Assignment 10

My score
100% (10/10)

Q1

10 / 10

Assignment 10 (due Tuesday, Dec. 5, 11pm; 1.5% of the course grade):

Chapter 7: Problems 5 8, 9, 14, 15, 18, 22, 23 (please submit solutions for any 6 problems)

~~9.~~
5.

(a) The renewal process $\{N(t), t\geq 0\}$. here gamma (r,λ) distribution.
So we can get that.

$$P(N(t)\geq n) = \sum F(x)$$

$F(x)$ is a possion. with rate λ.
and. $N(t)$ mean t times.

$$\therefore\ p(N(t)\geq n) = \sum^{\infty}[e^{-\lambda t}(\lambda t)^{n}]/n!$$

$$= \sum_{i=nr}^{\infty}[e^{-\lambda t}(\lambda t)^{i}]/i!$$

(b) And we know that.

$$N(t)\geq n \iff S_n\leq t.$$

$\therefore$ we can use this function to calculate the $m(t)$

$$m(t) = E[N(t)] = \sum_{n=1}^{\infty}p\{N(t)\geq n\} = \sum_{n=1}^{\infty}p\{S_n\leq t\} = \sum_{n=1}^{\infty}F_n(t)$$

$$= \sum_{n=1}^{\infty}\sum_{i=nr}^{\infty}[e^{-\lambda i}(\lambda t)^{i}]/i!$$

8.

lifetime.

A machine. $\xrightarrow{F}$ new.

fail. ageT

(a) we define that

X if lifetime of new machine is X, $X<T$.

T ... is $\geq T$

$\therefore$ E[long run average cost] $= \int_0^T x f(x) dx + T P(X>T)$

$= \int_0^T x f(x) dx + T(\overset{1-f(T)}{\cancel{\quad x}})$

(b)

The fail means the machine did not used.

~~EF~~

$E[F] = \int_0^T x f(x) dx + (1-F(T))(T+E[F])$

$E[F|x] = \begin{cases} X & x \leq T \\ T+E[F] & x > T \end{cases}$

14. From the wald equation.

~~E...~~ $E[X_1 + \cdots X_N] = E[N] \cdot E[X]$.

$E[X] = 2p-1$

$E[X_1 + \cdots + X_N] = E[N] \cdot E[X] = E[T] \cdot (2p-1)$

$= E[\sum_{j=1}^{T^{++}} X_j]$

$$= (-i)\, P\left\{\sum_{j=1}^{T} X_j = -i\right\} + (N-i)\, P\left\{\sum_{j=1}^{T} X_j = N-i\right\}$$

∴ ~~when $p = \frac{1}{2}$. we can find that~~

~~$E[T] = i(N-i)$~~

$$= (-i)\left[1 - \frac{(q/p)^i}{(q/p)^N}\right] + (N-i)\left[\frac{1-(q/p)^i}{1-(q/p)^N}\right]$$

$$= N\,\frac{1-((1-p)/p)^i}{1-((1-p)/p)^N} - i. \quad p \neq \frac{1}{2}.$$

14. when $p = \frac{1}{2}$

$$E\left[\sum_{j=1}^{T} X_j\right] = (-i)\, p\left\{\sum_{j=1}^{T} X_j = -i\right\} + (N-i)\, p\left\{\sum_{j=1}^{T} X_j = N-i\right\}$$

$$= (-i)\left[1 - \frac{i}{N}\right] + (N-i)\left[i/N\right]$$

$$= 0. \quad \text{when } p = 1/2 .$$

$j=1$

15

(a) $T = \sum_{i=1}^{N} X_i$

X means the ~~amount~~ time of he travel after ~~those~~ ith choice.

(b) The wald equation is $E[T] = E[N]E[X]$. and $E[N]=3$. ∵ N is geometric distribution with $p = 1/3$.

$\therefore E[T] = E[\sum_{i}^{N} X_i] = E[X]E[N]$.

$\therefore E[N] = 3$ ~~$E[X] = (2+4+6)/3 = 4$~~.

~~$\therefore E[T] = 3 \times 4 = 12$~~.

$\therefore E[X] = 14/3$ ∴ ~~$14/3$ ... $E[$~~

$\therefore E[T] = 14$.

(c)

$E[\sum_{i=1}^{N} X_i | N=n]$

$\because$ N is given to n. and X can be 4 or 6.

$E[\sum_{i=1}^{N} X_i | N=n] = E[\sum_{i=1}^{N} X_i | X_1 \neq 2 \cdots X_{n-1} \neq 2, X_n = 2]$.

$= 2 + (n-1) E[X_i | X_i \neq 2]$

$= 2 + (n-1) \times 6$

$= 6n - 4$.

$\therefore E[\sum_{i=1}^{n} X_i] = n E[X_i]$

$= 14n/3$.

(d) $E[T] = E[E[\sum_{i=1}^{N} X_i | N=n]] = E[6N-4]$

$= 14$.

22.

(a) 1 car $\to$ T time.

$\hookrightarrow$ Age T.

break with rate λ.

And each time a car is repaired the time until the next breakdown is rate μ.

$$E[X] = E[X \mid break](1-e^{-\lambda T}) + E[X \mid other] e^{-\lambda T}$$
$$= (T+\frac{1}{\mu})(1-e^{-\lambda T}) + (T+\frac{1}{\lambda})(e^{-\lambda T})$$

rate buy new car is $1/E[X]$

$$= 1 \Big/ [(T+\frac{1}{\mu})(1-e^{-\lambda T}) + (T+1/\lambda)(e^{-\lambda T})]$$

(b)

$$E[\text{long run cost}] = E[\text{cost incurred}] \Big/ E[X]$$

let Y is first break time

The expected length of a cycle is

$$E[\text{cost incurred}] = \int_0^\infty E[\text{cost} \mid Y=y] \lambda e^{-\lambda y} dy$$

Then we use $E[\text{long run cost}] = E[\text{cost incurrent}] / E[X]$

$$= \int_0^\infty E[\text{cost} \mid Y=y] \lambda e^{-\lambda y} dy \Big/$$

$$\left(T+\frac{1}{\mu}\right)\left(1-e^{-\lambda T}\right)+\left(T+\frac{1}{\lambda}\right)\left(e^{-\lambda T}\right).$$

23

A . B player .

serve A -> win -> P_a .

se B -> w -> $q_a = 1 - P_a$.

(a)

N equal to the number point in this cycle .

$$E[N] = 1 + q_a / p_b = \frac{P_b + q_a}{P_b}$$

The propotion by A) $= 1/E[N] = \dfrac{P_b}{P_b + q_a}$

(b)

They have some proportion .

$P = (P_a + P_b)/2$.

(c)

The condition is under A win higher under the winner service .

$\therefore \; P(A|W_n) > P$

$$\therefore \frac{P_b}{P_b + q_a} > \frac{(P_a + P_b)}{2}.$$

$$2P_b > (P_a + P_b)(P_b + q_a)$$

$$2P_b > P_aP_b + P_aq_a + P_b^2 + P_bq_a \implies \therefore \; P_b > P_aP_b + P_aq_a + P_bq_a.$$