PHYSICS-INFORMED NEURAL NETWORKS FOR MODAL ANALYSIS OF DIAPHRAGM-STRUCTURED MEMS WITH EXPERIMENTAL VALIDATION

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ABSTRACT

We present and experimentally validate a novel framework that leverages physics-informed neural networks (PINNs) for advanced MEMS eigenmode analysis. Our work advances the field in three key ways: (1) we demonstrate the successful application of PINNs to calculate multiple vibration modes in MEMS diaphragms of arbitrary geometry by proposing and implementing three key physical equation as constraints; (2) we validate our PINNs model through digital holographic microscopy (DHM) measurements on a fabricated Piezoelectric Micromachined Ultrasonic Transducer (PMUT) device; and (3) we develop a highly efficient computational platform that combines our PINNs and already existing analytical models to evaluate PMUT array performance using only geometrical configurations of the array.

KEYWORDS

Physics-informed Neural Networks; Diaphragm-structured MEMS; Eigenmodes; Micromachined Ultrasonic Transducers

INTRODUCTION

The performance of diaphragm-structured microelectromechanical systems (MEMS) depends heavily on their vibrational modal behavior, which influences key operational characteristics such as resonance, sensitivity, and structural integrity [1-3]. Accurate modal analysis is crucial for optimizing these devices in various applications, including micromachined ultrasonic transducers, resonators, microvalves, and micropumps. Traditionally, researchers rely on finite element analysis (FEA) and analytical methods to study eigenmodes. While these techniques have been widely used, they come with significant drawbacks. FEA, though heavily used, is computationally expensive and often requires appropriate software, limiting accessibility. Analytical solutions, on the other hand, are restricted to simple geometries and struggle to handle complex shapes [4] and boundary conditions commonly found in practical MEMS designs.

A promising alternative is emerging through physics-informed neural networks (PINNs), a deep learning framework that integrates physical laws into neural network training. PINNs have demonstrated their effectiveness in solving complex partial differential equations across various fields, including fluid dynamics [5], solid mechanics [6,7], and electromagnetics [8]. By embedding governing equations directly into the learning process, PINNs can provide accurate, data-efficient solutions without the need for large simulation datasets. Despite these advantages, their application to diaphragm-structured MEMS eigenmode analysis remains largely unexplored. One of the main challenges lies in defining appropriate physical constraints and ensuring that PINN-based results align with experimental or numerical benchmarks.

This study explores the potential of PINNs for modal analysis in diaphragm-based MEMS devices. We develop a framework that incorporates fundamental mechanical principles to accurately predict eigenmodes, addressing the limitations of conventional methods. Through comparative analysis, we evaluate the effectiveness of PINNs against established FEA and experimental results, demonstrating their potential to offer fast, reliable, and accessible alternatives for MEMS analysis, design, and optimization.

PINNS MODEL ARCHITECTURE AND SYSTEM WORKFLOW

The objective of this study is to develop a data-efficient and physics-driven framework capable of accurately predicting modal characteristics without relying solely on computationally expensive simulations or limited analytical models.

The first step in our approach involves defining the fundamental physics governing diaphragm vibrations. We identify three key physical constraints: the equivalence relation, the Helmholtz equation, and the orthogonality condition. These constraints, along with data-driven learning, form the basis of the total loss function, expressed as:

$$\mathcal{L} = \mathcal{L}^{\mathrm{Equ}} + \mathcal{L}^{\mathrm{Helm}} + \mathcal{L}^{Ortho} + \mathcal{L}^{\mathrm{data}} \quad (1)$$

where $\mathcal{L}^{\mathrm{E}qu}$, $\mathcal{L}^{\mathrm{H}elm}$, and $\mathcal{L}^{Ortho}$ enforce physical consistency of the three constraints mentioned above, and $\mathcal{L}^{\mathrm{data}}$ represents the error between model predictions and observed data. This composite loss ensures that the PINN solution remains faithful to both theoretical principles and empirical simulations. The model seeks to minimize this total loss function by optimizing the neural network parameters θ:

$$\theta^* = \underset{\theta}{\operatorname{argmin}} \mathcal{L}(\theta) \quad (2)$$

where θ^* represents the optimal set of network parameters that yield the most accurate predictions. This optimization process is performed using a gradient-based approach to refine model accuracy iteratively.

