TUDelft

TI2316 Lab Course
Solutions 7

deadline: June 17, 2019, 13:45

1. Suppose we have the following language:

$$E_{\mathsf{REX}} = \{\langle R \rangle \mid R \text{ is a regular expression such that } L(R) = \varnothing\}.$$

Show that E_{REX} is decidable. You are allowed to make use of the deciders implied by the text in the book (e.g., Section 4.1).

Solution: The following TM M decides E_{REX}:

$M = $ "On input $\langle R \rangle$:

1. Convert R into a DFA D.

2. Run the decider for E_{DFA} on $\langle D \rangle$.

3. If the decider accepts, accept. Otherwise, reject."

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2. Consider the language

$$L = \{\langle M, w \rangle \mid M \text{ halts on } w \text{ and visits only the first } |w|^2 \text{ tape positions}\}.$$

Is this language decidable? Explain.

Solution: L is decidable. We can show this much like the proof of Theorem 5.9 in Sipser.

Namely, given M and w, there are only a finite number of configurations for M that use the first $|w|^2$ tape positions, say k of them, and we can compute k based on M and w. We can let M run on w for k steps. If M has halted without using tape after position $|w|^2$, we accept. Else M either has looped, has used tape after position $|w|^2$, or will do so in the next step. In all these cases, we can reject with certainty.

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3. We define EQC_{TM} as:

$$EQC_{\mathsf{TM}} = \{\langle M_1, M_2 \rangle \mid M_1, M_2 \text{ are Turing machines with } L(M_1) = \overline{L(M_2)}\}.$$

Use direct reduction to show that $A_{\mathsf{TM}} \leq EQC_{\mathsf{TM}}$.

Solution: Suppose we have a decider R for EQC_{TM}. We construct a TM S that decides A_{TM} as follows:

$S = $ "On input $\langle M, w \rangle$:

1. Construct the TM M_1:

   $M_1 = $ "On input x :

   1. Run M on w.

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  2. If M accepts w, accept. Else reject."

2. Construct the TM M_2:

 $M_2 =$ "On input x :

  1. Reject."

3. Run R on $\langle M_1, M_2 \rangle$.

4. If R accepts, accept. Otherwise, reject."

We claim S is indeed a decider for A_{TM}. That is, we claim S accepts $\langle M, w \rangle$ if $\langle M, w \rangle \in A_{\mathsf{TM}}$ and rejects it otherwise.

First note that $L(M_2) = \varnothing$.

If $\langle M, w \rangle \in A_{\mathsf{TM}}$, then M accepts w. This implies $L(M_1) = \Sigma^*$. Hence, $L(M_1) = \overline{L(M_2)}$ and R accepts. Consequently, S accepts also, as required.

If $\langle M, w \rangle \notin A_{\mathsf{TM}}$, then M does not accept w: it either loops or rejects w. In both cases, this implies $L(M_1) = \varnothing$. Hence, $L(M_1) \neq \overline{L(M_2)}$, and R rejects. Consequently, S rejects also, as required.

Therefore, $A_{\mathsf{TM}} \leq EQC_{\mathsf{TM}}$, as desired.

(Neither M_1 nor M_2 use the input they get!)