Mott,

Machine Elements in Mech. Design

(KBB 85 MOT)

7 Springs

9hlz

7-1 OVERVIEW

A *spring* is a flexible element used to exert a force or a torque and, at the same time, to store energy. The force can be a linear push or pull, or it can be radial, acting similarly to a rubber band around a roll of drawings. The torque can be used to cause a rotation, for example, to close a door on a cabinet or to provide a counterbalance force for a machine element pivoting on a hinge.

Springs inherently store energy when they are deflected and return the energy when the force that causes the deflection is removed. When this is the primary design objective, the spring is frequently referred to as a *power spring* or *motor spring*.

Table 7-1 lists several types of springs and shows their uses.

7-2 HELICAL COMPRESSION SPRINGS

In the most common form of helical compression spring round wire is wrapped into a cylindrical form with a constant pitch between adjacent coils. This basic form is completed by a variety of end treatments, as shown in Figure 7-1.

For medium- to large-size springs used in machinery, the squared and ground end treatment provides a flat surface on which to seat the spring. The end coil is collapsed against the adjacent coil (squared), and the surface is ground until at least 270° of the last in contact with the bearing surface. Springs made from smaller wire (less than mately 0.020 in. or 0.50 mm) are usually squared only, without grinding. In l cases the ends may be ground without squaring or left with plain ends, simply cut th after coiling.

ou are probably familiar with many uses of helical compression springs. The able ballpoint pen depends on the helical compression spring, usually installed l the ink supply barrel. Suspension systems for cars, trucks, and motorcycles ntly incorporate these springs. Other automotive applications include the valve ;s in engines, hood linkage counterbalancing, and the clutch pressure plate springs. nufacturing, springs are used in dies to actuate stripper plates, in hydraulic control s, as pneumatic cylinder return springs, and in the mounting of heavy equipment for ; isolation. Many small devices such as electrical switches and ball check valves

Table 7-1 Types of Springs

Uses	*Types of Springs*
Push	Helical compression spring (Figures 7-1 and 7-26) Belleville spring (Figure 7-27) Torsion spring (Figure 7-19): force acting at the end of the torque arm Flat spring, such as a cantilever or leaf spring
Pull	Helical extension spring (Figures 7-14 and 7-17) Torsion spring (Figure 7-19): force acting at the end of the torque arm Flat spring, such as a cantilever or leaf spring Drawbar spring (special case of the compression spring) Constant force spring
Radial	Garter spring, elastomeric band, spring clamp
Torque	Torsion spring (Figure 7-19), power spring

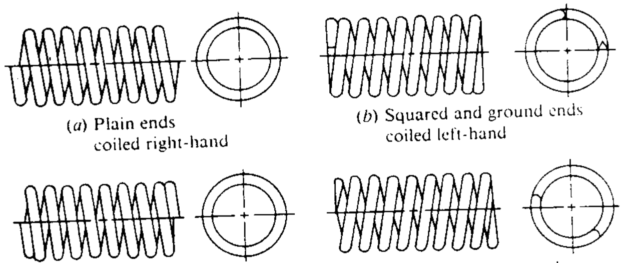


Figure 7-1 Appearance of Helical Compression Springs Showing End Treatments

incorporate helical compression springs. Desk chairs have stout springs to return the chair seat to its upright position. And don't forget the venerable pogo stick!

The following paragraphs define the many variables used to describe and to analyze the performance of helical compression springs.

Diameters

Figure 7-2 shows the notation used in referring to the characteristic diameters of helical compression springs. The outside diameter (OD), the inside diameter (ID), and the wire diameter (D_w) are obvious and can be measured with standard measuring instruments. In calculating the stress and deflection of a spring, the mean diameter, D_m, is used. Notice that

$$OD = D_m + D_w$$
$$ID = D_m - D_w$$

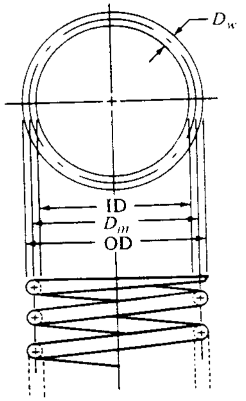


Figure 7-2 Notation for Diameters

Standard Wire Diameters

The specification of the required wire diameter is one of the most important outcomes of the design of springs. There are several types of materials typically used for spring wire, and the wire is produced in sets of standard diameters covering a broad range. Table 7-2 lists the most common standard wire gages. Notice that, except for music wire, the wire size gets smaller as the gage number gets larger. Also see the notes to the table.

