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BAYESIAN ESTIMATION OF A SMALL-SCALE NEW KEYNESIAN MODEL WITH HETEROGENEOUS EXPECTATIONS

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This paper uses Bayesian methods to estimate a small-scale New Keynesian model with heterogeneous expectations (HE). Agents form expectations via Euler equation adaptive learning (AL) and differ by the model they use to forecast. Type A agents use a correctly specified model, while type B and type C agents use misspecified models. Quarterly US data from the pre-Great Moderation and Great Moderation periods are used to jointly estimate the degree of agent heterogeneity, the AL parameters, and the deep model parameters. Results show that the data exhibit significant expectational heterogeneity, and that the HE model fits the data better than a model with homogeneous agent AL.

Keywords: Heterogenous Expectations, Adaptive Learning, Bayesian Econometrics, New Keynesian Model

1. INTRODUCTION

Dynamic stochastic general equilibrium modeling that assumes representative agent rational expectations has been the dominant paradigm within macroeconomics for decades. However, relatively recent work based on survey evidence suggests that this assumption may not be appropriate. For example, Mankiw et al. (2003) show significant disagreement among inflation forecasters, while Carroll (2003) finds evidence that agents are less than fully rational when forming expectations of inflation and unemployment. Branch (2004, 2007) and Pfajfar and Santoro (2010) find evidence of heterogeneity in inflation expectations, while Cole and Milani (2019) find that the rational expectations assumption is a primary reason the New Keynesian model fails to match the data well.

Lab experiments also provide evidence against the homogeneous agent rational expectations paradigm. Hommes (2011) finds evidence of heterogeneous expectations (HE) when experiment participants are asked to forecast a market

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price. Within a New Keynesian framework, Pfajfar and Zakelj (2014) find that inflation expectations show significant heterogeneity between subjects showing rational expectations and subjects displaying expectations that have adaptive learning (AL) properties.

This paper extends the theoretical work in Adam (2005), Guse (2005), Berardi (2007, 2009), Branch and McGough (2009, 2010), Branch and Evans (2011), Massaro (2013), Gasteiger (2014, 2017), Di Bartolomeo et al. (2016), Branch and Gasteiger (2019), Bazzana (2020), and others by incorporating HE into a small-scale New Keynesian model, where agents employ the Euler equation AL mechanism of Evans and Honkapohja (2001) by behaving like econometricians who estimate parameters of their model and use it to make forecasts. Similar to those theoretical papers, agent heterogeneity is incorporated by assuming agents differ in the model they employ to form expectations.[1] The relevant equilibrium concept is the heterogeneous expectations equilibrium (HEE) studied in Berardi (2007) and Berardi (2009), so the model setup is most similar to the work in those papers.

The small-scale New Keynesian model under consideration contains two state variables, an aggregate demand (AD) shock and an aggregate supply (AS) shock. Three agent types are considered. Agent type A uses a correctly specified model in the sense that it is of the same form as the minimum state variable (MSV) rational expectations equilibrium (REE) solution. Agent types B and C, however, use more parsimonious models that are misspecified versions of the MSV REE solution; agent type B only incorporates the AD shock and agent type C only incorporates the AS shock. The motivation for this setup is that since the AL mechanism assumes agents behave like econometricians, agents within the model should be subject to the same computational and cognitive limitations that real-world forecasters are subject to. It is feasible to assume that one result of these limitations could be model misspecification. With this in mind, it can be thought that cognitive and computation limitations contribute to agent type B and C's model misspecification. This setup is consistent with how other studies in the literature motivate HE (i.e., Berardi (2007, 2009), Branch and McGough (2009), and Massaro (2013)).

The paper differs from the previously mentioned theoretical work in that it employs the empirical methodology used in Milani (2007), Milani (2012), and Milani (2019), by using Bayesian methods to jointly estimate the degree of expectational heterogeneity, the AL parameters, and the deep model parameters. Estimation results using quarterly US data from three time periods (pre-Great Moderation, Great Moderation, and pre-Great Recession) show two main results. First, there is significant expectational heterogeneity in the data as evidenced by the existence of more than one agent type. Second, the HE model provides a better fit of the data than does a model that assumes homogeneous agent AL. These results represent the main contributions of the paper.

The paper is organized as follows: Section 2 introduces the New Keynesian model with HE, solves for the HEE solution, and provides the uniqueness and stability criteria for the solution. Section 3 discusses the estimation procedure, Section 4 presents the empirical results and provides some intuition, and Section 5 concludes. Appendix A discusses some robustness checks, and Appendix B describes the data.

2. A NEW KEYNESIAN MODEL WITH HE

The model under consideration is similar to a standard small-scale New Keynesian model with imperfect competition and sticky prices, as presented in Clarida et al. (1999). The log-linearized equations of the model are

$$x_t = \hat{E}_t x_{t+1} - \sigma^{-1}(i_t - \hat{E}_t \pi_{t+1}) + u_t^x \quad (1)$$

$$\pi_t = \beta \hat{E} \pi_{t+1} + \kappa x_t + u_t^\pi, \quad (2)$$

where equations (1) and (2) are the New Keynesian IS equation and Phillips curve equation, respectively. x_t is the output gap, π_t is the inflation rate, i_t is the nominal interest rate, σ is the power utility parameter in the household utility function, $\beta \in (0, 1)$ is the household discount factor, κ is a composite parameter that represents the slope of the New Keynesian Phillips curve, u_t^x is an AD shock, and u_t^π is an AS shock. The slight modification to the model is that $\hat{E}_t$ is a possibly non-rational economy-wide expectations operator.[2] All variables are expressed as log deviations from the steady state.

The model is completed by assuming a policy rule for the nominal interest rate. This paper assumes the central bank sets the interest rate as a linear combination of the private sector expectations of the output gap and the inflation rate,

$$i_t = i_x \hat{E}_t x_{t+1} + i_\pi \hat{E}_t \pi_{t+1} + \varepsilon_t^i, \quad (3)$$

where $\varepsilon_t^i \sim iid\ \mathcal{N}(0, \sigma_i^2)$ is a monetary policy shock.

The AD and supply shocks are assumed to be univariate AR(1) processes,

$$u_t^x = \rho_x u_{t-1}^x + \varepsilon_t^x \qquad \varepsilon_t^x \sim iid\ \ \mathcal{N}(0, \sigma_x^2) \quad (4)$$

$$u_t^\pi = \rho_\pi u_{t-1}^\pi + \varepsilon_t^\pi \qquad \varepsilon_t^\pi \sim iid\ \ \mathcal{N}(0, \sigma_\pi^2). \quad (5)$$

2.1. Heterogeneous Expectations

Following somewhat similar setups in Branch and McGough (2009) and Berardi (2007, 2009), assume there are three agent types, type A, type B, and type C, who are similar in all ways except in how they form expectations. Denote the agent types' expectations operators as $\hat{E}^{\mathrm{A}}$, $\hat{E}^{\mathrm{B}}$, and $\hat{E}^{\mathrm{C}}$. Furthermore, assume that the economy-wide expectations operator $\hat{E}$ appearing in equations (1) through (3) can be written as a weighted average of the expectation operators of the three agent types,

$$\hat{E} = \alpha_A \hat{E}^{\mathrm{A}} + \alpha_B \hat{E}^{\mathrm{B}} + (1 - \alpha_A - \alpha_B)\hat{E}^{\mathrm{C}}, \quad (6)$$

where α_A is the proportion of type A agents, α_B is the proportion of type B agents, $1-\alpha_A-\alpha_B$ is the implied proportion of type C agents, and it must be the case that $\alpha_A+\alpha_B \leq 1$.

