12. individual birth λ immigration θ (population $\leq N$)

die μ

(a)

if the t-time is birth and die model and n population

$\lambda_n = n\lambda \quad n < N$

$\mu_n = n\mu$

$\lambda_n = n\lambda + \theta \quad n \geq N$

(b)

$\lambda_n P_n = \mu_{n+1} P_{n+1}$

$P_1 = \frac{\lambda_0}{\mu_1} P_0 = \frac{\lambda}{\mu} P_0 \qquad \lambda_n = n\lambda \qquad \lambda_1 = \lambda \qquad \lambda_2 = 2\lambda$

$P_2 = \frac{\lambda_1}{\mu_2} P_1 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} P_0 = \frac{\lambda^2}{2\mu^2} P_0 \qquad \mu_n = n\mu \qquad \mu_1 = \mu \qquad \mu_2 = 2\mu$

$P_3 = \frac{\lambda_2}{\mu_3} P_2 = \frac{\lambda_2 \lambda_1 \lambda_0}{\mu_3 \mu_2 \mu_1} P_0 =$

$\vdots$

$P_{n+1} = \frac{\lambda_1}{\mu_{n+1}} P_n = \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1} P_0$

$$\sum_{n=0}^{\infty} \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1} = \sum_{n=0}^{n} \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \cdots \mu_1} + \sum_{n>N}^{\infty} \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1} \mu_n \cdots \mu_1}$$

$$= A + \sum_{n=N}^{\infty} \frac{\lambda^{n+1}}{\mu_{n+1} \cdots \mu_1} = \frac{(N\mu)^{N-1}}{\mu_{N-1}\mu_{N-2} \cdots \mu_1} \sum_{n=N+1}^{\infty} \frac{\lambda^{n+1}}{(N\mu)^{n+1}} + A$$

Then ~~so~~ using $\sum_n P_n = 1$

$$P_0 + P_0 \sum_{n=0}^{\infty} \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1}\mu_n \cdots \mu_1} = 1$$

$$P_0 = \frac{1}{1 + \frac{(N\mu)^{N-1}}{\mu_{N-1}\mu_{N-2}\cdots\mu_1} \sum_{n=N+1}^{\infty} \frac{\lambda^{n+1}}{(N\mu)^{n+1}} + A} \Rightarrow P_{n+1} = \frac{\lambda_n \lambda_{n-1} \cdots \lambda_0}{\mu_{n+1}\mu_n \cdots \mu_1 \left(1 + \frac{(N\mu)^{N-1}}{\mu_{N-1}\mu_{N-2}\cdots\mu_1} \sum_{n=N+1}^{\infty} \frac{\lambda^{n+1}}{(N\mu)^{n+1}}\right)}$$

13. → service time $1/4$

poten. new arrive 3 per hour.

room most have 2.

(a)

$\lambda_0 = \lambda_1 = 3$ in

$\mu_1 = \mu_2 = 4$ leave.

$P_1 = \frac{\lambda_0}{\mu_1} P_0 = \frac{3}{4} P_0$

$P_2 = (\frac{3}{4})^2 P_0$

Since $\sum_{0}^{2} P_i = 1$

$\therefore$ ~~$P_0 =$~~ $(\cancel{P_0} + \frac{3}{4} P_0 + (\frac{3}{4})^2 P_0) = 1$

$P_0 = (1 + \frac{3}{4} + (\frac{3}{4})^2)^{-1} = 16/37$

The average number is ~~$P_0 +$~~ ~~$P_1 +$~~

$N(\text{average number}) = 0 P_0 + 1 P_1 + 2 P_2 = 30/37$

(b) proportion of the potential customer in the barber is equal to barber has 0 or 1 customer.

$P_0 + P_1 = 16/37 + \frac{16}{37} \times \frac{3}{4} = \frac{28}{}$

$P_0 + P_1 = \frac{}{37} + \frac{}{37} = \frac{}{37}$

(C) if the barber work twice.

$P_1 = \frac{3}{8} P_0$

$P_2 = (\frac{3}{8})^2 P_0$

$P_0 = (1 + \frac{3}{8} + (\frac{3}{8})^2)^{-1} = \frac{64}{97}$

$P_1 = \frac{24}{97}$

$P_0 + P_1 = \frac{88}{97}$

$\frac{88}{97} - \frac{28}{37} = 0.907 - 0.7567 = 0.1503 \approx 0.15$

$0.15 \times \lambda = \underline{0.45}$

$\therefore$ improve to 0.45 per hous.

15. 2 serves $\rightarrow$ 2 service per h per people

arrives 3 per hour.

system capacity is 3.

(a)

$\lambda_0 = \lambda_1 = \lambda_2 = 3.$

$\mu_1 = 2$

$\mu_2 = \mu_3 = 4$

$P_1 = \frac{\lambda_0}{\mu_1} P_0 = \frac{3}{2} P_0$

$P_2 = \frac{9}{8} P_0.$

$P_3 = \frac{3}{4} P_2 = \frac{27}{32} P_0.$

$P_0 = (1 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32})^{-1} = 32/143.$

$\therefore P_1 = \frac{32}{143} \times \frac{3}{2} = \frac{48}{143}$

$P_2 = \frac{36}{143}.$

$P_3 = \cancel{\frac{3143}{143}}\ \frac{27}{143}$

$\cancel{P_0 + P_1 =}\ 1 \times P_0 + 1 \times P_1 + 1 \times P_2 + 0 \times P_3 = \frac{116}{143}$

(b) 1 serves. 4 serve per h.