Lengths

It is important to understand the relationship between the length of the spring and the force exerted by it (see Figure 7-3). The *free length*, L_f, is the length that the spring assumes when it is exerting no force as if it were simply sitting on a table. The *solid length*, L_s, is found when the spring is collapsed to the point where all coils are touching. This is obviously the shortest possible length the spring can have. The spring is usually not compressed to the solid length during operation.

The shortest length for the spring during normal operation is the *operating length*, L_o. At times, a spring will be designed to operate between two limits of deflection. Consider the valve spring for an engine, for example, as shown in Figure 7-4. When the valve is open, the spring assumes its shortest length, which is L_o. Then, when the valve is closed, the spring gets longer but still exerts a force to keep the valve securely on its seat. The length at this condition is called the *installed length*, L_i. So the valve spring length changes from L_o to L_i during normal operation as the valve itself reciprocates.

Forces

We will use the symbol F to indicate forces exerted by a spring with various subscripts to specify which level of force is being considered. The subscripts correspond to those used for the lengths. Thus

F_s = Force at solid length, L_s: The maximum force that the spring ever sees.

F_o = Force at operating length, L_o: The maximum force the spring sees in *normal operation*.

F_i = Force at installed length, L_i: The force varies between F_o and F_i for a reciprocating spring.

F_f = Force at free length, L_f: This force is zero.

Spring Rate

The relationship between the force exerted by a spring and its deflection is called its *spring rate*, k. Any change in force divided by the corresponding change in deflection can be used to compute the spring rate.

$$k = \Delta F/\Delta L \tag{7-1}$$

For example,

$$k = \frac{F_o - F_i}{L_i - L_o}$$

Table 7-2 Wire Gages and Diameters for Springs

Gage No.	*U.S. Steel Wire Gage (in)*[a]	*Music Wire Gage (in)*[b]	*Brown & Sharpe Gage (in)*[c]	*Preferred Metric Diameters (mm)*[d]
7/0	0.490 0	—	—	13.0
6/0	0.461 5	0.004	0.580 0	12.0
5/0	0.430 5	0.005	0.516 5	11.0
4/0	0.393 8	0.006	0.460 0	10.0
3/0	0.362 5	0.007	0.409 6	9.0
2/0	0.331 0	0.008	0.364 8	8.5
0	0.306 5	0.009	0.324 9	8.0
1	0.283 0	0.010	0.289 3	7.0
2	0.262 5	0.011	0.257 6	6.5
3	0.243 7	0.012	0.229 4	6.0
4	0.225 3	0.013	0.204 3	5.5
5	0.207 0	0.014	0.181 9	5.0
6	0.192 0	0.016	0.162 0	4.8
7	0.177 0	0.018	0.144 3	4.5
8	0.162 0	0.020	0.128 5	4.0
9	0.148 3	0.022	0.114 4	3.8
10	0.135 0	0.024	0.101 9	3.5
11	0.120 5	0.026	0.090 7	3.0
12	0.105 5	0.029	0.080 8	2.8
13	0.091 5	0.031	0.072 0	2.5
14	0.080 0	0.033	0.064 1	2.0
15	0.072 0	0.035	0.057 1	1.8
16	0.062 5	0.037	0.050 8	1.6
17	0.054 0	0.039	0.045 3	1.4
18	0.047 5	0.041	0.040 3	1.2
19	0.041 0	0.043	0.035 9	1.0
20	0.034 8	0.045	0.032 0	0.90
21	0.031 7	0.047	0.028 5	0.80
22	0.028 6	0.049	0.025 3	0.70
23	0.025 8	0.051	0.022 6	0.65
24	0.023 0	0.055	0.020 1	0.60 or 0.55
25	0.020 4	0.059	0.017 9	0.50 or 0.55
26	0.018 1	0.063	0.015 9	0.45
27	0.017 3	0.067	0.014 2	0.45
28	0.016 2	0.071	0.012 6	0.40
29	0.015 0	0.075	0.011 3	0.40
30	0.014 0	0.080	0.010 0	0.35
31	0.013 2	0.085	0.008 93	0.35
32	0.012 8	0.090	0.007 95	0.30 or 0.35
33	0.011 8	0.095	0.007 08	0.30
34	0.010 4	0.100	0.006 30	0.28
35	0.009 5	0.106	0.005 01	0.25
36	0.009 0	0.112	0.005 00	0.22
37	0.008 5	0.118	0.004 45	0.22
38	0.008 0	0.124	0.003 96	0.20
39	0.007 5	0.130	0.003 53	0.20
40	0.007 0	0.138	0.003 14	0.18

