

## TI2316 Lab Course Solutions 4

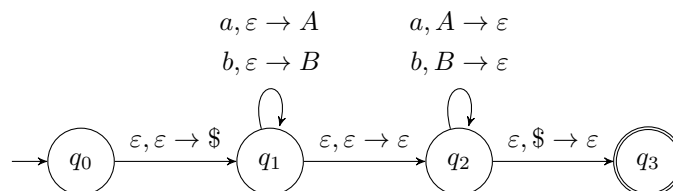
**deadline:** May 23, 2017, 13:45

### EXTRA, DRAFT

1. Suppose we have the context-free grammar  $G = (\{S\}, \{a, b\}, R, S)$  with  $R$  containing the following rules:

$$S \rightarrow abSba \mid baSab \mid \varepsilon$$

- (a) Consider the following PDA  $M'$ :



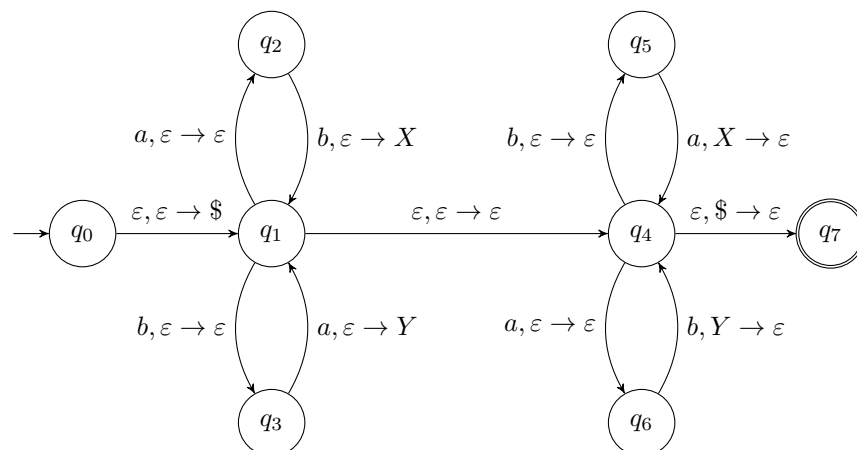
$M'$  is *not* a PDA for  $G$ . Give a set of rules  $R'$  such that  $L(G') = L(M')$  where  $G' = (\{S\}, \{a, b\}, R', S)$ , and briefly explain why  $M'$  generates  $G'$ .

**Solution:** In  $q_1$ , whenever  $M'$  reads a letter, it pushes information on the stack such that in  $q_2$ , it can match those with letters read there. Since a stack is LIFO,  $M'$  accepts iff the first half of the word is the reverse of the second half. Hence  $R'$  is:

$$S \rightarrow aSa \mid bSb \mid \varepsilon$$

- (b) Construct a PDA  $M$  such that  $L(M) = L(G)$ . Use no more than 8 states.

**Solution:**



- (c) Briefly explain (in no more than 5 lines) what you needed to change to the incorrect PDA  $M'$  to fix it.

**Solution:** The PDA cannot read  $as$  and  $bs$  independently, but needs to remember (in the left half of the PDA) that an  $a$  is part of  $ab$ , and that a  $b$  is part of  $ba$ . We need something similar in the right half. This is the purpose of the extra states.

2. Consider a CFG  $G = (\{S, V\}, \{\text{if}, \text{else}, \text{print}, (, ), \text{x}, \text{y}, \text{z}\}, R, S)$  with  $R$  containing the following rules:

$$\begin{aligned} S &\rightarrow \text{if}( V ) S \mid \text{if}( V ) S \text{ else } S \mid \text{print}( V ) \\ V &\rightarrow \text{x} \mid \text{y} \mid \text{z} \end{aligned}$$

Show that  $G$  is ambiguous by giving an appropriate word  $w$ . What are the different meanings of  $w$  when interpreted in a language like Java?

**Solution:** The word  $w = \text{if}(\text{x}) \text{ if}(\text{y}) \text{ print}(\text{y}) \text{ else print}(\text{z})$  (ignoring spaces) has two different leftmost derivations. The difference between its two interpretations lies in the question to what if the `else` belongs: if `x` is false, is `z` or nothing printed?

This issue is called the *dangling else problem*.