

$$\pi_t = \alpha_{t-1} + \tilde{\phi}_{t-1} \pi_{t-1} + \eta_t \tag{1}$$

$$\alpha_t = \alpha_{t-1} + \nu_{\alpha,t} \tag{2}$$

$$\phi_t = \phi_{t-1} + \nu_{\phi,t} \tag{3}$$

where $\tilde{\phi}_t = [1 + 0.1\exp(-\phi_t)]^{-1}$ is a monotonic transformation of ϕ_t with a range of $(0,1)$. π_t is the actual inflation rate, α_t and $\tilde{\phi}_t$ are the drift and autoregressive parameters in the AR(1) process. α_t and ϕ_t each follow a random walk. The idiosyncratic error terms are normally distributed as

$$v_t = \begin{bmatrix} \eta_t \\ \nu_{\alpha,t} \\ \nu_{\phi,t} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ 0 & \sigma_\alpha^2 & 0 \\ 0 & 0 & \sigma_\phi^2 \end{bmatrix} \right)$$

I employ a time-varying parameter model of inflation for two reasons. First, with a constant parameter model, the learning process implies that agents will eventually infer the parameters perfectly. The older cohorts would make better forecasts about inflation given their superior knowledge about model parameters, which is not consistent with the observations from the survey data. Second, time-varying drift and autoregressive parameters capture some essential features of the inflation process for the period of interest. A Wald test for a structural break in an AR(1) model of inflation rejects the null of no break at 1% significance level, with an estimated break date of 2008Q4. As inflation becomes less volatile in recent decades compared to the early 1980s, inflation persistence has declined over time. The time-varying AR(1) model accommodates such changes in dynamics and allows households' perception of model parameters to adapt accordingly.

Households do not monitor inflation perfectly. In each period, household i observes a noisy signal of inflation

$$s_{it} = \pi_t + \omega_{it}, \quad \omega_{it} \sim N(0, \sigma_{i,\omega}^2) \tag{4}$$

and updates his / her information set as $\mathcal{I}_{i,t} = \mathcal{I}_{i,0} \cup \{s_{i1}, \ldots, s_{it}\}$. The household's choice of signal structure is motivated by the rational inattention literature as discussed in Section 3.1. The size of noise, $\sigma_{i,w}$, is a direct measure of households information capacity. Households with larger information capacities obtain signals on inflation with better accuracy (i.e. with smaller $\sigma_{i,w}$). [2]

The household model can be summarized using a state-space representation

$$s_{it} = h\xi_t + \omega_{it} \tag{5}$$

$$\xi_t = \beta(\xi_{t-1}) + \nu_t \tag{6}$$

where $\xi_t = (\pi_t, \alpha_t, \phi_t)'$ represents the unobserved state variables characterizing the inflation dynamics. Equation (5) is the observation equation for household i, derived from Equation (4) with $h = (1, 0, 0)$; Equation (6) is the state equation with $\beta(\cdot)$ denoting a system of nonlinear functions of the state variable ξ as characterized in Equations (1)-(3), which in matrix form is:

$$\begin{pmatrix} \pi_t \\ \alpha_t \\ \phi_t \end{pmatrix} = \begin{bmatrix} \alpha_{t-1} + \tilde{\phi}_{t-1}\pi_{t-1} \\ \alpha_{t-1} \\ \phi_{t-1} \end{bmatrix} + \begin{pmatrix} \eta_t \\ \nu_{\alpha,t} \\ \nu_{\phi,t} \end{pmatrix}$$

In this model, household i's inflation expectation at period t is $E(\pi_{t+1}|\mathcal{I}_{i,t})$. Given that inflation π is one of the unobserved state variables, the household problem is to solve the state-space model in Equations (5)-(6) to make an optimal inference about the current state ξ_t given the information set $\mathcal{I}_{i,t}$. Due to the non-linearity of the state-space model, I assume households use the extended Kalman filter to update their beliefs at every period. Details of model solutions are presented in the next subsection.

3.3 Solution to the household problem

This section presents the extended Kalman filter to analytically solve the household problem in Section 3.2. The main goal here is to provide a strategy to describe the household's expectation updating procedure.

Let $\xi_{i,t|t-1} \equiv E(\xi_t|\mathcal{I}_{i,t-1})$ denote household i's prior about the state variables at period t before

[2] In this model, the inflation process is no longer standard Gaussian as it allows for time-varying parameters. The optimal signal structure could be different from Equation (4) if I allow for a more general structure. However, to my knowledge, no analytical solution exists for the more general case. Therefore, I consider the restricted case in which the signal structure takes the form as in Equation (4).

observing the signal s_{it}. After receiving s_{it}, the household updates his / her belief of the state ξ_t to the posterior $\xi_{i,t|t} \equiv E(\xi_t|\mathcal{I}_{i,t})$ using the extended Kalman filter

$$\xi_{i,t|t} = \xi_{i,t|t-1} + K_{it}(s_{it} - \pi_{i,t|t-1}) \tag{7}$$

where K_{it} is the Kalman gain. The extended Kalman filter is applied to a local linearization of the state equations at the posterior $\xi_{i,t|t}$. The difference $(s_{it} - \pi_{i,t|t-1})$ measures the new information household i obtains about ξ_t once it receives the new signal s_{it}. The Kalman gain is calculated as the mean-squared error of the new information relative to that of the existing forecast $\xi_{i,t|t-1}$. Other things equal, a household with a more precise signals would use a larger Kalman gain. The Kalman gain and the covariance matrix $P_{i,t|t-1} = E[(\xi_t - \xi_{i,t|t-1})(\xi_t - \xi_{i,t|t-1})']$, which is the mean-squared error of the existing forecast $\xi_{i,t|t-1}$, evolve as

$$K_{it} = P_{i,t|t-1}h'(hP_{i,t|t-1}h' + \sigma^2_{i,w})^{-1} \tag{8}$$

$$P_{i,t+1|t} = B_{it}(P_{i,t|t-1} - P_{i,t|t-1}h'(hP_{i,t|t-1}h' + \sigma^2_{i,w})^{-1}hP_{i,t|t-1})B'_{it} + Q^H \tag{9}$$

where $B_{it} = \frac{\partial \beta}{\partial \xi'}\Big|_{\xi_{i,t|t}} = \begin{bmatrix} \tilde{\phi}_{i,t|t} & 1 & \pi_{i,t|t}\frac{\partial \tilde{\phi}}{\partial \phi}|_{\phi_{i,t|t}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\tilde{\phi}_{i,t|t} = [1 + 0.1\exp(-\phi_{i,t|t})]^{-1}$, and $Q^H = \begin{bmatrix} \sigma^2_\eta & 0 & 0 \\ 0 & \sigma^2_\alpha & 0 \\ 0 & 0 & \sigma^2_\phi \end{bmatrix}$.

To forecast the state variables for period $t+1$, the household uses

$$\xi_{i,t+1|t} = \beta(\xi_{i,t|t}) = \begin{bmatrix} \alpha_{i,t|t} + \tilde{\phi}_{i,t|t}\pi_{i,t|t} \\ \alpha_{i,t|t} \\ \phi_{i,t|t} \end{bmatrix} \tag{10}$$

Each household i iterates on Equations (7)-(10) to update their inflation expectations period by period. To complete the process, I assume each household enters the sample at period 0 with some priors about ξ_0 specified as

$$\begin{bmatrix} \pi_0 \\ \alpha_0 \\ \phi_0 \end{bmatrix} \sim N\left(\begin{bmatrix} \pi_{i,0|0} \\ \alpha_{i,0|0} \\ \phi_{i,0|0} \end{bmatrix}, \begin{bmatrix} \sigma_{i,p}^2 & 0 & 0 \\ 0 & \sigma_{i,p}^2 & 0 \\ 0 & 0 & \sigma_{i,p}^2 \end{bmatrix} \right)$$

Under this scenario, households are allowed to form different priors. Their priors could depend on their past experience of inflation to account for both cohort and age effects, as well as various other demographic factors including levels of education.

To summarize, Equations (7)-(10) describe an updating rule from $\xi_{i,t|t-1}$ to $\xi_{i,t+1|t}$ given a realized signal s_{it}. This updating rule is household-specific, i.e. it varies with $\sigma_{i,\omega}$ and the household's prior, and is path-dependent, i.e. it depends on the history of observed signals.

4 The econometric framework

In this section, I build an econometric framework to estimate the model of household inflation expectations discussed in Section 3. Estimation of the household model is difficult for at least three reasons. The first challenge comes from the limitations in household expectation measurements. Household survey data suffers from various sampling and reporting errors. Second, each individual household's information set is not directly observable from an econometrician's perspective. Third, households update their inflation expectations in a nonlinear and time-varying manner. I construct an estimable state-space model that overcomes these challenges.

4.1 Accounting for survey sampling and reporting errors

Household survey data suffers from various sampling and reporting issues. Omitting these issues leads to inconsistent estimates of model parameters and state variables. I document two major sources of error as examples and propose a unified framework to account for these errors.

One source of error arises from the survey sample design. As discussed in Section 2.1, the Michigan Survey of Consumers (MSC) does not track individual households over time. I group households by demographics and consider the mean response from each group in every quarter as a measure for a representative household from that group. However, with a sample size of about 500 households, the MSC data suffers from a non-trivial sampling error. For example, let σ^i_{t,π^e} be the standard deviation of reported inflation expectations π^e in demographic group i at period t. An estimate of the average sampling error for the group of households with non-college education is $\frac{1}{T}\sum_t \frac{\sigma^i_{t,\pi^e}}{\sqrt{N_{i,t}}} = 0.21$, where $N_{i,t}$ is the sample size. For reference, the average inflation rates during this period is about 3%. Thus a significant amount of time-series variation in the reported mean expectations arises from pure sampling error.