$\therefore P_1 = \frac{3}{4} P_0$

$P_2 = \frac{9}{}\ P$

$P_2 = \frac{9}{16} P_0$

$P_3 = \frac{27}{64} P_0$

$P_0 = \left(1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64}\right)^{-1} = 64/175$

$1 \times P_0 + 1 \times P_1 + 1 \times P_2 + 0 \times P_3 = 1 - 1 \times P_3 = 1 - \frac{64}{175} \times \frac{27}{64} = 1 - \frac{27}{175} = \frac{148}{175}$

18. working rate is λ

and have k distinct phases.

phase 1 must $\rightarrow$ time rate μ_i.

(a) $\lambda P_0 = \mu_k P_k$

$\mu_1 P_1 = \lambda P_0$

also $\because$ $\mu_1 P_1 = \mu_2 P_2 \cdots\cdots = \mu_i P_i$. each are independent.

$\Downarrow$

$\therefore \mu_i P_i = \lambda P_0$

also $\sum P_k = 1$

$P_i = (\lambda / \mu_i) P_0$.

P_i is the proportion of machine undergoing a phase i repair.

(b)

$\because \sum P_j = 1$

$1 = P_0 + P_0 \sum_{j=1}^{k} (\lambda/\mu_j)$

$\therefore P_0 = [1 + \sum_{j=1}^{k} (\lambda/\mu_j)]^{-1}$

$\therefore$ P_0 is ~~the~~ the proportion of machine working.

20. 2 machine. one for ~~p~~ spare.

↳ one machine → rate λ.

↓

fail → ~~repir~~ repair μ.

$\lambda_0 = \lambda_1 = \lambda$

$\mu_1 = \mu_2 = \mu$

$$\begin{tikzcd} 0 \arrow[r, bend left, "\lambda"] & 1 \arrow[l, bend left, "\mu"] \arrow[r, bend left, "\lambda"] & 2 \arrow[l, bend left, "\mu"] \end{tikzcd}$$

$\because$ this is a exponential function.

$E(x) = 1/\lambda$.

(a) $\cancel{E[0 \to 1] = 1/\lambda}$. $\cancel{E[0 \to 2] = 1/\lambda} +$

$\cancel{E[1 \to 0] =}$ $E[T_0] = 1/\lambda$ $E[T_1] = \frac{1}{\lambda+\mu} + \frac{\lambda}{\lambda+\mu} \times 0 + \frac{\mu}{\lambda+\mu} \times [E[T_0] + E[T_1]]$

$E[T_1] = 1/\lambda + \mu/\lambda_2$.

(b) $Var = Var[T_0] + Var[T_1]$

(c) $\cancel{P_0 =}$ $F_0 + F_1 + F_2 = 1 + \lambda/\mu + \lambda^2/\mu^2 = \dfrac{\mu^2+\lambda\mu+\lambda^2}{\mu^2}$

Then we have

$\cancel{F_0 =}$ $P_0 = F_0/\Sigma F_n = \mu^2/(\mu^2+\lambda\mu+\lambda^2)$, $P_1 = F_1/\Sigma p_n = \lambda\mu/(\mu^2+\lambda\mu+\lambda^2)$

and $P_2 = F_2/\Sigma F_n = \lambda^2/(\mu^2+\lambda\mu+\lambda^2)$

$\therefore$ the proportion of machine is equal to $1-P_2 = (\mu^2+\lambda\mu)/\mu^2+\lambda\mu+\lambda^2$

22. Customers arrive → rate λ.

If n already in → join $1/(n+1)$

not join $n/(n+1)$

service μ.

The number in this is birth and die process.

$\lambda_n = (1/(n+1))\lambda$

$\mu_n = \mu$.

Due to this is poisson process, so we can get.

$$P_n = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$$

$$= \frac{e^{-\lambda/\mu}(\lambda/\mu)^n}{n!}$$

[illegible]

23. $X(t)$ is the number of machine breakdown at time t.

(a) This is a birth and die process

$\therefore$

$$\lambda_0 = 3/10 \qquad \lambda_1 = 2/10 \qquad \lambda_2 = 1/10$$

$$\mu_1 = 1/8 \qquad \mu_2 = 2/8 \qquad \mu_3 = 2/8$$

$$P_1 = \lambda_0/\mu_1 P_0 = \frac{3/10}{1/8} P_0 = 12/5\, P_0.$$

$$P_2 = \lambda_1\lambda_0/\mu_2\mu_1\, P_0 = 48/25\, P_0$$

$$P_3 = 192/250\, P_0.$$

and $\quad \Sigma P_j = 1$

$$P_0 = \left[1 + 12/5 + 48/25 + 192/250\right]^{-1} = 250/1522.$$

(b).

$$\therefore \quad P_1 = 12/5 \times 250/1522 = \frac{600}{1522}$$

$$P_2 = 48/25 \times 250/1522 = 480/1522$$

$$P_3 = 192/250 \times 250/1522 = 192/1522$$

$(480 + 192)/$

for both repairmen are busy is $P_2+P_3=$

$= 672/1522$

$= 0.44152$