4)

~~S_i is the service~~ time

The customer arrive this system with rate λ.

the time serve a group i with PDF of g_i.

X_n means the number of nth service batch.

X_n depends on the group i.

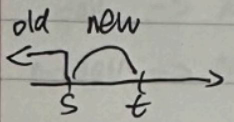
the transition probabilities is P_i

$$P_i = \int_0^\infty \frac{e^{-\lambda x}(\lambda x)^i}{i!} g_i \, dx$$

~~草~~ So this P_i is the transition probablities of X_n.

b3.

firstly we use $X(s)=n$ mean in times there are n customers ~~cos~~



(a) $\because$ the old customers means they are in time s.

and the new customers means they are in $[s, t+s]$

For old: $E_1[X(t+s) \mid X(s)=n] = n+e^{-t\mu}$

For new: $E_2[X(t+s) \mid X(s)=n] = \lambda t$

$\therefore E[X(t+s) \mid X(s)=n] = E_1 + E_2 = n+e^{-t\mu} + \lambda t$

(b)

we know that custome arrive has ~~probably~~ rate λ

the service with rate μ.

$\therefore$ each n custome have $e^{-\mu t}$ still in service.

~~and~~

also $\because Var(X) = E(X^2) - [E(X)]^2$.

$\therefore$ for old $Var_1[X(t+s) \mid X(s)=n] =$ ~~$nt e^{-t\mu} - (nt e^{-st\mu})$~~ $ne^{-\mu t}$

and for new the var is ~~λt~~ $Var_2[X(t+s) \mid X(s)=n] = \lambda t$

$\therefore Var[X(t+s) | X(s)=n] = Var_1 + Var_2 = ne^{-\mu t} + \lambda t$

(C) we define T_1 and T_2

T_1 : the time until current leave $\rightarrow \mu$

T_2 : the time next arrival. $\rightarrow \lambda$.

empty mean $T_1 < T_2$.

$$p(T_1 < T_2) = \frac{\mu}{\lambda + \mu}$$

73.

$p(\text{fail}) = p$

$p(\text{not fail}) = 1-p$

(a) we split the N to N_1 and N_2.

N_1: no fail shocks

N_2: fail shocks.

~~$p(N=n$ $p(T=t|N=n)=$~~

$$p(T|N=n) = \frac{e^{-\lambda T}(\lambda T)^n}{n!}$$

(b)

Firstly we give $N=n$.

$$p(N=n) = \frac{e^{-\lambda} \times \lambda^n}{n!}$$

Then

$p(N=n|T=t) = P(N=n, T=t) / p(T=t)$

So we can try to calculate $p(N=n, T=t)$, and this $= p(N=n) \times p(T=t|N=n)$

$\therefore$ we can try to calculate $p(T=t|N=n)$

and $p(T=t|N=n)$ is easy calculate like in ~~ex~~ problem (a).

$p(T=t|N=n) = [(\lambda T)^n \cdot e^{-\lambda T}] / n!$

$\therefore$ according to this

$$p(N=n, T=t) = [(\lambda T)^n \cdot e^{-\lambda T}] / n! \cdot p(N=n)$$

$$= [(\lambda T)^n \cdot e^{-\lambda T}] / n! \cdot \cancel{\frac{e^{-\lambda n}}{n!}} \frac{e^{-n\lambda}(\lambda)^n}{n!}$$

and $p(T=t) = \dfrac{e^{-\lambda t}(\lambda t)^t}{t!}$

$$p(N=n \mid T=t) = \left[[(\lambda T)^n \cdot e^{-\lambda T}] / n! \cdot \frac{e^{-\lambda}(\lambda)^n}{n!}\right] / \left(\frac{e^{-\lambda t}(\lambda t)^t}{t!}\right)$$

(C) Because this is poisson distribution.

fail $\to N_1 = \lambda p$
not fail $\to N_2 = \lambda(1-p)$.

and this is give $T=t$

$\therefore$ ~~we~~ when we calculate the conditional distribution of N. give $T=t$.
in not fail condition, they must have parameter with $\lambda(1-p)t$.

