DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

CSE2315 Lab Course
Solutions 4

deadline: March 10, 2020, 15:45

1. Suppose we have the following language over the alphabet $\Sigma = \{a, b, \#\}$:

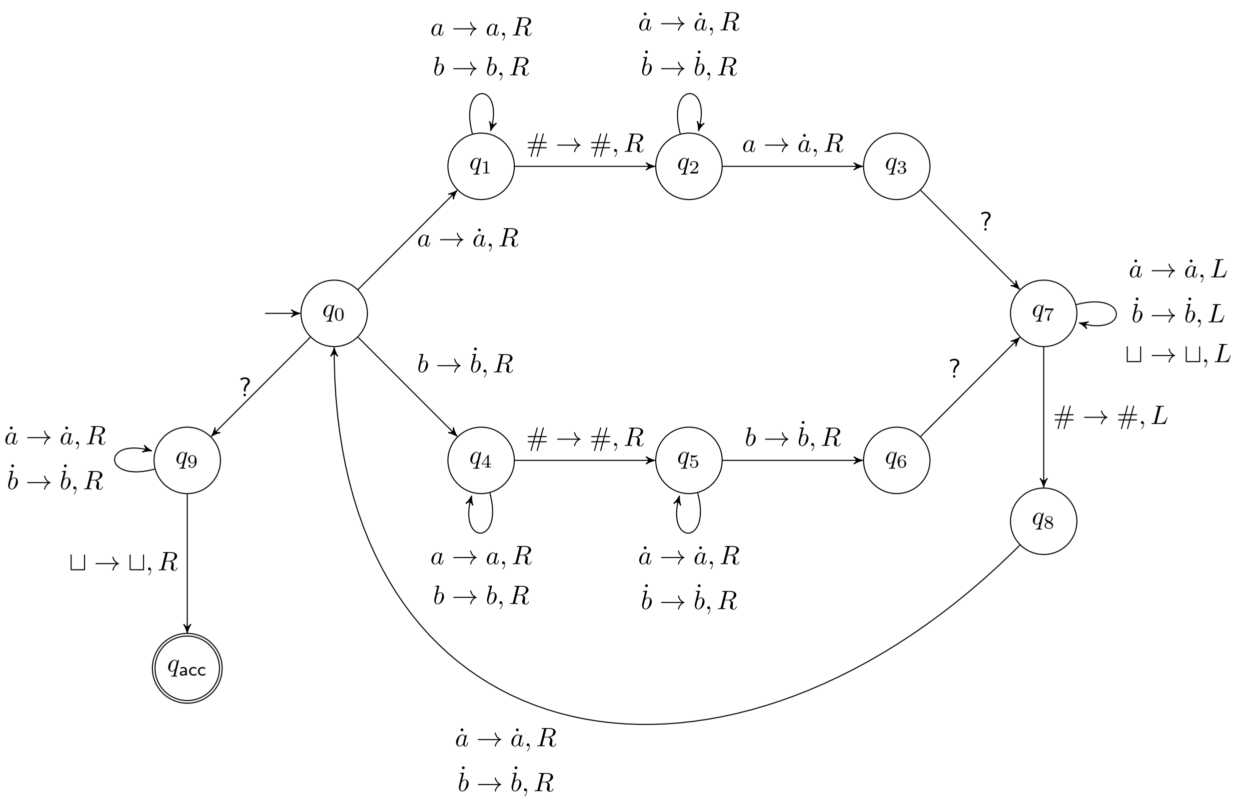
$$L = \{v\#w \mid v, w \in \{a,b\}^* \text{ and } w \text{ is } v \text{ with each letter repeated}\}.$$

Examples of words in L are:

$\#$
$ab\#aabb$
$babb\#bbaabbbb$

We have an incomplete transition diagram for a Turing machine deciding L. Assume that missing transitions lead to the reject state, with the head moving one place to the right. Question marks indicate transitions that you need to fill out in one of the questions.

