DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TI2316 Lab Course
Solutions 5

deadline: June 6, 2017, 13:45

EXTRA, DRAFT

1. Suppose we have the following language over the alphabet $\Sigma = \{a, b, \#\}$:

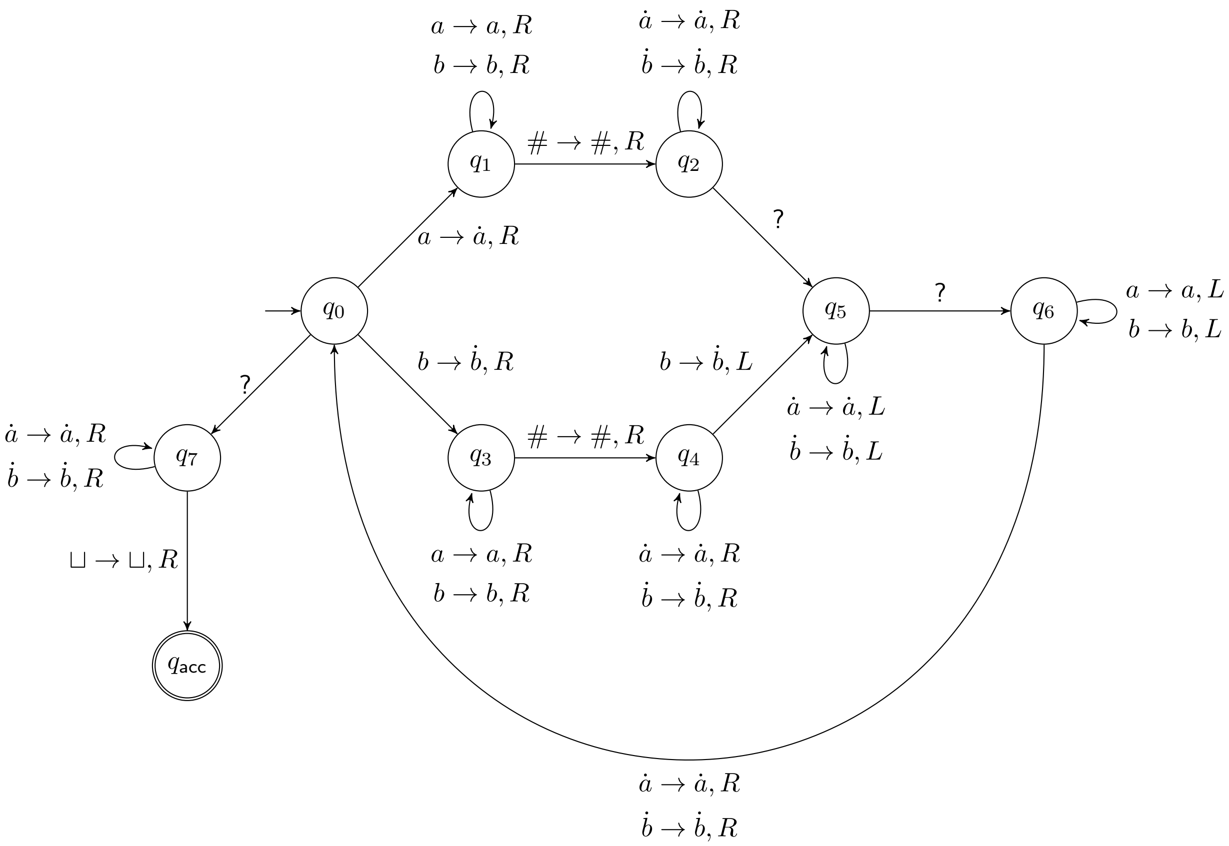
$$L = \{v\#w \mid v, w \in \{a,b\}^* \ \wedge \ |v| = |w| \ \wedge \\ \forall 1 \le i \le |v| : \text{if the } i\text{'th letter in } v \text{ is } a, \text{ the } i\text{'th letter in } w \text{ is } b\}.$$

Examples of words in L are:

$\#$
$ab\#bb$
$ab\#ba$
$babb\#abba$

We have an incomplete transition diagram for a Turing machine deciding L. Assume that missing transitions lead to the reject state, with the head moving one place to the right. Question marks indicate missing transitions.

TODO: lijkt op vorige



(a) Given the above Turing machine, determine for each of the following words whether they are accepted by the machine. Consider the transitions with a question mark to be missing.

i. $a\#a$

Solution:

$$q_0 a\#a$$
$$\dot{a}q_1\#a$$
$$\dot{a}\#q_2 a$$
$$\dot{a}\#a q_{\mathsf{rej}}$$

This word shows the machine correctly rejects the word.

ii. $b\#b$

Solution:

$$q_0 b\#b$$
$$\dot{b}q_3\#b$$
$$\dot{b}\#q_4 b$$
$$\dot{b}q_5\#\dot{b}$$
$$\dot{b}\#q_{rej}\dot{b}$$

This word shows the machine incorrectly rejects the word.

iii. $\#$

Solution:

$$q_0\#$$
$$\#q_{\mathsf{rej}}$$

This word shows the machine incorrectly rejects the word.

(b) Which **four** transitions need to be added such that that the Turing machine decides L? Three transitions are already marked with a question mark, the last one you have to fill in yourself.

Solution:
From q_0 to q_7, we need $\# \to \#, R$.
From q_2 to q_5, we need $b \to \dot{b}, L$.
From q_5 to q_6, we need $\# \to \#, L$.
From q_4 to q_5, we need $a \to \dot{a}, L$.

2. Suppose we have the following language over the alphabet $\Sigma = \{a, b, c, \#\}$:

$$L = \{v\#w \mid n_a(v) + n_b(v) = n_c(w)\},$$

where $n_x(y)$ denotes the number of occurrences of symbol x in word y. Give a high-level description of a deterministic Turing machine that decides L.

Solution: A high-level description for a machine M deciding L is given by:

$M =$ "On input $w \in \Sigma^*$:

1. Check if there is exactly one $\#$ symbol in w. If not, reject.
2. Go to the first character on the tape.
3. Check the current character on the tape.
   a. If it is $\#$, go right until the first unmarked c (if any). If there is such a c, reject. Else, accept.
   b. Else if it is a, mark it and go to step 4.
   c. Else if it is b, mark it and go to step 4.
   d. Else, mark it, go right and go to step 3.
4. Go right until the first character after the $\#$ symbol.
5. Check the current character on the tape.
   a. If it is marked, go right and go to step 5.
   b. Else if it is c, mark it and go to step 6.
   c. Else if it is the blank symbol, reject.
   d. Else, mark it, go right and go to step 5.
6. Go left to the first unmarked character on the tape and go to step 3.

3. Consider the type of Turing machines with the following property. An R transition moves the head three positions to the right, and an L transition moves the head two positions to the left. What class of languages do these TMs recognize? Explain.

Solution: Such Turing machines recognize the same class of languages as usual TMs. An R transition (in the usual sense) can be implemented by executing R followed by L (in the above sense). Similarly, an L transition (in the usual sense) can be implemented by executing R followed by two times L (in the above sense).

Conversely, an R transition (in the above sense) can be implemented by three times R (in the usual sense), and an L transition (in the above sense) can be implemented by two times L (in the usual sense).