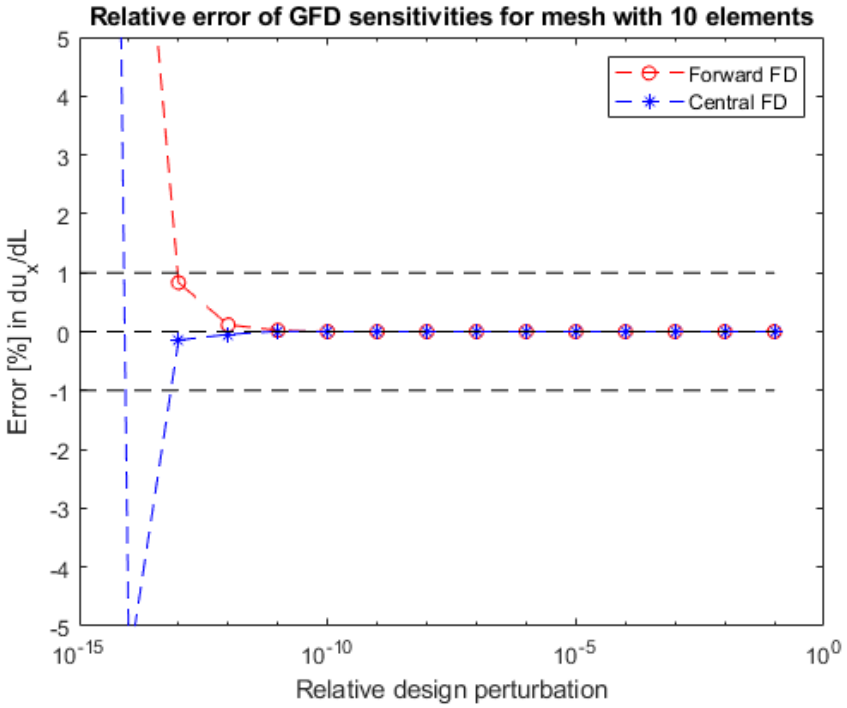
Exercise 8.1

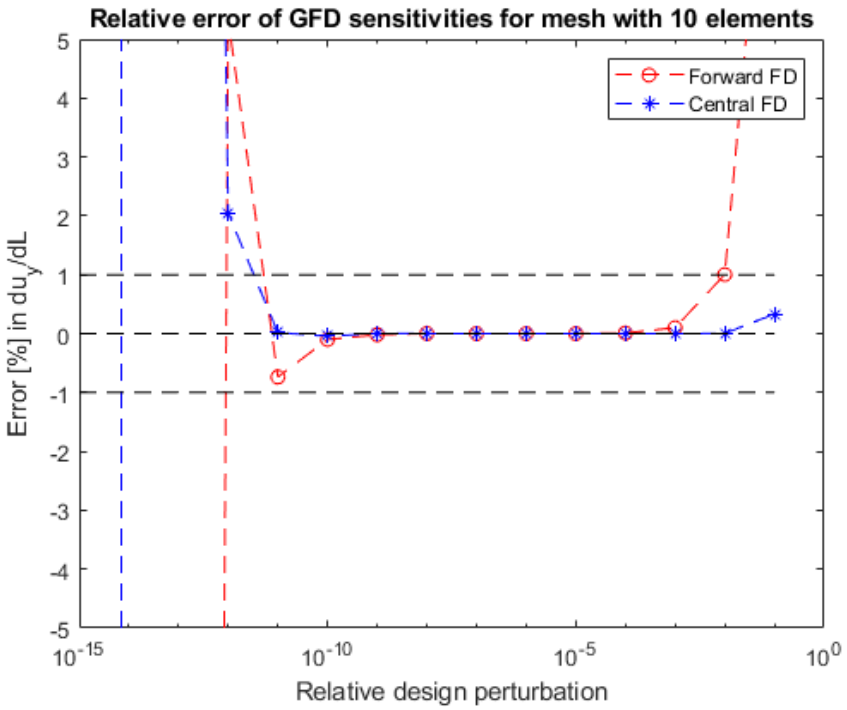
8.1a

Upon inspection, it is observed that the forward finite difference function “ffd” and central finite difference function "cfd” are implemented correctly: The FFD method evaluates the function at time step t and t+h, whereas CFD evaluates the function at t+h and t-h.

