Assignment 6

My score

98% (9.8/10)

Q1

9.8 / 10

Chapter 4: Problems 2, 3, 6-8, 14, 17-20, 22, 25, 28-30, 42, 44-46, 54, 57, 58, 67 [Please choose 10 problems and submit your solutions for marking.]

When submitting your solutions for assignment 6, please include the signed and dated Academic Integrity Checklist as the cover page, with your name clearly typed or printed.

2.

Because the offspring ~~is~~ of size is depend on the last one

$p(X_t)$ just depend on $p(X_{t-1})$. the other offspring will not affect them.

So they are Markov chain.

transition?

~~on the other hand they have mean > λ~~

6.

For the~~first~~ ~~step~~ step

$$P^{(1)} = \left\| \begin{matrix} \frac{1}{2}+\frac{1}{2}(2p-1) & \frac{1}{2}-\frac{1}{2}(2p-1) \\ \frac{1}{2}-\frac{1}{2}(2p-1) & \frac{1}{2}+\frac{1}{2}(2p-1) \end{matrix} \right\|$$

$$= \left\| \begin{matrix} p & 1-p \\ 1-p & p \end{matrix} \right\| = P$$

$$P^{(2)} = \left\| \begin{matrix} \frac{1}{2}+\frac{1}{2}(2p-1)^2 & \frac{1}{2}-\frac{1}{2}(2p-1)^2 \\ \frac{1}{2}-\frac{1}{2}(2p-1)^2 & \frac{1}{2}+\frac{1}{2}(2p-1)^2 \end{matrix} \right\| = \left\| \begin{matrix} 1+2p^2-2p & 2p-2p^2 \\ 2p-2p^2 & 1+2p^2-2p \end{matrix} \right\|$$

$$P^{(3)} = \left\| \begin{matrix} \frac{1}{2}+\frac{1}{2}(2p-1)^3 & \frac{1}{2}-\frac{1}{2}(2p-1)^3 \\ \frac{1}{2}-\frac{1}{2}(2p-1)^3 & \frac{1}{2}+\frac{1}{2}(2p-1)^3 \end{matrix} \right\|$$

in $P(2)$ $\quad a_{11} = 1+2p^2-2p = 1+2p(p-1) = p^2+(1-p)^2 = p\times p+(1-p)(1-p)$

$\therefore P^{(2)} = P^{(1)} \times \begin{Vmatrix} p & 1-p \\ 1-p & p \end{Vmatrix}$

and then

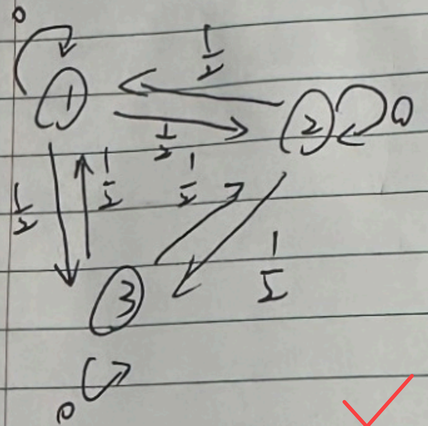
$$P^{(3)} = P^{(2)} \times \begin{Vmatrix} p & 1-p \\ 1-p & p \end{Vmatrix}$$

$$\therefore P^{(n+1)} = P^{(n)} \cdot P$$

should derive it

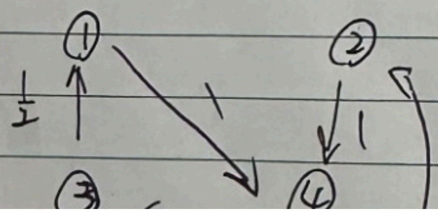
14.

$$P_1 = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & \frac{1}{2} & \frac{1}{2} \\ 2 & \frac{1}{2} & 0 & \frac{1}{2} \\ 3 & \frac{1}{2} & \frac{1}{2} & 0 \end{array}$$

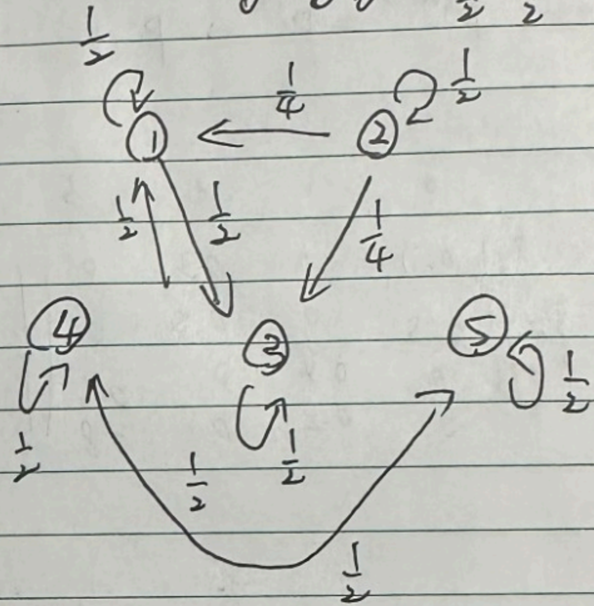


1.2.3 are recurrent

$$P_2 = \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{Vmatrix}$$



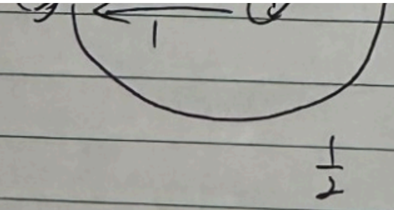
$$P_3 = \begin{Vmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 1/4 & \frac{1}{2} & 1/4 & 0 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{Vmatrix}$$



1,3,4,5 recurrent

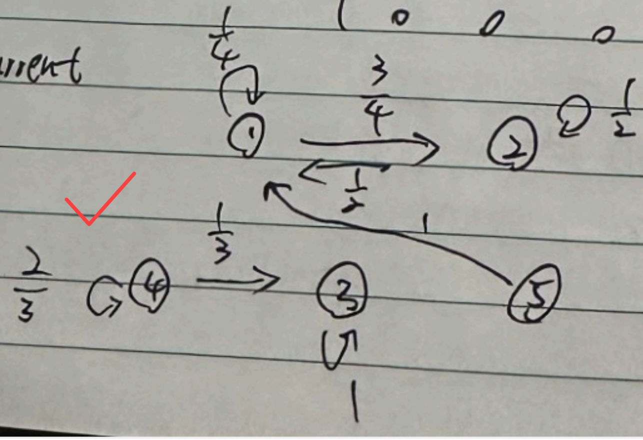
~~1~~ 2 transient

$$P_4 \begin{Vmatrix} \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \end{Vmatrix}$$



$$\begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

1, 2, 3, 4 are recurrent



5, 4 transient

1, 2, 3 recurrent

separate

17.

the strong ~~law~~ law of large number is

$$P\{\lim_{n\to\infty} A_n - \mu = 0\} = 1$$

And we already know that

$$P\{Y_i = 1\} = p = 1 - P\{Y_i = -1\}$$

$\because$ if $p > \frac{1}{2}$. $E(Y) > 0$ ~~so~~ converge of Y must > 0.

$\therefore \sum_{i=1}^{n} Y_i \to \infty$ when $n \to \infty$.

$p < \frac{1}{2}$ $E(Y) < 0$ converge Y must < 0

$\therefore p \neq \frac{1}{2}$. $\sum_{i}^{n} Y_i$ transient.

18. coin 1. $p(h) = 0.6$

$p(T) = 0.4$

$$P = \begin{array}{c|cc|} & 0 & 1 \\ 0 & 0.6 & 0.4 \\ 1 & 0.5 & 0.5 \end{array}$$

coin 2. $p(h) = p(T) = 0.5$

(a)

$\because \pi_j = \sum_{i=0}^{\infty} \pi_i P_{ij}$ $\quad \sum \pi_j = 1$

$$\therefore \begin{cases} \pi_0 = 0.6\pi_0 + 0.5\pi_1 \\ \pi_1 = 0.4\pi_0 + 0.5\pi_1 \\ \pi_0 + \pi_1 = 1 \end{cases}$$

$\pi_0 = 1 - \pi_1$

$1 - \pi_1 = 0.6(1-\pi_1) + 0.5\pi_1$

$1 = 0.6 - 0.6\pi_1 + 0.5\pi_1 + \pi_1$

$0.4 = 0.9\pi_1$

$\pi_1 = 4/9$ $\quad \pi_0 = 5/9$

18 (b)

we need to find P_{01}^{4}

$$P^2 = \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 0.6 & 0.4 \\ 0.5 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.56 & 0.44 \\ \underset{55}{0.\cancel{45}} & 0.45 \end{bmatrix}$$

$$P^4 = P^2 \times P^2 = \begin{bmatrix} 0.56 & 0.44 \\ 0.55 & 0.45 \end{bmatrix} \begin{bmatrix} 0.56 & 0.44 \\ 0.55 & 0.45 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5556 & 0.4444 \\ 0.5555 & 0.4445 \end{bmatrix}$$

$$P_{01}^{4} = 0.4444$$

(c) $p(H)$ for coin1 is 0.8

$p(H)$ for coin2 is 0.5

20.