After substituting equation (3) into equation (1), the system can be written in the condensed form

$$y_t = F_1\hat{E}_t^A y_{t+1} + F_2\hat{E}_t^B y_{t+1} + F_3\hat{E}_t^C y_{t+1} + F_4 u_t^x + F_5 u_t^\pi + F_6\varepsilon_t^i, \tag{7}$$

where $y_t = [x_t\ \pi_t]'$. The MSV REE solution is such that the endogenous variables can be written as a linear combination of the shock processes,

$$y_t = \bar{P}_1 u_t^x + \bar{P}_2 u_t^\pi + \bar{P}_3\varepsilon_t^i.$$

2.2. Adaptive Learning

This paper relaxes the assumption of rational expectations and assumes agents form expectations by employing the Euler equation AL mechanism of Evans and Honkapohja (2001) in that agents behave like econometricians who estimate the parameters of their model of the economy and use their estimated model to make forecasts.

Specifically, assume that agent type A uses a model for x_t and π_t that is of the same form as the MSV REE solution,

$$x_t = A_{1,1}u_t^x + A_{1,2}u_t^\pi + A_{1,3}\varepsilon_t^i$$
$$\pi_t = A_{2,1}u_t^x + A_{2,2}u_t^\pi + A_{2,3}\varepsilon_t^i,$$

where $A_{1,1}$, $A_{1,2}$, $A_{1,3}$, $A_{2,1}$, $A_{2,2}$, and $A_{2,3}$ are parameters that are estimated by the agent using econometric methods and are referred to in the learning literature as agent beliefs. Writing the previous equations more compactly,

$$y_t = A_x u_t^x + A_\pi u_t^\pi + A_i\varepsilon_t^i, \tag{8}$$

where $A_x = [A_{1,1}\ A_{2,1}]'$, $A_\pi = [A_{1,2}\ A_{2,2}]'$, and $A_i = [A_{1,3}\ A_{2,3}]'$. The AL literature refers to equation (8) as agent type A's perceived laws of motion (PLMs) of the economy.

Implicit in the AL mechanism is the idea that agents within the model are behaving like econometricians, which suggests that they are subject to the same computational and cognitive limitations as real-world econometricians. These limitations may lead real-world econometricians to use misspecified models that result from a lack of knowledge of the true data-generating process. To capture this idea, I therefore employ two more agent types, type B and type C, each of whom can be thought of as being subject to computational and cognitive limitations that result in them employing PLMs that are misspecified in the sense that they are not of the same form as the MSV REE solution. As in Berardi (2007, 2009) and Branch and McGough (2009), the different agent types employed here can be interpreted as having different computational and cognitive abilities.

Assume that agent type B's PLMs only include the AD shock u_t^x,

$$x_t = B_1 u_t^x$$
$$\pi_t = B_2 u_t^x$$

or more compactly,

$$y_t = B u_t^x, \tag{9}$$

where $B = [B_1 \ B_2]'$.

Assume that agent type C's PLMs only include the AS shock u_t^π,

$$x_t = C_1 u_t^\pi$$
$$\pi_t = C_2 u_t^\pi$$

or more compactly,

$$y_t = C u_t^\pi, \tag{10}$$

where $C = [C_1 \ C_2]'$.

It is assumed at time t that all agent types know the values of u_t^x, u_t^π, and ε_t^i, the form of the shock processes in equations (4) and (5), and the autoregressive (AR) coefficients of the shock processes ρ_x and ρ_π. They do not, however, know the current values of the endogenous variables x_t and π_t. Then, iterating equations (8), (9), and (10) one period forward and taking expectations gives the forecasting equations for each agent type,

$$\hat{E}_t^{\mathrm{A}} y_{t+1} = A_x \rho_x u_t^x + A_\pi \rho_\pi u_t^\pi \tag{11}$$
$$\hat{E}_t^{\mathrm{B}} y_{t+1} = B \rho_x u_t^x \tag{12}$$
$$\hat{E}_t^{\mathrm{C}} y_{t+1} = C \rho_\pi u_t^\pi. \tag{13}$$

Plugging in equations (11), (12), and (13) into equation (7) gives the actual law of motion (ALM) of the economy,

$$y_t = (F_1 A_x \rho_x + F_2 B \rho_x + F_4) u_t^x + (F_1 A_\pi \rho_\pi + F_3 C \rho_\pi + F_5) u_t^\pi + F_6 \varepsilon_t^i. \tag{14}$$

2.3. Heterogeneous Expectations Equilibrium

The relevant equilibrium concept is a HEE, the properties of which were originally studied in Berardi (2007, 2009).[3] Following the techniques in those papers and in Evans and Honkapohja (2001), the map from agent type A's PLMs to the ALM is

$$A_x \to F_1 A_x \rho_x + F_2 B \rho_x + F_4$$
$$A_\pi \to F_1 A_\pi \rho_\pi + F_3 C \rho_\pi + F_5$$
$$A_i \to F_6$$

and the associated ordinary differential equations (ODEs) are

$$\dot{A}_x = F_1 A_x \rho_x + F_2 B \rho_x + F_4 - A_x$$
$$\dot{A}_\pi = F_1 A_\pi \rho_\pi + F_3 C \rho_\pi + F_5 - A_\pi$$
$$\dot{A}_i = F_6 - A_i.$$

The map from agent type B's PLM to the ALM is

$$B \to F_1 A_x \rho_x + F_2 B \rho_x + F_4$$

and the associated ODE is

$$\dot{B} = F_1 A_x \rho_x + F_2 B \rho_x + F_4 - B.$$

The map from agent type C's PLM to the ALM is

$$C \to F_1 A_\pi \rho_\pi + F_3 C \rho_\pi + F_5$$

and the associated ODE is

$$\dot{C} = F_1 A_\pi \rho_\pi + F_3 C \rho_\pi + F_5 - C.$$

The HEE solution is the equilibrium point $\bar{A}_x$, $\bar{A}_\pi$, $\bar{A}_i$, $\bar{B}$, $\bar{C}$ of the system

$$\dot{A}_x = (F_1 \rho_x - I) A_x + F_2 \rho_x B + F_4$$
$$\dot{A}_\pi = (F_1 \rho_\pi - I) A_\pi + F_3 \rho_\pi C + F_5$$
$$\dot{A}_i = -A_i + F_6$$
$$\dot{B} = F_1 \rho_x A_x + (F_2 \rho_x - I) B + F_4$$
$$\dot{C} = F_1 \rho_\pi A_\pi + (F_3 \rho_\pi - I) C + F_5,$$

where I is a 2×2 identity matrix.