[a] Use the U.S. Steel Wire Gage for steel wire, except music wire. This gage has also been called the *Washburn and Moen Gage (W&M)*, the *American Steel Wire Co. Gage*, and the *Roebling Wire Gage*.

[b] Use the Music Wire Gage only for music wire (ASTM A228).

[c] Use the Brown & Sharpe Gage for nonferrous wires such as brass, phosphor bronze, etc.

[d] The preferred metric sizes are from Associated Spring, Barnes Group, Inc., and are listed as the nearest preferred metric size to the U.S. Steel Wire Gage. The gage numbers do not apply.

Source: Associated Spring, Barnes Group, Inc. *Engineering Guide to Spring Design*. Bristol, Conn., 1981; Carlson, Harold. *Spring Designer's Handbook*. New York: Marcel Dekker, 1978; Oberg, E., et al. *Machinery's Handbook*, 22d ed. New York: Industrial Press, 1984.

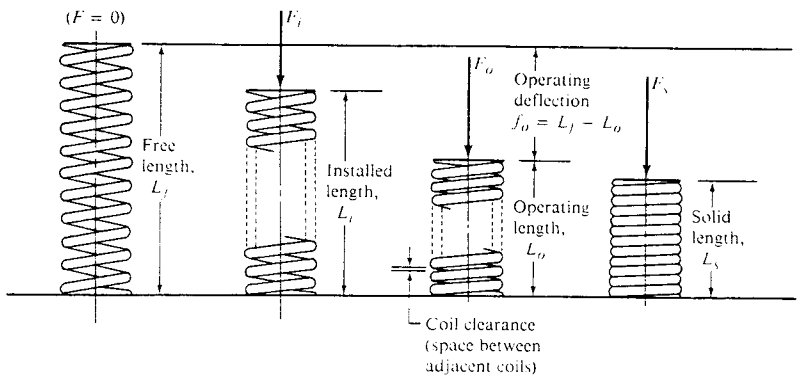


Figure 7-3 Notation for Lengths and Forces

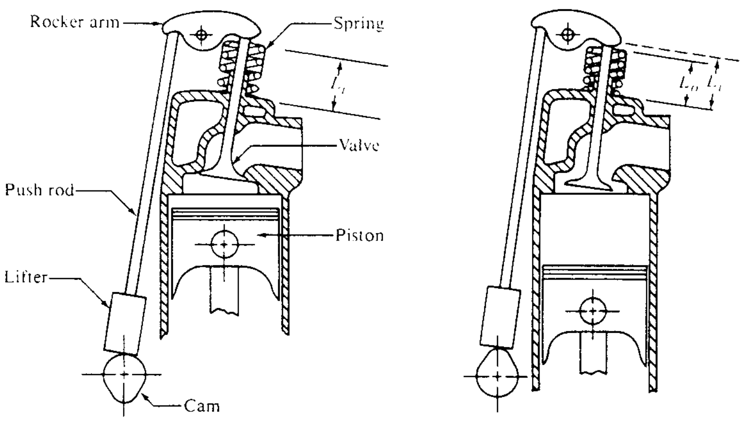
or

$$k = \frac{F_o}{L_f - L_o}$$

or

$$k = \frac{F_i}{L_f - L_i}$$

In addition, if the spring rate is known, the force at any deflection can be computed. For example, if a spring has a rate of 42.0 pounds per inch (lb/in), the force exerted at



(*a*) Valve closed: spring length, L_i (*b*) Valve open: spring length, L_o

Figure 7-4 Illustration of Installed Length and Operating Length

a deflection from free length of 2.25 in would be

$$F = k(L_f - L) = (42.0 \text{ lb/in})(2.25 \text{ in}) = 94.5 \text{ lb}$$

Spring Index

The ratio of the mean diameter of the spring to the wire diameter is called the *spring index, C*.

$$C = D_m/D_w$$

It is recommended that C be greater than 5.0, with typical machinery springs having C values ranging from 5 to 12. For C less than 5, the forming of the spring will be very difficult, and the severe deformation required may create cracks in the wire. The stresses and deflections in springs are dependent on C, and a larger C will help to eliminate the tendency for a spring to buckle.