$\because \sum_{i} P_{ij} = 1$ for all j

and $\pi_j = 1/m+1$

$$\begin{cases} \pi_j = \sum_{i=1}^{m} \pi_i P_{ij} \\ \sum_{j}^{m} \pi_j = 1. \end{cases}$$

$\therefore$ this is a long run proportion.

22. ~~22~~

	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	$\frac{1}{6}$	$\frac{1}{6}$	……			..$\frac{1}{6}$						
1	0	0	$\frac{1}{6}$	$\frac{1}{6}$	……			$\frac{1}{6}$					
2													
3	0	0	0	…			….		$\frac{1}{6}$				
4						⋮							
5													
6													
7													
8													
9													
10													
11													
12	0	0	0	…									

because this is rolling a die for all number is $\frac{1}{6}$

X_n is a new space which mean the number divide 13.

$X_n = Y_n/13$.

according to the Ex 20.

$\pi_y = 1/_{M+1} = 1/13.$

$\therefore$ in here $\pi = (1/13, \cdots 1/13)$

$\pi_0 = 1/13.$

$\lim_{n\to\infty} P\{Y_n \text{ is multiple } 13\} = 1/13$

28. team win game $\rightarrow p(\text{next win}) = 0.8 \rightarrow \text{dinner} = 0.7$

loss game $\rightarrow p(\text{next win}) = 0.3 \rightarrow \text{dinner} = 0.2$.

π mean the proportion of games the team win.

$$\pi = \pi \times 0.8 + (1-\pi) \times 0.3.$$

$$\pi = 0.8\pi + 0.3 - 0.3\pi$$

$$0.5\pi = 0.3$$

$$\pi = 3/5$$

$$p(\text{dinner}) = \frac{3}{5} \times 0.7 + \frac{2}{5} \times 0.2$$

$$= 1/2.$$

29.
$$\begin{cases} 0.7\pi_1 + 0.2\pi_2 + 0.1\pi_3 = \pi_1 \\ 0.2\pi_1 + 0.6\pi_2 + 0.4\pi_3 = \pi_2 \\ 0.1\pi_1 + 0.2\pi_2 + 0.5\pi_3 = \pi_1 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

$$\pi_1 = 1 - \pi_2 - \pi_3$$

$$(1-\pi_2-\pi_3)0.7 + 0.2\pi_2 + 0.1\pi_3 = 1-\pi_2-\pi_3$$

$$0.7 - 0.7\pi_2 - 0.7\pi_3 + 0.2\pi_2 + 0.1\pi_3 - 1 + \pi_2 + \pi_3 = 0$$

$$0.5\pi_2 + 0.4\pi_3 = 0.3$$

$$(1 - \pi_2 - \pi_3)0.2 + 0.6\pi_2 + 0.4\pi_3 - \pi_2 = 0.$$

$$0.2 - 0.2\pi_2 - 0.2\pi_3 + 0.6\pi_2 + 0.4\pi_3 - \pi_2 = 0.$$

$$-0.6\pi_2 + 0.2\pi_3 = -0.2$$

$$\Rightarrow \quad \pi_1 = 6/17, \quad \pi_2 = 7/17, \quad \pi_3 = 4/17$$

30

3 of 4 truck follow a car.

5 car follow a truck.

0 mean car

1 mean truck

	0	1
0	4/5	1/5
1	$\frac{3}{4}$	$\frac{1}{4}$

$$\begin{cases} \pi_0 = \frac{4}{5}\pi_0 + \frac{3}{4}\pi_1 \\ \pi_1 = \frac{1}{5}\pi_0 + \frac{1}{4}\pi_1 \end{cases}$$

$\pi_0 + \pi_1 = 1$

$\therefore \quad \pi_0 = 15/19$

$\pi_1 = 4/19$