Calculus
Problems 1.1
Jasper Ty

A1.4.1: Let a be the side of length 4. Let the adjacent side be b and the hypotenuse c. The altitude perpendicular to the hypotenuse splits the triangle into two similar triangles (proof via the angle sum) that are similar to the original triangle. s is the hypotenuse of one of them, a is the hypotenuse of the other. Hence, if the length of the altitude is x, we have the relationship

$$x/a = b/c$$

Simplifiying

$$x = \frac{4\sqrt{c^2 - 16}}{c}$$

A1.4.2: The perimeter of a triangle is

$$P = a + b + c$$

Then

$$P^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

Replacing $a^2 + b^2 = c^2$,

$$P^2 = c^2 + c^2 + 2ab + 2ac + 2bc = 2c^2 + 2ab + 2ac + 2bc$$

Collecting a common c,

$$P^2 = 2c(a + b + c) + 2ab$$

Replacing $a + b + c = P$

$$P^2 = 2Pc + 2ab$$

Using the previously found identity for the altitude perpendicular to the hypotenuse $A = ab/c$,

$$P^2 = 2Pc + 2(12c)$$

Then

$$P^2 = 2c \cdot P + 2c \cdot 12$$

$$P^2 = 2c(P + 12)$$

Solving for c,

$$\frac{P^2}{2(P + 12)} = c$$

A1.4.3: We find the different regions of interest: