Assignment 7

My score
75% (7.5/10)

Q1

7.5 / 10

Assignment 7 (due Tuesday, Nov. 14, 11pm; 1.5% of the course grade):

Chapter 5: Problems 11, 17, 19, 21, 22, 34, 43, 44, 49, 62

When submitting your solutions for assignment 7, please include the signed and dated Academic Integrity Checklist as the cover page, with your name clearly typed or printed.

11.

we already know $p = p(A_1 \cdots A_n)$

$$= p(A_1)\, p(A_2 | A_1) \cdots p(A_n | A_1 \cdots A_{n-1})$$

and $p = p\{X > \max_i Y_i\}$

$$= p\{X > Y_1\} \cdot p\{X > Y_2\} \cdots p\{X > Y_n\}$$

and Because

$\cancel{X \sim}$ $X \sim \exp(\lambda)$

$$F_X(x) = P_X(X < x) = 1 - e^{-\lambda x}$$

$$\therefore p(X > y) = 1 - (1 - e^{-\lambda y})$$
$$= e^{-\lambda y}$$

$$\therefore p(X > Y_i) = P_X(X > Y_i) \cdot p(Y_i = y_i)$$

$$= e^{-\lambda y_i} \cdot \mu e^{-\mu y_i}$$

$$\therefore p(X > Y_n) = e^{-\lambda y_n} (\mu e^{-\mu y_n})$$

$\therefore p = P\{X > \max_i Y_i\} = \prod_{i=1}^{n} e^{-\lambda y_i} (\mu e^{-\mu y_n})$

when $n = 2$.

$p = \prod_{i=1}^{2} e^{-\lambda y_i} (\mu e^{-\mu y})$

$= e^{-\lambda y} (\mu e^{-\mu y})$

17.

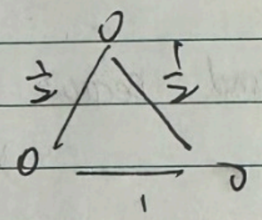
we already know that there are n cities need to be Connected

and the cost to construct between city i to j is C_{ij}.

and we know that all $\binom{n}{2}$ costs C_{ij} are independent. with mean 1.

(a) when n=3

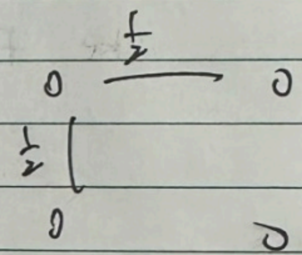
Expect cost = 1+1+1 = 3.



(b) when n=4

Expect cost = 1+1+1+1

= $1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}+1 = 4$



19.

$E(X_{Alice}) = E(X \mid T_A < T_B)\cdot P(T_A < T_B) + E(X \mid T_A > T_B)\cdot P(T_A \geq T_B)$

$= E$

Because the winner and receives $Re^{-\alpha t}$ if the winning time is t.
where R and α are constants.

$\therefore E(A \text{ win}) = E(Re^{-\alpha t} \mid T_A < T_B)$

21.

the time ~~wait~~ ~~for~~ server 1 is $1/\mu_1 (\frac{\mu_2}{\mu_1+\mu_2}) + (\frac{1}{\mu_2}+\frac{1}{\mu_1})(\frac{\mu_1}{\mu_1+\mu_2})$

and we know that.

The service times at server i are exponential with rate μ_i.

and suppose that when you arrive there is one customer in the system and that customer is being served by server 1.

$E(\text{spend}) = \frac{1}{\mu_1} + \frac{\mu_1}{\mu_1+\mu_2}$.

22

S ~~ince from the~~ we suppose that when you arrive to find two others in the systems. one being served by 1 and other 1 by 2.

the ~~So expected time spend is $\frac{1}{\mu_1} + \frac{\mu_1}{\mu_1+\mu_2}$~~.

~~theis is that we get from 21~~.

the $E(X) = \frac{1}{\mu_1} + \frac{\mu_1}{\mu_1+\mu_2}$.

