DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TI2316 Lab Course
Solutions 2

deadline: May 9, 2017, 13:45

EXTRA, DRAFT

1. Let Σ be an alphabet with $a \in \Sigma$. Further, let $L \subseteq \Sigma^*$ be a regular language.

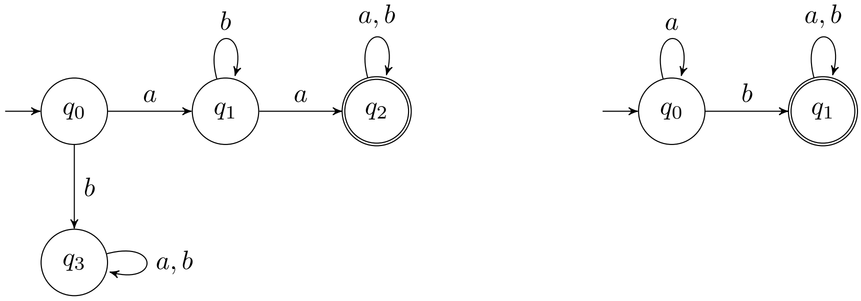
   Show that $L' = \{w \mid aw \in L\}$ is regular.

   Solution: Let D be a DFA such that $L(D) = L$, and let q_0 be the start state of D. Let $q_0 \to q_i$ be the transition for the letter a from the start state to some state q_i. Create DFA D' that is identical to D, except that q_i is the start state (rather than q_0). We claim that $L(D') = L'$.

   A word w is in L', iff aw is in L, iff D accepts aw, iff D accepts w when starting in q_i, iff D' accepts w.

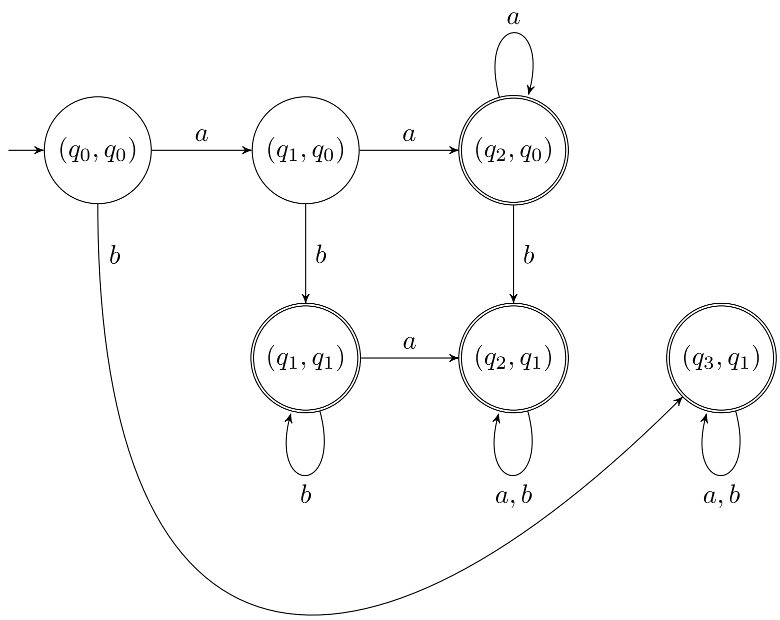
   (Possibly, $q_0 = q_i$. This does not invalidate the proof! What is L' in this case?)

2. Suppose we have the following DFAs D_1 (left) and D_2 (right):

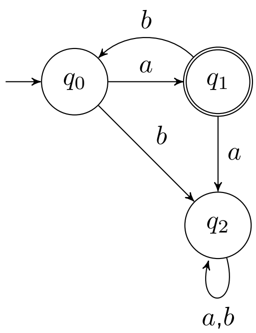


   Create a DFA D such that $L(D) = L(D_1) \cup L(D_2)$ using the construction in Sipser. Please give the resulting DFA only and remove unreachable states.

   Solution: The following DFA D has the desired language.



3. Let $\Sigma = \{a, b\}$ be an alphabet. Further, let $L_1 \subseteq \Sigma^*$ be a regular language, and let $L_2 \subseteq \Sigma^*$ be the language of the following DFA:



Show that $L = L_1(\overline{L_2} \cup L_1)$ is regular. Use no more than three lines.

Solution:

By definition, languages generated by DFAs are regular. We also know that regular languages are closed under concatenation, complement and union. Thus L must be regular.