8.1b

Using 10 elements, the following errors are computed for du_x/L and du_y/L respectively:





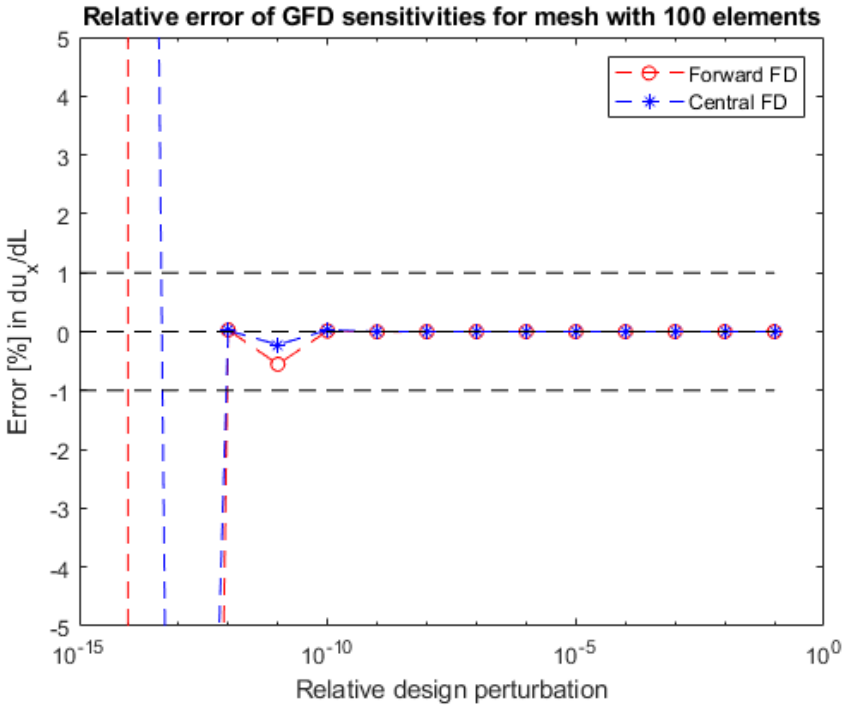
It is observed that for the horizontal displacement, a larger range of relative design perturbations is allowed, as the error very quickly approaches zero at 1e-11 and higher. For the vertical displacement, it is seen that only the range of perturbations between 1e-11 to 1e-2 is allowed for the cfd method and 1-10 to 1e-3 for the ffd method. This is because geometrical nonlinearities become large sooner for the vertical displacements compared to the horizontal displacements, which could be an explanation for the increased error at high relative design perturbations.

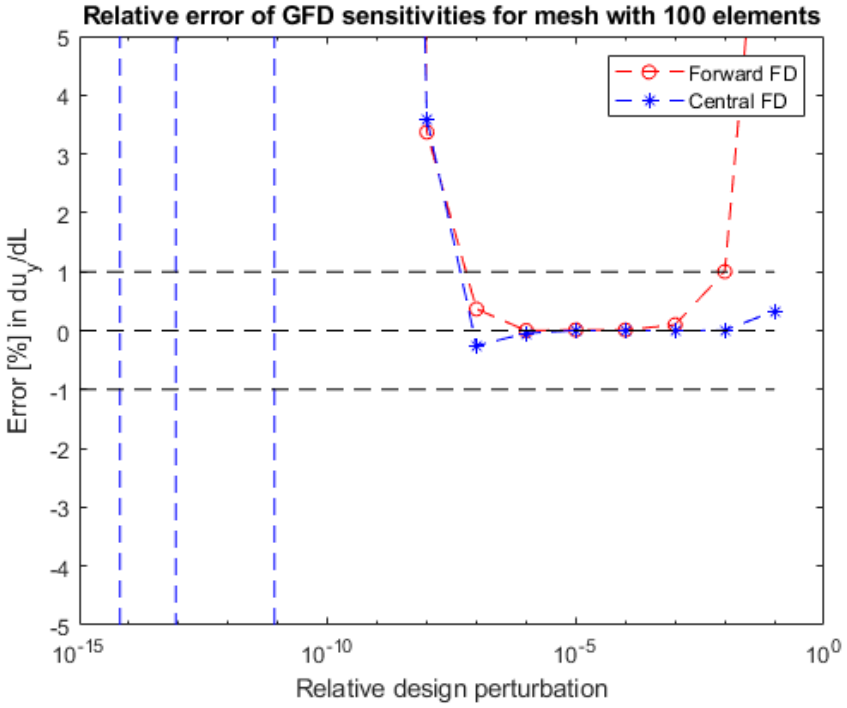
As for the small perturbations below 1e-11, the high errors can be attributed to the fact that the displacements are near zero and the resulting error is mostly numerical noise.