In this [illegible],

34.

(a)

$p(\text{A obtain a new kidney}) = \frac{\lambda}{\lambda+\mu_A}$

(b)

$p(\text{B obtain a new kidney}) = \frac{\lambda}{\lambda+\mu_B}$

(c) ~~ned~~ A or B. new kidney

$p(\text{neither A or B}) = p(\text{A not}) \times p(\text{B not})$

$= p(1 - \text{A get}) \times p(1 - \text{B get})$

$= (1 - \frac{\lambda}{\lambda+\mu_A}) \times (1 - \frac{\lambda}{\lambda+\mu_B})$

(D) $p(\text{both A and B}) = p(\text{A get}) \times p(\text{B get})$

$$= \frac{\lambda}{\lambda+\mu_A} + \frac{\lambda}{\lambda+\mu_B}.$$

43.

we already know that the poisson process with rate λ.

And given that

$X_1 \sim exp(\mu_1)$

$X_2 \sim exp(\mu_2)$

So we know the PDF of X_1 and X_2.

$PDF(X_1) = \mu_1 e^{-\mu_1 X_1}$

$PDF(X_2) = \mu_2 e^{-\mu_2 X_2}$

$P(X_1 + X_2 < T) = \int_0^{\alpha} p(X_1 + X_2 < T) f(X_1) dx_1 dX_2.$

$= \mu_1 \mu_2 e^{-\mu_1 X_1} e^{-\mu_2 X_2} dx_1 dX_2$

$= \cancel{\mu_1 \mu_2}$

44

(a) the probability that her waiting time is 0.

and. poisson rate is λ.

$$P(T) = e^{-\lambda T}$$

(b) For the expect waiting time.

$\cancel{P(E)} =$ $P(E) = 1 - P(\text{NO car})$

$= 1 - P(\text{waiting time} = 0)$

49.

we already know that if no event occur by time T.
the we lose.

~~and~~

(a) ~~$p(\ldots)$~~ =
the strategy stop at first event after time S

and
$p(\ldots) = p(N(T) - N(S)) = \lambda(T-S)e^{-\lambda(T-S)}$

(b) we need to maximizes the probability of win.

$e^{-\lambda(T-S)}\lambda = \lambda(T-S)e^{-\lambda(T-S)}$

~~when S~~ = so we need to max $e^{-\lambda(T-S)}$
$\therefore S=0$

(c) $f = e^{-\lambda t}$ ... t

(c) from $e = e$

we know that

$$xT = 1$$

$$T = \frac{1}{x}$$

$\therefore$ the probability of ~~the~~ win when using the preceding with value of s.

62.

(a)

X_i is independent poisson random veriable with means.

$E(X_1) = \lambda p_1(1-p_2)$

$E(X_2) = \lambda(1-p_1)p_2$

$E(X_3) = \lambda p_1 p_2$

$E(X_4) = \lambda(1-p_1)(1-p_2)$.

(b). $\frac{E(X_1)}{E(X_2)} = \lambda p_1(1-p_2) / \lambda(1-p_1)p_2 = 1-p_2 / p_2$.

$\frac{E[X_2]}{E[X_3]} = \lambda(1-p_1)p_2 / \lambda p_1 p_2 = 1-p_1 / p_1$

(c), $E[X_1] / E[X_2] = \frac{1}{p_2} - 1$.

$\frac{1}{p_2} - 1 = \frac{X_1+X_3}{X_2}$

$$\beta_2 = \frac{2X_3}{X_1+X_3+X_3}$$

(d) $\because$ λ is the expect value

$\cancel{E(X_4)=}$ $\lambda = E(X_1+X_2+X_3+X_4)$

$E(X_4) = \lambda - E(X_1+X_2+X_3)$.

$\cancel{E(X_4)=}$ $\therefore$ we can use λ to estimate the $E(X_4)$