Assignment 3

My score

90% (9/10)

Q1

9 / 10

Chapter 2: Problems 28, 37, 40, 42, 45, 54, 59, 63, 68, 70

When submitting your solutions for assignment 3, please include the signed and dated Academic Integrity Checklist as the cover page, with your name clearly typed or printed.

cover page?

28, ~~the~~

(a) the whole process is

$O_1 \to H/T \qquad O_2 \to H/T$

$O_1 = O_2$ HH, TT $\to$ step 1

$O_1 \neq O_2$ HT $\to x=1$

TH $\to x=0$

$p(x=0) = p(x=1) = \frac{1}{2}$

(b)

NO,

~~final~~ final flip is head $p^n(1-p)$ $\sum_{n=1}^{\infty} p^n(1-p) = p$

final flip is tail $(1-p)^n p$ $\sum_{n=1}^{\infty} (1-p)^n \cdot p = (1-p)$

So. their p is different.

~~27.~~

~~$P_n(X) = P(X_1) + P(X_2) \dots + P(X_n)$~~

$= \cancel{x}$

37.

$$p\{M\} = p\{\max(x_1, x_2 \cdots x_n) \leq x\}$$
$$= p\{x_1 \leq x, \cdots x_n \leq x\}$$
$$= \prod_{i=1}^{n} p\{x_i \leq x, \cdots x_n \leq x\}$$
$$= x^n$$

The PDF of M

$$f_X(x) = dF_X(x)/dx$$

$$= d\,x^n/dx$$

$$= n\,x^{n-1}$$

4.

$P(X=4) = p^4 + (1-p)^4$

$P(X=5) = A_4^3\,p^3 \cdot (1-p)p + A_4^3\,(1-p)^3 \cdot p(1-p)$

$P(X=6) = A_5^3\,p^3 \cdot (1-p)^2 \cdot p + A_5^3\,(1-p)^3 \cdot p^2(1-p)$

$P(X=7) = A_6^3\,p^3(1-p)^3$

$$E(X) = \sum_{i=4}^{7} i\,P(X=i)$$

when $p = 1/2$

$$E(x) = 4 \times \left(\frac{1}{2}^4 + \frac{1}{2}^4 \right) + 5 \times 4 \cdot \left(\frac{1}{2}\right)^3 \frac{1}{2} \frac{1}{2} \times 2) + 6 \times 20 \times 2 \times \left(\frac{1}{4}\right)^6$$

$$+ 7 \times 120 \times \left(\frac{1}{2}\right)^6 = 93/16$$

42.

we can suppose they already have i type coupon

$\therefore p(\text{have}) = \frac{m-i}{m}$, $i = 0, 1 \cdots, m-1$

$X = \sum_{i=1}^{m} X_i$

and X_i is the number needed

~~$\therefore E(X) = \sum_{i=0}^{m-1} \Sigma X_i = \sum_{i=0}^{m-1} \frac{m-i}{m}$~~

$E(X) = \sum_{i=0}^{m-1} \Sigma X_i$

$\because$ p is geometric distribution

$\therefore E(X) = \sum_{i=0}^{m-1} \frac{m}{m-i}$

45

Total r keys and k box

2key in box $C_r \Sigma p_i$

3key in box $C_r^3 \Sigma p_i^3$

⋮

n key in box $C_r^n \Sigma p_i^n$

$C_r^1 \Sigma p_i^1 = r$

$C_r^0 \Sigma p_i^0 = k$

$$C_r^0 \Sigma p_i^0 - C_r^1 \Sigma p_i^1 + C_r^2 \Sigma p_i^2 + \cdots + C_r^n \Sigma p_i^n (-1)^{r-1}$$

$$= \Sigma C_r^n p_i^n (-1)^{r-1}$$

and then

$$E(\text{collisions}) = \underline{\Sigma C_r^n p_i^n (-1)^{r-1}} - k + r$$

54

(a)

type 1 type 2

P_1 $P_2 = 1 - P_1$

same α

~~x~~ diff β

$P_1 = \alpha P_1 + (1-P_1)\beta$

$P_2 = \beta P_1 + (1-P_1)\alpha$

(b)

