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A1.1.1:

(a) $f(-1) = -2$

(b) $f(2) \approx 1.8$

(c) $f(x) = 2$ at $x = 1$ and $x = -3$

(d) $f(x) = 0$ at $x \approx -2.5$ and $x \approx 0.4$

(e) The domain of f is $[-3, 3]$ and the range of f is $[-2, 3]$

(f) f is increasing on $[-1, 3]$

A1.1.2:

(a) $f(-4) = -2$ and $g(3) = 4$

(b) $f(x) = g(x)$ at $x = -2$ and $x = 2$

(c) $f(x) = -1$ at $x = -3$ and $x = 4$

(d) f is decreasing on $[0, 4]$

(e) The domain of f is $[-4, 4]$ and the range of f is $[-2, 3]$

(f) The domain of g is $[-4, 3]$ and the range of g is $[0.5, 4]$

A1.1.3: The range of the vertical ground acceleration function is about $[-70, 120]$
The range of the north-south ground acceleration function is about $[-300, 420]$
The range of the east-west ground acceleration function is about $[-200, 200]$

A1.1.4: Skip

A1.1.5: Is not a function. Fails the vertical line test.

A1.1.6: Is a function. The domain is $[-2, 2]$ and the range is $[-1, 2]$

A1.1.7: Is a function. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$

A1.1.8: Is not a function. Fails the vertical line test.

A1.1.9: Skip

A1.1.10: Skip

A1.1.11: Skip

A1.1.12: Skip

A1.1.13: Skip

A1.1.14: Skip

A1.1.15: Skip

A1.1.16: Skip

A1.1.17: Skip

A1.1.18: Skip

A1.1.19:

- $f(2) = 3(2)^2 - 2 + 2 = 3 \cdot 4 = 12$
- $f(-2) = 3(-2)^2 - (-2) + 2 = 3 \cdot 4 + 4 = 16$
- $f(a) = 3a^2 - a + 2$
- $f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2$
- $f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a + 3 = 3a^2 + 5a + 6$
- $2f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4$
- $f(2a) = 3(2a)^2 - 2a + 2 = 3 \cdot 4a^2 - 2a + 2 = 12a^2 - 2a + 2$
- $f(a^2) = 3(a^2)^2 - 2a^2 + 2 = 3a^4 - 2a^2 + 2$
- $[f(a)]^2 = [3a^2 - a + 2]^2 = 9a^4 + a^2 + 4 - 3a^3 + 6a^2 - 2a = 9a^4 + 7a^2 - 3a^3 - 2a + 4$
- $f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 - 2ah + h^2 - a - h + 2$

A1.1.20: Let $f(r) := V(r+1) - V(r)$

$$\begin{aligned} f(r) &= \frac{4}{3}\pi(r+1)^3 - \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi\Big((r+1)^3 - r^3\Big) \\ &= \frac{4}{3}\pi\Big(r^3 + 3r^2 + 3r + 1 - r^3\Big) \\ &= \frac{4}{3}\pi\Big(3r^2 + 3r + 1\Big) \end{aligned}$$

A1.1.21:

$$\begin{aligned} f(2+h) &= (2+h) - (2+h)^2 \\ &= 2 + h - 4 + 4h + h^2 \\ &= h^2 + 5h - 2 \end{aligned}$$

$$\begin{aligned} f(x+h) &= (x+h) - (x+h)^2 \\ &= x + h - x^2 + 2hx + h^2 \\ &= h^2 + h + 2hx - x^2 + x \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(h^2 + h + 2hx - x^2 + x) - (x - x^2)}{h} \\ &= \frac{h^2 + h + 2hx}{h} \\ &= h + 1 + 2x \end{aligned}$$