To implement this framework, we employ an attention-gated residual U-Net architecture. As shown in Figure 1, the left side of the model represents the deep learning network, while the right side integrates the

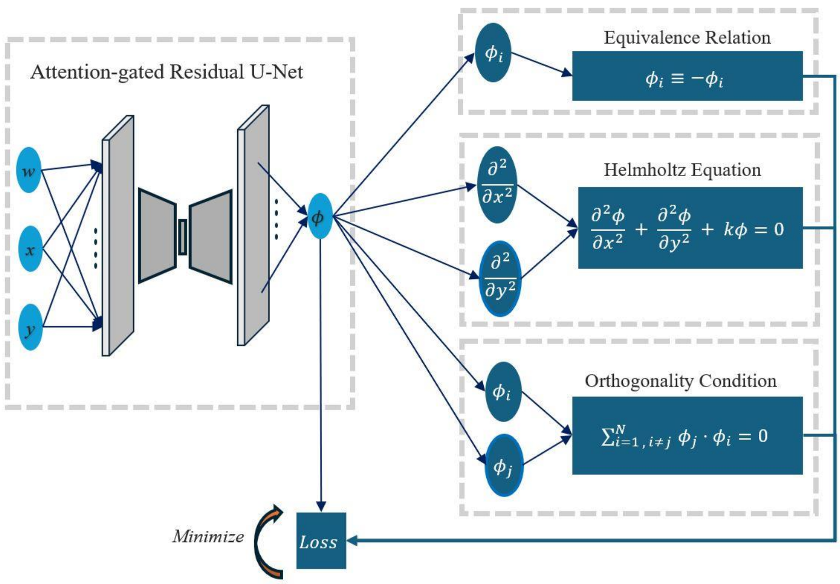

Figure 1: PINNs model incorporating three physical constraints.

governing physics equations. This hybrid approach combines data-driven optimization with embedded physics constraints, allowing the model to capture complex modal behaviors more effectively. The attention mechanism further enhances learning efficiency by selectively weighting critical features during training.

Training the model involves computing the derivatives of the network output with respect to spatial coordinates in an automated way, ensuring that the solution satisfies the underlying differential equations. The training process minimizes the total loss function using the Adam optimizer, an adaptive gradient descent method that adapts the step size for each parameter based on past gradient information to improve convergence. By iteratively refining the network weights, the model learns to approximate eigenmodes with high accuracy while adhering to physical laws.

Model validation is a crucial component of this study. To assess the reliability of our PINN-based approach, we compare its predictions against experimental measurements obtained from a fabricated PMUT device. This validation process ensures that the model not only captures theoretical behaviors but also accurately represents real-world device performance.

Beyond model validation, we develop a comprehensive computational platform that integrates the trained PINN model with established analytical calculations [9,10]. This platform provides a powerful tool for PMUT analysis, enabling rapid and precise evaluation of device behavior based solely on geometrical configurations. One key advantage of this system is its ability to efficiently calculate output power for PMUT arrays, facilitating the design and optimization of transducer arrays without requiring extensive numerical simulations.

RESULTS AND DISCUSSION

Comparison between PINNs and non-physics model

The proposed PINNs model transforms binary matrices of MEMS diaphragm shapes into eigenmode vibration patterns while adhering to fundamental principles of vibration physics. Unlike conventional neural networks, which lack explicit physical constraints, the PINN model achieves significantly lower validation loss values—more than 5 times less, as shown in Figure 2. This improvement highlights the advantage of embedding physics-based experiences into the learning process, enabling more accurate predictions with the same number of training samples. By enforcing physical consistency, the PINN model generalizes better across varying diaphragm geometries, reducing errors in eigenmode estimation.

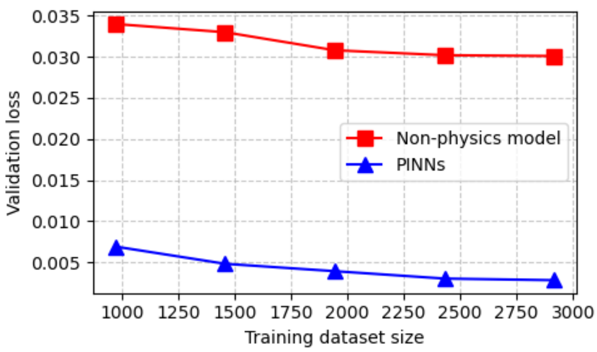

Figure 2: Comparison of validation losses between our PINNs model and non-physics neural networks.

