DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TI2316 Lab Course
Solutions 3

deadline: May 16, 2017, 13:45

EXTRA, DRAFT

1. Suppose we have the following language L over the alphabet $\Sigma = \{a, b, c\}$:

$$L = \{w \in \Sigma^* \mid |w| \geq 2 \ \wedge \ \text{if } w \text{ contains a } b\text{, the last two letters of } w \text{ are equal}\}.$$

   (a) Give a regular expression R such that $L(R) = L$.

Solution:

$$R = (a \cup c)(a \cup c)(a \cup c)^* \cup (a \cup b \cup c)^* b(a \cup b \cup c)^*(aa \cup bb \cup cc) \cup bb$$

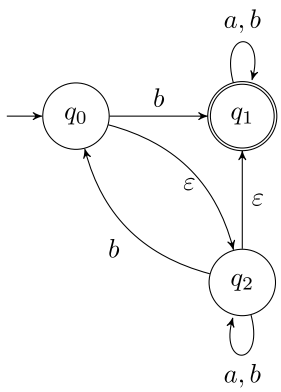
   (b) Explain how you arrived at your answer.

Solution: Words without a b are no problem so long as they are at least size 2. If w does contain a b, then the last two letters need to be aa, bb or cc. The regular expression $(a \cup b \cup c)^*$ represents any word, so the case $|w| \geq 3$ is represented by

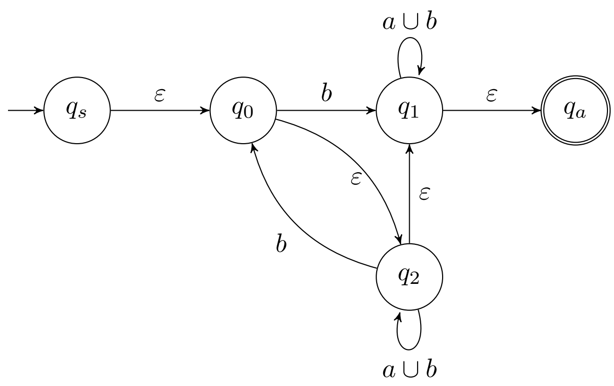
$$(a \cup b \cup c)^* b(a \cup b \cup c)^*(aa \cup bb \cup cc).$$

The case $|w| = 2$ only applies when $w = bb$, and this word is represented separately.

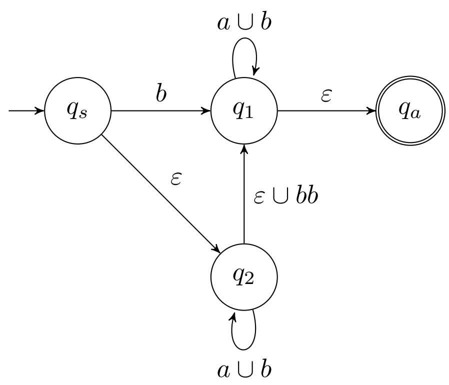
2. Convert the following NFA N to a regular expression R such that $L(R) = L(N)$. Please show all the intermediate steps and eliminate the states in the following order: q_0, q_1, q_2.



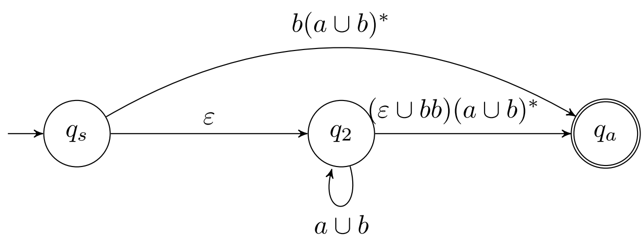
Solution: Rewrite to a GNFA by adding a new start state q_s and accept state q_a and turning the labels into regular expressions:



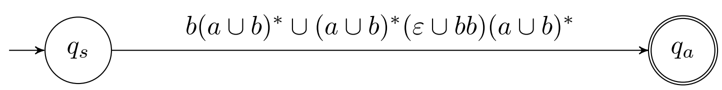
Eliminate q_0:



Eliminate q_1:



Eliminate q_2:



The desired regular expression can be seen on the last remaining transition.

3. Suppose we have the following language L over the alphabet $\Sigma = \{1, ;\}$:

$$L = \{a; b; c \mid a, b, c \in \{1\}^* \wedge |a|^2 + |b|^2 = |c|^2\}.$$

Use the pumping lemma to show that L is not regular.

Hint: use that $3^2 + 4^2 = 5^2$, and that the equality also holds if you multiply the left and right hand side with the same factor.

Solution:

Proof. Suppose L is regular. Then there exists a pumping length $p > 0$, in accordance with the pumping lemma. The lemma says all words $w \in L$ with $|w| \geq p$ can be split up into $w = xyz$ with $|xy| \leq p$ and $|y| > 0$ such that $xy^iz \in L$ for all $i \geq 0$.

Now, take $w = 1^{3p}; 1^{4p}; 1^{5p}$. We have $w \in L$ because $|a|^2 + |b|^2 = (3p)^2 + (4p)^2 = p^2(3^2 + 4^2) = p^2 5^2 = (5p)^2 = |c|^2$. If we split this word according to the above requirements, we get $x = 1^m$, $y = 1^n$ and $z = 1^{3p-m-n}; 1^{4p}; 1^{5p}$ for some $m \geq 0, n > 0$.

Now, take $i = 2$. Then $xy^iz = a^m a^{2n} a^{3p-m-n}; 1^{4p}; 1^{5p} = a^{3p+n} 1^{4p} 1^{5p}$. Since $n > 0$, we have $|a|^2 + |b|^2 = (3p + n)^2 + (4p)^2 \neq (5p)^2 = |c|^2$. Hence $xy^2z \notin L$. This contradicts the pumping lemma, so L is not regular. $\square$