A1.1.22:

$$\begin{aligned} f(2+h) &= \frac{2+h}{2+h+1} \\ &= \frac{2+h}{3+h} \end{aligned}$$

$$f(x+h) = \frac{x+h}{x+h+1}$$

$$\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\frac{x+h}{x+h+1} - \frac{x}{x+1}}{h} \\
&= h^{-1}\left(\frac{(x+h)(x+1)}{(x+h+1)(x+1)} - \frac{x(x+h+1)}{(x+h+1)(x+1)}\right) \\
&= h^{-1}\left(\frac{x^2+hx+x+h-x^2-hx-x}{x^2+hx+x+x+h+1}\right) \\
&= h^{-1}\left(\frac{h}{x^2+hx+2x+h+1}\right) \\
&= \frac{1}{x^2+2x+hx+h+1}
\end{aligned}$$

A1.1.23: The domain of f is $\mathbb{R} - \{1/3\}$

A1.1.24: The domain of f is $\mathbb{R} - \{-2, -1\}$

A1.1.25: The domain of f is $[0, \infty)$

A1.1.26: The domain of g is $[0, 4]$

A1.1.27: The domain of h is all x for which $x^2 - 5x > 0$, so either $x < 0$ or $x > 5$. The domain is $(-\infty, 0) \cup (5, \infty)$

A1.1.28: The domain of this function is $[-2, 2]$, the range of this function is $[0, 2]$. I can't sketch it, but the graph is the upper half of a circle of radius 2 centered at the origin.

A1.1.29: Domain: $\mathbb{R}$

A1.1.30: Domain: $\mathbb{R}$

A1.1.31: Domain: $\mathbb{R}$

A1.1.32: Domain: $\mathbb{R} - \{2\}$

A1.1.33: Domain: $[5, \infty)$

A1.1.34: Domain: $\mathbb{R}$

A1.1.35: Domain: $\mathbb{R} - \{0\}$

A1.1.36: Domain: $\mathbb{R} - \{0\}$

A1.1.37: Domain: $\mathbb{R}$

A1.1.38: Domain: $\mathbb{R}$

A1.1.39: Domain: $\mathbb{R}$

A1.1.40: Domain: $\mathbb{R}$

A1.1.41: The slope is $(-6-1)/4-(-2) = -7/6$. The domain is $[-2, 4]$. So

$$f(x) = -7/6(x+2)+1 \qquad -2 \leq x \leq 4$$

A1.1.42:

$$f(x) = -5/9(x+3)+1 \qquad -3 \leq x \leq 6$$

A1.1.43:

$$f(x) = -\sqrt{-x} + 1 \qquad x \leq 0$$

A1.1.44:

$$f(x) = +\sqrt{(1 - (x-1)^2}$$

A1.1.45:

$$f(x) = \begin{cases} x+1 & -1 \leq x < 2 \\ -\frac{3}{2}(x-2) & 2 \leq x \leq 4 \end{cases}$$

A1.1.46:

$$f(x) = \begin{cases} -2x+2 & 0 \leq x < 1 \\ x-1 & 1 \leq x \end{cases}$$

A1.1.47: Let l denote length and w denote width. Then a perimeter of 20 means

$$20 = 2l + 2w$$

The area of a rectangle is $A = lw$. We eliminate w by solving for w in the constraint above. We have that $w = (20 - 2l)/2 = 10 - l$. So our function is

$$f(l) = l(10 - l)$$

A1.1.48: Same as above, but solve for w using the area constraint: $w = 16/l$. Then

$$f(l) = 2l + 2\frac{16}{l} = 2l + \frac{32}{l}$$

A1.1.49: The area of any triangle is the length of a side times the length of the altitude whose base is that side, divided by two. Let the length of a side be s. By symmetry, any altitude will bisect a side. Hence an altitude's height can be found by the Pythagorean theorem, with side lengths s and $s/2$. The relationship between s and h is then

