Assignment 2

My score

95% (9.5/10)

Q1

9.5 / 10

Chapter 2: Problems 9, 10, 12, 15, 20, 33, 34

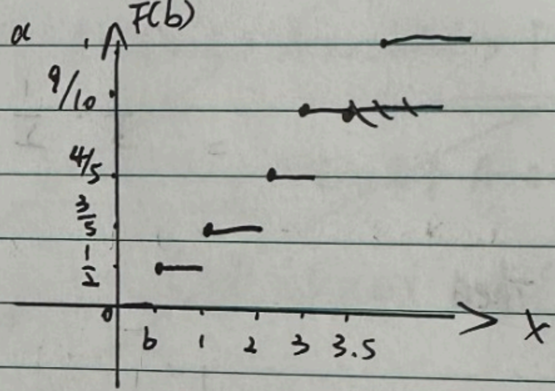
When submitting your solutions for assignment 2, please include the signed and dated Academic Integrity Checklist as the cover page, with your name clearly typed or printed.

Exercise

9.

F is given by $F(b) = \begin{cases} 0 & b<0 \\ \frac{1}{2} & 0 \le b < 1 \\ \frac{3}{5} & 1 \le b < 2 \\ \frac{4}{5} & 2 \le b < 3 \\ \frac{9}{10} & 3 \le b < 3.5 \\ 1 & b \ge 3.5 \end{cases}$

So we can plot a



So, $p(0) = 0$.

$p(1) = p(1 \le b < 2) - p(0 \le b < 1) = \frac{3}{5} - \frac{1}{2} = \frac{1}{10}$

$p(2) = p(2 \le b < 3) - p(1 \le b < 2) = 4/5 - 3/5 = \frac{1}{5}$

$p(3) = p(3 \le b < 3.5) - p(2 \le b < 3) = 9/10 - 4/5 = \frac{1}{10}$

$p(3.5) = p(b \ge 3.5) - p(3 \le b < 3.5) = 1 - 9/10 = \frac{1}{10}$

10. we have three dice and at most 1 six appear, so we use 1 minus the twice and three time. six appear.

$$1 - P(\text{twice } 6) - P(\text{three } 6) = 1 - \binom{3}{2}\frac{5}{6} \times \left(\frac{1}{6}\right)^2 - \left(\frac{1}{6}\right)^3 \binom{3}{3}$$

$$= 200/216$$

12.

we have multi-choice exam. with three possible answers
and 5 questions.

if a student they want to get 4 or more correct
~~so~~

they must get 4 and 5 correct

in one multy choice. the sample ~~space~~ space is

AB. AC, BC. ABC

So. in each multi choice they have $\frac{1}{4}$ to choice correct.

$\therefore$ $p(4 \text{and more}) = p(4) + p(5)$

$= p \binom{5}{4} (\frac{1}{4})^4 (\frac{3}{4}) + \binom{5}{5} (\frac{1}{4})^5$

$=$

15.

(a) when $k = (n+1)p$

$$\frac{P(X=k)}{P(X=k-1)} = \binom{n}{k} p^k (1-p)^{n-k} \Big/ \binom{n}{k-1} p^{k-1} (1-p)^{n-k+1}$$

$$= \frac{\cancel{n!}}{(n-k)!\,\cancel{k!}_{\,k}} p^{\cancel{k}}(1-p)^{\cancel{n-k}} \Big/ \frac{\cancel{n!}}{\cancel{(n-k+1)!}_{\,n-k+1}\,\cancel{(k-1)!}} p^{\cancel{k-1}}\cancel{(1-p)^{n-k+1}}$$

$$= \frac{p}{k} \Big/ \frac{1-p}{n-k+1} = \frac{p}{k} \cdot \frac{n-k+1}{1-p} = \frac{p}{1-p} \cdot \frac{n-k+1}{k}$$

So $P(X=k) / P(X=k-1) \ge 1 \Rightarrow P(X=k) \ge P(X=k-1)$

$p \cdot (n-k+1) \ge (1-p) \cdot k \Rightarrow p(n+1) \ge k$

So $(n+1)p$ is a integer

(b) $k = (n+1)p$

$~~p(X=(n+1)p-1)~~ = \binom{n}{n+1} p^{n+1} (1-p)^{n-n+1}$

$p(X = k-1) = \binom{n}{k-1} p^{(k-1)} (1-p)^{n-k+1}$

$= \frac{n!}{(n-k+1)!(k-1)!} \cdot p^{(k-1)} (1-p)^{n-k+1}$

$p(X=k) = \binom{n}{k} p^{k} (1-p)^{n-k}$

and we already know $p(X=k) \geq p(X=k-1)$

So. $(n+1)p > (n+1)p - 1$

~~and when~~

$\therefore \frac{p}{k} > \frac{1-p}{n-k+1}$

$p(n+1) > k$

~~so k is~~ So $p(n+1)$ is not an integer.

20.

