

Assignment 5



My score

92% (9.2/10)

Q1

9.2 / 10

[ch3_conv_ass.pdf](#)

Question 2: In the condition "If the number of balls of this color is not less than the number of balls of the other color", the number of balls means the number of balls left in the urn after the draw.

1.

we know $P(X_n(S)=0) = 0.5 = P(X_n(S)=1)$.

$\therefore X_n$ is not $\rightarrow 0$.

and doesn't converge in probability, a.e. e.

and this is a $(0,1)$ distribution.

so.

$$F_n(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & x \in [0,1] \\ 1 & x \geq 1 \end{cases} \quad \checkmark$$

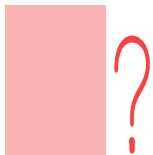
\therefore this converge to distribution when $n \rightarrow \infty$

2.

$$E[X_1(S)] = \frac{2}{3} \quad E[X_2(S)] = \frac{X_1(S)^2 + 3X_1(S)}{3}$$

\therefore when the n is $\rightarrow \infty$ the number of ball is from 1 to 3.

$\therefore E[X_n(S)]$ not converge to.



3.

$$(a) U_n(s) = s/n$$

$$|U_n(s) - V(s)| = |U_n(s) - 0| = |U_n(s)| = s/n$$

and $s \in [0, 1]$

$$\therefore s/n < \epsilon$$



$$\therefore \lim_{n \rightarrow \infty} \{ |U_n(s) - V(s)| < \epsilon \} = 1$$

$$U_n(s) - V(s) < \epsilon$$

$\therefore U_n(s)$ converges to 0

$$(b) V_n(s) = s(1 - 1/n)$$

$$\text{When } n \rightarrow \infty \quad 1 - 1/n = 1 \quad V_n(s) = s$$

$$\lim_{n \rightarrow \infty} |V_n(s) - V(s)| = \lim_{n \rightarrow \infty} |s/n| = 0$$

$$\therefore \lim_{n \rightarrow \infty} \{ |V_n(s) - V(s)| = 0 \} = 1$$

$$\therefore V_n(s) \xrightarrow{\text{a.e.}} s \quad \text{as } n \rightarrow \infty$$

$$(c) W_n(s) = \cos(2\pi n s)$$

$$\text{When } n \rightarrow \infty \quad \cos(2\pi s) = 1 \text{ or } -1 \text{ or } 0$$

$W_n(s)$ doesn't converge because different s will cause different ~~value~~ value.

4.

$$n = 2^j + k$$

$$k < n - 2^j \Rightarrow k/2^j = n/2^j - 1$$

$$k+1/2^j = \frac{n+1}{2^j} - 1$$

$$\therefore \lim_{n \rightarrow \infty} P(|Y_n(s) - Y(s)| > \epsilon) = \lim_{n \rightarrow \infty} P(|Y_n(s)| > \epsilon)$$

when $\epsilon \geq 1$

$$\lim_{n \rightarrow \infty} P(|Y_n(s)| > \epsilon) = 0.$$

$$\text{when } \epsilon \in (0, 1) \quad n \rightarrow \infty. \quad k+1/2^j = \frac{n+1}{2^j} - 1 \rightarrow 0 \quad \checkmark$$

$$k/2^j = n/2^j - 1 \rightarrow 0.$$

$$\therefore \lim_{n \rightarrow \infty} P(|Y_n(s)| > \epsilon) = 0.$$

$$\therefore Y_n(s) \xrightarrow{P} 0 \quad \text{as } n \rightarrow \infty.$$

But from $n/2^j - 1$ to $(n+1)/2^j - 1$, there still have some $Y_n(s) \neq 0$.

$\therefore Y_n(s)$ not converge in a.e.