Another source of error arises from people's preference for reporting certain numbers over others. A large share of respondents to the MSC report inflation expectations that are multiples of five. As documented in Binder (2017), households' preference for reporting multiples-of-five is associated with uncertainty in their inflation expectations. Inflation uncertainty varies more in cross-section than over time with lower uncertainty for more educated. Her findings suggest that such digit preference in reporting on average leads to an upward bias of 0.7% in households' reported inflation expectations. A sizable fraction of inflation forecast error documented in Section 2.3 can be attributed to this reporting error.

Sampling errors and reporting errors are prevalent in household surveys but are difficult to identify. I summarize these unobserved measurement errors in household inflation expectations in terms of two components: 1) a demographic-specific component θ_i that captures the persistent upward bias caused by reporting errors; 2) an idiosyncratic component $e_{i,t} \sim N(0, \sigma^2_{i,e})$ that captures time-varying sampling errors. The size of both type of errors is allowed to vary across different demographic groups.

Let $\hat{\pi}_{i,t+1|t}$ denote the mean inflation expectations of group i at period t reported in the MSC. Let $\pi_{i,t+1|t}$ be the actual inflation expectations free from observational errors. The relationship between $\hat{\pi}_{i,t+1|t}$ and $\pi_{i,t+1|t}$ can be summarized as

$$\hat{\pi}_{i,t+1|t} = \pi_{i,t+1|t} + \theta_i + e_{i,t+1}$$

The econometrician observes $\hat{\pi}_{i,t+1|t}$ and is interested in uncovering the unobserved state variable

$\pi_{i,t+1|t}$. The dynamics of $\pi_{i,t+1|t}$ are described in Section 3.3. In the following subsection, I present the full framework for estimation.

4.2 The state-space representation

Here I present the state-space model used for estimation. The information available to the econometrician is different from the household's information set. Therefore the state-space model in the household problem could not be used for the purpose of estimation. To construct the state-space model for estimation, I establish the relationship between observables available to the econometrician and information available to the households. I use the updating rule implied by the solution to the household problem to construct the state equations in this econometric framework.

Observation Equations:

$$\hat{\pi}_t = \pi_t + e_t \tag{11}$$

$$\hat{\pi}_{i,t+1|t} = \pi_{i,t+1|t} + \theta_i + e_{i,t+1} \tag{12}$$

Let $\hat{\pi}_t$ denote the inflation rates used for estimation by the econometrician. Inflation is measured in a variety of ways depending on the goods and services considered. Some popular measures in the US include CPI, PPI, and GDP deflator. The inflation measure used by the econometrician is a proxy for the actual inflation rate π_t that is relevant for the households. Equation (11) is the observation equation for inflation in which the inflation measure $\hat{\pi}_t$ equals the actual inflation rate π_t plus some idiosyncratic noise $e_t \sim N(0, \sigma_e^2)$.

Equation (12) summarizes the observation equations of inflation expectations for demographic groups $i \in \{1, \dots, n\}$ as discussed in Section 4.1. This setup accounts for both the persistent and idiosyncratic errors associated with survey reporting and sampling issues.

State Equations:

$$\zeta_{t+1} = F_t(\zeta_t) + \tau_{t+1} \tag{13}$$

where $F_t(\cdot)$ denotes a system of time-varying nonlinear equations. The vector ζ_t is composed of a total of $4n+3$ state variables including the ones governing the law of motion for inflation, $\{\pi_t, \alpha_t, \phi_t\}$, and the ones involved in household expectations formation, $\{s_{it}, \pi_{i,t+1|t}, \alpha_{i,t+1|t}, \phi_{i,t+1|t}, \ \forall i \in (1, \ldots, n)\}$.

I now specify each state equation. The law of motion for inflation, determined by Equations (1)-(3), mean that the state variables $\{\pi_t, \alpha_t, \phi_t\}$ satisfy

$$\pi_{t+1} = \alpha_t + \tilde{\phi}_t \pi_t + \eta_{t+1}$$

$$\alpha_{t+1} = \alpha_t + \nu_{\alpha,t+1}$$

$$\phi_{t+1} = \phi_t + \nu_{\phi,t+1}$$

Let $(x_{i,\pi,t}, x_{i,\alpha,t}, x_{i,\phi,t}) \equiv (\pi_{i,t|t-1}, \alpha_{i,t|t-1}, \phi_{i,t|t-1})$ denote household i's forecast of the state variables $\{\pi_t, \alpha_t, \phi_t\}$ in period $t-1$. This information, as well as each household's signal s_{it}, are unobservable from the econometrician's perspective. Equations (7)-(10) in Section 3.3 imply a law of motion for $(x_{i,\pi,t}, x_{i,\alpha,t}, x_{i,\phi,t})$ given by

$$s_{i,t+1} = \alpha_t + \tilde{\phi}_t \pi_t + \eta_{t+1} + w_{i,t+1} \tag{14}$$

$$\begin{aligned} x_{i,\pi,t+1} = x_{i,\alpha,t} + x_{i,\phi,t} x_{i,\pi,t} + (K_{i,\alpha,t} + K_{i,\phi,t} x_{i,\pi,t} + K_{i,\pi,t} x_{i,\phi,t})(s_{it} - x_{i,\pi,t}) \\ + K_{i,\phi,t} K_{i,\pi,t} (s_{it} - x_{i,\pi,t})^2 \end{aligned} \tag{15}$$

$$x_{i,\alpha,t+1} = -K_{i,\alpha,t} x_{i,\pi,t} + x_{i,\alpha,t} + K_{i,\alpha,t} s_{it} \tag{16}$$

$$x_{i,\phi,t+1} = -K_{i,\phi,t} x_{i,\pi,t} + x_{i,\phi,t} + K_{i,\phi,t} s_{it} \tag{17}$$

where $K_{it} \equiv (K_{i,\pi,t}, K_{i,\alpha,t}, K_{i,\phi,t})'$ is the Kalman gain matrix specified in Equation (8). Equation (14) describes how household i's information about inflation updates in period $t+1$. Equations (15)-(17) characterize the way households update their forecasts about π, α, and ϕ over each period. To view this process in two steps, the first step is a signal extraction process where household i updates the belief about ξ_t after observing new information s_{it}. In the second step, the household makes a forecast about ξ_{t+1} using information up to period t. Note that the state equations are nonlinear time-varying functions of the state variables since K_{it} is time-varying. Appendix B provides detailed derivations of Equations (14)-(17).

5 Structural estimation

I conduct the structural estimation in a computationally efficient fashion by employing the extended Kalman Filter. This involves 1) linearizing the nonlinear state-space model iteratively by locally approximating the state equations around the econometrician's inference about the state variables; 2) constructing the log-likelihood function of the state-space model to perform numerical optimization. I describe the details of these procedures in Section 5.1. In Section 5.2, I discuss the identification of model parameters in a simplified version of this model. Estimation results and inferences are presented in Section 5.3 - 5.4.

The econometric framework presented in Section 4 could be written in a nonlinear state-space representation as follows:

$$\zeta_{t+1} = F_t(\zeta_t) + \tau_{t+1} \tag{18}$$

$$y_t = \theta + H\zeta_t + \epsilon_t \tag{19}$$

The error terms are white noise and mutually uncorrelated, such that

$$E\left[\begin{pmatrix}\epsilon_t \\ \tau_t\end{pmatrix}\begin{pmatrix}\epsilon_t' & \tau_t'\end{pmatrix}\right] = \begin{bmatrix} R & 0 \\ 0 & Q\end{bmatrix}$$

Equation (18) is the state equation. ζ_t, the vector of state variables, contains π_t, α_t, ϕ_t, and the household's knowledge about these variables. Equation (19) is the observation equation. y_t, the vector of observed variables, contains noisy measures of inflation and household inflation expectations, which are information available to the econometrician. Specifically,

$$\underset{(4n+3)\times 1}{\zeta_t} = (\pi_t \ \alpha_t \ \phi_t \ s_{1t} \ x_{1,\pi,t} \ x_{1,\alpha,t} \ x_{1,\phi,t} \ \dots \ s_{nt} \ x_{n,\pi,t} \ x_{n,\alpha,t} \ x_{n,\phi,t})'$$

$$\underset{(n+1)\times 1}{y_t} = (\hat{\pi}_t \ \hat{\pi}_{1,t+1|t} \ \dots \ \hat{\pi}_{n,t+1|t})'$$

The detailed specification of error terms τ_t and η_t, coefficient matrices θ and H, and covariance matrices R and Q are presented in Appendix A.

5.1 The likelihood function

The model parameters are estimated with maximum likelihood estimation. In the case of linear state space models, the Kalman filter provides a convenient algorithm to construct the likelihood function when the initial state ζ_0 and the innovations $\{\epsilon_t, \tau_t\}_{t=0}^T$ are multivariate Gaussian. However, the standard Kalman filter is not directly applicable in this case due to the nonlinearity in the state equations. To approximate the likelihood of this nonlinear state space model in closed form, I apply the extended Kalman filter to linearize the state equations around the inference $\xi_{t|t}$.