NOTE: We actually missed **six** transitions for question b. We have already added one extra self-loop in q_7 the figure below.



(a) Given the above Turing machine, determine for each of the following words whether they are accepted by the machine. Consider the transitions with a question mark to be missing.

i. $a\#a$

Solution:

$$q_0 a\#a$$
$$\dot{a}q_1\#a$$
$$\dot{a}\#q_2 a$$
$$\dot{a}\#\dot{a}q_3$$
$$\dot{a}\#\dot{a}\sqcup q_{\text{rej}}$$

This word shows the machine correctly rejects the word.

ii. $b\#bb$

Solution:

$$q_0 b\#bb$$
$$\dot{b}q_4\#bb$$
$$\dot{b}\#q_5 bb$$
$$\dot{b}\#\dot{b}q_6 b$$
$$\dot{b}\#\dot{b}bq_{\text{rej}}$$

This word shows the machine incorrectly rejects the word.

iii. $\#$

Solution:

$$q_0\#$$
$$\#q_{\text{rej}}$$

This word shows the machine incorrectly rejects the word.

(b) Which **five** transitions need to be added such that that the Turing machine decides L? Three transitions are already marked with a question mark, the last two you have to fill in yourself. Hint: the last two transitions are loops at the same node.

Solution:
From q_0 to q_9, we need $\# \to \#, R$.
From q_3 to q_7, we need $a \to \dot{a}, L$.
From q_6 to q_7, we need $b \to \dot{b}, L$.
From q_8 to q_8, we need $a \to a, L$.
From q_8 to q_8, we need $b \to b, L$.

2. Suppose we have the following language over the alphabet $\Sigma = \{1, \ldots, 9, \#\}$:

$$L = \{v\#w \mid v, w \in \{1, \ldots, 9\}^* \wedge \exists a, b \in \{1, \ldots, 9\} : c_a(v) = c_b(w)\},$$

where $c_x(y)$ denotes the number of occurrences of symbol x in word y. Give a high-level description of a nondeterministic Turing machine that decides L.

Solution: A high-level description for a machine M deciding L is given by:

$M =$ "On input $w \in \Sigma^*$:

1. Check if there is exactly one $\#$ symbol in w. If not, reject.
2. Nondeterministically decide on symbols a and b from $\{1, \ldots, 9\}$.
3. Go to the first character on the tape.
4. Check the current character on the tape.
   a. If it is $\#$, go right until the first unmarked b (if any). If there is such a b, reject. Else, accept.
   b. Else if it is a, mark it and go to step 5.
   c. Else, mark it, go right and go to step 4.
5. Go right until the first character after the $\#$ symbol.
6. Check the current character on the tape.
   a. If it is marked, go right and go to step 6.
   b. If it is b, mark it and go to step 7.
   c. Else, mark it, go right and go to step 6.
7. Go left to the first unmarked character on the tape and go to step 4.

3. Consider the following language:

$$L = \{\langle v_1, v_2, \ldots, v_k, D\rangle \mid v_1, v_2, \ldots, v_k \in \{0,1\}^* \text{ and } D \text{ is a DFA with } L(D) = \{v_1, v_2, \ldots, v_k\}\}.$$

Is the following Turing machine M a decider for L? Explain.

$M =$ "On input $w = \langle v_1, v_2, \ldots, v_j, D\rangle$:

1. For $i = 1, \ldots, j$:
   a. Run D on v_i.
   b. If D rejects, reject.
2. If D has accepted all v_i, accept."

Solution: No, M does not decide L. It only checks whether $\{v_1, \ldots, v_j\}$ is a *subset* of $L(D)$. For example, M always accepts if $L(D) = \Sigma^*$. Alternatively: Take a DFA D' such that $L(D') = \{0, 01\}$. As an input to M take $\langle 0, D'\rangle$. M accepts this input, but $L(D') \neq \{0\}$, so it should reject this input. Therefore M is not a decider for this language.