Ordinary differential equations

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What is this?

These are notes I am taking for the class MATH-623, *Ordinary Differential Equations*, at Drexel University, taught by Yixin Guo.

Contents

1 Basic theory

1.1 Definitions

Definition 1.1.1. Let $J \subseteq \mathbb{R}, U \subseteq \mathbb{R}, \Lambda \subseteq \mathbb{R}^k$ be open sets, and let $f : J \times U \times \Lambda \to \mathbb{R}^n$ is a smooth function. An **ordinary differential equation (ODE)** is an equation of the form

$$\dot{\mathbf{x}} = f(t, \mathbf{x}, \lambda) \tag{1}$$

where the dot denotes differentiation with respect to the independent variable t.

Definition 1.1.2. A **solution** of an ODE (1) is a function $\mathbf{F} : J_0 \to U$, where $J_0 \subseteq J \subseteq R$, such that

$$\frac{d}{dt}\mathbf{F}(t) = f(t, \mathbf{F}(t), \lambda), \qquad \forall t \in J_0$$

i.e, a function for which we can put $\mathbf{x} = \mathbf{F}(t)$ in (1). The **orbit** of the solution $\mathbf{F}$ is the set

$$\{\mathbf{F}(t) \in U : t \in J_0\} \subseteq \mathbb{R}^n.$$

This is also called the **trajectory**, **integral curve**, or **solution curve**

Example 1.1.3. The **forced Van der Pol** equation is defined

$$\begin{cases} \dot{x_1} = x_2, \\ \dot{x_2} = b(1 - x_1^2)x_2 - \omega^2 x_1 + a\cos\Omega t. \end{cases} \tag{2}$$