When comparing the cfd and ffd methods, they behave similarly for the horizontal displacement, whereas the cfd suffers less from increased error at high perturbations in the vertical displacement direction. This may be explained by the fact that the derivatives are based on two neighboring points instead of one, resulting in a much smaller truncation error.

8.1c

Upon increasing the number of elements to 100, the following plots are generated:





It is observed that the increased number of elements does not affect the relative error in the horizontal direction that much, but it is very clear that the feasible range of perturbation sizes is significantly decreased for the vertical direction. This may be attributed to the fact that the error for each individual element may be small, but summed over the 100 elements it adds up to a very large error. This error propagation may result in a very large tip displacement error.

8.1d

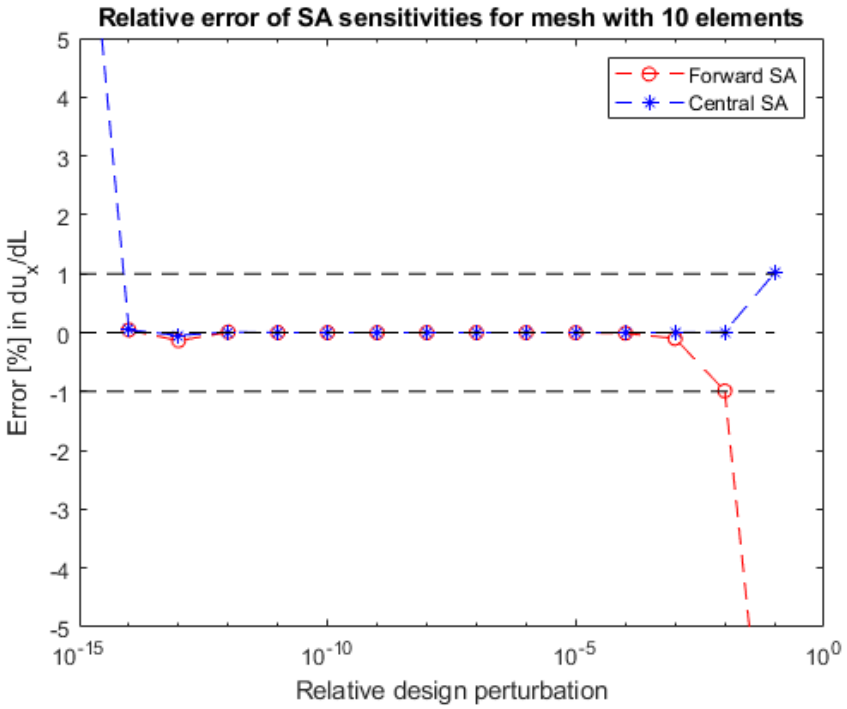
Using 450 elements, the elapsed time for the ffd method is 2.94 seconds and 4.85 seconds for the cfd method.