Let $\mathcal{Y}_t = \{y_t, y_{t-1}, \ldots, y_1\}$ denote the econometrician's information set at date t and $\zeta_{t|t}$ denote the locally optimal linear projection of ζ_t given $\mathcal{Y}_t$. The matrix $P_{t|t} = E[(\zeta_t - \zeta_{t|t})(\zeta_t - \zeta_{t|t})']$ is the mean square error of the projection $\zeta_{t|t}$. To iteratively calculate $\zeta_{t+1|t+1}$ and $P_{t+1|t+1}$, suppose that

$$\zeta_t | \mathcal{Y}_t \sim N(\zeta_{t|t}, P_{t|t}) \tag{20}$$

The state equation, Equation (18), after linearization around $\zeta_{t|t}$ yields

$$\begin{aligned}
\zeta_{t+1} &= F_t(\zeta_{t|t}) + \Phi_t(\zeta_t - \zeta_{t|t}) + \tau_{t+1} \\
\Phi_t &= \left.\frac{\partial F_t(\zeta_t)}{\partial \zeta_t'}\right|_{\zeta_t = \zeta_{t|t}}
\end{aligned}$$

To forecast the state vector

$$\begin{aligned}
\zeta_{t+1|t} &= F_t(\zeta_{t|t}) \\
P_{t+1|t} &= \Phi_t P_{t|t} \Phi_t + Q
\end{aligned}$$

To forecast the observation variables

$$y_{t+1|t} = \theta + H\zeta_{t+1|t}$$

$$E[(y_t - y_{t+1|t})(y_t - y_{t+1|t})'] = HP_{t+1|t}H' + R$$

Given that $\{\epsilon_t, \tau_t\}$ are multivariate Gaussian, the conditional likelihood of y_{t+1} is

$$y_{t+1}|\mathcal{Y}_t \sim N(\theta + H\zeta_{t+1|t},\ HP_{t+1|t}H' + R) \tag{21}$$

To update the inference

$$\begin{aligned}\zeta_{t+1|t+1} &= \zeta_{t+1|t} + K_{t+1}(y_{t+1} - y_{t+1|t})\\ K_{t+1} &= P_{t+1|t}H'(HP_{t+1|t}H' + R)^{-1}\end{aligned}$$

Assume that the econometrician start with a prior $\zeta_0 \sim N(\zeta_{0|0}, P_{0|0})$, the approximate log-likelihood for the data $\mathcal{Y}_T$ is

$$\begin{aligned}\log \mathcal{L} = \sum_{t=1}^{T} \log f_{Y_t}(y_t|\mathcal{Y}_{t-1}) = -\frac{(n+1)T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|HP_{t|t-1}H' + R|\\ -\frac{1}{2}\sum_{t=1}^{T}(y_t - \theta - H\zeta_{t|t-1})'(HP_{t|t-1}H' + R)^{-1}(y_t - \theta - H\zeta_{t|t-1})\end{aligned} \tag{22}$$

This can now be maximized over the unknown parameters $\{\sigma_\eta, \sigma_\alpha, \sigma_\phi, \sigma_e, \{\theta_i, \sigma_{i,\omega}, \sigma_{i,e}, \xi_{i,0|0}, \sigma_{i,p}\}_{i=1}^{n}\}$.

5.2 Identification

The primary source of heterogeneity in household inflation expectations arises from the unobserved differences in information capacity as measured by the size of noise $\sigma_{i,\omega}$ in the signal. Another important source of heterogeneity in expectations reported in surveys comes from the sampling error $\sigma_{i,e}$. If two different values for $\sigma_{i,\omega}$ and $\sigma_{i,e}$ imply the identical dynamics of inflation expectations for group i, there is no way to use observable data to choose between the two. I consider a simplified case of this model to demonstrate the identifiability of this framework.

Consider a case with one representative household ($n = 1$) and a constant parameter AR(1) process for inflation. This is a restricted case of the model with $\sigma_\alpha = \sigma_\phi = 0$. The household problem in Section 3.2 simplifies to the following state-space representation:

$$s_t = \pi_t + \omega_t \tag{23}$$

$$\pi_t = \alpha + \phi\pi_{t-1} + \eta_t \tag{24}$$

where Equation (23), the observation equation of the household, states that the representative household receives a noisy signal of inflation s_t where $\omega_t \sim i.i.d.N(0, \sigma_\omega^2)$. Equation (24), the state equation of the household, describes the law of motion perceived by the household. The analysis in Section 3.1 implies that s_t is the optimal signal under an information flow constraint and the size of noise σ_ω is inversely related to the size of the information capacity.

In this setup, with constant drift and autoregressive parameters, the household could update inflation expectations using the Kalman filter optimally. For simplicity, I assume that the household uses the steady-state Kalman filter to abstract from the influence of the prior belief. Applying the standard Kalman filter techniques (see Hamilton (1994) for details), household inflation expectations evolve as the following:

$$\pi_{t+1}^e = \alpha + \phi K \ s_t + \phi(1-K) \ \pi_t^e \tag{25}$$

where $\pi_{t+1}^e = E(\pi_{t+1}|\mathcal{I}_t)$ denotes the household's inflation expectation for period $t+1$ using signals received up to period t. The steady-state Kalman gain, K, is a decreasing function of σ_ω^2 with range $[0, 1]$.

Next, I apply the econometric framework in Section 4 to estimate this simplified model. The observation equations of the econometrician in elaborated form are

$$\begin{pmatrix} \hat{\pi}_t \\ \hat{\pi}_t^e \end{pmatrix} = \begin{pmatrix} 0 \\ \theta_h \end{pmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} \pi_t \\ s_t \\ \pi_t^e \end{pmatrix} + \begin{pmatrix} e_t \\ e_{h,t} \end{pmatrix} \tag{26}$$

where the observed vector is $y_t = (\hat{\pi}_t, \hat{\pi}_t^e)'$ and the state vector is $\xi_t = (\pi_t, s_t, \pi_t^e)'$. The parameter θ_h measures the persistent reporting error and $e_{h,t} \sim i.i.d.N(0, \sigma_{h,e}^2)$ is the idiosyncratic sampling error.

The state equations in the econometric framework are

$$\begin{pmatrix} \pi_{t+1} \\ s_{t+1} \\ \pi_{t+1}^e \end{pmatrix} = \begin{pmatrix} \alpha \\ \alpha \\ 0 \end{pmatrix} + \begin{bmatrix} \phi & 0 & 0 \\ \phi & 0 & 0 \\ 0 & \phi K & \phi(1-K) \end{bmatrix} \begin{pmatrix} \pi_t \\ s_t \\ \pi_t^e \end{pmatrix} + \begin{pmatrix} \eta_{t+1} \\ \eta_{t+1} + \omega_{t+1} \\ 0 \end{pmatrix} \tag{27}$$

Note that the size of noise in the signal, σ_ω, appears in the covariance matrix Q of the state equations, as well as in the transition matrix F through K. A shock to ω_t leads to persistent changes in both household inflation expectation π^e and the econometrician's observation $\hat{\pi}^e$. However, a shock to $e_{h,t}$ has no impact on household inflation expectation π^e but only a contemporary effect on the econometrician's observation $\hat{\pi}_t^e$ without any persistence.

Furthermore, Equations (26)-(27) imply a law of motion for $\hat{\pi}_t^e$ of

$$\hat{\pi}_{t+1}^e = c_h + \phi K \; \hat{\pi}_t + \phi(1-K) \; \hat{\pi}_t^e + e_{h,t+1} + \phi(1-K) \; e_{h,t} + \epsilon_{h,t} \tag{28}$$

where $c_h = \alpha + (1 - \phi + \phi K)\theta_h$ and $\epsilon_{h,t} = \phi K(\omega_t - e_t) \sim N(0, \phi^2 K^2(\sigma_\omega^2 + \sigma_e^2))$, given that $\{\omega_t\} \perp \{e_t\} \perp \{e_{h,t}\}$.

Consider an extreme case where $\sigma_\omega = 0$, which implies $K = 1$. This means the household is free from information flow constraint and is capable of tracking inflation perfectly well. In particular, the household's reported inflation expectation is equivalent to the full-information rational expectation with a persistent bias θ_h and an idiosyncratic error $e_{h,t}$. In other words, the conditional distribution of $\hat{\pi}_{t+1}^e | \hat{\pi}_t$ is independent from $\{\hat{\pi}_t^e, \ldots, \hat{\pi}_0^e\}$ under this scenario. This is easily testable by regressing $\hat{\pi}_{t+1}^e$ on a constant, $\hat{\pi}_t^e$, and $\hat{\pi}_t$. Under the null hypothesis that $\sigma_\omega = 0$, the coefficient of $\hat{\pi}_t^e$ equals zero, which is rejected by the survey data.

5.3 Estimation results

I obtain the maximum-likelihood estimates of the structural parameters by maximizing Equation (22) using numerical optimization methods. The estimation involves three groups of forecasters ($n = 3$): 1) professionals; 2) households without a college degree; 3) households with a college degree, as discussed in Section 2. The professional forecasters are included as a benchmark and are assumed to be free from constraints of information flow capacity. The basic information being used is the actual dynamics of inflation and average expectations reported from each group.

At the aggregate level, the law of motion for inflation is governed by three state variables π_t, α_t, and ϕ_t, as specified in Equations (1) - (3). π_t is a time-varying AR(1) process with the idiosyncratic error $\sigma_\eta = 0.23$. α_t and ϕ_t are modeled as random walks. $\sigma_\alpha = 0.27$ measures the size of steps in the drift parameter α_t. Given that the autoregressive coefficient $\tilde{\phi}_t$ is a monotonic transformation of ϕ_t with a range $(0, 1)$, the size of steps in ϕ_t, $\sigma_\phi = 0.26$, translates into a one standard deviation step of about 0.06 at the peak when $\tilde{\phi}_t = 0.5$ and smaller steps as $\tilde{\phi}_t$ approaches boundaries. Panel A1 of Table 1 reports the estimated AR parameters.