The system can be written compactly as

$$\begin{pmatrix} \dot{A}_x \\ \dot{A}_\pi \\ \dot{A}_i \\ \dot{B} \\ \dot{C} \end{pmatrix} = G_1 \begin{pmatrix} A_x \\ A_\pi \\ A_i \\ B \\ C \end{pmatrix} + G_2.$$

Setting the system equal to zero,

$$G_1 \begin{pmatrix} \bar{A}_x \\ \bar{A}_\pi \\ \bar{A}_i \\ \bar{B} \\ \bar{C} \end{pmatrix} + G_2 = 0$$

implies the HEE solution,

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_\pi \\ \bar{A}_i \\ \bar{B} \\ \bar{C} \end{pmatrix} = -G_1^{-1} G_2, \tag{15}$$

which is unique if G_1^{-1} exists and is stable under least-squares learning (i.e., Expectationally stable or E-stable) if G_1 has all eigenvalues with negative real part.

2.4. AL Algorithms

Following the AL literature, agents use historical data and update the parameters of their model as new data become available using a constant gain learning (CGL) algorithm. The CGL algorithm for agent type A is

$$A_{j,t} = A_{j,t-1} + gn_A {R_{j,t}^{A}}^{-1} z_{t-1}^{A} \left(y_{t-1}^{obs} - A'_{j,t-1} z_{t-1}^{A} \right)$$
$$R_{j,t}^{A} = R_{j,t-1}^{A} + gn_A \left(z_{t-1}^{A} {z_{t-1}^{A}}' - R_{j,t-1}^{A} \right),$$

where $j = 1, 2$, $A_{j,t}$ is the agent's beliefs, gn_A is the AL gain parameter, $R_{j,t}^{A}$ is a 3×3 moment matrix for the agent beliefs, z_{t-1}^{A} is a vector of regressors, $z_{t-1}^{A} = [u_{t-1}^{x} \ u_{t-1}^{\pi} \ \varepsilon_{t-1}^{i}]'$, and y_{t-1}^{obs} is the most recent observation of the dependent variable in the PLM, $y_{t-1}^{obs} = x_{t-1}, \pi_{t-1}$. With this notation, $j = 1$ corresponds with $y_{t-1}^{obs} = x_{t-1}$, $j = 2$ corresponds with $y_{t-1}^{obs} = \pi_{t-1}$, and

$$A_{1,t} = \left[A_{1,1} \ A_{1,2} \ A_{1,3} \right]'$$
$$A_{2,t} = \left[A_{2,1} \ A_{2,2} \ A_{2,3} \right]'.$$

The AL gain parameter represents how much past data agents use when estimating the parameters of their model. Specifically, $1/gn_A$ is how many past observations the agent is using to estimate its model, so that as the gain increases, the agent is using less historical data.

The CGL algorithm for agent type B is

$$B_{j,t} = B_{j,t-1} + gn_B {R_{j,t}^{B}}^{-1} z_{t-1}^{B} \left(y_{t-1}^{obs} - B_{j,t-1} z_{t-1}^{B} \right)$$
$$R_{j,t}^{B} = R_{j,t-1}^{B} + gn_B \left(z_{t-1}^{B} z_{t-1}^{B} - R_{j,t-1}^{B} \right),$$

where $j = 1, 2$, $B_{j,t}$ is the agent's beliefs, gn_B is the AL gain parameter, $R_{j,t}^{B}$ is a moment scalar for the agent beliefs, and z_{t-1}^{B} is a regressor, $z_{t-1}^{B} = u_{t-1}^{x}$.

The CGL algorithm for agent type C is

$$C_{j,t} = C_{j,t-1} + gn_C {R_{j,t}^{C}}^{-1} z_{t-1}^{C} \left(y_{t-1}^{obs} - C_{j,t-1} z_{t-1}^{C} \right)$$
$$R_{j,t}^{C} = R_{j,t-1}^{C} + gn_C \left(z_{t-1}^{C} z_{t-1}^{C} - R_{j,t-1}^{C} \right),$$

where $j = 1, 2$, $C_{j,t}$ is the agent's beliefs, $R^C_{j,t}$ is a moment scalar for the agent beliefs, and z^C_{t-1} is a regressor, $z^C_{t-1} = u^\pi_{t-1}$.

It is important to note that under CGL, the agent's parameters will only converge to a distribution centered about the HEE solution values of equation (15), assuming the uniqueness and stability criteria are met.[4]

3. BAYESIAN ESTIMATION

The proportion of type A agents (α_A), proportion of type B agents (α_B), AL gain parameters (gn_A, gn_B, and gn_C), and other model parameters are jointly estimated using Bayesian methods. The parameters estimated are stacked in a column vector,

$$\theta = [\sigma_x, \sigma_\pi, \sigma_i, \sigma, \beta, \kappa, i_x, i_\pi, \rho_x, \rho_\pi, gn_A, gn_B, gn_C, \alpha_A, \alpha_B]'.$$

The posterior distribution is obtained by Bayes theorem and summarizes all of the information about the parameters,

$$p(\theta|Y^T) = \frac{p(Y^T|\theta)p(\theta)}{p(Y^T)},$$

where $p(Y^T|\theta)$ is the likelihood function, $p(\theta)$ is the prior distribution of the parameters, and Y^T is the data.

The parameters are estimated using US data on the output gap, inflation rate, and the nominal interest rate. The data are quarterly and are segmented into two time periods. The first period runs from 1954Q3 to 1979Q4 (i.e., the pre-Great Moderation period), and the second period runs from 1980Q1 to 2007Q4 (i.e., the Great Moderation period). The estimation is also run over the entire sample period (i.e., from 1954Q3 to 2007Q4, the pre-Great Recession period). The segmentation into the pre-Great Moderation and Great Moderation periods recognizes the existence of a significant structural change in the US economy occurring around 1980 due to the disinflation monetary policy regime of Paul Volcker, while the pre-Great Recession period allows comparison of estimation results with those of Milani (2007).

The output gap is the log difference between real gross domestic product (RGDP) and potential RGDP (Congressional Budget Office (CBO) estimate) multiplied by 100. The inflation rate is the annualized quarterly growth rate of the GDP implicit price deflator. The nominal interest rate is the effective federal funds rate. All data were obtained from the Federal Reserve Economic Data website (FRED). Appendix B provides information on the data.

Equations from (1) to (6), (11) to (13), and the equations in Section 2.4 can be written in the form

$$s_t = J_1 s_{t-1} + J_2 \varepsilon_t, \tag{16}$$

where s_t is a column vector that includes the three endogenous variables (x_t, π_t, and i_t), the two shock processes (u^x_t and u^π_t), and the six expectation variables

($\hat{E}_t^A x_{t+1}$, $\hat{E}_t^B x_{t+1}$, $\hat{E}_t^C x_{t+1}$, $\hat{E}_t^A \pi_{t+1}$, $\hat{E}_t^B \pi_{t+1}$, and $\hat{E}_t^C \pi_{t+1}$). $\varepsilon_t = [\varepsilon_t^x \ \varepsilon_t^\pi \ \varepsilon_t^i]'$ represents the three exogenous shocks. Moreover, the output gap (GAP_t), the inflation rate ($INFL_t$), and the effective federal funds rate (INT_t) are related to the endogenous variables in the model with a series of measurement equations,

$$GAP_t = x_t \tag{17}$$

$$INFL_t = \pi_t \tag{18}$$

$$INT_t = i_t, \tag{19}$$

so that equation (16) along with measurement equations from (17) to (19) make the state-space representation of the model.