$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

So

$$h = \sqrt{s^2 - \frac{s^2}{4}} = \sqrt{\frac{3s^2}{4}} = \frac{s}{2}\sqrt{3}$$

Then the area of the triangle is

$$f(s) = \frac{sh}{2} = \frac{s^2}{4}\sqrt{3}$$

A1.1.50: The volume of a cube with edge length s is $V = s^3$. The surface area is $6s^2$. We have that

$$6s^2 = 6 \cdot (s^3)^{\frac{2}{3}}$$

So

$$f(V) = 6V^{\frac{2}{3}}$$

A1.1.51: Let s be the length of a side of the base. Then its volume is $s^2 \cdot h$, where h is the height of the box, so we have that

$$2 = s^2 h$$

The surface of this box, since it is open topped, is $s^2 + 4sh$. We can write h in terms of s by solving the volume constraint for it. This gives us $h = 2/s^2$. Then

$$f(s) = s^2 + 4s\left(\frac{2}{s^2}\right) = s^2 + \frac{8}{s}$$

A1.1.52: The perimeter of the window is $30 = x + 2h + x\pi/2$, where h is the height of the rectangular region. Then $h = 30 - x(1 + \pi/2)$. The area of the window is $xh + \pi(x/2)^2$. Then

$$\begin{aligned} f(x) &= x(30 - x(1+\pi/2)) + \pi\left(\frac{x}{2}\right)^2 \\ &= 30x - \frac{2+\pi}{2}x^2 + \frac{\pi}{4}x^2 \\ &= 30x + \frac{\pi - 2\pi - 4}{4}x^2 \\ &= 30x - \frac{\pi+4}{4}x^2 \end{aligned}$$

A1.1.53: The volume of the box is xlw, where $l = 20 - 2x$ and $w = 12 - 2x$. Then our volume function is

$$f(x) = x(20 - 2x)(12 - 2x)$$

A1.1.54:

$$f(x) = \begin{cases} 2 & 0 < x \leq 1 \\ 2.2 & 1 < x \leq 1.1 \\ 2.4 & 1.1 < x \leq 1.2 \\ 2.6 & 1.2 < x \leq 1.3 \\ 2.8 & 1.3 < x \leq 1.4 \\ 3 & 1.4 < x \leq 1.5 \\ 3.2 & 1.5 < x \leq 1.6 \\ 3.4 & 1.6 < x \leq 1.7 \\ 3.6 & 1.7 < x \leq 1.8 \\ 3.8 & 1.8 < x \leq 1.9 \\ 4 & 1.9 < x < 2 \end{cases} = \begin{cases} 2 & 0 < x \leq 1 \\ 2 + 0.2 \cdot \text{ceil}(10(x-1)) & 1 < x < 2 \end{cases}$$

A1.1.55:

(a)

$$R(I) = \begin{cases} 0 & I \leq 10000 \\ \frac{I - 0.1\cdot(I-10000)}{I} & 10000 < I \leq 20000 \\ \frac{I - 0.1\cdot(10000) - 0.15(I-20000)}{I} & 20000 < I \end{cases}$$

(b) The amount of tax for 14000 is $4000 \cdot 0.1 = 400$ dollars. For 26000 it's $10000 \cdot 0.1 + 6000 \cdot 0.15 = 1000 + 900 = 1900$ dollars.

(c)

$$T(I) = \begin{cases} 0 & I \leq 10000 \\ 0.1(I - 10000) & 10000 < I \leq 20000 \\ 1000 + 0.15(I - 20000) & 20000 < I \end{cases}$$

A1.1.56: Skip

A1.1.57:

(a) g is even and f is odd.

(a) f is neither and g is odd.

A1.1.58: The point $(-5, 3)$

A1.1.59: The point $(-5, -3)$

A1.1.60:

(a) Mirror along y axis

(b) Mirror along y axis and flip (the copy) along x axis.

A1.1.61: Even. $f(-x) = (-x)^{-2} = ((-x)^2)^{-1} = (x^2)^{-1} = x^{-2} = f(x)$.

A1.1.62: Odd. $f(-x) = (-x)^{-3} = ((-x)^3)^{-1} = (-(x^3))^{-1} = -x^{-3} = -f(x)$.

A1.1.63: Neither. $f(1) = 2$, but $f(-1) = 0$.

A1.1.64: Even.

A1.1.65: Odd.

A1.1.66: Neither.