At the household level, the estimation results reveal heterogeneities in the size of information frictions and measurement errors between households with different levels of education. Households with a college degree or above monitor inflation near perfectly ($\sigma_{2,\omega} = 0.00$), while households without a college degree observe inflation with standard error 0.22 percent ($\sigma_{3,\omega} = 0.22$). The model allows for the actual inflation observed by the households to be different from the CPI. The estimates suggest that there is an idiosyncratic difference of about half percent per quarter ($\sigma_e = 0.49$).

Both groups of households report their expectations with sizable persistent biases. Column (1) in Panel C of Table 1 reports the estimated θ_i with measures this persistent reporting bias. Column (2) in Panel D of Table 1 reports the size of error due to rounding preference inferred from the estimates in Binder (2017). Rounding error can explain about half of this persistent bias.

Panel C of Table 1 summarizes the size of idiosyncratic sampling errors $\sigma_{i,e}$ in Column (1). The sizes of sampling errors are comparable between two groups at around 0.25 percent, which is in line with the model-free average sampling errors, calculated as $\frac{1}{T}\sum_t \frac{\sigma^i_{t,\pi^e}}{\sqrt{N_{i,t}}}$, and reported in Column (2)

Table 1: Estimated parameters

Parameters	(1) Structural estimates	(2) Calibration
Panel A1: The law of motion for inflation		
σ_η	0.23	
σ_α	0.27	
σ_ϕ	0.26	
Panel A2: The observation error in inflation		
σ_e	0.49	
Panel B: Size of information frictions $\sigma_{i,\omega}$		
non-college	0.22	
college	0.00	
Panel C: Persistent reporting bias θ_i		
non-college	2.28	0.95
college	1.54	0.71
Panel D: Idiosyncratic sampling errors $\sigma_{i,e}$		
non-college	0.26	0.21
college	0.24	0.20

of Panel C of Table 1.

5.4 Inference

In this section, I use the estimated parameters to make inferences about the state variables ζ_t by applying the Kalman smoothing to obtain $E(\zeta_t|\mathcal{Y}_T)$.

The first variable of interest is the household inflation expectations. Figure 2 plots the model-implied household inflation expectations, with the dashed red line for households with non-college education and the solid blue line for households with college degrees or above. After correcting for measurement errors, inflation expectations from both groups of households reveal to be moving rather closely. The dashed cyan line in Figure 2 plots the full-information rational expectation (FIRE) inferred from the model estimates, which is the optimal inflation forecast using the time-varying parameter AR(1) model.

Knowing FIRE requires information not only about inflation π_t itself but also information about the drift α_t and autoregressive parameters ϕ_t. In this model, α_t and ϕ_t are both unobserved state variables from the perspectives of the households. The estimated model has estimates of 1) the actual α_t and ϕ_t that drives the inflation dynamics, and 2) the understanding of the households about these two state variables. Figures 3 and 4 plot the estimated actual dynamics and the inference from three groups of forecasters on α and ϕ respectively. A close look at Figure 4 suggests that inferences about ϕ from all three groups of households has declined substantially and decoupled from the actual ϕ_t after the Great Recession. I discuss the implications of this observation in Section 6.2.

To visualize the contribution from each source of error, I perform a decomposition of the household forecasting errors in Figure 1 using the model inferences. The heterogeneities in forecasting errors are driven by three major sources: 1) the information flow capacity, 2) the idiosyncratic sampling error $e_{i,t}$, and 3) the persistent reporting error θ_i. Figure 5 plots the inferred values of $\theta_i + e_{i,t}$, which amounts to the measurement errors arising from sampling and reporting. The measurement errors for households with non-college education (dashed red line) are mostly higher compared to the ones for households with college degrees or above (solid blue line) due to their larger persistent error θ_i.

Figure 6 plots the deviations from FIRE which measures the inflation forecasting error due to rational inattention. The two groups of households do not exhibit significant differences in forecast errors due to heterogeneity in information flow constraints. Due to the time-varying nature of the inflation process, the forecast errors are path-dependent. The differences resulting from information capacity appears to be small compared to high volatility in inflation. But there is a systematic discrepancy between the two series.

6 Applications

In this section, I apply my model estimates to answer some important policy questions. First, I find a robust Phillips curve relationship by using model-implied household inflation expectations in the estimation. Next, I examine movements in inflation expectations after a change in the path of inflation in two scenarios. In an analysis for the period of 1986Q1, I show that the household inattention leads to higher unemployment rates when the central bank lowers the inflation target. For periods after the Great Recession, I provide evidence that it takes longer for the Fed to raise household inflation expectations since they are more anchored.

6.1 Fitting the Phillips curve

To evaluate the usefulness of this household inflation expectation model, I explain the role of model-implied inflation expectations in an expectation-augmented Phillips curve. Economists have raised concerns about a breakdown in the Phillips curve after the Great Recession. Inflation in the past ten years has been higher than predicted by the historical Phillips Curve, using either rational expectations, or adaptive expectations, or professional forecasts. Coibion and Gorodnichenko (2015b) find that using household inflation expectations makes up for the missing disinflation in the Phillips curve. I reproduce their results in Table 2 by estimating Equation (29) using the raw survey data from MSC on average household inflation expectations.

$$\pi_t - E_t\pi_{t+1} = c + \kappa \, UE_t^{gap} + \upsilon_t \tag{29}$$

where $E_t\pi_{t+1}$ is household inflation expectations, UE^{gap} is the unemployment gap.

Table 2: Replication of Coibion and Gorodnichenko (2015b)

	non-college	college	CG(2015b)
κ	-0.36**	-0.29**	-0.34**
R^2	0.24	0.18	**0.24**

However, the raw data from MSC are coarse measures of household inflation expectations due to measurement errors. I re-estimate Equation (29) using model-implied household inflation expectations to see if correcting for these errors improves the fit of the Phillips curve. Four different measures of inflation expectations ($E_t\pi_{t+1}$) are considered: 1) the model-implied rational expectation, 2) the professional forecasts, 3) inflation expectations for households without college degree, and 4) for households with a college degree. The results are reported in Table 3.

Both sets of estimates are based on the sample of 1982Q1 - 2018Q4. By using model-implied

Table 3: Phillips curve with model-implied inflation expectations

	rational	professional	non-college	college
κ	-0.06	-0.16**	-0.37**	-0.30**
R^2	0.01	0.06	**0.27**	0.20

household inflation expectations, the R^2 of the regression improves from 0.24 to 0.27. This reassuring as the Phillips curve relationship remains robust when proper inflation expectations are used in the estimation. In contrast, if model-implied rational expectations are used for estimating the Phillips curve instead, the comovement between unemployment and inflation simply disappears.

My analysis suggests that based on the Phillips curve estimation with the best R^2, a 1% increase in unemployment rate is correlated with a decrease in inflation by 0.37% when controlling for household inflation expectations. In the following section, I apply this result to a model-based quantitative analysis to study the outcome of a change in the inflation target.

6.2 Monetary policy implications

I provide implications to monetary policy by considering two different scenarios. The first analysis is based on a decrease in inflation target in the 1980s, where I show that household inattention to inflation leads to higher unemployment rates than what rational expectation model predicts. The second analysis studies an increase in inflation target after the Great Recession. Compared to the 1980s, household inflation expectations are more anchored during this period, making it more difficult for the Fed to achieve its target quickly.

The objective of the Federal Reserve is to achieve both full employment and stable prices, which is known as the "dual mandate". The presence of the Phillips curve suggests that the central bank sometimes faces a trade-off in achieving both goals. When the Fed tries to lower the inflation target, if inflation expectations move sluggishly, unemployment rate would increase as predicted by the Phillips curve. I provide estimates on the changes in unemployment rate when the Fed seeks to alter the path of inflation.

The time variation in my model parameters implies that effects of monetary policy are time-dependent. Consider a monetary policy change that aims to lower the inflation target by 1%.

Given that the inflation process follows an AR(1) process, the long-run inflation is determined by $\alpha/(1-\phi)$. Using my estimates of α_t and ϕ_t, I calculate the current level of long-run inflation as $\pi_t^{LR} = \alpha_t/(1-\phi_t)$ for any given period in the sample. I assume that the change in the target is implemented by a one-time negative shock to the drift parameter, lowering α_t to the level of $\tilde{\alpha} = (1-\phi_t) \times (\pi_t^{LR} - 1)\%$, with no other subsequent shocks.

First, consider a monetary policy that aims to decrease the inflation target in 1986Q1. Households start with their inferences about α_t, ϕ_t and π_t as estimated in the model. I simulate new paths of signals $\{s_{i,t}^j, s_{i,t+1}^j, \ldots, s_{i,t+8}^j\}$, $j = 1, \ldots, 1000$, where $s_{i,t+h}^j = \tilde{\pi}_{t+h} + \omega_{i,t+h}^j$ and $\omega_{i,t+h}^j$ is randomly drawn from the distribution $N(0, \sigma_{i,\omega}^2)$. $\tilde{\pi}_{t+h} = \tilde{\alpha} + \phi_t \tilde{\pi}_{t+h-1}$ is the deterministic hypothetical path of inflation after the policy shock. This assumes that the central bank commits to this new inflation target and manages to keep inflation at a deterministic converging path. In each simulation, household i updates inflation expectations according to the procedure described in Section 3.3. Rational expectations are calculated by starting with the model-implied values of $(\pi_t, \alpha_t, \phi_t)$, and update based on the new path of inflation $\{\pi_{t+h}\}_{h=0}^8$. Figure 7 plots the path of inflation, rational expectations, and the medians of household expectations from each education group. It shows that expectations from households without a college degree, households with a college degree and rational expectations converge to the new inflation target at speeds inversely related to the degrees of their inattention.