The estimation method is similar to that developed in Milani (2007). Draws from the posterior distribution are generated using the random walk Metropolis–Hastings algorithm, where the scaling parameter is set to ensure an acceptance rate between 20 and 40%, and any draws that produce an indeterminate REE solution and/or a non-unique and/or non-stable HEE solution are discarded. The likelihood function is calculated using the Kalman filter at each iteration. Five hundred thousand draws are initially taken, and the first 50% are discarded as a burn-in.

3.1. Prior Distributions of the Parameters

Table 1 summarizes the priors used in the estimation. The main parameters of interest are the proportion of type A agents (α_A), the proportion of type B agents (α_B), and the three AL gains (gn_A, gn_B, and gn_C). Because one of the main goals of the analysis is to find estimates of these parameters, I assign them relatively diffuse priors and let the data decide. Therefore, the prior distributions for the proportion of type A agents and the proportion of type B agents are Uniform (0,1), with the restriction that $\alpha_A + \alpha_B \leq 1$. The prior distributions for the AL gains are Uniform (0,0.15).[5]

Distributions for the other parameters are set in a manner consistent with the literature. Similar to An and Schorfheide (2007), the power utility paramater (σ) is given a Gamma distribution with a mean of 2 and a standard deviation of 0.5. Following Milani (2007), the discount factor (β) follows a beta distribution with a mean of 0.99 and a standard deviation of 0.01, while normal distributions are used for both of the interest rate policy parameters (i_π and i_x); i_π is given a mean of 1.5 and a standard deviation of 0.25 and i_x has a mean of 0.5 and a standard deviation of 0.25. The slope of the Phillips curve (κ) is a Uniform distribution with the interval (0,1), as in Herbst and Schorfheide (2016). Inverse Gamma distributions with means of 1 and standard deviations of 0.5 are used for the standard deviations of the shock process innovations (σ_x, σ_π, and σ_i), while Uniform distributions in the interval (0,0.97) are used for the autoregressive parameters ρ_x and ρ_π, as in Milani (2007).

TABLE 1. Prior distributions

Description	Parameter	Range	Prior dist.	Prior mean	Prior std. dev.
Demand shock	σ_x	$\mathbb{R}^+$	Inverse Gamma	1	0.5
Supply shock	σ_π	$\mathbb{R}^+$	Inverse Gamma	1	0.5
Monetary policy (MP) shock	σ_i	$\mathbb{R}^+$	Inverse Gamma	1	0.5
Power utility	σ	$\mathbb{R}^+$	Gamma	2	0.5
Discount factor	β	[0,1]	Beta	0.99	0.01
Phillips curve slope	κ	$\mathbb{R}^+$	Uniform	0.5	0.29
Feedback output gap	i_x	$\mathbb{R}$	Normal	0.5	0.25
Feedback inflation	i_π	$\mathbb{R}$	Normal	1.5	0.25
AR demand shock	ρ_x	(0,0.97)	Uniform	0.485	0.28
AR supply shock	ρ_π	(0,0.97)	Uniform	0.485	0.28
Gain coeff. agent A	gn_A	$\mathbb{R}^+$	Uniform	0.075	0.04
Gain coeff. agent B	gn_B	$\mathbb{R}^+$	Uniform	0.075	0.04
Gain coeff. agent C	gn_C	$\mathbb{R}^+$	Uniform	0.075	0.04
Proportion agent A	α_A	[0,1]	Uniform	0.5	0.29
Proportion agent B	α_B	[0,1]	Uniform	0.5	0.29

4. EMPIRICAL RESULTS

Table 2 displays the posterior means, as well as the 5th and 95th percentiles (i.e., 90% credible intervals) of the posterior distribution of each parameter for the three time periods.

The degree of expectational heterogeneity is about 44% type A (α_A), 15% type B (α_B), and 41% type C (α_C) in the pre-Great Moderation period and about 39% type A, 24% type B, and 37% type C in the Great Moderation period. These findings suggest that more than one agent type was prevalent in those periods, which implies a significant amount of expectational heterogeneity in the data.[6]

The estimates of the AL gains (gn_A, gn_B, and gn_C) in the pre-Great Moderation and Great Moderation periods show that agent type A was using the least amount of historical data in its CGL algorithm (i.e., it had the largest gain) compared to the other agent types. Agent type B has a gain that decreases in the Great Moderation period relative to the pre-Great Moderation period. Interestingly, type B's gain is nearly zero in the Great Moderation period, implying that its beliefs were nearly constant.

Turning to the other parameters, the estimates of the standard deviation of the demand shock (σ_x), the standard deviation of the supply shock (σ_π), power utility parameter (σ), and the coefficient on the expected output gap in the monetary policy rule (i_x) show some variability between the pre-Great Moderation and Great Moderation periods, while the other estimates are relatively stable.

The estimated coefficients of the monetary policy rule are of some interest. Note that the estimated coefficient on expected inflation in the monetary policy rule (i_π) is larger in the pre-Great Moderation period than it is in the Great

TABLE 2. Posterior estimates

Description	Parameter	Pre-Great Moderation Posterior mean	Pre-Great Moderation 90% post. cred. interval	Great Moderation Posterior mean	Great Moderation 90% post. cred. interval	Pre-Great Recession Posterior mean	Pre-Great Recession 90% post. cred. interval
SD demand shock	σ_x	0.627	[0.467,0.826]	0.931	[0.823,1.039]	0.505	[0.417,0.613]
SD supply shock	σ_π	1.257	[1.069,1.464]	0.940	[0.849,1.011]	1.010	[0.902,1.126]
SD MP shock	σ_i	0.811	[0.676,0.958]	0.796	[0.682,0.938]	1.156	[1.021,1.307]
Power utility	σ	1.385	[1.029,1.822]	1.049	[0.931,1.174]	1.256	[1.061,1.491]
Discount factor	β	0.990	[0.970,0.999]	0.989	[0.971,0.999]	0.990	[0.972,1.000]
Phillips curve slope	κ	0.952	[0.869,0.997]	0.974	[0.933,0.999]	0.982	[0.947,0.999]
Feedback output gap	i_x	0.709	[0.465,0.924]	0.210	[0.030,0.549]	0.778	[0.589,0.986]
Feedback inflation	i_π	1.600	[1.435,1.777]	1.464	[1.363,1.601]	1.805	[1.652,1.962]
AR demand shock	ρ_x	0.937	[0.892,0.967]	0.959	[0.944,0.969]	0.949	[0.919,0.968]
AR supply shock	ρ_π	0.830	[0.751,0.906]	0.903	[0.871,0.938]	0.893	[0.837,0.945]
Gain coeff. agent A	gn_A	0.069	[0.022,0.133]	0.089	[0.074,0.102]	0.015	[0.008,0.023]
Gain coeff. agent B	gn_B	0.042	[0.000,0.116]	0.000	[0.000,0.000]	0.065	[0.003,0.142]
Gain coeff. agent C	gn_C	0.030	[0.012,0.057]	0.025	[0.015,0.040]	0.054	[0.008,0.098]
Proportion agent A	α_A	0.435	[0.199,0.608]	0.385	[0.364,0.409]	0.640	[0.537,0.711]
Proportion agent B	α_B	0.150	[0.035,0.296]	0.244	[0.229,0.258]	0.037	[0.002,0.120]

Moderation period, which seems at odds with a long line of literature that suggests monetary policy was less aggressive with respect to inflation in the pre-Great Moderation period than it was in the Great Moderation period.[7] Specifically, Milani (2008), analyzing a small-scale New Keynesian model with homogeneous agent AL, finds that the estimated coefficient on expected inflation in the monetary policy rule is smaller in the pre-Great Moderation period than it is in the Great Moderation period. The source of the conflict between that author's result and the result here is unclear.[8] However, there is no conflict with respect to the estimates of the coefficient on the expected output gap in the monetary policy rule; in both Milani (2008) and the current paper, the estimate in the pre-Great Moderation period is larger than the estimate in the Great Moderation period.

