Calculus
Chapter 1.3
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A1.3.1:

(a) $f(x) + 3$

(b) $f(x) - 3$

(c) $f(x - 3)$

(d) $f(x + 3)$

(e) $-f(x)$

(f) $f(-x)$

(g) $3f(x)$

(h) $f(x)/3$

A1.3.2:

(a) Vertical stretch by a factor of 5

(b) Shift right by 5

(c) Flip along x axis

(d) Flip along x axis and stretch by a factor of 5

(e) Horizontal shrink by a factor of 5

(f) Vertical stretch by a factor of 5 then shift down 3

A1.3.3:

(a) 3

(b) 1

(c) 4

(d) 5

(e) 2

A1.3.4: Skip

A1.3.5: Skip

A1.3.6: $y = 2\sqrt{3(x-2) - (x-2)^2}$

A1.3.7: $y = -\sqrt{3(-x-1) - (-x-1)^2} - 1$

A1.3.8:

(a) Vertical stretch by factor of two

(b) Shift up by one

A1.3.9: Skip

A1.3.10: Skip

A1.3.11: Skip

A1.3.12: Skip

A1.3.13: Skip

A1.3.14: Skip

A1.3.15: Skip

A1.3.16: Skip

A1.3.17: Skip

A1.3.18: Skip

A1.3.19: Skip

A1.3.20: Skip

A1.3.21: Skip

A1.3.22: Skip

A1.3.23: Skip

A1.3.24: Skip

A1.3.25: Skip

A1.3.26: Skip

A1.3.27:

(a) The graph of $f\big(|x|\big)$ takes the part of f's graph on the positive half of the plane and mirrors it onto the negative half of the plane.

(b)

(c)

A1.3.28: Skip

A1.3.29: Skip

A1.3.30: Skip

A1.3.31: Domain of $f+g$: $\mathbb{R}$

$$f+g = x^3 + 2x^2 + 3x^2 - 1 = x^3 + 5x^2 - 1$$

Domain of $f-g$: $\mathbb{R}$

$$f-g = x^3 - x^2 + 1$$

Domain of fg: $\mathbb{R}$

$$fg = 3x^5 + 6x^4 - x^3 - 2x^2$$

Domain of f/g: $\mathbb{R} - \{+\sqrt{1/3}, -\sqrt{1/3}\}$

$$f/g = \frac{x^3 - 2x^2}{3x^2 - 1}$$

A1.3.32: Domain of $f+g$: $[-1,1]$

$$f+g=\sqrt{1+x}+\sqrt{1-x}$$

Domain of $f-g$: $[-1,1]$

$$f-g=\sqrt{1+x}-\sqrt{1-x}$$

Domain of fg: $[-1,1]$

$$fg=\sqrt{1-x^2}$$

Domain of f/g: $[-1,1)$

$$f/g=\frac{\sqrt{1+x}}{\sqrt{1-x}}$$

A1.3.33: Skip

A1.3.34: Skip

A1.3.35: The domain of f is $\mathbb{R}$, the range of f is $[-1/8,\infty)$. The domain of g is $\mathbb{R}$, and the range of g is $\mathbb{R}$
Firstly, the domain of $f\circ g$ is $\mathbb{R}$, but the range of $f\circ g$ is $[-1/8,\infty)$

$$\begin{aligned}(f\circ g)(x)&=2[g(x)]^2-g(x)\\&=2(3x+2)^2-(3x+2)\\&=2(9x^2+6x+4)-3x-2\\&=18x^2+12x-3x+8-2\\&=18x^2+9x-6\end{aligned}$$

The domain of $g\circ f$ is $\mathbb{R}$, and the range of $g\circ f$ is $[13/8,\infty)$

$$\begin{aligned}(g\circ f)(x)&=3f(x)+2\\&=3(2x^2-x)+2\\&=6x^2-2x+2\end{aligned}$$

The domain of $f\circ f$ is $\mathbb{R}$, and its range is again $[-1/8,\infty)$

$$\begin{aligned}(f\circ f)(x)&=2[f(x)]^2-f(x)\\&=2(2x^2-x)^2-(2x^2-x)\\&=2(4x^4-4x^3+x^2)-2x^2+x\\&=8x^4-4x^3+2x^2-2x^2+x\\&=8x^4-4x^3+x\end{aligned}$$