I impute the changes in unemployment gap using the model predictions on household inflation expectation movements following the decrease in inflation target. My Phillips curve estimation in Section 6.1 suggests that increasing the gap between inflation expectations of households without a college degree and the actual level of inflation by one percent corresponds to a 2.70 (= 1/0.37) percent increase in the unemployment gap. As a result, the unemployment gap widens by 0.54% on impact if household inflation expectations follows the one with non-college educated. Alternatively, if I use the estimates from households with a college degree, the gap increases by only 0.03%. Therefore, household inattention makes it more costly for the Fed to lower inflation rate than would be the case if the households are perfectly informed and following the Fed closely.

The second analysis considers a monetary policy that aims to increase the inflation target in

Table 4: Inflation parameters and households inferences

	1986Q1			2018Q4		
	actual	non-college	college	actual	non-college	college
π_t	3.25	3.50	3.48	2.57	1.70	1.97
α_t	0.78	1.37	1.48	1.18	1.60	1.91
ϕ_t	0.54	0.51	0.47	0.44	0.06	0.01

2018Q4. During periods when the short-term interest rate was at the zero lower bound, an objective of central banks was to raise inflation expectations directly in order to stimulate the economy. Much of the recent policy debate has been over whether the Fed should increase its target (Blanchard et al. (2010)). I simulate the new paths of inflation expectations in the same way as discussed in the first analysis, except that the shock raises α_t to the level of $(1-\phi_t)\times(\pi_t^{LR}+1)\%$ in this case. Figure 8 plots the path of inflation, rational expectations, and medians of household expectations from each education group after the increase in inflation target. The rational inattention by households to what the Fed is doing makes it more difficult for the Fed to accomplish its objectives quickly.

Household inflation expectations converge at different speeds during different sample periods. To see this, consider the average behavior of household inflation expectations during a period after the Great Inflation (1982Q1 - 1989Q4) versus a period after the Great Recession (2011Q1 - 2018Q4). For each period of the earlier sample, I consider a decrease in inflation target by 1% and simulate the paths of inflation expectations for households with college education. The average gap between household expectations and actual inflation is 0.08% at the three quarters horizon. When I conduct the same analysis for the later sample but instead consider an increase in inflation target by 1%, the average gap is as large as 0.22% at the three quarters horizon. The difference in the speeds of convergence results from households perceptions about the inflation process. Table 4 presents the parameters of the inflation process and households inferences in two specific periods. Households perception about the autoregressive parameter ϕ_t is close to zero in 2018Q4 while its actual value is 0.44. Households perceptions about the autoregressive parameter ϕ_t has remained low since the Great Recession. As a result, households inflation expectations are more anchored in this period.

7 Conclusions

I develop a new framework to understand the heterogeneous household expectation updating process and construct new measures of their inflation expectations. This framework is based on a Bayesian learning model of inflation, subject to the household's information flow constraint. The household model extends the dynamic rational inattention model by incorporating time-varying parameters. The solution to this model implies a time-varying inflation updating procedure dependent on the information processing capacity of the household.

To estimate the household model, I develop an econometric framework to adjust for reporting and sampling errors in the data from the Michigan Survey of Consumers. My estimation results show heterogeneities in the size of information frictions and measurement errors among households with different levels of education. I provide new estimates of household inflation expectations by correcting the measurement errors. My analysis demonstrates that using model-implied inflation expectations further improves the fit of the expectation-augmented Phillips curve.

My results also provide implications to the monetary policy. Household expectations respond more sluggishly to changes in the monetary policy due to inattention. As a result, it is more costly in terms of unemployment rate for the Fed to lower the inflation target. Household inflation expectations are more anchored after the Great Recession. Consequently, it takes longer for the Fed to raise inflation expectations compared to earlier periods. These results help to rethink how policies should be designed to effectively manage household inflation expectations.

References

Adam, K. and Weber, H. (2019). Optimal trend inflation. *American Economic Review*, 109(2):702–37.

Ball, L. M. (1994). What determines the sacrifice ratio? In *Monetary policy*, pages 155–193. The University of Chicago Press.

Ball, L. M. (2014). *The case for a long-run inflation target of four percent.* Number 14-92. International Monetary Fund.

Bhandari, A., Boroviika, J., and Ho, P. (2019). Survey Data and Subjective Beliefs in Business Cycle Models. *Working paper.*

Binder, C. C. (2017). Measuring uncertainty based on rounding: New method and application to inflation expectations. *Journal of Monetary Economics*, 90:1–12.

Blanchard, O., Dell'Ariccia, G., and Mauro, P. (2010). Rethinking macroeconomic policy. *Journal of Money, Credit and Banking*, 42:199–215.

Carroll, C. D. (2003). Macroeconomic expectations of households and professional forecasters. *the Quarterly Journal of economics*, 118(1):269–298.

Cavallo, A., Cruces, G., and Perez-Truglia, R. (2017). Inflation expectations, learning, and supermarket prices: Evidence from survey experiments. *American Economic Journal: Macroeconomics*, 9(3):1–35.

Coibion, O. and Gorodnichenko, Y. (2012). What Can Survey Forecasts Tell Us about Information Rigidities? *Journal of Political Economy*, 120(1):116–159.

Coibion, O. and Gorodnichenko, Y. (2015a). Information rigidity and the expectations formation process: A simple framework and new facts. *American Economic Review*, 105(8):2644–2678.

Coibion, O. and Gorodnichenko, Y. (2015b). Is the phillips curve alive and well after all? inflation expectations and the missing disinflation. *American Economic Journal: Macroeconomics*, 7(1):197–232.

Coibion, O., Gorodnichenko, Y., Kumar, S., and Pedemonte, M. (2018). Inflation expectations as a policy tool? *NBER Working Paper*, No. 24788.

Coibion, O., Gorodnichenko, Y., and Wieland, J. (2012). The optimal inflation rate in new keynesian models: should central banks raise their inflation targets in light of the zero lower bound? *Review of Economic Studies*, 79(4):1371–1406.

D'Acunto, F., Hoang, D., Paloviita, M., and Weber, M. (2019). IQ, Expectations, and Choice. *NBER Working Paper*, No. 25496.

Galí, J. (2015). *Monetary policy, inflation, and the business cycle: an introduction to the new Keynesian framework and its applications.* Princeton University Press.

Hamilton, J. D. (1994). *Time Series Analysis.* Princeton University Press, Princeton, NJ.

Hébert, B. M. and Woodford, M. (2019). Rational inattention when decisions take time. *NBER Working Paper*, No. 26415.

Hobijn, B. and Lagakos, D. (2005). Inflation inequality in the United States. *Review of Income and Wealth*, 51(4):581–606.

Kamdar, R. (2018). The Inattentive Consumer: Sentiment and Expectations. *Working paper.*

Kaplan, G. and Schulhofer-Wohl, S. (2017). Inflation at the household level. *Journal of Monetary Economics*, 91:19–38.

Krugman, P. (2014). Inflation targets reconsidered. In *ECB Forum on Central Banking, Conference Proceedings*, pages 110–122.

Maćkowiak, B., Matějka, F., and Wiederholt, M. (2018). Dynamic rational inattention: Analytical results. *Journal of Economic Theory*, 176:650–692.

Mackowiak, B. and Wiederholt, M. (2009). Optimal sticky prices under rational inattention. *American Economic Review*, 99(3):769–803.

Malmendier, U. and Nagel, S. (2016). Learning from Inflation Experiences. *The Quarterly Journal of Economics*, 131(1):53–87.

Mankiw, N. G., Reis, R., and Wolfers, J. (2003). Disagreement about inflation expectations. *NBER Macroeconomics Annual*, 18:209–248.

Matějka, F. and McKay, A. (2015). Rational inattention to discrete choices: A new foundation for the multinomial logit model. *American Economic Review*, 105(1):272–98.

Nakamura, E., Steinsson, J., Sun, P., and Villar, D. (2018). The Elusive Costs of Inflation: Price Dispersion during the U.S. Great Inflation*. *The Quarterly Journal of Economics*, 133(4):1933–1980.

Schmitt-Grohé, S. and Uribe, M. (2010). Chapter 13 - the optimal rate of inflation. volume 3 of *Handbook of Monetary Economics*, pages 653 – 722. Elsevier.

Sims, C. A. (2003). Implications of rational inattention. *Journal of Monetary Economics*, 50(3):665–690.

Woodford, M. (2003). Imperfect common knowledge and the effects of monetary policy. *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, pages 25–58.