The estimates for the pre-Great Recession period provide an opportunity to compare the results of the HE model with those of Milani (2007), who estimates a small-scale New Keynesian model with a homogeneous adaptively learning agent that employs a PLM of the same form as the MSV REE solution. The agent in that paper is similar to the type A agent in the current paper. The estimate of the AL gain of 0.0183 found by that author is quite comparable to the estimate of the AL gain for agent type A of 0.015. Furthermore, Milani (2007) finds an implied power utility parameter of 1.33, which is similar to the current estimate of σ of 1.26.[9] Moreover, the estimates for the discount factor (β), monetary policy rule coefficient on the output gap, and autoregressive coefficient of the supply shock process (ρ_π) are similar to those in Milani (2007). The other comparable parameter estimates show significant differences, which is probably attributable to the differences in the structures of the two models and the time periods of the data used in the estimations.[10,11]

4.1. Model Comparison

Bayesian criteria are used to compare two variants of the heterogeneous expecations model. The first is a restricted version of the HE model in that all agents are of type A, so it is a model of homogeneous agent AL and is somewhat comparable to the model estimated in Milani (2007) (see footnote 10).[12] The second version of the model is the HE model with all three agent types described in Section 2. Table 3 displays the results.

The HE model generates a larger log-marginal likelihood than the AL model for both sub-periods and the full sample, suggesting it is a better fit of the data. Moreover, the Bayes factor for the pre-Great Moderation and pre-Great Recession periods are both over 14, which is decisive support in favor of the HE model. The Bayes factor of 0.22 in the Great Moderation period provides weak evidence in favor of the HE model.[13]

4.2. Economic Intuition and Forecast Performance

To shed some economic light on the estimation results, note that in the pre-Great Moderation period the estimate for the standard deviation of the supply shock is

TABLE 3. Model comparison

	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
	AL	HE	AL	HE	AL	HE
Log-marginal likelihood	−551.6	−518.5	−531.4	−530.9	−1123.7	−1090.1
Log Bayes factor		14.4		0.22		14.6

Notes: AL is the model with homogeneous agent AL and HE is the model with heterogeneous expectations. The log-marginal likelihood is calculated using Geweke's modified harmonic mean approximation. The Bayes factor compares the HE model to the AL model and is in $\log_{10}$ terms.

significantly larger than that of the demand shock. Intuitively, this suggests that supply shocks were relatively more important drivers of economic activity than were demand shocks during the pre-Great Moderation period, which can at least partially explain the relatively low proportion of type B agents in that period. Note also that in the Great Moderation period, the size differential between the standard deviations of the supply and demand shocks is much smaller than in the pre-Great Moderation period, implying an increased importance of demand shocks relative to supply shocks in driving economic activity, which may explain the relatively larger proportion of type B agents and lower proportion of type C agents found in that period compared to the pre-Great Moderation period.

Further intuition can be discerned by analyzing the relative forecast performance of the three agent types. Table 4 shows the mean squared forecast error (MSE) for the agent type forecasts for the two subperiods and the full sample. To calculate these numbers, the model parameters were set to the estimates in Table 2, the initial values of the AL beliefs were set to the HEE solution values, and values of agent forecasts were calculated at each iteration of the Kalman filter.

In the pre-Great Moderation period, agent type A is the best forecaster of both the output gap and inflation rate. Agent type B is better at forecasting inflation than it is at forecasting the output gap, while type C is better at forecasting the output gap than it is at forecasting inflation. Furthermore, since type B's MSE for inflation is lower than type C's and type C's MSE for the output gap is lower than type B's, it can be concluded that the demand shock was a relatively better predictor of inflation (than the supply shock) and the supply shock was a relatively better predictor of the output gap (than the demand shock) during the pre-Great Moderation period.

In the Great Moderation period, agent type A is the best forecaster of inflation but forecasts the output gap slightly worse than type C. Type B is the worst forecaster of the three, mainly because its AL gain is very small, which leads to high persistence in its beliefs and apparently poor forecasts. The fact that type C forecasts the output gap slightly better than type A suggests that the supply shock was a particularly good predictor of that variable during the Great Moderation period.[14]

In the pre-Great Recession period, agent type A is the best forecaster of inflation. Type B is better at forecasting inflation than type C, but type C is better

TABLE 4. Forecast performance

	Pre-Great Moderation					
	Agent type A		Agent type B		Agent type C	
	Output gap	Inflation	Output gap	Inflation	Output gap	Inflation
Mean squared error	1.63	1.84	4.95	3.30	2.06	6.84
	Great Moderation					
	Agent type A		Agent type B		Agent type C	
	Output gap	Inflation	Output gap	Inflation	Output gap	Inflation
Mean squared error	1.39	1.54	8.19	21.66	1.34	3.67
	Pre-Great Recession					
	Agent type A		Agent type B		Agent type C	
	Output gap	Inflation	Output gap	Inflation	Output gap	Inflation
Mean squared error	1.98	2.07	4.98	2.42	1.87	5.20

at forecasting the output gap than the other two. This suggests that the demand shock was a relatively better predictor of inflation (than the supply shock) and that the supply shock was a relatively better predictor of the output gap (than the demand shock) during this period.

The fact that the demand shock was an important predictor of inflation in the pre-Great Moderation and pre-Great Recessions periods is an interesting finding that can be at least somewhat explained by the relatively large estimated value of the slope of the New Keynesian Phillips curve (κ) in those periods (0.952 and 0.982, respectively).[15] The fact that the supply shock was an important predictor of the output gap in both subperiods and the full sample can be explained by the relatively large estimated values of the coefficient on expected inflation in the monetary policy rule (i_π).[16] The fact that both the supply shock and the demand shock have some explanatory power may explain why the estimation results show significant expectational heterogeneity. As a consequence, it is not surprising that the HE model fits the data better than a model with homogeneous agent AL.

In summary, the empirical results suggest two main conclusions. First, there is significant expectational heterogeneity in the data, as evidenced by the high prevalence of more than one agent type in both subperiods and the full sample. Second, the HE model fits the data better than the homogeneous agent AL model.