(a) $p = C_{365}^{i} C_{n}^{2i} \left(C_{2i}^{2} \cdot C_{2i-2}^{2} \cdots C_{4}^{2} \right) \cdot C_{365-i}^{n-2i} \cdot A_{n-2i}^{n-2i} \Big/ 365^{n}$

$$P = \frac{365!}{i!(\cancel{365-i})!} \cdot \frac{n!}{\cancel{2i}!(\cancel{n-2i})!} \cdot \frac{\cancel{2i}!}{2!\,(\cancel{2i-2})!} \cdot \frac{(\cancel{2i-2})!}{2!\,(\cancel{2i-4})!} \cdots \frac{\cancel{4}!}{2!\,2!} \cdot$$

$$\frac{(\cancel{365-i})!}{(\cancel{n-2i})!\,(365-i-n+2i)!} \cdot \frac{(\cancel{n-2i})!}{(\cancel{0})!} \Big/ 365^{n}$$

$$P = 365! \times n! \times 2i! \times 365^{n} \Big/ (365-n+i)! \times 2^{n}$$

(b) For first set number is C_{365}^{i}

For second ~~set~~ number is C_{365-i}^{n-2i}

(C) $p(\text{no at least 3 same})$ means ~~we only~~ have 0. and 2 people same birthday

$$p(\text{no at least 3 same}) = \sum_{i\in[0,\frac{n}{2}]} C_{365}^{i} C_n^{2i} (C_{2i}^{2} \cdot C_{2i-2}^{2} \cdots C_4^{2}) \cdot C_{365-i}^{n-2i} \cdot A_{n-2i}^{n-2i} \Big/ 365^n$$

So. $P(\text{no at least 3 same}) = P_1 + P_2 + P_3$ when $i = 0, 1, 2$.

when $i=0$

$$p_1 = \cancel{365 \times n! \times 365}\ 365! \times n! \times 365^n \Big/ (365-n)! \times 2^n$$

when $i = 1$

$$\cancel{P_2 = 365! \times n! \times n! \times 365^n \Big/ (365 - \tfrac{n}{2})! \times 2^n}$$

$$\cancel{= 365! \times 2n! \times 365^n \Big/ (365 - \tfrac{n}{2})! \times 2^n}$$

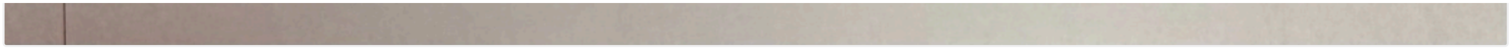
$$P_2 = \frac{365! \times n! \times 2! \times 365^n}{(365-n+1)! \times 2^n}$$

when $i = 2$

~~p(no at least 3 same) = $P_1 + P_2$~~

$$P_3 = 365! \times n! \times 4! \times 365^n \Big/ (365-n+2)! \times 2^n$$

$$p(\text{no at least 3 same}) = P_1 + P_2 + P_3$$



33.

(a) we know the PDF.

Because in pdf $\int_{-\infty}^{\infty} f_X(x)dx = 1$

So $\int_{-\infty}^{\infty} c(1-x^2)dx = 1$

$c\int_{-1}^{1}(1-x^2)dx = 1$

$c(x - x^3/3)\Big|_{-1}^{1} = 1$

$c\left(\frac{3-1}{3} - \frac{-3+1}{3}\right) = 1$

$c = 3/4$

(b)

由 (a) 问可知

$f(x) = 3/4\,(1-x^2) \quad , -1<x<1$

Because cdf need

$$\lim_{t \to -\infty} F_X(t) = 0$$

$$f(x) = 3/4\,(1 - x^2)$$

$$F(x) = \int f(x)\,dx + C = \frac{3}{4}x - \frac{x^3}{4} + C$$

$\because$ $F(-1) = 0$ $F(1) = 1$

$\therefore$ $F(x) = \frac{3}{4}x - \frac{x^3}{4} + \frac{1}{2}$

range

34.

(a) $\int_0^2 f(x)dx = 1$

$\int_0^2 c(4x-2x^2)dx = 1$

$c \cancel{\int_0^2} \left(\frac{4 \cdot x^2}{2} - 2 \cdot \frac{x^3}{3}\right)\Big|_0^2 = 1$

$c\left(2x^2 - 2 \cdot \frac{x^3}{3}\right)\Big|_0^2 = 1$

$c\left(2 \cdot 4 - 2 \cdot \frac{8}{3} - 0\right) = 1$

$c = \frac{3}{8}$

(b) $p\left\{\frac{1}{2} < x < \frac{3}{2}\right\} = \int_{1/2}^{3/2} \frac{3}{8}(4x-2x^2)dx = \frac{3}{8}\left(2x^2 - 2 \cdot x^3/3\right)\Big|_{\frac{1}{2}}^{\frac{3}{2}}$

$= 3/8\left(\frac{8}{2} - \frac{26}{12}\right) = \left(\frac{48-26}{12}\right) \cdot 3/8 = 11/16$