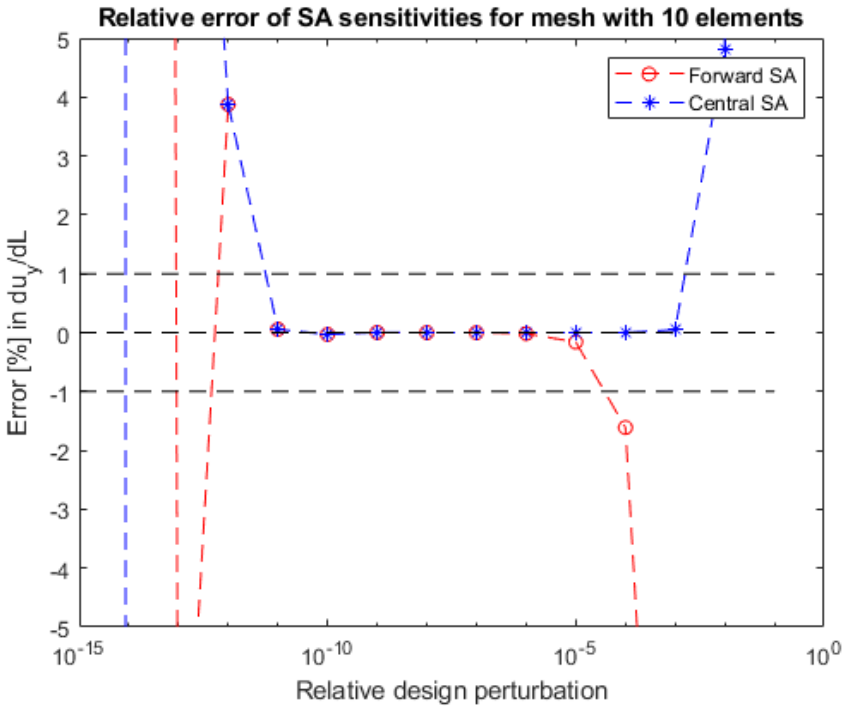
8.2a

A quick scan through the code verified that the script indeed uses a semi-analytical approach.

8.2b

Just as in 8.1b, the the following errors are computed for du_x/L and du_y/L using the semi-analytical approach:



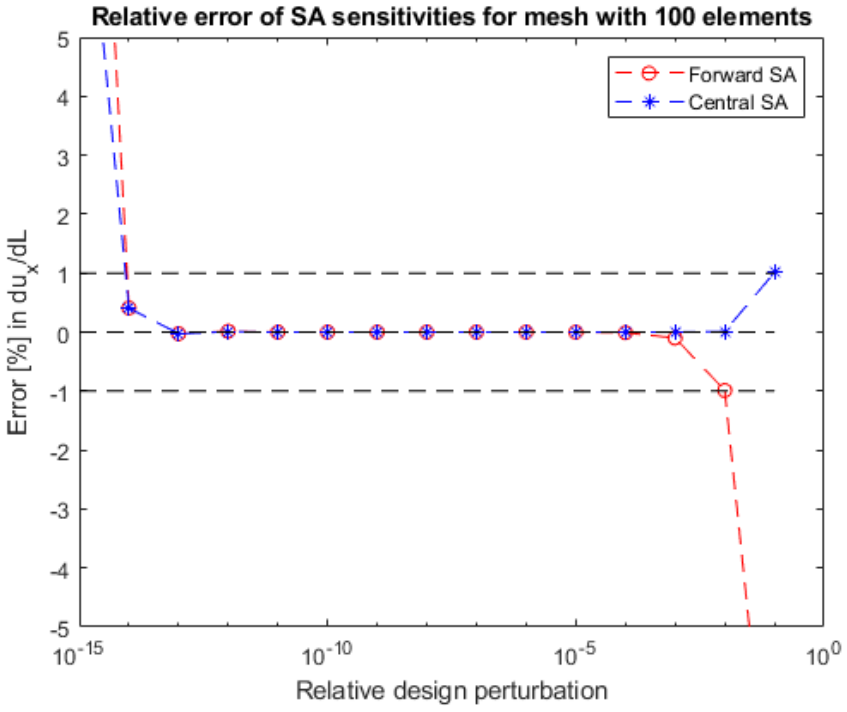


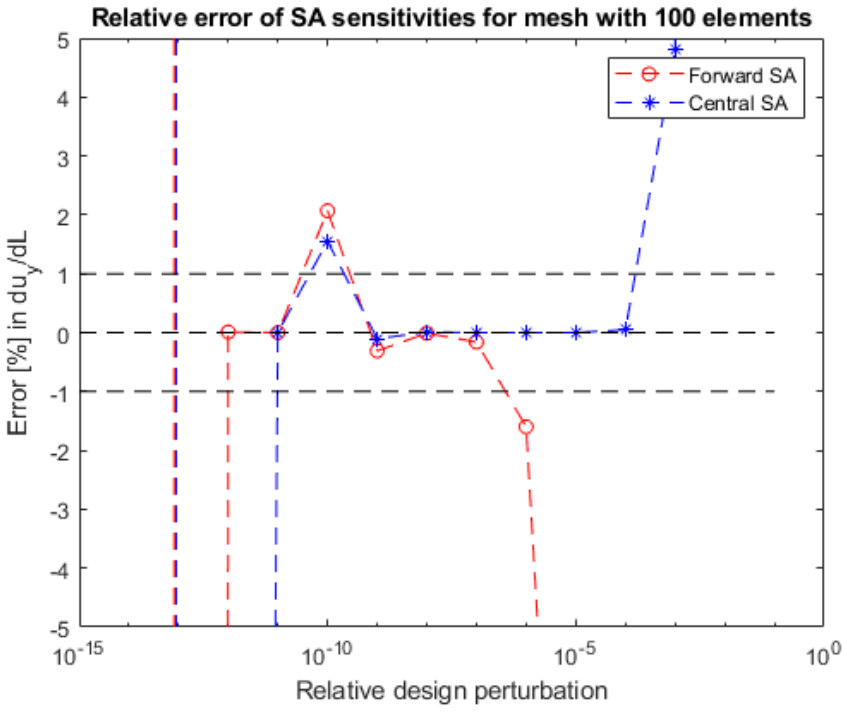
Again, it is observed that the condition error decreases rapidly for perturbations larger than 1e-14 for the horizontal displacement, and 1e-9 for vertical displacement. Truncation errors become significantly large for perturbations larger than 1e-3 and 1e-5 for horizontal and vertical

displacements respectively. Using Central SA, the relative perturbations can be increased to 1e-2 and 1e-3 before the truncation error becomes significant.

Just as in 8.1b, these results are as expected: the condition error is roughly the same for both methods, but the truncation error remains small for larger perturbations in the case of Central SA as the truncation error is of third order.

8.2c





The study of the SA model is repeated with a beam consisting of 100 elements. Just as in 8.1c, there are little noticeable differences in the error plots of the horizontal displacement. For the vertical

displacement however, it is hard to discern whether the error in Forward SA is condition or truncation based, as the error is zero hardly anywhere. It may be concluded that Forward SA is less suitable for computations with a higher number of elements. Central SA still has a feasible region between 1e-9 and 1e-4 where both the condition and truncation errors are small.

8.2d

Using 450 elements, the runtime is 0.495 seconds for Forward SA and 0.565 seconds for Central SA. Both these times are significantly faster than the Global Finite Differences approaches used in 8.1. This may be attributed to the fact that the GFD functions invert the system matrix for each perturbation size, whereas the SA approaches only do this for the unperturbed case.

8.3a

Comparing the range of design perturbations between CFD and SA, it can be observed that the range is slightly larger for the CFD approach in the horizontal direction and considerably larger in the vertical direction. Especially in the vertical direction, both the condition and truncation errors are significantly large for a larger range of perturbation sizes.

8.3b

Comparing the computation times, SA approaches are almost 8 times faster than the CFD approaches. This implies that a trade-off is to be made between accuracy and computation times, depending on the requirements of the model.