Number of Coils

The total number of coils in a spring will be called N. But in the calculation of stress and deflections for a spring, some of the coils are inactive and are neglected. For example, in a spring with squared and ground ends or simply squared ends, each end coil is inactive and the number of *active coils*, N_a, is $N - 2$. For plain ends all coils are active: $N_a = N$. For plain coils with ground ends, $N_a = N - 1$.

Pitch

Pitch, p, refers to the axial distance from a point on one coil to the corresponding point on the next adjacent coil. The relationships among the pitch, free length, wire diameter, and number of active coils are given next.

Squared and ground ends:	$L_f = pN_a + 2D_w$
Squared ends only:	$L_f = pN_a + 3D_w$
Plain and ground ends:	$L_f = p(N_a + 1)$
Plain ends:	$L_f = pN_a + D_w$

Pitch Angle

Figure 7-5 shows the pitch angle, λ; it can be seen that the larger the pitch angle, the steeper the coils appear to be. Most practical spring designs produce a pitch angle less than about 12°. If the angle is greater than 12°, undesirable compressive stresses develop in the wire and the formulas presented later are inaccurate. The pitch angle can be computed by the formula

$$\lambda = \tan^{-1}\left[\frac{p}{\pi D_m}\right] \tag{7-2}$$

The logic of this formula can be seen by taking one coil of a spring and unwrapping it onto a flat surface, as illustrated in Figure 7-5. The horizontal line is the mean circumference of the spring, and the vertical line is the pitch, p.

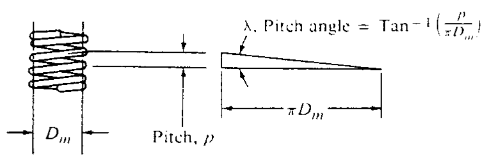


Figure 7-5 Pitch Angle

Installation Considerations

Frequently, a spring is installed in a cylindrical hole or around a rod. When it is, adequate clearances must be provided. When a compression spring is compressed, its diameter gets larger. Thus the inside diameter of a hole enclosing the spring must be greater than the outside diameter of the spring to eliminate rubbing. An initial diametral clearance of one-tenth of the wire diameter is recommended for springs having a diameter of 0.50 in (12 mm) or greater. If a more precise estimate of the actual outside diameter of the spring is required, the following formula can be used for the OD at the solid length condition:

$$OD_s = \sqrt{D_m^2 + \frac{p - D_w^2}{\pi^2}} + D_w \tag{7-3}$$

Even though the spring ID gets larger, it is also recommended that the clearance at the ID be approximately $0.1D_w$.

Springs with squared ends or squared-and-ground ends are frequently mounted on a button-type seat or in a socket with a depth equal to the height of just a few coils for the purpose of locating the spring.

Coil Clearance

The term *coil clearance* refers to the space between adjacent coils when the spring is compressed to its operating length, L_o. The actual coil clearance can be estimated from

$$cc = (L_o - L_s)/N_a$$

One guideline is that the coil clearance should be greater than $D_w/10$, especially in springs loaded cyclically. Another recommendation relates to the overall deflection of the spring:

$$(L_o - L_s) > 0.15(L_f - L_s)$$

Materials Used for Springs

Virtually any elastic material can be used for a spring. However, most mechanical applications use metallic wire, either high-carbon steel (most common), alloy steel, stainless steel, brass, bronze, beryllium copper, or nickel base alloys. Most spring materials are made according to specifications of the ASTM. Table 7-3 lists some common types.

Types of Loading and Allowable Stresses

The allowable stress to be used for a spring depends on the type of loading, the material, and size of the wire. Loading is usually classified into three types:

Light service: Static loads or up to 10 000 cycles of loading with a low rate of loading (nonimpact).

Average service: Typical machine design situations; moderate rate of loading and up to 1 million cycles.