Modal analysis of a fabricated device

To validate the model experimentally, we fabricate a piezoelectric micromachined ultrasonic transducer (PMUT) with a droplet-shaped diaphragm. Figure 3(a) presents a top-view image of the fabricated PMUT cell, while Figure 3(b) illustrates its cross-sectional schematic, showing the customizable diaphragm structure. A dual-electrode configuration is employed to enhance vibration efficiency, ensuring a well-defined response across multiple modes. This physical prototype provides a reliable benchmark for evaluating the accuracy of PINN-predicted eigenmodes.

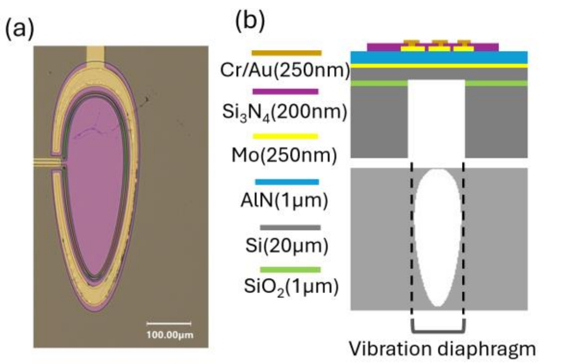

Figure 3: (a) Top-view of fabricated droplet-shaped PMUT. (b) Cross-sectional schematic of PMUT cell.

A direct comparison between PINN predictions and experimental results is shown in Figure 4. The first row presents digital holographic microscopy (DHM) measurements of the PMUT's 1st to 4th eigenmodes, while the second row displays the corresponding PINN predictions. The strong agreement between experimental and predicted modal shapes confirms the model's ability to accurately capture vibrational behavior. This validation demonstrates the effectiveness of PINNs in MEMS modal analysis, particularly in scenarios where traditional analytical solutions are impractical due to complex geometries.

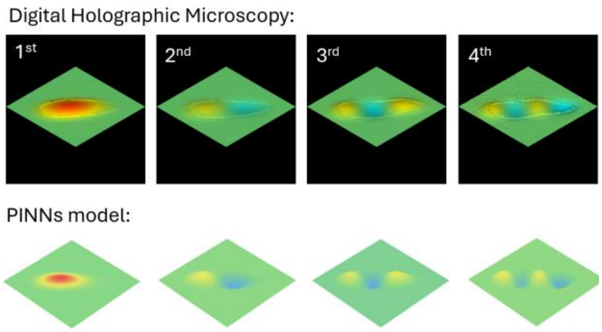

Figure 4: Comparison of experimental DHM measurements (top row) and PINNs predictions (bottom row) for the 1st to 4th eigenmode.

Modal analysis of a device with irregularly shaped diaphragm

To further assess the model's robustness, we extend the evaluation to an irregularly shaped diaphragm and compared PINNs predictions against FEA simulations. Figure 5 shows the eigenmodes obtained from both methods for the 1st to 6th mode. We plot the examined device diaphragm in the top inset, which depicts a binary image of the input diaphragm. The first row shows the FEA results, while the second row displays the corresponding PINN predictions. The two sets of results exhibit strong visual agreement, with the PINN model accurately capturing the expected modal shapes. The residuals, shown in the third row and calculated by normalized prediction minus normalized simulation (with phase reversed if they are in opposite phases), remain low overall but exhibit a slight upward trend with increasing mode number, indicating minor discrepancies at higher modes. Notably, the model correctly predicts mode reversals in the 2nd, 3rd, and 6th—demonstrating physical accuracy as these modes differ only in phase while maintaining identical spatial patterns. This agreement underscores the capability of PINNs to generalize across diverse MEMS configurations, even for complex and irregular geometries where traditional analytical models would be insufficient.