$P(T_1, 2)$

1	2
Type 1	Type 1
Type 1	Type 2
Type 2	Type 1
Type 2	Type 2

$$p(F_1, 2) = p_1^2 a + p_2^2 \overset{a}{\beta} + p_1(1-p_1)(a+\beta^2)$$

$$= p_1^2 a + (1-p_1)^2 a + p_1(1-p_1)(a+\beta^2)$$

(C)

$F_{1,3}$

1	2	3
T_1	T_1	T_1
T_2	T_2	T_2
T_1	T_2	T_2
T_2	T_1	T_1

$P(F_{1,2} | F_{1,3}) = p_1^3 a + (1-p_1)^3 \beta a + p_1(1-p_1)^2 \beta + \cancel{p}(1-p_1) p_1^2 \beta$

$p(F_{1,2}) = p_1^2 a + \cancel{p}(1-p_1)^2 a + p_1(1-p_1)(\cancel{\alpha}\beta)^2$

5?.

(a) Because they are ~~t~~independent

So.

their CDF is $F(X_1)$ $F(X_2)$ $F(X_3)$ and $F(X_4)$

their P is the same ?

(b)

$$P = \int p(x)\,dx$$

$$P = \int\int\int\int dx_4\,dx_3\,dx_2\,dx_1$$

$$= \int_0^1 \int_{x_1}^1 \int_0^{x_2} \int_{x_3}^1 dx_4\,dx_3\,dx_2\,dx_1$$

$$= \int_0^1 \int_{x_1}^1 \int_0^{x_2} (1 - x_3)\,dx_3\,dx_2\,dx_1$$

$$= \int_0^1 \int_{x_1}^1 (x_2 - x_2^2/2)\,dx_2\,dx_1 \quad \cancel{= \int_0^1 \frac{x_1^2}{2} - \frac{1}{2}\cdot\frac{1}{3}x_1^3}$$

$$= \int_0^1 \frac{1}{2}x_2^2 - \frac{1}{2} \times \frac{1}{3}x_2^3 \Big|_{x_1}^{1} \, dx_1$$

$$= \int_0^1 \frac{1}{2}x_2^2 - \frac{1}{6}x_2^3 \Big|_{x_1}^{1} \, dx_1$$

$$= \int_0^1 \left[\left(\frac{1}{2} - \frac{1}{6}\right) - \frac{1}{2}x_1^2 + \frac{1}{6}x_2^3\right] dx_1$$

$$= \int_0^1 \left(\frac{1}{3} - \frac{1}{2}x_1^2 + \frac{1}{6}x_2^3\right) dx_1$$

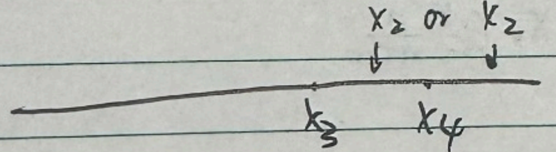
$$= \left(\frac{1}{3}x_1 - \frac{1}{2}\times\frac{1}{3}x_1^3 + \frac{1}{6}\times\frac{1}{4}x_2^4\right)\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{6} + \frac{1}{24} = \frac{8}{24} - \frac{4}{24} + \frac{1}{24} = 5/24$$

(c)

$4! = 24$

Because we already know $p = P\{x_1 < x_2 > x_3 < x_4\}$



so. if $x_3 < x_2 < x_4$

the order will like $x_4 > x_2 > x_3 > x_1$

$x_4 > x_2 > x_1 > x_3$

if $x_3 < x_4 < x_2$

the order will like $\cancel{x_3}\ x_2 > x_4 > x_3 > x_1$

$x_2 > x_1 > x_4 > x_3$

$x_2 > x_4 > x_1 > x_3$

63.