5. CONCLUSION

This paper empirically analyzes a small-scale New Keynesian model with HE. The structure of the New Keynesian model is such that the MSV REE solution contains two variables, an AD shock and an AS shock. Agents employ the Euler equation AL mechanism of Evans and Honkapohja (2001), where they behave like econometricians who estimate the parameters of their model and use the model to make forecasts. Expectational heterogeneity arises by allowing three different agent types that are similar in all ways except in the structure of their forecasting models; type A agents use a model which is consistent with the MSV REE solution, type B agents use a model that only includes the AD shock, and type C agents use a model that only includes the AS shock.

The relevant equilibrium concept in the model is a HEE. The paper derives the equilibrium's uniqueness and stability criteria and uses Bayesian methods with quarterly US data to estimate the model parameters for the pre-Great Moderation, Great Moderation, and pre-Great Recession periods. Results show that, for both subsamples and the full sample, there is significant expectational heterogeneity, as seen by the relatively high prevalence of more than one agent type, and that the HE model fits that data better than a model with homogeneous agent AL.

The results of this paper add to the body of evidence that suggests that expectational heterogeneity is important in macroeconomic models. With this in mind, future work should empirically evaluate HE in a more state-of-the-art medium-scale New Keynesian model.

NOTES

1. A similar setup is found in Elias (2016), although in that paper the setting is an exchange rate model and the estimation methodology is not Bayesian.

2. Note that this approach is similar to the one employed in Milani (2007) in that the rational expectations operator is replaced with a non-rational expectations operator.

3. The heterogeneous expectations equilibirum concept is a special case of the restriced perceptions equilibrium concept originally studied in Evans and Honkapohja (2001).

4. See Evans and Honkapohja (2001) for more details regarding convergence of constant gain AL algorithms.

5. The upper bounds for the learning gains are set at 0.15 because values greater than these lead to instability in the model in that moment matrices in the AL algorithms become nearly non-singular.

6. These results are somewhat consistent with existing literature. For example, Branch (2004) finds about 48% of inflation forecasters can be considered to be using rational expectations. This is similar to the percentages of type A agents found in the current paper. However, the results are not completely comparable because type A agents are not using rational expectations but can be assumed to be the most rational of the three agent types.

7. Two studies of this issue are Clarida et al. (2000) and Lubik and Schorfheide (2004); however, neither of those studies considers a model with AL.

8. The result is not robust to other variants of the model, as explained in Appendix A.

9. Under rational expectations, Giannoni and Woodford (2004) and Milani (2007) find values for σ of 0.66 and 3.18, respectively. Note that the elasticity of intertemporal substitution in Milani (2007) is equivalent to $1/\sigma$ in the current paper.

10. The model in Milani (2007) includes habit formation and inflation indexation, both of which are absent from the model analyzed in the current paper. Another significant difference is that Milani (2007) uses current values of the output gap and inflation in the monetary policy rule. The time span of the data in Milani (2007) is from 1960Q1 to 2004Q2.

11. Note that in Table 2, some parameter estimates in the full sample do not fall in between the estimates for the two subsamples. However, this is not uncommon in empirical research. For example, the same phenomenon occurs in the parameter estimates of Smets and Wouters (2007).

12. The specific parameter restrictions are $\alpha_A = 1$, $\alpha_B = 0$, $gn_B = 0$, and $gn_C = 0$.

13. Interpretation of the Bayes factor is based on Jeffreys' guidelines. See Jeffreys (1961).

14. The estimation and forecasting results in this section imply that in the Great Moderation, a significant fraction of the population (i.e., type B agents) learns very slowly (i.e., has a low AL gain) and becomes worse at forecasting (i.e., higher MSE compared to that of the pre-Great Moderation). This is contrary to evidence from surveys, for example Croushore (2010), which suggests that forecasters got better at forecasting inflation after the 1970s. However, the results of Table 4 show that both agent types A and C got better at forecasting (i.e., lower MSE) in the Great Moderation period compared to the pre-Great Moderation period, so the results are not completely at odds with survey evidence.

15. A value of κ near 1 implies that demand shocks pass through to inflation in an almost one-to-one manner. See equations (1) and (2).

16. This fact was pointed out to me by an anonymous referee.

17. In general, Bayesian estimation of dynamic stochastic general equilibrium models requires that the number of observeables in the state-space measurement equation is less than or equal to the number of stochastic shocks in the model. Because of this, in this robustness exercise the vector of observeables is modified, rather than expanded as it is Milani (2011), since expanding the vector would necessitate adding more stochastic shocks to the model.

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A: ROBUSTNESS

Several robustness checks are performed. Specifically, an alternative measure of the output gap is considered, expectation data from the Survey of Professional Forecasters (SPF) is utilized, and an alternative monetary policy rule is employed.

A.1. ROBUSTNESS—ALTERNATIVE MEASURE OF THE OUTPUT GAP

The first robustness check performed replaces the CBO estimate of potential output with Hodrick–Prescott (HP)-filtered RGDP in calculating the output gap in measurement equation (17). All other aspects of the estimation remain the same as described in Section 3. Table A1 shows the estimation results.

The degree of agent heterogeneity is about 33% type A, 22% type B, and 45% type C in the pre-Great Moderation period, 34% type A, 29% type B, and 37% type C in the Great Moderation period, and 18% type A, 29% type B, and 53% type C in the pre-Great Recession period. Consistent with original results is that agent type B remains the least prevalent agent type, and the fact that the standard deviation of the AS shock is larger than that of the AD shock in the pre-Great Moderation and Great Moderation periods.

The major change in the estimates of the AL gains is that agent type A is no longer using the least amount of data in its CGL algorithm in both subperiods, and the estimates for type A and type C are not as stable between the pre-Great Moderation and Great Moderation periods. Agent type B's gain estimate is near zero in the Great Moderation period, which is consistent with the original result.

TABLE A1. Posterior estimates—HP filtered potential output

Description	Parameter	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
		Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval
SD demand shock	σ_x	0.734	[0.524,0.974]	0.738	[0.613,0.875]	0.961	[0.858,1.045]
SD supply shock	σ_π	1.165	[0.942,1.406]	0.894	[0.772,1.027]	0.943	[0.859,1.036]
SD MP shock	σ_i	0.801	[0.680,0.937]	0.986	[0.835,1.186]	0.959	[0.855,1.097]
Power utility	σ	1.477	[1.146,1.872]	1.477	[1.206,1.831]	1.112	[0.998,1.242]
Discount factor	β	0.989	[0.967,0.999]	0.987	[0.963,0.999]	0.992	[0.981,1.000]
Phillips curve slope	κ	0.888	[0.685,0.994]	0.921	[0.792,0.996]	0.971	[0.938,0.998]
Feedback output gap	i_x	0.991	[0.578,1.351]	0.536	[0.237,0.833]	1.545	[1.381,1.706]
Feedback inflation	i_π	1.416	[1.204,1.600]	1.790	[1.626,1.974]	1.399	[1.259,1.545]
AR demand shock	ρ_x	0.940	[0.896,0.968]	0.946	[0.914,0.968]	0.950	[0.926,0.967]
AR supply shock	ρ_π	0.724	[0.627,0.816]	0.853	[0.760,0.935]	0.824	[0.763,0.884]
Gain coeff. agent A	gn_A	0.112	[0.061,0.147]	0.081	[0.018,0.133]	0.035	[0.016,0.058]
Gain coeff. agent B	gn_B	0.027	[0.000,0.110]	0.000	[0.000,0.001]	0.000	[0.000,0.000]
Gain coeff. agent C	gn_C	0.022	[0.010,0.039]	0.082	[0.055,0.137]	0.018	[0.011,0.024]
Proportion agent A	α_A	0.330	[0.082,0.555]	0.339	[0.272,0.403]	0.177	[0.117,0.215]
Proportion agent B	α_B	0.220	[0.054,0.370]	0.286	[0.250,0.324]	0.289	[0.263,0.312]

TABLE A2. Model comparison—HP-filtered potential output

	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
	AL	HE	AL	HE	AL	HE
Log-marginal likelihood	−535.4	−510.3	−528.7	−518.3	−1116.3	−1073.0
Log Bayes factor		10.9		4.5		18.8

Notes: AL is the model with homogeneous agent AL and HE is the model with heterogeneous agent AL. The log-marginal likelihood is calculated using Geweke's modified harmonic mean approximation. The Bayes factor compares the HE model to the AL model and is in $\log_{10}$ terms.