The domain of $g\circ g$ is $\mathbb{R}$, and the range of $g\circ g$ is $\mathbb{R}$

$$\begin{aligned}(g\circ g)(x)&=3[g(x)]+2\\&=3(3x+2)+2\\&=9x+6+2\\&=9x+8\end{aligned}$$

A1.3.36: The domain of f is $\mathbb{R}$ and the range of f is $\mathbb{R}$. The domain of g is $\mathbb{R}-\{0\}$ and the range of g is $\mathbb{R}-\{0\}$.
The domain of $f\circ g$ is $\mathbb{R}-\{0\}$, and the range of $f\circ g$ is $\mathbb{R}-\{1\}$

$$\begin{aligned}(f\circ g)(x)&=1-[g(x)]^3\\&=1-\left(\frac{1}{x}\right)^3\\&=1-\frac{1}{x^3}\end{aligned}$$

The domain of $g \circ f$ is $\mathbb{R} - \{1\}$. The range of $g \circ f$ is $\mathbb{R} - \{0\}$.

$$\begin{aligned}(g \circ f)(x) &= \frac{1}{f(x)} \\ &= \frac{1}{1 - x^3}\end{aligned}$$

The domain of $f \circ f$ is $\mathbb{R}$ and the range of $f \circ f$ is $\mathbb{R}$

$$\begin{aligned}(f \circ f)(x) &= 1 - [f(x)]^3 \\ &= 1 - (1 - x^3) \\ &= x^3\end{aligned}$$

The domain of $g \circ g$ is $\mathbb{R} - \{0\}$. The range of $g \circ g$ is $\mathbb{R} - \{0\}$

$$\begin{aligned}(g \circ g)(x) &= \frac{1}{g(x)} \\ &= \frac{1}{\frac{1}{x}} \\ &= x\end{aligned}$$

A1.3.37: $f : \mathbb{R} \to [-1, 1]$, and $g : [0, \infty) \to (-\infty, 1]$
$f \circ g : [0, \infty) \to [-1, 1]$

$$\begin{aligned}(f \circ g)(x) &= \sin(g(x)) \\ &= \sin(1 - \sqrt{x})\end{aligned}$$

$g \circ f : \{x \in R \,:\, 2k\pi < x < (2k+1)\pi\} \to [0, 1]$

$$\begin{aligned}(g \circ f)(x) &= 1 - \sqrt{f(x)} \\ &= 1 - \sqrt{\sin x}\end{aligned}$$

$f \circ f : \mathbb{R} \to [\sin(-1), \sin(1)]$

$$\begin{aligned}(f \circ f)(x) &= \sin(f(x)) \\ &= \sin \sin x\end{aligned}$$

$g \circ g : [0, \infty) \to [0, 1]$

$$\begin{aligned}(g \circ g)(x) &= 1 - \sqrt{g(x)} \\ &= 1 - \sqrt{1 - \sqrt{x}}\end{aligned}$$

A1.3.38: $f : \mathbb{R} \to \mathbb{R}$, and $g : \mathbb{R} \to [31/20, \infty)$
$f \circ g : \mathbb{R} \to (-\infty, -73/20]$

$$\begin{aligned}(f \circ g)(x) &= 1 - 3g(x) \\ &= 1 - 3(5x^2 + 3x + 2) \\ &= 1 - 15x^2 - 9x - 6 \\ &= -15x^2 - 9x - 5\end{aligned}$$

$g \circ f : \mathbb{R} \to [31/20, \infty)$

$$\begin{aligned}(g \circ f)(x) &= 5[f(x)]^2 + 3f(x) + 2 \\ &= 5(1 - 3x)^2 + 3(1 - 3x) + 2 \\ &= 5(1 - 6x + 9x^2) + 3 - 9x + 2 \\ &= 5 - 30x + 45x^2 + 3 - 9x + 2 \\ &= 45x^2 - 39x + 10\end{aligned}$$

$f \circ f : \mathbb{R} \to \mathbb{R}$

$$\begin{aligned}(f \circ f)(x) &= 1 - 3f(x) \\ &= 1 - 3(1 - 3x) \\ &= 1 - (3 - 9x) \\ &= 1 - 3 + 9x \\ &= 9x - 2\end{aligned}$$

