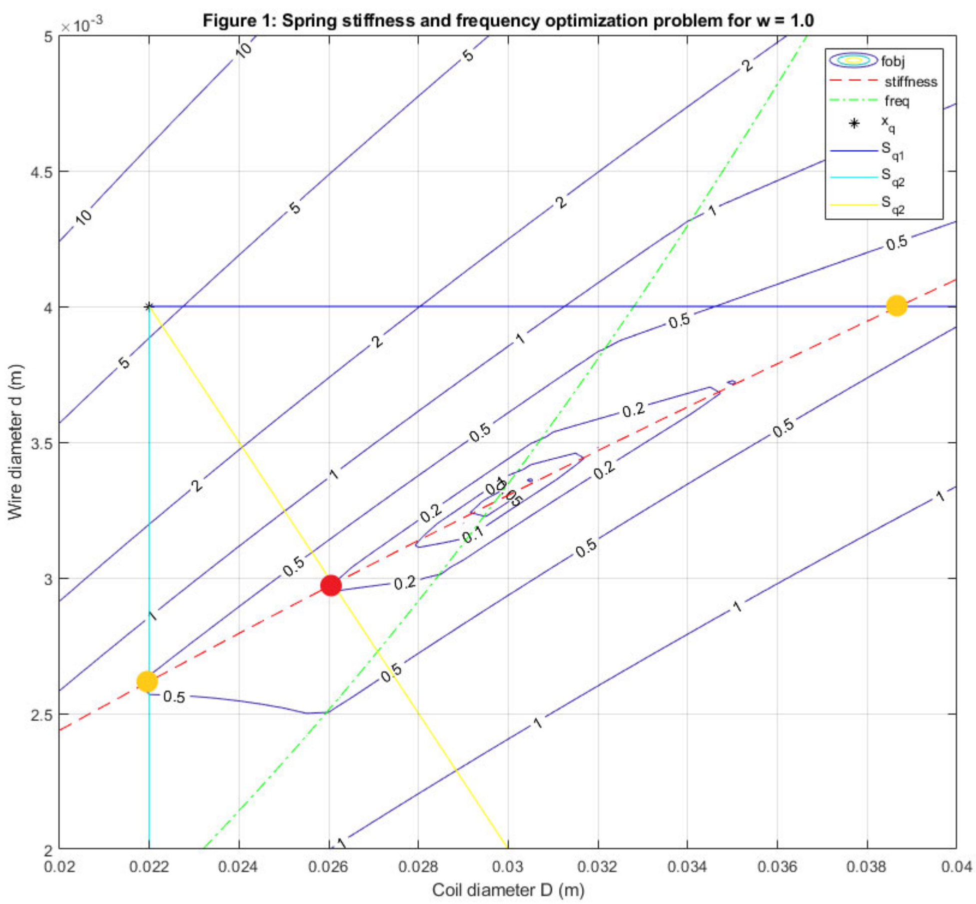
Exercise 3.1

The lowest value for f(alpha) is reached with S_q3, approximately 0.2 at alpha=2. This point is denoted in red in the graph below. The horizontal S_q1 line reaches its lowest point at the yellow point, this point is outside of the feasible area (it violates the frequency constraint) at alpha=8.5 and f(alpha) = 0.4. The vertical line S_q2 reaches its lowest point by the yellow dot, at approximately alpha=2.8 and f(alpha)=0.5.



Exercise 3.2

The anonymous function called by fminbnd is: @(a) springobjw3(a, xq, sq, kt, ft, w). This is repeated three times for Sq1, Sq2, and Sq3 respectively. The results and parameters used for each line search are found in the table below.

	Sq1	Sq2	Sq3
Lower bound	-1	-2	-1
Upper bound	9	4	4
Optimal value of alpha	8.35	2.76	2.04
Line search minimum	0.28	0.45	0.18
Resulting coil diameter D (m)	0.039	0.022	0.026
Resulting wire diameter d (m)	0.0040	0.0026	0.0030

Compared to 3.1, the estimated values of alpha and f(alpha) correspond quite well to the computed values above, except for the minimum function value along S_q2, which was harder to estimate as it did not lie close to any contour line compared to the other two points.

When changing the tolerance on alpha from 1e-4 to 1e-8, the number of iterations needed for convergence increases from 22 to 38 for the line search along S_q2. The resulting differences are presented in the following table:

	Tolerance 1e-4	Tolerance 1e-8	(Absolute) difference
Optimal value of alpha	2.762406307554135	2.762389126637516	1.7181e-5
Objective function value	0.454900255797457	0.454891911381886	8.3444e-6
Resulting coil diameter D (m)	0.022	0.022	0
Resulting wire diameter d (m)	0.002618796846223	0.002618805436681	8.5905e-9

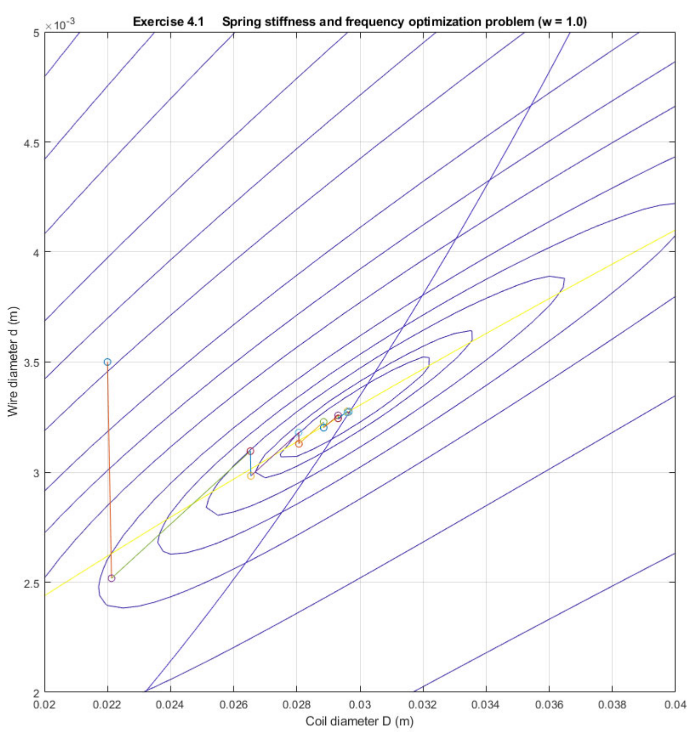
The zero difference in the resulting coil diameter can be explained by the fact that Sq2 is in a direction of constant coil diameter, as the first element in Sq2 is zero. The differences in alpha and objective function value both result in a very small final difference in the resulting wire diameter, which we would consider negligible.

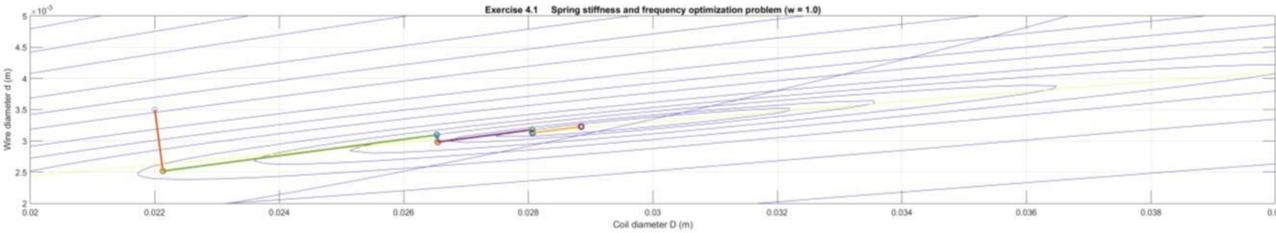
Exercise 4.1

Sq is of order 1e4, resulting in a very small value of alpha, which is of order 1e-7, as only a very small step along the sq direction is needed to reach the optimum. Such enormous differences do not contribute to a credible result. Normalizing the search vector sq already increases the order of alphaq to a factor 1e-2, which is still quite small. Therefore, sq can be scaled again by a factor 1e-3 to set the search vector to millimetre scale, which is the smallest order of magnitude of the design variables, in this case the wire diameter. This results in an alphaq of order 1, which is a desireable result.

As a next step, the feasible bounds on alpha must be re-evaluated, as the initial bounds are [0.0, 1.0], which is too tight. Theses bounds are dependent on the direction of the search vector and have to be calculated for each iteration.

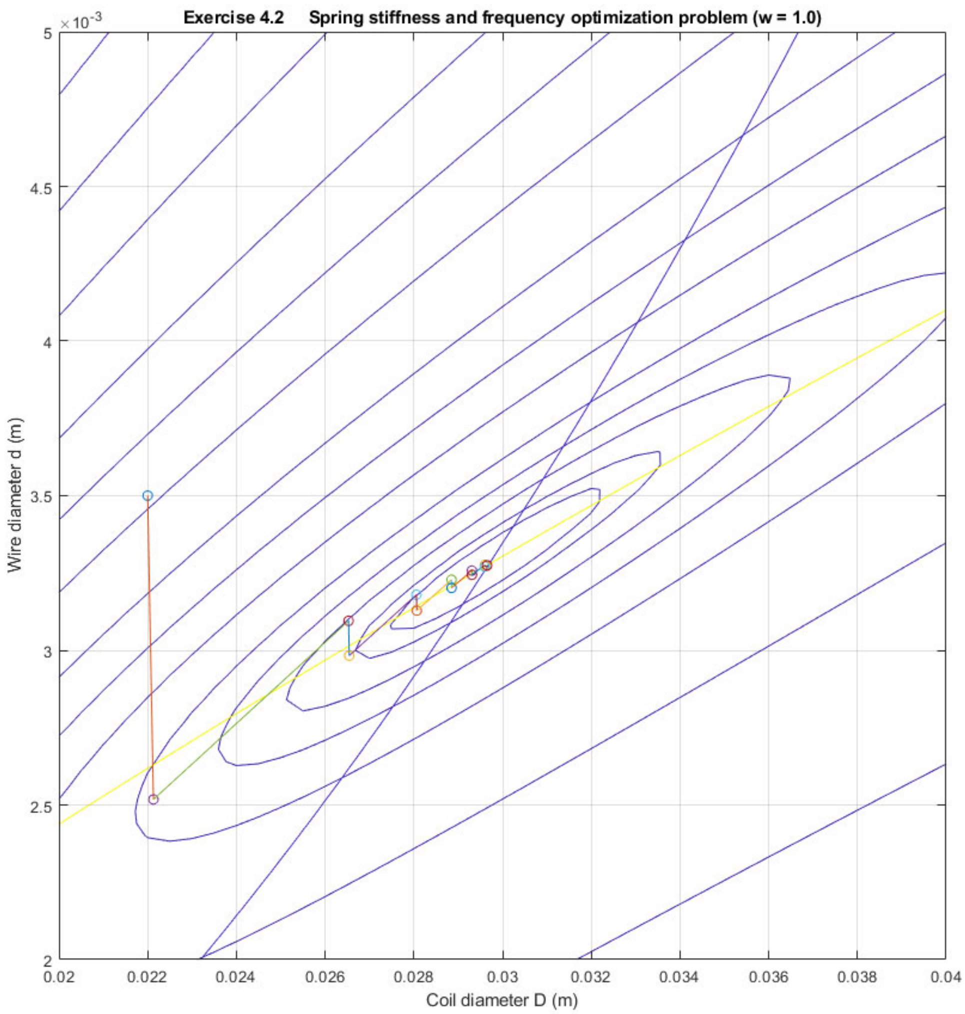
Lastly, it is observed that the steepest decent iterations are not orthogonal to each other, which is not as expected, because each iteration should end in a minimum along the search vector direction, thus the derivative in that direction should be zero. The reason the lines do not appear to be orthogonal is because the x and y axis do not have the same scale. When the axis are set equal the lines are orthogonal.




Exercise 4.1 Spring stiffness and frequency optimization problem (w = 1.0)
Wire diameter d (m)
×10^-3
Coil diameter D (m)
5
4.5
4
3.5
3
2.5
2
0.02
0.022
0.024
0.026
0.028
0.03
0.032
0.034
0.036
0.038
0.04

Exercise 4.2

To automate the process used in Exercise 4.1, a convergence criterion is defined as the absolute value between two successive objective function evaluations. If this difference is less than 1.0e-6, the optimisation is terminated. Moreover, the maximum number of iterations is limited to 20. The resulting steepest decent process is presented in the Figure below and seems very similar to the result of Exercise 4.1. A total of 14 iterations were needed to satisfy the convergence criterion.



Exercise 4.3

For this last exercise, a new objective function is needed, as fminunc does not require the user to supply a starting point and search direction for each iteration, just one initial point. The default value of Tolfun is 1.0e-6, which is the same as the function evaluation tolerances used in Exercise 4.2. This makes sure that the results can be compared, as the termination criteria are the same.

Using the BFGS Hessian Update option, the minimum is found within 10 iterations based on the tolerance on the gradient size. However, the steepest descent option takes a lot of function evaluations and iterations to reach the same minimum. This seems to be due to the step size being of order 1e-7, which results in very small decrements of the objective function in each iteration.

We tried to remedy this by increasing the DiffMinChange (Minimum change in variables for finite-difference gradients) from zero to 1e-6. Now the function returns: Local minimum possible. When comparing the optimal values found the x values are close to each other for both algorithms. However, the steepdesc needed more then 5000 function evaluations.

X_opt_bfgs = [0.0297   0.00328]

X_opt_steepdsc = [0.0290   0.00322]

X_opt_ bfgs - X_opt_steepdsc = [0.66   0.062]*1e-3