Appendices

A Specification of the econometric framework

I specify the elements in the econometric framework used for estimation in Section 5. The nonlinear state-space model is

$$\zeta_{t+1} = F_t(\zeta_t) + \tau_{t+1}$$
$$y_t = \theta + H\zeta_t + \epsilon_t$$

where ζ_t denotes the vector of state variables and y_t denotes the vector of observed variables. Specifically,

$$\underset{(4n+3)\times 1}{\zeta_t} = \begin{bmatrix} \pi_t \\ \alpha_t \\ \phi_t \\ s_{1t} \\ x_{1,\pi,t} \\ x_{1,\alpha,t} \\ x_{1,\phi,t} \\ \vdots \\ s_{nt} \\ x_{n,\pi,t} \\ x_{n,\alpha,t} \\ x_{n,\phi,t} \end{bmatrix}, \quad \underset{(4n+3)\times 1}{\tau_{t+1}} = \begin{bmatrix} \eta_{t+1} \\ \nu_{\alpha,t+1} \\ \nu_{\phi,t+1} \\ \eta_{t+1} + \omega_{1,t+1} \\ 0 \\ 0 \\ 0 \\ \vdots \\ \eta_{t+1} + \omega_{n,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\underset{(n+1)\times 1}{y_t} = \begin{bmatrix} \hat{\pi}_t \\ \hat{\pi}_{1,t+1|t} \\ \vdots \\ \hat{\pi}_{n,t+1|t} \end{bmatrix}, \quad \underset{(n+1)\times 1}{\theta} = \begin{bmatrix} 0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}, \quad \underset{(n+1)\times 1}{\epsilon_t} = \begin{bmatrix} e_t \\ e_{1,t} \\ \vdots \\ e_{n,t} \end{bmatrix}$$

$$
\underset{(n+1)\times(4n+3)}{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 0 & 0 \end{bmatrix}
$$

$F_t(\zeta_t)$ is a $(4n+3)\times 1$ matrix such that for $i = 1, \dots, n$

$$
\begin{aligned}
F_t(\zeta_t)_{(1,1)} &= \alpha_t + \tilde{\phi}_t \pi_t \\
F_t(\zeta_t)_{(2,1)} &= \alpha_t \\
F_t(\zeta_t)_{(3,1)} &= \phi_t \\
F_t(\zeta_t)_{(4i,1)} &= \alpha_t + \tilde{\phi}_t \pi_t \\
F_t(\zeta_t)_{(4i+1,1)} &= x_{i,\alpha,t} + x_{i,\phi,t} x_{i,\pi,t} + (K_{i,\alpha,t} + K_{i,\phi,t} x_{i,\pi,t} + K_{i,\pi,t} x_{i,\phi,t})(s_{it} - x_{i,\pi,t}) \\
&\qquad + K_{i,\phi,t} K_{i,\pi,t} (s_{it} - x_{i,\pi,t})^2 \\
F_t(\zeta_t)_{(4i+2,1)} &= -K_{i,\alpha,t} x_{i,\pi,t} + x_{i,\alpha,t} + K_{i,\alpha,t} s_{it} \\
F_t(\zeta_t)_{(4i+3,1)} &= -K_{i,\phi,t} x_{i,\pi,t} + x_{i,\phi,t} + K_{i,\phi,t} s_{it}
\end{aligned}
$$

The error terms are white noise and mutually uncorrelated, such that

$$
E\left[\begin{pmatrix} \epsilon_t \\ \tau_t \end{pmatrix} \begin{pmatrix} \epsilon_t' & \tau_t' \end{pmatrix}\right] = \begin{bmatrix} R & 0 \\ 0 & Q \end{bmatrix}
$$

where

$$
\underset{(n+1)\times(n+1)}{R} = \begin{bmatrix} \sigma_e^2 & 0 & \dots & 0 \\ 0 & \sigma_{1,e}^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_{n,e}^2 \end{bmatrix}
$$

$$
\underset{(4n+3)\times(4n+3)}{Q} = \begin{bmatrix}
\sigma_\eta^2 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & \sigma_\alpha^2 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & \sigma_\phi^2 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
\sigma_\eta^2 & 0 & 0 & \sigma_\eta^2 + \sigma_{1,w}^2 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\sigma_\eta^2 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & \sigma_\eta^2 + \sigma_{n,w}^2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0
\end{bmatrix}
$$

B Derivation of state equations

To obtain Equation (14), recall that households obtain a signal of inflation in period $t+1$ as

$$s_{i,t+1} = \pi_{t+1} + \omega_{i,t+1} \tag{30}$$

where π_{t+1} is determined by the law of motion for inflation as

$$\pi_{t+1} = \alpha_t + \tilde{\phi}_t \pi_t + \eta_{t+1} \tag{31}$$

Substituting π_{t+1} in Equation (30) by expression in Equation (31) produces Equation (14).

To obtain Equations (15)-(17), recall that households update their beliefs from $\xi_{i,t|t-1}$ to $\xi_{i,t+1|t}$ in two steps as described in Equations (7) and (10)

$$
\begin{aligned}
&\xi_{i,t|t} = \xi_{i,t|t-1} + K_{it}(s_{it} - \pi_{i,t|t-1}) \\
&\xi_{i,t+1|t} = \beta(\xi_{i,t|t})
\end{aligned}
$$

The first step is a signal extraction process where household i updates the belief about ξ_t after observing new information s_{it}. The Kalman gain $K_{it} \equiv (K_{i,\pi,t}, K_{i,\alpha,t}, K_{i,\phi,t})$ relies on information up to period $t-1$, as stated in Equations (8)-(9). In the second step, the household makes a forecast about ξ_{t+1} using information up to period t. In elaborated form, the two system of equations are

$$\pi_{i,t|t} = \pi_{i,t|t-1} + K_{i,\pi,t}(s_{it} - \pi_{i,t|t-1}) \tag{32}$$

$$\alpha_{i,t|t} = \alpha_{i,t|t-1} + K_{i,\alpha,t}(s_{it} - \pi_{i,t|t-1}) \tag{33}$$

$$\phi_{i,t|t} = \phi_{i,t|t-1} + K_{i,\phi,t}(s_{it} - \pi_{i,t|t-1}) \tag{34}$$

and

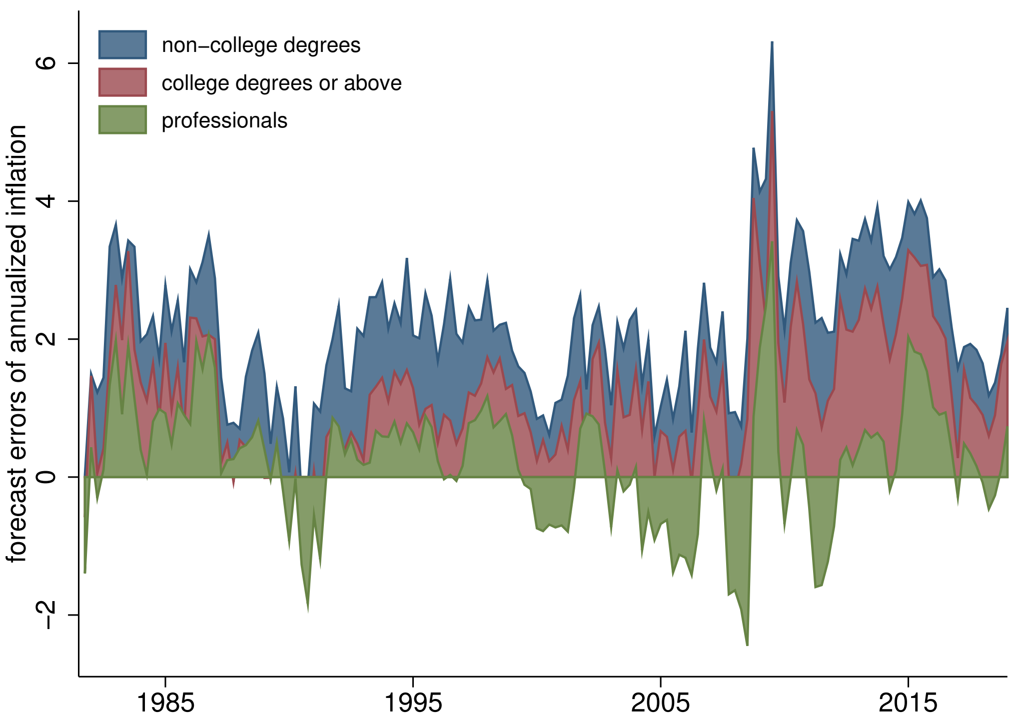
$$\pi_{i,t+1|t} = \alpha_{i,t|t} + \phi_{i,t|t}\pi_{i,t|t} \tag{35}$$

$$\alpha_{i,t+1|t} = \alpha_{i,t|t} \tag{36}$$

$$\phi_{i,t+1|t} = \phi_{i,t|t} \tag{37}$$

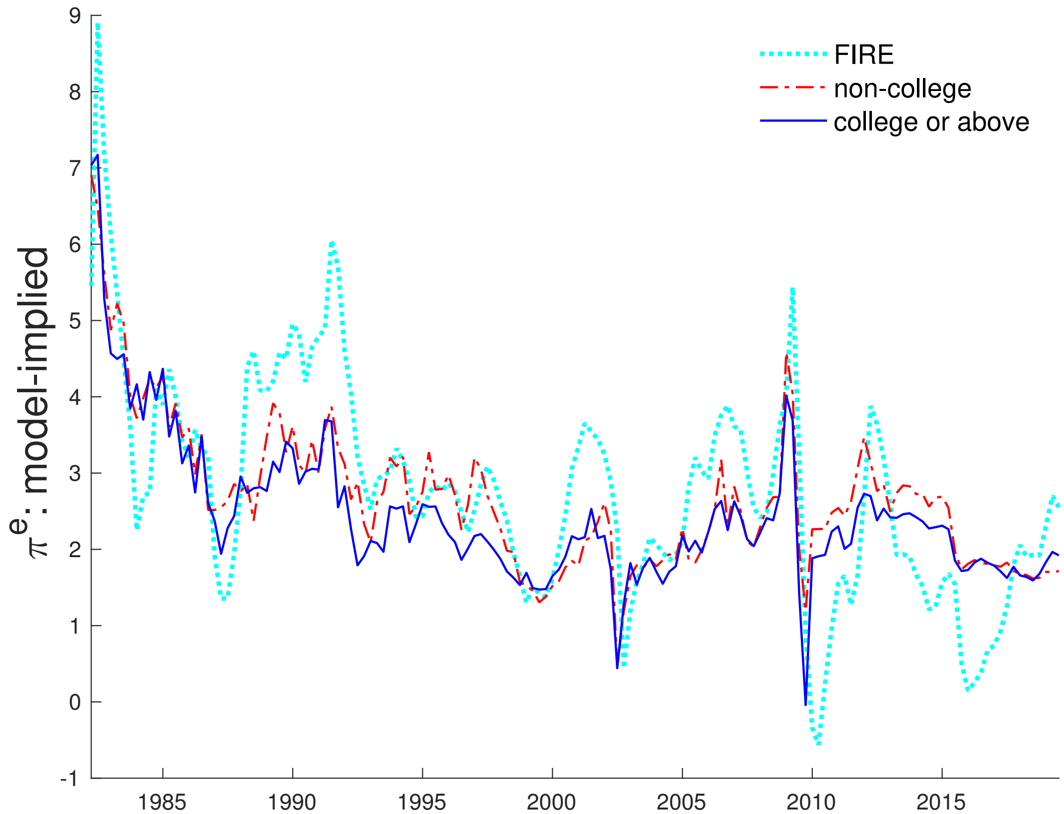
Substituting $\pi_{i,t|t}$, $\alpha_{i,t|t}$, and $\phi_{i,t|t}$ in Equations (35)-(37) into Equations (32)-(34) produces Equations (15)-(17)