$g \circ g : \mathbb{R} \to [1493/80, \infty)$

$$\begin{aligned}(g \circ g)(x) &= 5[g(x)]^2 + 3g(x) + 2 \\ &= 5(5x^2 + 3x + 2)^2 + 3(5x^2 + 3x + 2) + 2 \\ &= 5(25x^4 + 30x^3 + 29x^2 + 12x + 4) + 15x^2 + 9x + 6 + 2 \\ &= 125x^4 + 150x^3 + 145x^2 + 60x + 20 + 15x^2 + 9x + 6 + 2 \\ &= 125x^4 + 150x^3 + 160x^2 + 69x + 28\end{aligned}$$

A1.3.39: $f : \mathbb{R} - \{0\} \to (-\infty, -2] \cup [2, \infty),\ g : \mathbb{R} - \{-2\} \to \mathbb{R} - \{1\}$
$f \circ g : \mathbb{R} - \{0\} \to \{2\}$

$$\begin{aligned}(f \circ g)(x) &= g(x) + \frac{1}{g(x)} \\ &= \frac{x+1}{x+2} + \frac{x+2}{x+1} \\ &= \frac{(x+1)(x+2) + (x+2)(x+1)}{(x+1)(x+2)} \\ &= 2\end{aligned}$$

$g \circ f : \mathbb{R} - \{0\} \to [3/4, \infty)$

$$\begin{aligned}(g \circ f)(x) &= \frac{f(x) + 1}{f(x) + 2} \\ &= \frac{x + \frac{1}{x} + 1}{x + \frac{1}{x} + 2} \\ &= \frac{\frac{x^2}{x} + \frac{1}{x} + \frac{x}{x}}{\frac{x^2}{x} + \frac{1}{x} + \frac{2x}{x}} \\ &= \frac{\frac{x^2+x+1}{x}}{\frac{x^2+2x+1}{x}} \\ &= \frac{x^2 + x + 1}{x^2 + 2x + 1}\end{aligned}$$

$f \circ f : \mathbb{R} - \{0\} \to$

$$\begin{aligned}(f \circ f) &= f(x) + \frac{1}{f(x)} \\ &= \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{x}} \\ &= \frac{x^2 + 1}{x} + \frac{x}{x^2 + 1} \\ &= \frac{(x^2 + 1)^2 + x^2}{x(x^2 + 1)} \\ &= \frac{x^4 + 2x^2 + 1 + x^2}{x^3 + x} \\ &= \frac{x^4 + 3x^2 + 1}{x^3 + x}\end{aligned}$$

$g \circ g : \mathbb{R} \to$

$$
\begin{aligned}
(g \circ g) &= \frac{g(x)+1}{g(x)+2} \\
&= \frac{\frac{x+1}{x+2}+1}{\frac{x+1}{x+2}+2} \\
&= \frac{\frac{x+1+x+2}{x+2}}{\frac{x+1+2x+4}{x+2}} \\
&= \frac{2x+3}{3x+5}
\end{aligned}
$$

A1.3.40:

Domain: $\mathbb{R}$

$$
\begin{aligned}
(f \circ g) &= \sqrt{2g(x)+3} \\
&= \sqrt{2(x^2+1)+3} \\
&= \sqrt{2x^2+2+3} \\
&= \sqrt{2x^2+5}
\end{aligned}
$$

Domain: $[-3/2, \infty)$

$$
\begin{aligned}
(g \circ f) &= [f(x)]^2 + 1 \\
&= (\sqrt{2x+3})^2 + 1 \\
&= 2x+3+1 \\
&= 2x+4
\end{aligned}
$$

Domain: $[-3/2, \infty)$

$$
\begin{aligned}
(f \circ f) &= \sqrt{2f(x)+3} \\
&= \sqrt{2\sqrt{2x+3}+3}
\end{aligned}
$$

Domain: $\mathbb{R}$

$$
\begin{aligned}
(g \circ g) &= [g(x)]^2 + 1 \\
&= (x^2+1)^2 + 1 \\
&= (x^4+2x^2+1) + 1 \\
&= x^4+2x^2+2
\end{aligned}
$$

A1.3.41:

$$
\begin{aligned}
(f \circ g \circ h)(x) &= (g \circ h)(x) + 1 \\
&= 2h(x) + 1 \\
&= 2(x-1) + 1 \\
&= 2x - 1
\end{aligned}
$$