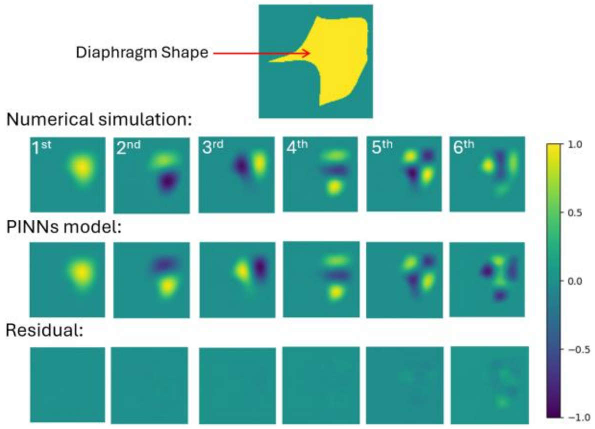

Figure 5: Comparison of numerical simulations and PINNs predictions for the 1st to 6th eigenmode. Top inset: irregularly shaped diaphragm. First row: numerical simulation results. Second row: PINNs model prediction results. Third row: residuals (normalized prediction minus normalized simulation under same phase condition) at different modes.

PMUT computation platform empowered by PINNs

Based on modal shape predictions and already existing analytical models [9,10], we evaluate the computational efficiency of our PINNs-enhanced approach in practical MEMS device analysis. Figure 6(a) presents the output power frequency response of a PMUT array, while Figure 6(b) illustrates the corresponding device configuration.

Our PINNs-enhanced computational platform works without the need for traditional numerical simulations. The process includes:

1. Analyzing the PMUT array's geometric structure, such as the diaphragm shape, array layout, material properties (like density and sound speed), and layer thicknesses.
2. Using PINNs models to predict the first to sixth eigen spatial patterns of each cell, based on its diaphragm shape. These patterns are essential for calculating the acoustic and mechanical impedances of all cells analytically, together with the remaining geometric features. The acoustic impedances cover both self-radiation and mutual radiation components.
3. Applying the calculated impedances in an equivalent circuit model to determine the array's output power.

The proposed PINNs-enhanced computational platform achieves significant speed improvements over conventional numerical solvers. Specifically, the PINN-based method completes power output calculations within 5 minutes, compared to approximately 10 hours required for COMSOL Multiphysics simulations. Despite this drastic reduction in computation time, the PINN-enhanced approach maintains accuracy comparable to FEM-based methods, making it an efficient alternative for large-scale MEMS analysis.

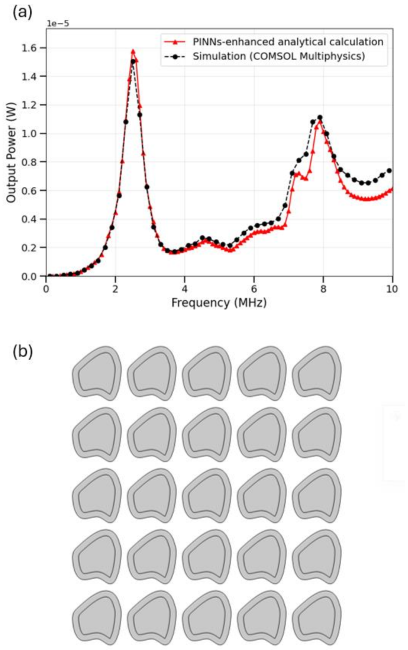

Figure 6. (a) Output power of the PMUT array with irregularly shaped diaphragms depicted in (b): comparison between PINNs-enhanced analytical model and finite element simulation.

CONCLUSION

This study presents a novel approach for determining the eigenmodes of diaphragm-structured MEMS devices using PINNs, providing an efficient alternative to FEA and analytical methods. By embedding fundamental physical laws into the learning process, the proposed framework achieves high predictive accuracy while significantly reducing computational costs. Compared to purely data-driven neural networks, PINNs exhibit 5 times lower training losses and require far fewer training samples collected from FEM simulations, making them a scalable solution for complex geometries. To validate the method, we apply it to a fabricated PMUT cell, where vibration mode predictions closely matched experimental measurements obtained through DHM. Additionally, the model successfully predicts the spatial patterns of the first 6 eigenmodes of an irregularly shaped diaphragm, demonstrating its robustness in handling non-standard structures. These findings highlight PINNs as a powerful tool for MEMS design and optimization, offering faster computations and greater flexibility while maintaining accuracy. By reducing reliance on computationally expensive simulations, PINNs can streamline MEMS development, enabling more efficient performance evaluation and design iterations. Future work could explore integrating PINNs with experimental feedback for adaptive refinement, further enhancing their applicability across a wider range of MEMS devices and complex geometries.

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