

Assignment 10



My score

100% (10/10)

Q1

10 / 10

Assignment 10 (due Tuesday, Dec. 5, 11pm; 1.5% of the course grade):

Chapter 7: Problems 5 8, 9, 14, 15, 18, 22, 23 (please submit solutions for any 6 problems)

4.

5.

- (a) The renewal process. $\{N(t), t \geq 0\}$ hence gamma (r, λ) distribution.
So we can get that.

$$P(N(t) \geq n) = \sum F(x_i)$$

$F(x)$ is a posson. with rate λ .

and. $N(t)$ mean t times

$$\therefore P(N(t) \geq n) = \sum_{i=n}^{\infty} [e^{-\lambda t} (\lambda t)^i] / i!$$

5.

$$= \sum_{i=n}^{\infty} [e^{-\lambda t} (\lambda t)^i] / i!$$

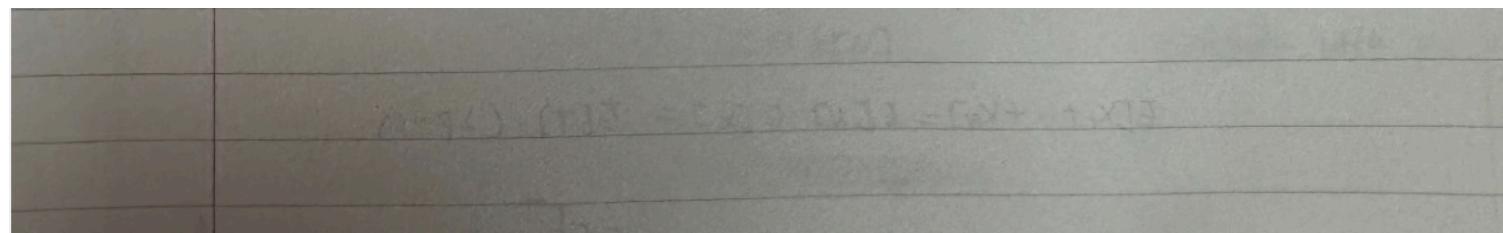
(b) And we know that.

$$N(t) \geq n \Leftrightarrow S_n \leq t.$$

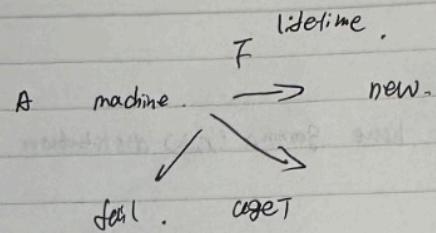
\therefore we can use this function to calculate the mlt.

$$m(t) = E[N(t)] = \sum_{n=1}^{\infty} P\{N(t) \geq n\} = \sum_{n=1}^{\infty} P\{S_n \leq t\} = \sum_{n=1}^{\infty} F_n(t)$$

$$= \sum_{n=1}^{\infty} \sum_{i=n}^{\infty} [e^{-\lambda t} (\lambda t)^i] / i!$$



8.



(a) we define that

$$\begin{aligned} X & \text{ if lifetime of new machine } \Rightarrow X, X \leq T \\ T & \quad \quad \quad \text{is } \geq T \end{aligned}$$

$$\therefore E[\text{long run average cost}] = \int_0^T x f(x) dx + T P(X > T)$$

$$= \int_0^T x f(x) dx + T \left(\frac{1 - F(T)}{f(T)} \right) \checkmark$$

(b)

The fail means the machine did not used.

~~E~~F

$$E[F] = \int_0^T x f(x) dx + (1 - F(T)) (T + E[F])$$

$$\begin{aligned} E[F|x] &= x & x \leq T \\ &= T + E[F] & x > T \end{aligned}$$

14. From the Wald equation.

~~$E[X_1 + \dots + X_N]$~~ $= E[N] \cdot E[X]$

$E[X] = 2P - 1$

$E[X_1 + \dots + X_N] = E[N] \cdot E[X] = E[T] \cdot (2P - 1)$

$$= E\left[\sum_{j=1}^{T+} X_j\right]$$

$$\begin{aligned}
 &= (-i) P\left\{\sum_{j=1}^N X_j = i\right\} + (N-i) P\left\{\sum_{j=1}^N X_j = N-i\right\} \\
 &= (-i) \left[1 - \frac{(q/p)^i}{(q/p)^N} \right] + (N-i) \left[\frac{1 - (q/p)^i}{1 - (q/p)^N} \right] \\
 &= N \frac{1 - ((1-p)p)^i}{1 - ((1-p)p)^N} - i \cdot p \neq \frac{1}{2}. \quad \checkmark
 \end{aligned}$$

~~$E(Z) = (N-i)$~~

←

~~∴ when $p \neq \frac{1}{2}$, we can find that~~

14. when $p = \frac{1}{2}$

$$E\left[\sum_{j=1}^T x_j\right] = (-i) p \left\{ \sum_{j=1}^T x_j = -i \right\} + (N-i) p \left\{ \sum_{j=1}^T x_j = N-i \right\}$$

$$=(-i) \left[1 - \frac{i}{N} \right] + (N-i) \left[\frac{i}{N} \right]$$

$$= 0 \quad \text{when } p = \frac{1}{2}.$$

$j=1$

15

$$(a) T = \sum_{i=1}^N X_i$$

X means the amount time of he travel after those ~~those~~ i th choice.