1. $N_1(t)$ $N_2(t)$, means in t ~~one~~ times. male and female number.

$X(t) = \{N_1(t), N_2(t)\}$.

suppose that i. j. male and female mating

$(i, j) \Rightarrow (i+1, j)$ male $+1$
$(i, j) \Rightarrow (i, j+1)$ female $+1$

there are same probability $p = 1/2$.

2. $A \xrightarrow{\text{rate } a} B$

let $N_A(t)$ $N_B(t)$ means the type A. B number in time t.

$B \xrightarrow{\text{rate } \beta} A$

$\therefore$ we need to calculat the $\{N_A(t), N_B(t)\}$

we suppose A. B in State $(n.m)$

~~Va~~ $V(n.m) = an + \beta m$

$\therefore P_{(n,m)(n-1,\, m+1)} = \frac{an}{an+\beta m}$

$P_{(n,m)(n+2\ m-1)} = \frac{\beta m}{an+\beta m}$

~~2.~~
3. 2 machine. single repairman

↳ with function rate μ_i.

repair time with rate μ.

we can not analyse this as a birth and death process.
Because we do not know the birth and death machine and their ~~status~~ states.
~~status~~

So, if we define the states of each machine, we can analyse it.
we have 4 states.

States 1 : Both working.
states 2 : 1 working 2 X
state 3 : 1 X 2 working.
state 4 : both X, 1 being repair
state 5 : both X, 2 being repair.

$$\text{State1} = \mu_1 + \mu_2$$
$$\text{State2} = \mu + \mu_1$$
$$\text{State3} = \mu + \mu_2$$
$$\text{State4} = \text{State5} = \mu$$

$$P_{1,2} = \frac{\mu_2}{\mu_1+\mu_2} \qquad P_{2,1} = \frac{\mu}{\mu+\mu_1}$$

$$P_{1,3} = \frac{\mu_1}{\mu_1+\mu_2} \qquad P = \frac{\mu}{}$$

$\mu_1+\mu_2$ $\quad$ $_{3,1} = \mu+\mu_2$

$P_{4,2} = \frac{\mu}{\mu} = 1$ $\qquad$ $P_{5,3} = \frac{\mu}{\mu} = 1$

4. potential customers $\longrightarrow$ rate λ.

arrive n customer in station.

probab a_n.

service rate is μ.

first, we assume that there are $N(t)$ customer at ttime.
and n customer already in this station. with new in rate a_n.

$\therefore$ $N(t)$ with birth and death process with.

birth rate $\lambda_n = \lambda a_n$
death rate $\mu_n = \mu$

5. (a) Yes. Because $X(t)$ means the number of infected number at t times
this is always change with time t.

(b) this is a birth process. Because ~~there are~~ the infected number always increased.

~~(c) assume there ~~one are~~ m are infected.~~

~~P(if contact then infected) $= \cancel{\#}(n-m)/\binom{N}{2}$~~

~~P(if contact then not infected) $= \cdots (n-m)$~~

~~$\therefore$ this is the birth rate of this infection.~~

~~$E[T] = E[\text{all are inflected}] = (n-m)/(N) \sum_{m=1}^{N} (n-m)/\binom{n}{2}$~~

5.

(c) assume there are m are inflected.

p(if contact then inflected) $= (n-m)/\binom{n}{2}$

$\because$ there are m are indlected.

