Schaum's Outline of Abstract Algebra Notes and Exercises

Jasper Ty

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Part I

Sets and Relations

Chapter 1

Sets

I don't have much to write here as far as notes go.

Exercises

Q1.1. Exhibit in tabular form:

(a) $A = \{a : a \in \mathbb{N}, 2 < a < 6\}$

(b) $B = \{p : p \in \mathbb{N}, p < 10, p \text{ is odd}\}$

(c) $C = \{x : x \in \mathbb{Z}, 2x^2 + x - 6 = 0\}$

A1.1.

(a) $A = \{3, 4, 5\}$

(b) $B = \{9, 7, 5, 3, 1\}$

(c) $C = \{-2\}$

Q1.2. Let $A = \{a, b, c, d\}$, $B = \{a, c, g\}$, $C = \{c, g, m, n, p\}$. Then $A \cup B = \{a, b, c, d, g\}$, $A \cup C = \{a, b, c, d, g, m, n, p\}$, $B \cup C = \{a, c, g, m, n, p\}$;

A1.2. WHAT IS THE QUESTION?

Q1.3. Consider the subsets $K = \{2, 4, 6, 8\}$, $L = \{1, 2, 3, 4\}$, $M = \{3, 4, 5, 6, 8\}$ of $U = \{1, 2, 3, \ldots, 10\}$.

(a) Exhibit K', L', M' in tabular form.

(b) Show that $(K \cup L)' = K' \cap L'$

A1.3.

(a) $K' = \{1, 3, 5, 7, 9, 10\}$
$L' = \{5, 6, 7, 8, 9, 10\}$
$U' = \{1, 2, 7, 9, 10\}$

(b) $K \cup L = \{1, 2, 3, 4, 6, 8\}$, so $(K \cup L)' = \{5, 7, 9, 10\}$. Using the above, $K' \cap L' = \{5, 7, 9, 10\}$.

Q1.4.
Q1.5.

Q1.6.

Q1.7.

Q1.8. Prove $(A \cup B) \cup C = A \cup (B \cup C)$

A1.8. Any element x belongs to the left hand side if $(x \in A \vee x \in B) \vee x \in C$. It belongs to the right hand side if $x \in A \vee (x \in B \vee x \in C)$. Both expressions are logically equivalent.

Q1.9. Prove $(A \cap B) \cap C = A \cap (B \cap C)$

A1.9. Similar answer as the previous– follows from the associativity of $\wedge$

Q1.10. Prove $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

A1.10. Follows from $\wedge$ distributing over $\vee$

Q1.11. Prove $(A \cap B)' = A' \cap B'$

A1.11. Follows from DeMorgan's laws

Q1.12. Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

A1.12. Follows from $\vee$ distributing over $\wedge$

Q1.13. Prove $A - (B \cup C) = (A - B) \cap (A - C)$

A1.13. Follows again from DeMorgan's laws

Chapter 2

Relations and Operations

Part II

Number Systems

Chapter 3

The Natural Numbers

Chapter 4

The Integers

Chapter 5

Some Properties of Integers

Chapter 6

The Rational Numbers

Chapter 7

The Real Numbers

Chapter 8

The Complex Numbers

Part III

Groups, Rings, and Fields

Chapter 9

Groups

9.1 Groups

DEFINITION 9.1: A non-empty set $\mathcal{G}$ equipped with a binary operation $\circ$ is a *group* if

P$_1$: $(a \circ b) \circ c = a \circ (b \circ c)$ (Associativity)

P$_2$: There exists an element $1 \in \mathcal{G}$ such that $1 \circ a = a \circ 1 = a$ for all $a \in \mathcal{G}$ (Unit)

P$_3$: For all $a \in \mathcal{G}$, there exists an element a^{-1} such that $a \circ a^{-1} = a^{-1} \circ a = 1$ (Inverse)

9.2 Simple Properties of Groups

THEOREM I. (Left Cancellation) Let $a, b, c \in \mathcal{G}$. Then $a \circ b = a \circ c$ implies $b = c$.

THEOREM II. (Latin Square Property)

Chapter 10

Further Topics on Group Theory

Chapter 11

Rings

Chapter 12

Integral Domains, Division Rings, and Fields

Chapter 13

Polynomials

Chapter 14

Vector Spaces

Chapter 15

Matrices

Chapter 16

Matrix Polynomials

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Linear Algebras

Chapter 18

Boolean Algebras