Assignment 1

My score

98% (98/100)

A1

98 / 100

Chapter 1: Problems 3, 10, 14, 20, 24, 33, 36, 37, 39, 46

Exercise

3. The space is

$S=\{(A_1, A_2, \cdots, A_n), n\geq 2\}$
$A_n \in (\text{heads}, \text{tails})$

Because this will be tossed exactly four times and head is twice

So they must be (T, T, H, H)
(H, T, H, H)

$\therefore$ for this two result the
$P(4\text{times}) = \cancel{\frac{1}{2}\times 4} \times (\frac{1}{2})^4 \times 2 = \frac{1}{8}$

10.
$P(\bigcup_{i=1}^{n} E_i) = P(E_1 \cup E_2 \cdots \cup E_i)$

$= P(E_1) + P(E_2) + \cdots + P(E_i) - P(E_1 E_2) - P(E_2 E_3) \cdots$

$- \cdots \Sigma P(E_i E_j) \quad \cdots + (-1)^{n+1} P(E_1 E_2 \cdots E_n)$

But

$$\sum_{i=1}^{n} P(E_i) = P(E_1) + P(E_2) + \cdots + P(E_i)$$

So, $\sum_{i=1}^{n} P(E_i) \geq P\left(\bigcup_{i=1}^{n} E_i\right)$

14.

Because this is tossing the dice forth.

If we assume

A win 1th the sample space is { S, FFS, FFFFS, FFFFFFS }

B win first the sample space is

{ FS, FFFS, FFFFFS, FFFFFFFS }

So $p(A\ win) = p(S) + p(FFS) + p(FFFFS) + p(FFFFFFS)$

$= p + (1-p)^2 p + (1-p)^4 p + (1-p)^6 p$

$= p \sum_{i=0}^{\infty} (1-p)^{2i}$

For $\sum_{i=1}^{\infty}$ is $\to \infty$. $\therefore p(A\ win) = p \cdot \lim_{i \to \infty} \sum_{i=0}^{\infty} (1-p)^{2i} = p \cdot \frac{1-(1-p)^{2n}}{1-(1-p)^2} = \frac{1}{2-p}$

$p(B\ win) = p(FS) + p(FFFS) + p(FFFFFS) + p(FFFFFFFS)$

$= (1-p)p + (1-p)^3 p + (1-p)^5 p + (1-p)^7 p$

$= (1-p)p \sum_{i=0}^{\infty} (1-p)^{2i} = 1-p/2-p$

20.

Becaus we have three dice

the sample space like { (1, 1, 0), ... {6, 6, 0} }

(1, 1, 1), ... {6, 6, 1} }

~~One dice~~

⋮

(1, 1, 6), ⋮ ... {6, 6, 6}

For each dice we just need two dice is same, so we need not calculate (1, 1, 1), ... (6, 6, 6). this six.

So, for each ~~dice~~ cycle, we have $6 \times 6 - 6 = 30$

We have three dice, So $30 \times 3 = 90$ ways that exactly two of dice is same number.

Then for three dice our sample space number is $6 \times 6 \times 6 = 216$

$\therefore$ we can calculate this probability the same number

$$p(\text{two same}) = 90/216 = 5/12$$

24.

Because $P_{n,m}$ denote the probability from first vote on A is always in lead.

$\therefore$ $P_{2,1} = \cancel{\{A, A\}}\ P\{A, A, B\} = \frac{1}{3}$

$P_{3,1} = P\{(A, A, B, A\}, (A, A, A, B)\} = \frac{1}{2}$

$P_{3,2} = P\{(A, A, B, A, B, B), (A, A, B, A, B)\} = \frac{2}{10} = \frac{1}{5}$

