## Exercise 1.1

The following relations were investigated:

**Axial spring stiffness**: Steinhilfer, 7.66, directly implemented on line 82

**Shear stress**: Mott (7-4)(7-5). Directly implemented on line 123-130

**Linear spring deflection**: Mott (7-6). Implemented in the spring constant k on line 82. $k = \frac{F}{f}$ Where F is the applied force and f is the linear deflection.

**Spring force**: incorporated in axial spring stiffness, Roloff 10.49

**Mechanical work**: not implemented

**First eigenfrequency**: Roloff 10.52, directly implemented on line 137

**Spring buckling**: not implemented, because it is not regarded as a critical failure mode due to the short spring length

**Blocking length**: Roloff 10.40 directly implemented on line 85.

**Installation consideration**: Mott (7-3). Not really considered but related to build-in volume, which is implemented on line 73. It should be $OD = \sqrt{D_m^2 + \frac{p-D_w^2}{\pi^2}} + D_w$ Where $D_w$ is the wire diameter and $D_m$ the mean diameter.

Based on geometric input parameters and fixed material and operation properties, the spring characteristics such as axial stiffness (Steinhilfer, 7.66), first eigenfrequency (Roloff, 10.52), spring elongation (Mott, 7.6) and shear stresses in the open and closed positions (Mott, 7.4&7.5) are calculated. Performance specifications such as mechanical work and spring force are not computed directly but can be inferred from the calculated stiffness and spring elongation. Lastly, some practical properties such as build-in volume (Mott, 7.3), spring mass and cost of materials and manufacturing are calculated. Buckling is not considered, as it is not regarded a critical failure mode due to the relatively short spring length and fixed spring ends. The resulting properties can be used in a later stage to determine constraint equations.

## Exercise 1.2

Design variables:

- mean winding diameter, continuous and deterministic
- wire diameter, continuous and deterministic
- number of active windings, discrete or continuous (depending if you take half windings into account) and deterministic

Objective: minimize cost, this is equivalent to minimizing mass unless other material selection is allowed, (this would add a discrete stochastic design variable)

Equality constraints:

- Unloaded spring length

- Spring length at closed valve position

Inequality constraints

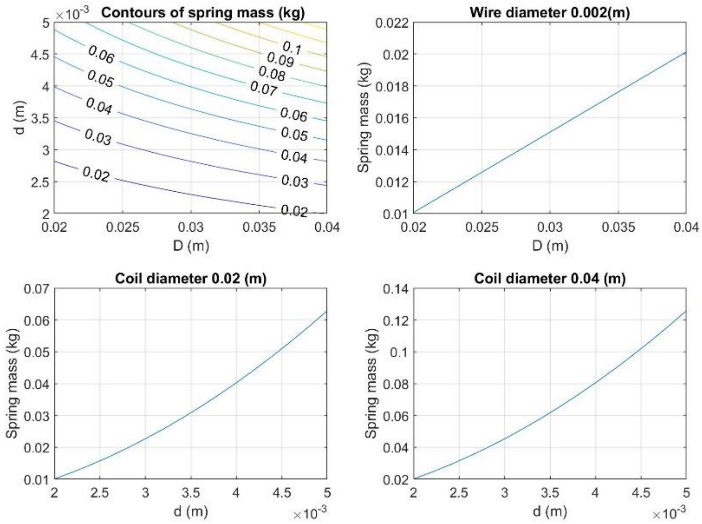
- Lower bound on lifetime
    - Tau1 & Tau2
- Upper and lower bounds on build-in volume, and thus on mean winding diameter
- Closed force F1>F1 min
- Fully opened force F2>F2min
- Not below lower bound on eigenfrequency: Freq1 > Freq1LB, which is determined by the maximum rotor speed multiplied by a safety factor.
- Lmin: minimum length of spring at maximum compression (m).
- L2: length of spring at opened valve position (m)