Table 7-3 Spring Materials

Material Type	*ASTM No.*	*Relative Cost*	*Temperature Limits, °F*
High-carbon steels:			
Hard drawn	A227	1.0	0–250
General-purpose steel with 0.60–0.70% carbon; low cost			
Music wire	A228	2.6	0–250
High-quality steel with 0.80–0.95% carbon; very high strength; excellent surface finish; hard drawn; good fatigue performance; used mostly in smaller sizes up to 0.125 in			
Oil-tempered	A229	1.3	0–350
General-purpose steel with 0.60–0.70% carbon; used mostly in larger sizes above 0.125 in; not good for shock or impact			
Alloy steels:			
Chromium-vanadium	A231	3.1	0–425
Good strength, fatigue resistance, impact strength, high-temperature performance; valve spring quality			
Chromium-silicon	A401	4.0	0–475
Very high strength and good fatigue and shock resistance			
Stainless steels:			
Type 302	A313(302)	7.6	<0–550
Very good corrosion resistance and high-temperature performance; nearly nonmagnetic; cold drawn; types 304 and 316 also fall under this ASTM class and have improved workability but lower strength			
Type 17-7 PH	A313(631)	11.0	0–600
Good high-temperature performance			
Copper alloys: All have good corrosion resistance and electrical conductivity			
Spring brass	B134	High	0–150
Phosphor bronze	B159	8.0	<0–212
Beryllium copper	B197	27.0	0–300
Nickel-base alloys: All are corrosion-resistant, have good high- and low-temperature properties, are nonmagnetic or nearly nonmagnetic (trade names of the International Nickel Company)			
Monel	—	—	−100–425
K-Monel	—	—	−100–450
Inconel	—	—	Up to 700
Inconel-X	—	44.0	Up to 850

Source: Associated Spring, Barnes Group, Inc. *Engineering Guide to Spring Design*. Bristol, Conn., 1981; Carlson, Harold. *Spring Designer's Handbook*. New York: Marcel Dekker, 1978; Oberg, E., et al. *Machinery's Handbook*, 22d ed. New York: Industrial Press, 1984.

Severe service: Rapid cycling for above 1 million cycles; possibility of shock or impact loading; engine valve springs are a good example.

The strength of a given material is greater for the smaller sizes. Figures 7-6 through 7-11 show the allowable working stresses for six different materials. Note that some curves can be used for more than one material by the application of a factor. As a conservative approach to design, we will use the average service curve for most design examples, unless true high cycling conditions exist. We will use the light service curve as the upper limit on stress when the spring is compressed to its solid height. If the stress exceeds the light service value by a small amount, the spring will undergo permanent set because of yielding.

7-3 STRESSES AND DEFLECTION FOR HELICAL COMPRESSION SPRINGS

As a compression spring is compressed under an axial load, the wire is twisted. Therefore, the stress developed in the wire is *torsional shear stress*, and it can be derived from the classical equation $\tau = Tc/J$.

When applied specifically to a helical compression spring, some modifying factors are needed to account for the curvature of the spring wire and for the direct shear stress created as the coils resist the vertical load. Also, it is convenient to express the shear stress in terms of the design variables encountered in springs. The resulting equation for stress is attributed to Wahl (7). The maximum shear stress, which occurs at the inner surface of the wire, is

$$\tau = \frac{8KFD_m}{\pi D_w^3} = \frac{8KFC}{\pi D_w^2} \qquad (7\text{-}4)$$

These are two forms of the same equation as the definition of $C = D_m/D_w$ demonstrates. The shear stress for any applied force, F, can be computed. Normally we will be concerned about the stress when the spring is compressed to solid length under the influence of F_s and when the spring is operating at its normal maximum load, F_o. Notice that the stress is inversely proportional to the *cube* of the wire diameter. This illustrates the great effect that variation in the wire size has on the performance of the spring.

The Wahl factor, K, in equation (7-4) is the term that accounts for the curvature of the wire and the direct shear stress. Analytically, K is related to C:

$$K = \frac{4C - 1}{4C - 4} + \frac{0.615}{C} \qquad (7\text{-}5)$$

Figure 7-12 shows a plot of K versus C for round wire. Recall that $C = 5$ is the recommended minimum value of C. The value of K rises rapidly for $C < 5$.