Figure 1: Inflation forecast errors: households v.s. professionals



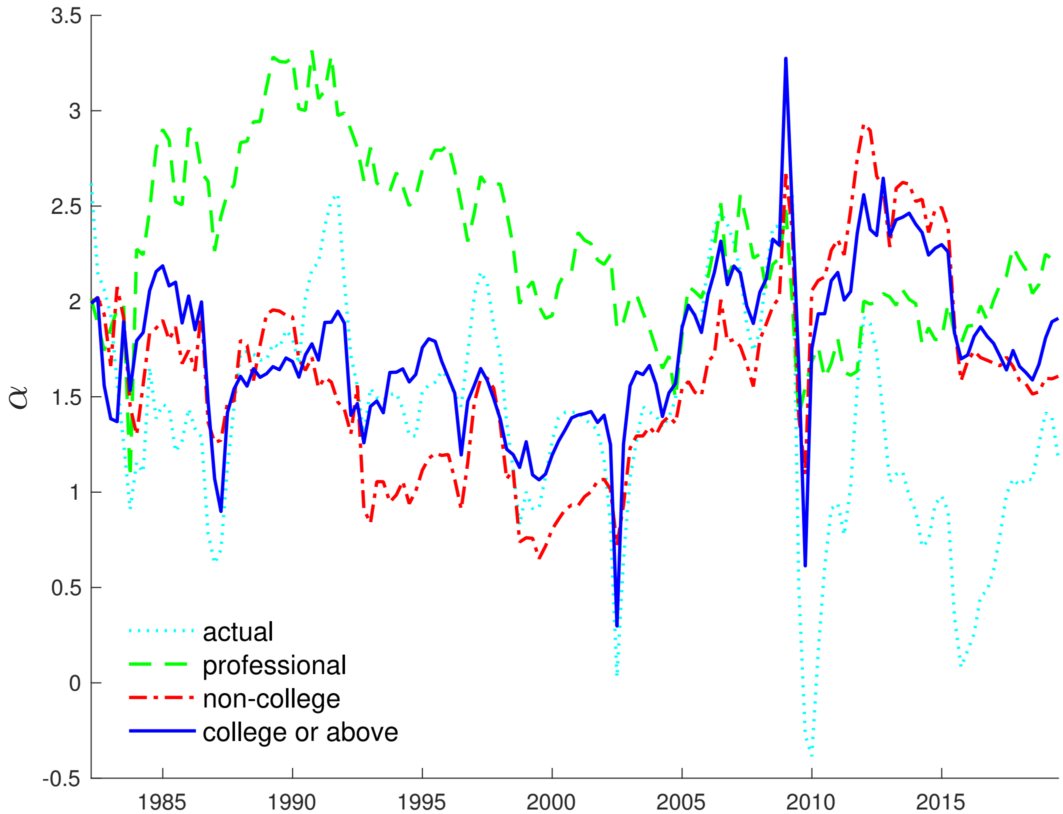
This figure plots average inflation forecast errors for 1) professional forecasters, 2) households with a college degree, and 3) households without a college degree. See Section 2.3 for details.

Figure 2: Model-implied inflation expectations



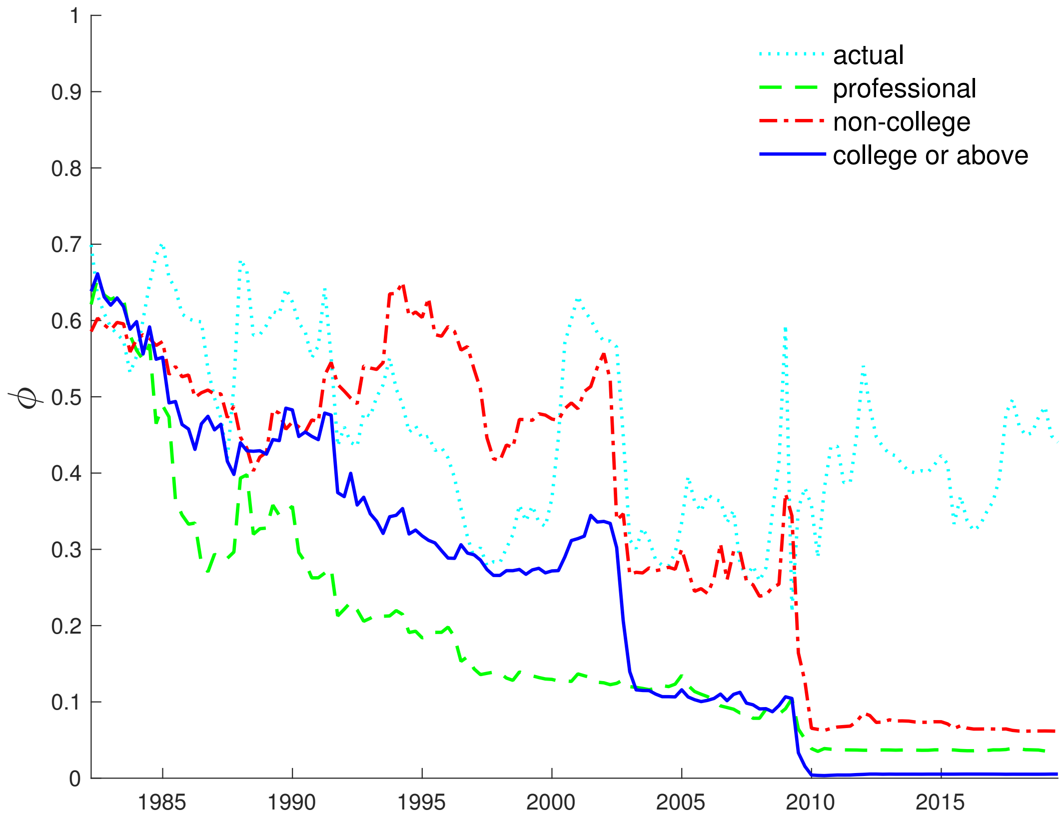
This figure plots estimates of model-implied 1) full-information rational expectations of inflation and 2) household inflation expectations corrected for measurement errors. See Section 5.4 for details.

Figure 3: Drift parameter α_t



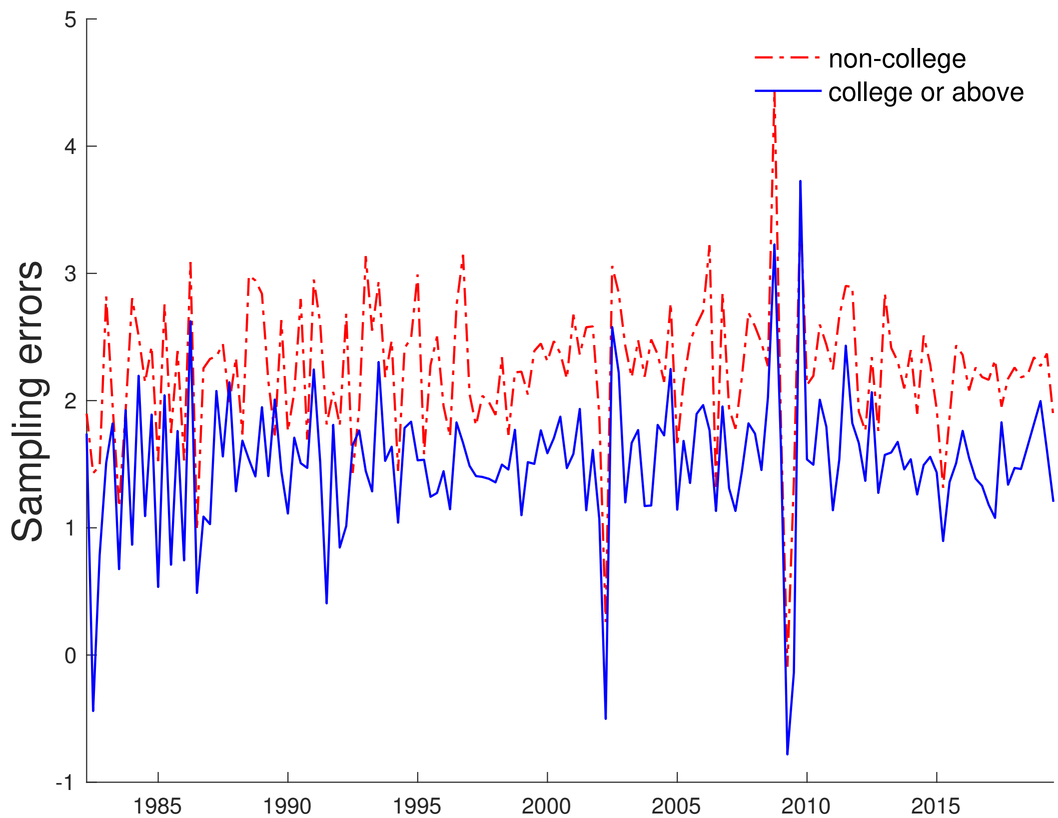
This figure plots estimates of α_t for 1) its actual value, 2) the perceptions of professional forecasters, 3) the perceptions of households without a college degree, and 4) household with a college degree. See Section 5.4 for details.