$\therefore \quad P = m(n-m)/\binom{n}{2}$

$\therefore$ the birth rate of inflected is $\lambda_i = [\lambda m(n-m)]/\binom{n}{2}$

p

T means every one inflected

$E[T] = \sum_{i=1}^{N} 1/\lambda_i$

$= \sum_{i=1}^{N} \{[\lambda m(n-m)]/\binom{n}{2}\}$

6.(a) state 0 $= 1/\lambda$ $\qquad E[T_i] = \frac{1}{\lambda_i} + \frac{\mu_i}{\lambda_i} E[T_{i-1}]$

state 1 $= 1/\lambda_1 + \frac{\mu_1}{\lambda_1} E[T_0]$.

state 2 $= 1/\lambda_2 + \frac{\mu_2}{\lambda_2} E[T_1]$

state 3 $= 1/\lambda_3 + \frac{\mu_3}{\lambda_3} E[T_2]$

state 4 $= 1/\lambda_4 + \frac{\mu_4}{\lambda_4} E[T_3]$

and we know $\lambda_i = (i+1)\lambda$.

$\mu_i = i\mu$.

then we can use these above to calculate the state 0 to 4.

$E[\text{State } 0 \text{ to } 4] = E[1] + E[2] + E[3] + \cancel{E[4]} + E[0]$.

(b)

$\text{state}5 = 1/\lambda_5 + \frac{\mu_5}{\lambda_5} E[T_4]$

$E[\text{state } 2 \text{ to } 5] = E[2] + E[3] + E[4]$

(c)

$$Var(T_j) = \frac{1}{\lambda_j(\lambda_j+\mu_j)} + \frac{\mu_j}{\lambda_j} Var(T_{j-1}) + \frac{\mu_j}{\lambda_j+\mu_j}[E(T_{j-1}) + E(T_j)]^2 .$$

$$Var[0 \text{ to } 4] = Var[0] + Var[1] + Var[2] + Var[3] .$$

$$Var[2 \text{ to } 5] = Var[2] + Var[3] + Var[4]$$

$$Var[0] = \frac{1}{\lambda_0(\lambda_0+\mu_0)} + \cancel{\frac{\mu_0}{\lambda_0}} Var$$

$$Var[1] = \frac{1}{\lambda_1(\lambda_1+\mu_1)} + \frac{\mu_1}{\lambda_1} Var(0) + \frac{\mu_1}{\lambda_1+\mu_1}[E(0) + E(1)]^2 .$$

$$Var[2] = \frac{1}{\lambda_2(\lambda_2+\mu_2)} + \frac{\mu_2}{\lambda_2} Var(1) + \frac{\mu_2}{\lambda_2+\mu_2}[E(1) + E(2)]^2 .$$

$$Var[3] = \frac{1}{\lambda_3(\lambda_3+\mu_3)} + \frac{\mu_3}{\lambda_3} Var(2) + \frac{\mu_3}{\lambda_3+\mu_3}[E(2) + E(3)]^2$$

$$Var[4] = \frac{1}{\lambda_4(\lambda_4+\mu_4)} + \frac{\mu_4}{\lambda_4} Var(3) + \frac{\mu_4}{\lambda_4+\mu_4}[E(3) + E(4)]^2 .$$

Then we can calculate the variances in parts(a) and (b).

7.

(a) Yes. Because. each individual's pass the stage is ~~indi~~ independent. and the pass time for each stage is exp distribution.

(b) state $n = (n, \dots, n_{k-1})$

because ~~we~~ we consider the infinitesimal

so we can assume only one event happens .

① new customer join.

② a customer from one stage to next .

③ a customer becomes a members .

7.(b) So. our stage are .

① $S_0 = (n_1+1, n_2 \cdots, n_{k-1})$

② $S_1 = (n_1-1, n_2+1 \cdots n_{k-1})$

$S_j = \{(n_1, n_2, \ldots n_j-1, n_{j+1}+1 \cdots n_j-1)$

③ $S_{k-1} = (n_1, n_2 \cdots n_{k-2}, n_{k-1}-1)$.

Fo S_0. rate $= \lambda$. ∵ this new customer .

Fo S_1 rate $=$ $n_1\mu$.

⋮

S_j rate $= n_j\mu$.