DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TI2316 Lab Course
Solutions 1

deadline: May 3, 2017, 13:45

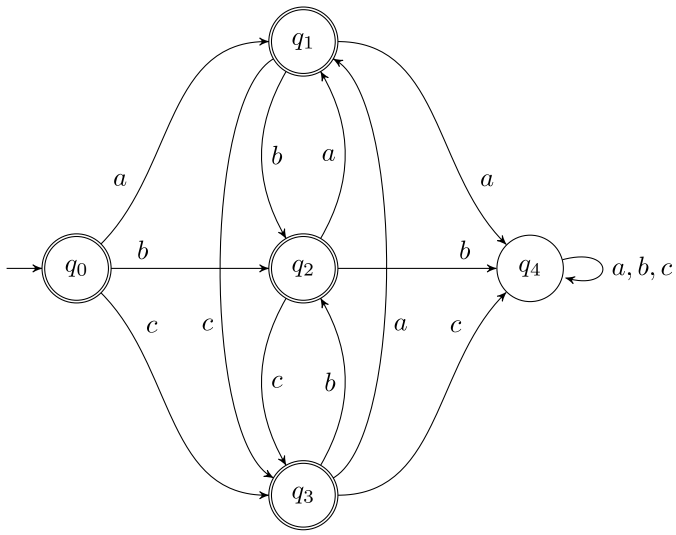
EXTRA, DRAFT

1. Suppose we have an alphabet $\Sigma = \{a, b, c\}$. Construct a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that recognizes the following language $L \subseteq \Sigma^*$:

$$L = \{w \mid \text{no two consecutive letters in } w \text{ are equal}\}.$$

(a) Give the transition graph of M. Use no more than 5 states.

Solution: The following transition graph gives a DFA that recognizes L:



(b) Describe briefly how M works.

Solution: The states q_1, q_2, q_3 correspond to the word having no consecutive equal characters and the last character read being a, b, c, respectively. In these states we accept, because the word adheres to the predicate of L.

If we read an a, b, c in q_1, q_2, q_3, respectively, then the word doesn't adhere to the predicate any more, and never will. Hence we go to trap state q_4.

The initial state q_0 is special because we haven't read any letter yet. We do accept there because $\varepsilon \in L$.

2. Suppose we have a DFA $M = (Q, \Sigma, \delta, q_0, F)$ that has states q_x, q_y and q_z such that:

$$\begin{array}{lll} \delta(q_x, a) = q_z, & \delta(q_y, a) = q_z, & \delta(q_z, a) = q_y, \\ \delta(q_x, b) = q_y, & \delta(q_y, b) = q_x, & \delta(q_z, b) = q_x, \end{array}$$

with $q_x, q_y, q_z \in Q$ and $a, b \in \Sigma$. Use induction over n to show that for all $n \geq 0$:

$$\delta^*(q_y, (ab)^n a) = q_z.$$

Clearly indicate the 3 steps of the inductive proof.

Solution:
Proof. We use a proof by induction over n, where $n \geq 0$.

Basis. If $n = 0$, then $(ab)^n a = a$. This means we get:

$$\begin{aligned} \delta^*(q_y, (ab)^n a) &= \delta^*(q_y, a) \\ &= \delta(\delta^*(q_y, \varepsilon), a) \\ &= \delta(q_y, a) \\ &= q_z \end{aligned}$$

So the claim holds for the base case.

Inductive hypothesis (IH). Suppose for some $k \geq 0$:

$$\delta^*(q_y, (ab)^k a) = q_z.$$

Inductive step. Under the above assumption, we now need to show that for $n = k + 1$, the claim holds as well. That is, we have to show

$$\delta^*(q_y, (ab)^{k+1} a) = q_z.$$

Filling in $k + 1$ gives:

$$\begin{aligned} \delta^*(q_y, (ab)^{k+1} a) &= \delta^*(q_y, (ab)^k aba) \\ &= \delta(\delta^*(q_y, (ab)^k ab), a) \\ &= \delta(\delta(\delta^*(q_y, (ab)^k a), b), a) \\ &\overset{\text{IH}}{=} \delta(\delta(q_z, b), a) \\ &= \delta(q_x, a), \\ &= q_z \end{aligned}$$

According to the principle of induction, we now conclude that for every $n \geq 0$:

$$\delta^*(q_y, (ab)^n a) = q_z.$$

This proves the claim. □

3. Consider the following claims (a) and (b). For each claim, verify whether it is true for arbitrary languages L_1, L_2, L_3 over a common alphabet Σ with $\{a, b, c\} \subseteq \Sigma$. If a claim is true, give a proof; if it is not true, give a counterexample with an explanation how the counterexample shows the claim is false.

   (a) If $\Sigma = \{a, b, c\}$, then $\Sigma^* = (\{a\}\ \Sigma^*) \cup (\{b\}\ \Sigma^*) \cup (\{c\}\ \Sigma^*)$.

   Solution: This claim is false. The empty string ε is in Σ^* but not in $(\{a\}\ \Sigma^*) \cup (\{b\}\ \Sigma^*) \cup (\{c\}\ \Sigma^*)$. (The latter set contains all words of length ≥ 1.)

   (b) If $\{a, b, c\} \subseteq L_2 - L_3$, then $L_1\ \{a, b, c\}\ \overline{\{a, b, c\}} \subseteq L_1 L_2 \overline{L_3}$.

   Solution: This claim is false. Let $L_1 = \{\varepsilon\}, L_2 = \{a, b, c\}, L_3 = \{\varepsilon\}$. Then indeed $\{a, b, c\} \subseteq L_2 - L_3$, but the word $a = \varepsilon a \varepsilon \in L_1\{a, b, c\}\overline{\{a, b, c\}}$ is not in $L_1 L_2 \overline{L_3}$. Indeed, since $\varepsilon \in L_3$, each word in $\overline{L_3}$ has length at least 1, and this also holds for each word in L_2. Hence words in $L_1 L_2 \overline{L_3}$ have length at least 2.