# Exercise 2.1

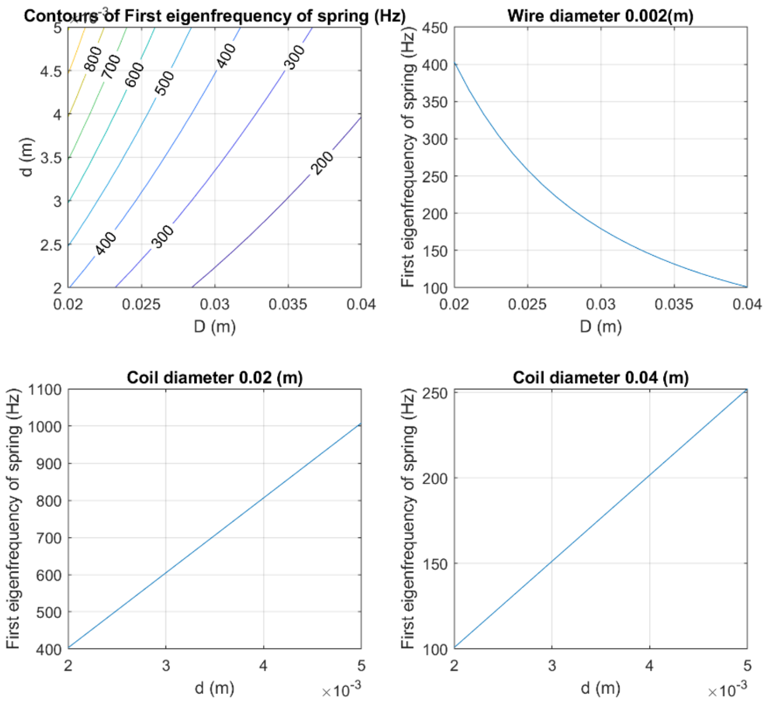
The contours of the spring stiffness are convex, non-linear and monotonic

**Smass**

The spring mass is linear in the wire diameter, nonlinear in the coil diameter. The contours are convex and monotonic.



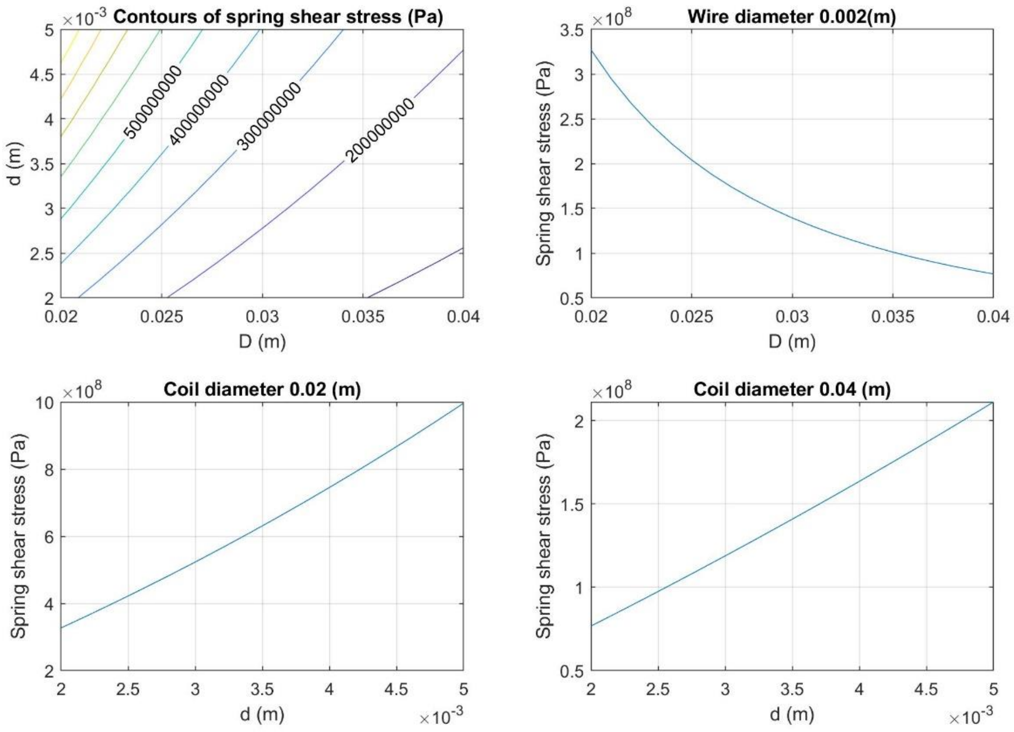

**Freq1**




The frequency is convex, non-linear and monotonic. However it is linear with the coil diameter.

**Tau2**

The contours of the spring shear stress are nonlinear, convex and monotonic.




## Exercise 2.2

The eigenfrequency can be calculated using `freq1 = 363*d/(n*D^2)`. The lower bound on the eigenfrequency is determined by multiplying the engine rotor speed by the cam shaft factor. As the cam shaft speed is half the rotor speed, the minimum frequency is halved as well and determined to be 520 Hz.

Negative null form

**Design variables**:

D and d

**Objective**:

$$\min_{d,D} smass$$

**Constraints**:

C1: 1 - L2/Lmin <= 0

C2: 1– F1/F1min <= 0

C3: 1 – F2/F2min <= 0

C4: Tau2/Tau12max – 1 <= 0

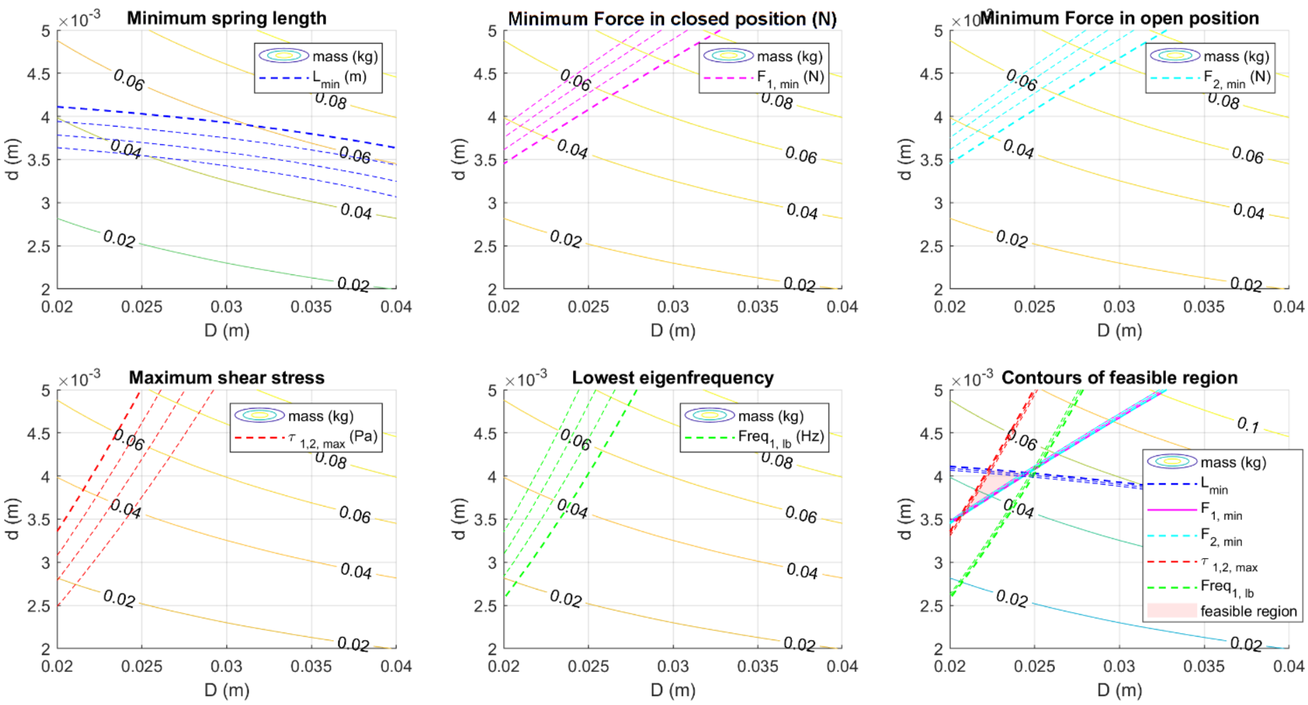
C5: 1 – freq1/Freq1lb <= 0

C6a: D/0.04 - 1 <= 0

C6b: 1 – D/0.02 <= 0

C7a: d/0.005 – 1 <= 0

C7b: 1 – d/0.002 <= 0




In the plots above the contours of the objective function are plot with a constraint. The 'fat' line indicates where the constraint is satisfied (i.e. equal to zero) the thinner lines indicate the feasible side. In the bottom right plot al constraints are plotted at the same time and the final feasible region is shown in red. Constraint C6 and C7 are not plotted since they coincide with the axis limits, the whole plotted area is feasible for these constraints. It is observed that constraint C2 and C3 (the force constraints) are redundant since they overlap. For the chosen lower bound of freq1 of 520 Hz this constraint is also redundant. From the contours it can be deduced that the mass is minimal when constraint C2 (F1) (and thus C3 (F2)) and C4 (tau) are satisfied, so a rough estimate for the optimal mass is between 0.03kg and 0.04kg.