~~$P_{4,2} = P\{(A, A, A, B, B, A), (A, A, B, A, B, A)$,~~

~~$(A, A, A, A, B, B), (A, A$~~ $.A$

$P_{4,1} = P\{(A, A, A)\} = \frac{4}{5} \times \frac{3}{4} = \frac{3}{5}$

$P_{4,2} = P\{(A, A, A) + (A, A, B, A)\} = \frac{4}{6} \times \frac{3}{5} \times (\frac{2}{4} + \frac{2}{4} \times \frac{2}{3})$

$$= \frac{1}{3}$$

$$P_{n,m} = \frac{n-m}{n+m}$$

33 Because we need to calculate at least one set is golden.

$\therefore$ ~~p{24}~~

$p\{\text{at least 1 golden}\} = 1 - p\{\text{No golden}\}$

then, Because the winner of tennis match is win 2 sets

So. $p(\text{total 2 sets}) = \frac{1}{2}$

$p(\text{total 3 sets}) = \frac{1}{2}$

$p(\text{have golden}) = (\frac{1}{2})^{24}$

$p(\text{No golden}) = 1 - (\frac{1}{2})^{24}$

Because $p\{\text{at least 1 golden}\} = 1 - p\{\text{No golden}\}$

~~$= 1 - p\{$~~

$= 1 - (p\{\text{No golden} \mid \text{total 2 sets}\} \cdot p\{\text{total 2 set}\}$

$+ p\{\text{No golden} \mid \text{total 3 sets}\} \cdot p\{\text{total 3 set}\}$

$= 1 - (\frac{1}{2} \times (1 - (\frac{1}{2})^{24})^2) - (\frac{1}{2} \times (1 - (\frac{1}{2})^{24})^3)$

36. Let A_i = Box we selected
B_i = marble is black

Because we need to calculate the marble black probability.

$P(B_i|A_i) = P(B_1|A_1) + P(B_2|A_2) = \frac{1}{2} + \frac{2}{3}$

$\cancel{P(B_1|A_1) = \frac{P(B_1)\cdot P(A_1|B_1)}{P(B_1)\cdot P(A_1|B_1) + P(B_1^c)\cdot P(A_1|B_1^c)} = \frac{1}{2}}$

Then Because we have two box. each box selected probability is $\frac{1}{2}$

So finally, the $P(\text{marble black}) = (\frac{1}{2} + \frac{2}{3}) \times \frac{1}{2}$

$= \frac{7}{12}$

37. $$P(A\,\text{box} \mid w) = \frac{P(w|A)\,P(A)}{P(w|A)\cdot P(A) + P(w|A^c)\,P(A^c)}$$

$$= \frac{1}{2}\times\frac{1}{2} \Big/ \frac{1}{2}\times\frac{1}{2} + \frac{1}{2}\times\frac{1}{3}$$

$$= \frac{1}{4} \Big/ \frac{1}{4}+\frac{1}{6}$$

$$= 3/5$$

39.

$$P(C \mid w) = \frac{P(w|\overset{C}{\cancel{A}})\,P(C)}{P(w|C^c)\,P(C^c) + P(w|C)\cdot P(C)}$$

$$= 0.7\times 0.44 \Big/ 0.44\times 0.7 + 0.56\times 0.58$$

$$= 0.308 \Big/ 0.308 + 0.3248$$

$= 0.4867 \approx 0.5$

46. Let $E = \{B, C\}$. is the answer by Sailer to A

$$P\{A.\text{exec} \mid E = B\} = \frac{P\{E=B \mid A\text{ exec}\}P\{A\text{ exec}\}}{P\{E=B\}}$$

$$= \frac{\frac{1}{3} \cdot P\{E=B \mid A\text{ exec}\}}{\frac{1}{2}}$$

$$= \frac{1}{3} \cdot \frac{1}{2} \Big/ \frac{1}{2} = \frac{1}{3}$$

~~Because~~ Then

$$P\{A\text{ exec} \mid E=C\} = \frac{1}{3}$$

So this probability of executed will not change

the jailer's reasoning is wrong.