(a) X = number of w balls seleted

n w m b

$$p(w) = \frac{n}{n+m}$$

$$p(b) = \frac{m}{n+m}$$

$P\{X=i\}$ mean the w ball seleten when k ball chose

$$p(X=i) = C_k^x \cdot \frac{n}{n+m}$$

(b) $E(X) = \sum x_i p(x_i)$

$$= \sum_i x_i C_k^{x_i} \frac{n}{n+m}$$

Because this is 0 or 1 distribution

So. ~~P(x)~~ $p(y) = 1 - p(x)$

$p(y) = 1 - C_k^x \cdot \frac{n}{n+m}$

$p(x) = 1 - p(y)$

$p(y=j) = C_k^y \left(\frac{m}{n+m}\right)$

$E(x) = \sum x_i \, p(x_i) = \sum x_i (1 - p(y_i))$

$= \sum x_i \left(1 - C_k^y \frac{m}{n+m}\right)$

68.

X: working time

W: repair time.

$Y = X + W$

(a) $\because$ X and W are independent.

$\therefore$ ~~$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y) = \lambda^2 e^{-\lambda y}$~~

~~$f_X(x) = \lambda$~~

~~$f_Y(y) = \lambda e^{-\lambda y}$~~

$f_X(x) = \int_0^{\infty} f_{X,Y}(x,y)\,dy = \int_0^{\infty} \lambda^2 e^{-\lambda y}\,dy = \lambda \int_0^{\infty} e^{-\lambda y}\,dy$

$= \lambda^2 \cdot \frac{1}{\lambda} e^{-\lambda x}$

$= \lambda e^{-\lambda x}$

(b) $f_Y(y) = \int_0^{\infty} f_{X,Y}(x,y)\,dx = \int_0^{y} \lambda^2 e^{-\lambda y}\,dx = \lambda e^{-\lambda y} \int_0^{y} dx$

$= \lambda e^{-\lambda y} \times \Big|_0^{y}$

$= \lambda e^{-\lambda y}\, y$

6f (c)

Because w and x are independent.

$\therefore$ joint density of w and x

means ~~$f(w) \cdot f(x)$~~

$f_{w,x}(w,x) = f_w(w) \cdot f_x(x)$

$f_w(w) = e^{-\lambda w} - 1$

$f_x(x) = \lambda e^{-\lambda x}$

$f_{w,x}(w,x) = \cancel{\lambda e^{-\lambda w - 1 + \lambda x}} \quad \lambda e^{-\lambda x - \lambda w} - 1$

6f(d)

$y = x + w$

$w = y - x$

$0 < x < y < \infty$

$w + x = y$

$w \geq y - x$

$F_W(w) = p(w = y - x) = p(w + x \geq y)$

$$= \int_0^{\infty} \int_x^{w+x} f(x,y)\, dy\, dx$$

$$= \int_0^{\infty} \int_x^{w+x} \lambda^2 e^{-\lambda y}\, dy\, dx$$

$$= \int_0^{a} \lambda^2 \frac{1}{\lambda} e^{-\lambda y} \Big|_x^{w+x} dx$$

$$= \lambda \int_0^{a} \left(e^{-\lambda(w+x)} - e^{-\lambda x}\right) dx$$

$$= \lambda \frac{1}{\lambda} e^{-\lambda(w+x)} - \frac{1}{\lambda} e^{-\lambda x} \Big|_0^{a}$$

$$= e^{-\lambda w} - 1$$

To .

$\because$ we need to calculate the mgf is exist

$\therefore$ we need to prove $E[X^k] < \infty$

First, we suppose. have a random variable, and P.D.f is f:

$$\therefore E[|X|^j] = \int_{-\infty}^{\infty} |x|^j f(x)dx$$

$$= \int_{|x|\leq 1} |x|^j f(x)dx + \int_{|x|>1} |x|^j f(x)dx$$

$j<k$

$$\leq \int_{|x|\leq 1} 1 \cdot f(x)dx + \int_{|x|>1} |x|^k f(x)dx$$

$$\leq \int_{|x|\leq 1} 1 \cdot f(x)dx + E(|X|^k)$$