(b) The wodd equation is $E[T] = E[N]E[X]$. and $E[N] = 3$. $\therefore N$ is geometric distribution with $p = 1/3$.

$$\therefore E[N] = 3 \quad \cancel{E[X] = (2+4+7)/3 = 4}$$

$$\cancel{\therefore E[T] = 3 \times 4 = 12}$$

$$\therefore E[X] = 14/3 \quad \therefore \cancel{14/3 = E[X]}$$

$$\therefore E[T] = 14$$

(C)

$$\text{Ex} \quad E\left[\sum_{i=1}^N X_i \mid N=n\right]$$

$\therefore N$ is given to n . And X can be 4 or 6.

$$E\left[\sum_{i=1}^N X_i \mid N=n\right] = E\left[\sum_{i=1}^N X_i \mid X_1 \neq 2 \dots X_{n-1} \neq 2, X_n = 2\right].$$

$$= 2 + (n-1)E[X_i \mid X_i \neq 2]$$

$$= 2 + (n-1) \times 6$$

$$= 6n - 4.$$

$$\therefore E\left[\sum_{i=1}^n X_i\right] = nE[X_i]$$

$$= 14n / 3.$$

$$(d) \quad E[T] = E\left[E\left[\sum_{i=1}^N X_i \mid N=n\right]\right] = E[6n - 4]$$

$$= 14.$$

22.

(a) If car \rightarrow T time.

Age T.
 ↳ break with rate λ .

And each time a car is repaired the time until the next breakdown is rate μ .

$$\begin{aligned} E[X] &= E[X \mid \text{break}] (1 - e^{-\lambda T}) + E[X \mid \text{other}] e^{-\lambda T} \\ &= (T + \frac{1}{\mu}) (1 - e^{-\lambda T}) + (T + \frac{1}{\lambda}) (e^{-\lambda T}) \quad \checkmark \end{aligned}$$

rate buy new car is $1/E[X]$

$$= \frac{1}{[(T + \frac{1}{\mu})(1 - e^{-\lambda T}) + (T + \frac{1}{\lambda})(e^{-\lambda T})]}$$

b,

$$E[\text{long run cost}] = E[\text{cost incurred}] / E[X]$$

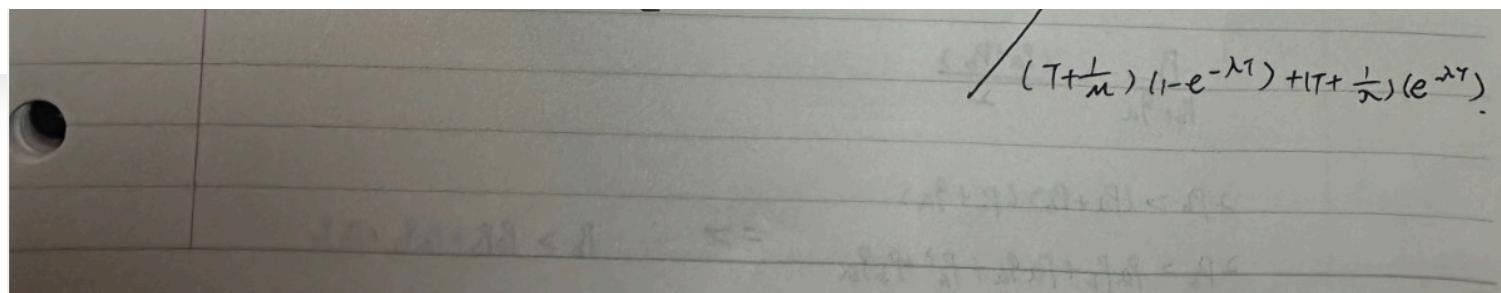
let γ is first break time

the expected length of a cycle is

$$E[\text{cost incurred}] = \int_0^\infty E[\text{cost} \mid \gamma=y] \lambda e^{-\lambda y} dy$$

Then we use $E[\text{long run cost}] = E[\text{cost incurred}] / E[X]$

$$= \int_0^\infty E[\text{cost} \mid \gamma=y] \lambda e^{-\lambda y} dy /$$



A handwritten mathematical equation on lined paper. The equation is:

$$\cancel{(T + \frac{1}{\mu}) (1 - e^{-\lambda T}) + (T + \frac{1}{\mu}) (e^{-\lambda T})}$$

23

A . B player .

Send A \rightarrow win $\rightarrow P_a$ Send B \rightarrow win $\rightarrow P_b = 1 - P_a$

(a)

N equal to the number point in this cycle.

$$E[N] = 1 + \frac{P_b + P_a}{P_b}$$

$$\text{The proportion by A} = \frac{1}{E[N]} = \frac{P_b}{P_b + P_a}$$

(b)

They have some proportion.

$$P = \frac{(P_a + P_b)}{2}$$

(c)

The condition is under A win higher under the winner sende.

$$\begin{aligned} P_{AWB} &> P \\ \therefore \frac{P_b}{P_a + g_a} &> \frac{(P_a + P_b)}{2} \\ 2P_b &> (P_a + P_b)(P_b + g_a) \\ 2P_b &> P_a P_b + P_a g_a + P_b^2 + P_b g_a \Rightarrow P_b > P_a P_b + P_a g_a + P_b g_a \end{aligned}$$