A1.3.42:

$$
\begin{aligned}
(f \circ g \circ h)(x) &= 2(g \circ h)(x) - 1 \\
&= 2[h(x)]^2 + 1 \\
&= 2(1-x)^2 + 1 \\
&= 2(x^2-2x+1) + 1 \\
&= 2x^2 - 2x + 2
\end{aligned}
$$

A1.3.43:

$$\begin{aligned}(f \circ g \circ h)(x) &= \sqrt{(g \circ h)(x) - 1} \\ &= \sqrt{[h(x)]^2 + 2 - 1} \\ &= \sqrt{(x+3)^2 + 1} \\ &= \sqrt{x^2 + 6x + 10}\end{aligned}$$

A1.3.44:

$$\begin{aligned}(f \circ g \circ h)(x) &= \frac{2}{(g \circ h)(x) + 1} \\ &= \frac{2}{\cos h(x) + 1} \\ &= \frac{2}{\cos \sqrt{x+3} + 1}\end{aligned}$$

A1.3.45: Let $f(x) := x^{10}$, $g(x) := x^2 + 1$

A1.3.46: Let $f(x) := \sin x$, $g(x) := \sqrt{x}$

A1.3.47: Let $f(x) := x/(x+4)$, $g(x) := x^2$

A1.3.48: Let $f(x) := 1/x$, $g(x) := x + 3$

A1.3.49: Let $f(t) := \sqrt{t}$, $g(t) := \cos t$

A1.3.50: Let $f(t) := t/(1+t)$, $g(t) := \tan t$

A1.3.51: Let $f(x) := 1 - x$, $g(x) := 3^x$, $h(x) := x^2$

A1.3.52: Let $f(x) := \sqrt{x}$, $g(x) := x - 1$, $h(x) := \sqrt{x}$

A1.3.53: Let $f(x) := x^4$, $g(x) := \sec x$, $h(x) := \sqrt{x}$

A1.3.54:

(a) $f(g(1)) = f(6) = 5$

(b) $g(f(1)) = g(3) = 2$

(c) $f(f(1)) = f(3) = 4$

(d) $g(g(1)) = g(6) = 3$

(e) $(g \circ f)(3) = g(f(3)) = g(4) = 1$

(f) $(f \circ g)(6) = f(g(6)) = f(3) = 4$

A1.3.55:

(a) $f(g(2)) = f(5) = 4$

(b) $g(f(0)) = g(0) = 3$

(c) $(f \circ g)(0) = f(g(0)) = f(3) = 0$

(d) $(g \circ f)(6) = g(f(6)) = g(6)$ which does not exist

(e) $(g \circ g)(-2) = g(g(-2)) = g(1) = 4$

(f) $(f \circ f)(4) = f(f(4)) = f(2) = -2$

A1.3.56: Skip

A1.3.57:

(a) $r(t) = 60t$

(b) $(A \circ r)(t) = \pi \cdot (60t)^2 = 3600\pi t^2$

A1.3.58:

(a) $d(t) = 350t$

(b) $s(d) = \sqrt{1 + d^2}$

(c) $s(d(t)) = \sqrt{1 + 350^2 t^2}$

A1.3.59: Skip

A1.3.60: Skip

A1.3.61:

(a) Let $f(x) := x^2 + 6$. Then $(f \circ g)(x) = (2x + 1)^2 + 6 = 4x^2 + 4x + 1 + 6 = 4x^2 + 4x + 7 = h(x)$

(b) Let $g(x) := x^2 + x + 1$. Then $(f \circ g)(x) = 3(x^2 + x + 1) + 2 = 3x^2 + 3x + 3 + 2 = 3x^2 + 3x + 5 = h(x)$

A1.3.62: Let $g(x) := 4x - 17$. Then $(g \circ f)(x) = 4(x + 4) - 17 = 4x + 16 - 17 = 4x - 1 = h(x)$

A1.3.63: Yes. $h(x) = f(g(x)) = f(g(-x)) = h(-x)$

A1.3.64: No. Cheap proof: let f be a nonzero constant. If f is odd, then h is odd, since $h(-x) = f(g(-x)) = f(-g(x)) = -f(g(x)) = -h(x)$. If f is even, then h is actually even, since $h(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = h(x)$