Figure 4: Autoregressive coefficient ϕ_t



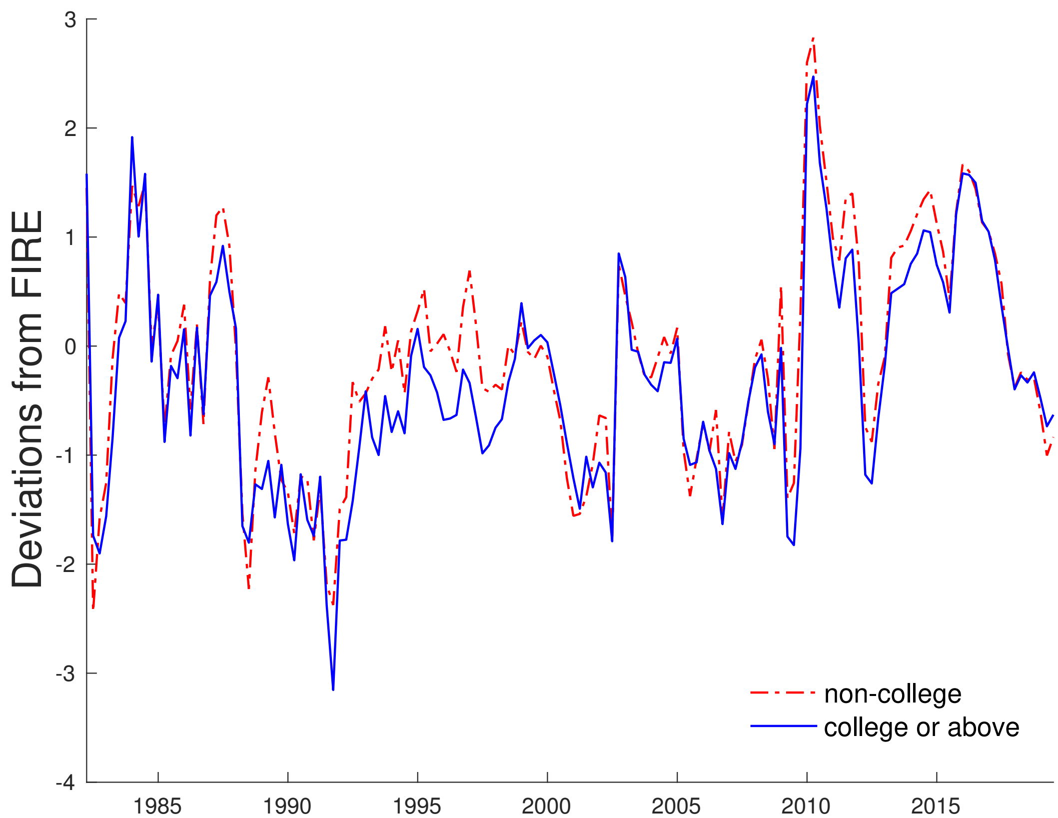
This figure plots estimates of ϕ_t for 1) its actual value, 2) the perceptions of professional forecasters, 3) the perceptions of households without a college degree, and 4) household with a college degree. See Section 5.4 for details.

Figure 5: Sampling and reporting errors



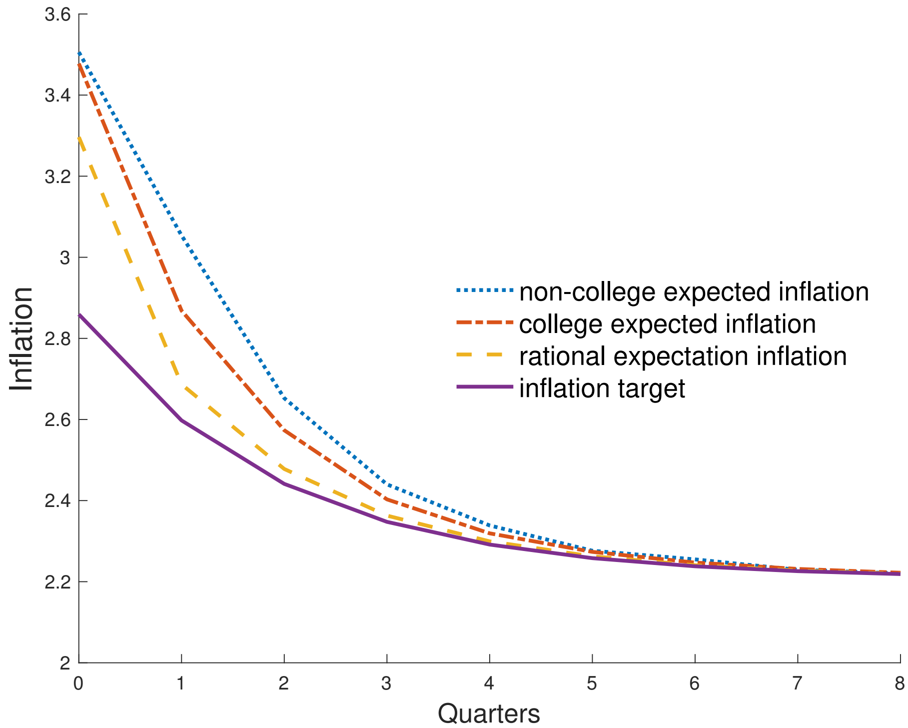
This figure plots the sum of idiosyncratic sampling errors and persistent reporting errors for two groups of households at different education levels. See Section 5.3 - 5.4 for details.

Figure 6: Deviations of household inflation expectations from full-information rational expectations



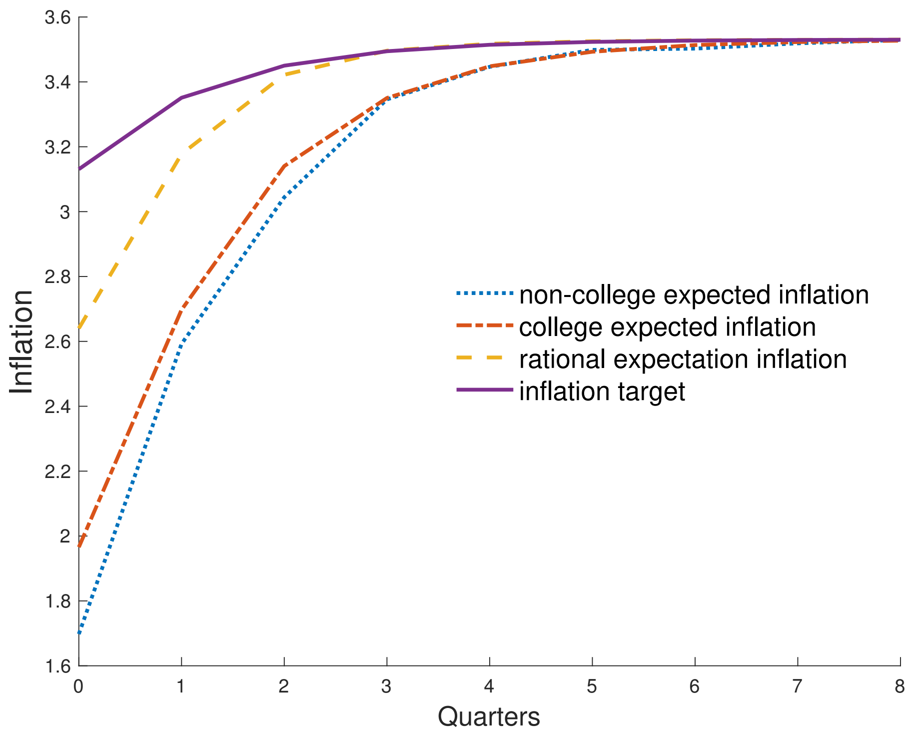
This figure plots deviations of household inflation expectations from model-implied full-information rational expectations. See Section 5.4 for details.

Figure 7: Paths of inflation expectations after a decrease in the inflation target in 1986Q1



This figure plots paths of inflation and inflation expectations assuming that the Fed aims for a 1% decrease in inflation target in 1986Q1. The lines represent the actual path of inflation, inflation expectations from rational expectations, and inflation expectations from two groups of households, one without a college degree and one with a college degree. See Section 6.2 for details.

Figure 8: Paths of inflation expectations after an increase in the inflation target in 2018Q4



This figure plots paths of inflation and inflation expectations assuming that the Fed increases the inflation target by 1% in 2018Q4. The lines represent the actual path of inflation, inflation expectations from rational expectations, and inflation expectations from two groups of households, one without a college degree and one with a college degree. See Section 6.2 for details.