Turning to the other parameter estimates and comparing them to the original results, the power utility parameter and the coefficient on the expected output gap in the monetary policy rule are significantly larger in the Great Moderation period, the standard deviation of the demand shock and the coefficient on the expected output gap in the monetary policy rule are significantly larger in the pre-Great Recession period, and the coefficient on the expected inflation rate in the monetary policy rule is significantly smaller in the pre-Great Recession period. All other estimates are relatively similar. Note that the coefficient on expected inflation in the monetary policy rule is larger in the Great Moderation period than it is in the pre-Great Moderation period, which conflicts with the original findings.

The model comparison results, shown in Table A2, remain the same in that the HE model fits the data better than the AL model in both subperiods and in the full sample.

A.2. ROBUSTNESS—EXPECTATION DATA

The second robustness check replaces measurement equations (17) and (18) with the following measurement equations:

$$EGAP_t = \hat{E}_t x_{t+1} \tag{A1}$$

$$EINFL_t = \hat{E}_t \pi_{t+1}, \tag{A2}$$

where Appendix B explains how $EGAP_t$ and $EINFL_t$ are constructed from the SPF data. All other aspects of the estimation remain the same as described in Section 3.[17] Table A3 shows the estimation results.

The degree of agent heterogeneity is about 37% type A, 42% type B, and 21% type C in the pre-Great Moderation period, 31% type A, 59% type B, and 10% type C in the Great Moderation period, and 36% type A, 47% type B, and 17% type C in the pre-Great Recession period. The major change compared to the original results is that now type B is no longer the least prevalent agent type, which can probably be explained by the fact that the estimates of the standard deviation of the supply shock are smaller in all sample periods, suggesting an increased importance of the demand shock in driving the output gap and inflation.

The estimates for the AL gains are consistent with the original findings in that agent type A is using the least amount of data in its CGL algorithm in the pre-Great Moderation and Great Moderation periods. The gain for type A is relatively stable between the pre-Great Moderation and Great Moderation periods. Furthermore, agent type B's gain, albeit small, is no longer nearly zero.

TABLE A3. Posterior estimates—SPF data

Description	Parameter	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
		Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval
SD demand shock	σ_x	0.864	[0.536,1.378]	0.571	[0.424,0.753]	0.694	[0.466,1.057]
SD supply shock	σ_π	0.781	[0.472,1.287]	0.781	[0.493,1.209]	0.585	[0.376,0.861]
SD MP shock	σ_i	1.876	[1.577,2.239]	1.593	[1.428,1.776]	2.051	[1.875,2.246]
Power utility	σ	1.904	[1.200,2.797]	2.230	[1.204,3.485]	1.735	[0.962,2.832]
Discount factor	β	0.987	[0.965,0.999]	0.989	[0.966,0.999]	0.989	[0.967,0.999]
Phillips curve slope	κ	0.040	[0.002,0.124]	0.017	[0.001,0.043]	0.027	[0.003,0.069]
Feedback output gap	i_x	0.442	[0.290,0.601]	0.367	[0.217,0.526]	0.434	[0.325,0.548]
Feedback inflation	i_π	1.012	[0.809,1.209]	1.637	[1.507,1.762]	1.228	[1.102,1.352]
AR demand shock	ρ_x	0.859	[0.762,0.946]	0.801	[0.707,0.894]	0.855	[0.796,0.920]
AR supply shock	ρ_π	0.895	[0.802,0.961]	0.957	[0.933,0.969]	0.948	[0.921,0.968]
Gain coeff. agent A	gn_A	0.083	[0.011,0.143]	0.100	[0.032,0.147]	0.118	[0.069,0.147]
Gain coeff. agent B	gn_B	0.026	[0.001,0.086]	0.007	[0.001,0.018]	0.009	[0.000,0.032]
Gain coeff. agent C	gn_C	0.072	[0.007,0.141]	0.044	[0.001,0.144]	0.027	[0.000,0.095]
Proportion agent A	α_A	0.372	[0.164,0.612]	0.306	[0.217,0.415]	0.357	[0.218,0.518]
Proportion agent B	α_B	0.424	[0.261,0.605]	0.589	[0.476,0.706]	0.472	[0.353,0.585]

TABLE A4. Model comparison—SPF data

	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
	AL	HE	AL	HE	AL	HE
Log-marginal likelihood	−240.8	−232.7	−463.0	−458.2	−735.6	−707.1
Log Bayes factor		3.5		2.1		12.4

Notes: AL is the model with homogeneous agent AL and HE is the model with heterogeneous agent AL. The log-marginal likelihood is calculated using Geweke's modified harmonic mean approximation. The Bayes factor compares the HE model to the AL model and is in $\log_{10}$ terms.

Comparing the other estimates to the original results, the Phillips curve slope is significantly smaller in both subperiods, while the standard deviation of the monetary policy shock and the power utility parameter are both significantly larger in both subperiods. All other parameter estimates are relatively similar to the original findings. Note, again, that the estimated coefficient on expected inflation in the monetary policy rule is larger in the Great Moderation period, compared to that of the pre-Great Moderation period, which conflicts with the original results.

With respect to model comparison, the result that the HE model fits the data better than the AL model in both subperiods and the full sample continues to hold, as shown in Table A4.

A.3. ROBUSTNESS—ALTERNATIVE MONETARY POLICY RULE

Consider the contemporaneous interest rate rule,

$$i_t = i_x x_t + i_\pi \pi_t + \varepsilon_t^i. \tag{A3}$$

As is apparent from equation (1), the current period output gap is dependent (partly) on the current period nominal interest rate. However, monetary policy rule (A3) implies that the monetary authority uses the current period output gap to (partly) determine the current period nominal interest rate. This creates a simultaneity problem which makes monetary policy rule (A3) non-operational. However, its use does provide an opportunity to check for robustness. With this mind, the last robustness check is to replace equation (3) with (A3) and leave all other aspects of the model and estimation the same as in Sections 2 and 3. Table A5 provides the estimation results.