Exercise 9

9.1

Mesh	Obtained design	Iterations	Compliance
15x5		51	374.05
30x10		72	225.59
45x15		78	212.47
60x20		94	203.31
75x25		196	201.54

Using the same volume of 0.5 and minimum change of 1%, the results in the table above were computed using different mesh sizes. For larger mesh sizes, a more complex structure becomes visible, but at the cost of a steep increase in number of iterations. Comparing the compliances, the increase in stiffness is small compared to the increase in number of iterations.

9.2

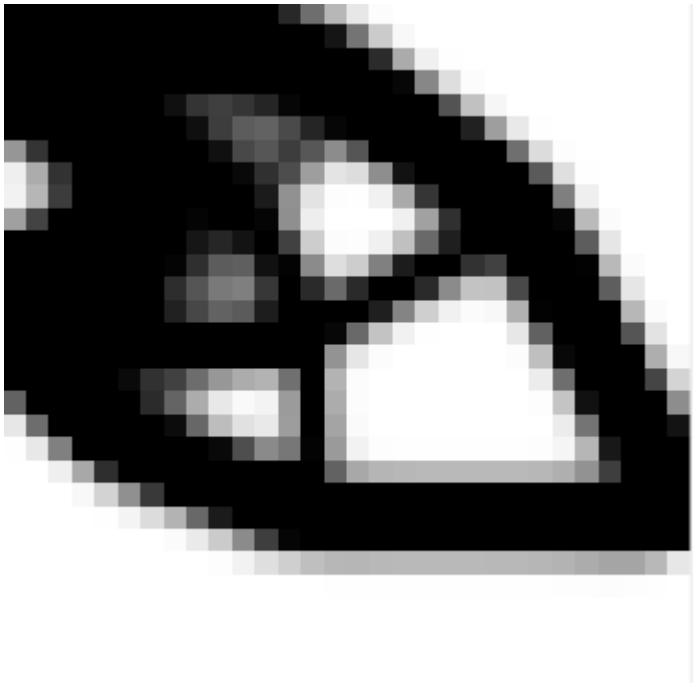
Penalization	Obtained design	Iterations	Compliance
1		9	164.16

2		48	213.95
3		78	212.47
4		117	216.83

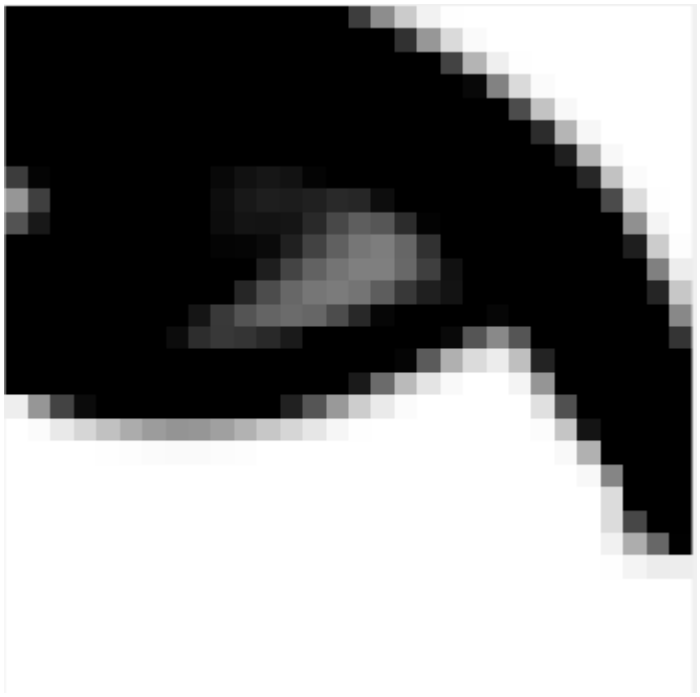
The penalization power may be used to provide a sharper contrast between places where material should and shouldn't be placed. A penalization power of around 3 is recommended, as this provides a feasible result at a relatively low number of iterations.

9.3

Using F = (0.5, -1) and a 30x30 mesh, the following result is computed:



For F = (0.5, 1), the result is presented below:



In both plots it is clear that the part of the structure that is loaded in tension contains more material. The plots therefore do seem to make sense.