The degree of agent heterogeneity is about 7% type A, 35% type B, and 58% type C in the pre-Great Moderation period, about 16% type A, 4% type B, and 80% type C in the Great Moderation period, and about 2% type A, 95% type B, and 3% type C in the pre-Great Recession period. These results contradict the original result that type A was the most prevalent in the pre-Great Moderation and Great Moderation periods. Moreover, the relatively large proportion of type B agents in the pre-Great Recession period is much different than what was found in the original results. Therefore, it can be concluded that the proportion of agent types is relatively sensitive to specification of the monetary policy rule.

The estimates of the AL gains show that agent type A is utilizing the least amount of data in its CGL algorithm in the Great Moderation and pre-Great Recession periods and that

TABLE A5. Posterior estimates—alternative monetary policy rule

		Pre-Great Moderation		Great Moderation		Pre-Great Recession	
Description	Parameter	Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval	Posterior mean	90% post. cred. interval
SD demand shock	σ_x	0.880	[0.666,1.127]	0.897	[0.683,1.153]	1.999	[1.598,2.461]
SD supply shock	σ_π	1.353	[1.012,1.698]	0.725	[0.614,0.842]	1.661	[1.532,1.802]
SD MP shock	σ_i	1.665	[1.463,1.899]	2.655	[2.344,3.008]	3.856	[3.402,4.406]
Power utility	σ	1.459	[1.003,2.001]	1.552	[1.116,2.062]	0.237	[0.183,0.301]
Discount factor	β	0.989	[0.968,0.999]	0.990	[0.969,0.999]	0.985	[0.956,0.999]
Phillips curve slope	κ	0.887	[0.651,0.995]	0.966	[0.901,0.998]	0.983	[0.949,0.999]
Feedback output gap	i_x	0.418	[0.301,0.546]	0.449	[0.241,0.671]	0.867	[0.668,1.099]
Feedback inflation	i_π	1.043	[0.998,1.123]	1.943	[1.750,2.138]	1.785	[1.604,1.978]
AR demand shock	ρ_x	0.916	[0.858,0.963]	0.940	[0.899,0.967]	0.949	[0.922,0.968]
AR supply shock	ρ_π	0.819	[0.728,0.904]	0.927	[0.875,0.965]	0.798	[0.731,0.863]
Gain coeff. agent A	gn_A	0.078	[0.009,0.143]	0.087	[0.024,0.142]	0.085	[0.011,0.145]
Gain coeff. agent B	gn_B	0.106	[0.039,0.147]	0.076	[0.007,0.144]	0.081	[0.059,0.107]
Gain coeff. agent C	gn_C	0.052	[0.013,0.127]	0.001	[0.000,0.003]	0.072	[0.008,0.138]
Proportion agent A	α_A	0.069	[0.004,0.196]	0.163	[0.010,0.330]	0.023	[0.001,0.075]
Proportion agent B	α_B	0.346	[0.149,0.526]	0.038	[0.002,0.129]	0.949	[0.875,0.991]

TABLE A6. Model comparison—alternative monetary policy rule

	Pre-Great Moderation		Great Moderation		Pre-Great Recession	
	AL	HE	AL	HE	AL	HE
Log-marginal likelihood	−555.6	−535.5	−532.2	−523.8	−1126.5	−1106.2
Log Bayes factor		8.7		3.6		8.8

Notes: AL is the model with homogeneous agent AL and HE is the model with heterogeneous agent AL. The log-marginal likelihood is calculated using Geweke's modified harmonic mean approximation. The Bayes factor compares the HE model to the AL model and is in $\log_{10}$ terms.

type C's gain is not very stable between the pre-Great Moderation and Great Moderation periods. Note also that type B's gain is not nearly zero in any of the periods. These results suggest that the estimates of the AL gains are sensitive to the monetary policy rule.

Comparing the estimates of the other parameters to the original results, the main differences, unsurprisingly, are the standard deviation of the monetary policy shock and the coefficients on the monetary policy rule. The standard deviations of the demand shock and supply shock, as well as the power utility parameter, are significantly different in the pre-Great Recession period. All other estimates are relatively similar to the original results. Furthermore, note that the estimate of the coefficient on inflation in the monetary policy rule is larger in the Great Moderation period than it is in the pre-Great Moderation period, which, again, conflicts with the original results.

Lastly, Table A6 shows the model comparison results and supports the original result that the HE model fits the data better than the AL model.

A.4. ROBUSTNESS SUMMARY

The two main results found in the original estimation are

1. There is a significant degree of expectational heterogeneity in the data
2. The HE model fits the data better than the AL model

These results continue to hold in all of the alternative specifications considered in this section, with the caveat that the degree of agent heterogeneity in the pre-Great Recession period with the alternative monetary policy rule is somewhat less pronounced than what was found in the original estimation.

B. DATA DESCRIPTION

The data used in the measurement equations are quarterly observations of the US output gap (GAP_t), inflation ($INFL_t$), and the interest rate (INT_t).

The output gap is calculated as

$$GAP_t = (\ln RGDP_t - \ln PRGDP_t) \times 100,$$

where $RGDP$ is real gross domestic product (GDP) and $PRGDP$ is potential real GDP.

TABLE B1. Data ID and source

Series	Data ID	Source
RGDP	GDPC1	Federal Reserve Economic Data (FRED)
PRGDP(*CBO*)	GDPPOT	Federal Reserve Economic Data (FRED)
PRGDP(*HP*)		HP-Filtered Real GDP (*RGDP*)
GDPDEF	GDPDEF	Federal Reserve Economic Data (FRED)
FEDFUNDS	FEDFUNDS	Federal Reserve Economic Data (FRED)
*NGDP*2	NGDP2	Survey of Professional Forecasters (SPF)
*PGDP*2	PGDP2	Survey of Professional Forecasters (SPF)
*PGDP*3	PGDP3	Survey of Professional Forecasters (SPF)

The inflation rate is annualized inflation and is calculated as

$$INFL_t = ((1 + (\ln GDPDEF_t - \ln GDPDEF_{t-1}))^4 - 1) \times 100,$$

where *GDPDEF* is the implicit price deflator for GDP.

The interest rate is the effective federal funds rate.

The forecast data are obtained from the SPF (mean across forecasters). The expected one-period ahead nominal GDP is the one-quarter-ahead forecast for the quarterly level of nominal GDP at an annual rate (*NGDP*2), the expected one-period ahead price level is the one-quarter-ahead forecast for the chain-weighted GDP price index (*PGDP*2), and the expected two-period ahead price level is the two-quarter-ahead forecast for the chain-weighted GDP price index (*PGDP*3). Due to data availability, the pre-Great Moderation and pre-Great Recession periods start in 1968Q1.

The expected one-period-ahead real GDP (*ERGDP*) is calculated as

$$ERGDP_t = \frac{NGDP2_t}{PGDP2_t},$$

where *PGDP*2 is transformed into the appropriate base-year value. The expected potential real GDP (*EPRGDP*) is found by applying the HP filter to the *ERGDP* series. The expected-one-period ahead output gap is calculated as

$$EGAP_t = (\ln ERGDP_t - \ln EPRGDP_t) \times 100.$$

The expected one-period-ahead inflation rate (*EINFL*) is calculated as

$$EINFL_t = ((1 + (\ln PGDP3_t - \ln PGDP2_t))^4 - 1) \times 100.$$

All data are demeaned before the estimation. Table B1 gives the source and identification code of the data.