Deflection

Because the primary manner of loading on the wire of a helical compression spring is torsion, the deflection is computed from the angle of twist formula,

$$\theta = TL/GJ$$

where θ is the angle of twist in radians, T is the applied torque, L is the length of the wire, G is the modulus of elasticity of the material in shear, and J is the polar moment of inertia

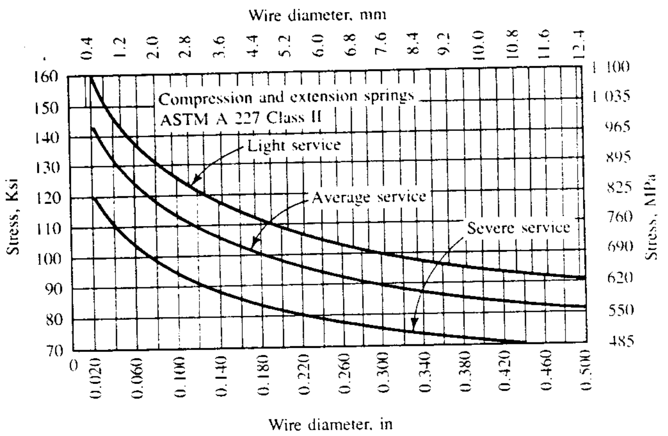
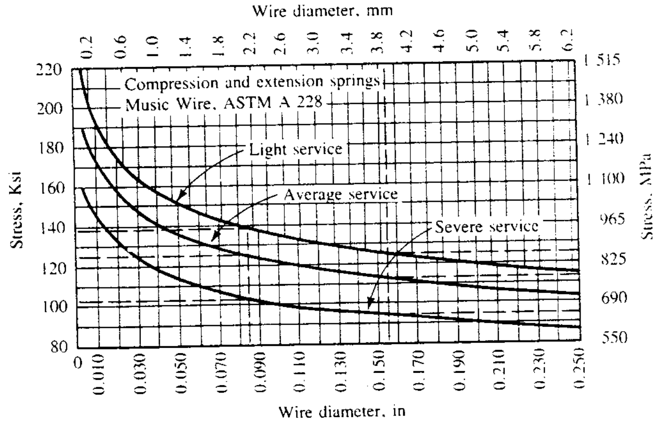


Figure 7-6 Design Stresses, ASTM A227 Steel Wire, Hard Drawn (Reprinted from Carlson, Harold. *Spring Designer's Handbook,* p. 144, by courtesy of Marcel Dekker, Inc.)



Similar design stresses for ASTM A 313, type 631 stainless steel wire, 17-7 PH

Figure 7-7 Design Stresses, ASTM A228 Steel Wire (Music Wire) (Reprinted from Carlson, Harold. *Spring Designer's Handbook,* p. 143, by courtesy of Marcel Dekker, Inc.)

of the wire. Again, for convenience, we will use a different form of the equation in order to calculate the linear deflection, f, of the spring from the typical design variables of the spring. The resulting equation is

$$f = \frac{8FD_m^3N_a}{GD_w^4} = \frac{8FC^3N_a}{GD_w} \tag{7-6}$$

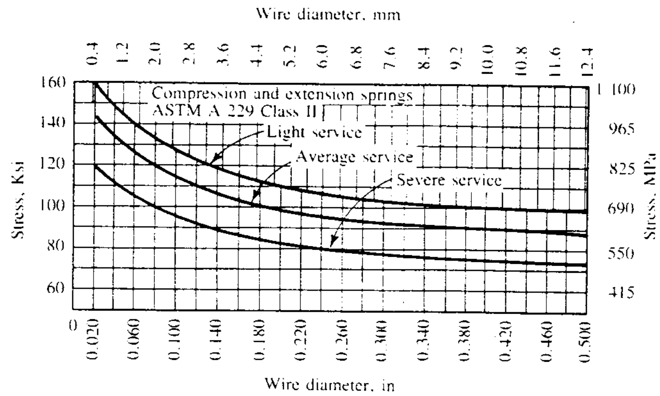


Figure 7-8 Design Stresses, ASTM A229 Steel Wire, Oil-tempered (Reprinted from Carlson, Harold. *Spring Designer's Handbook,* p. 146, by courtesy of Marcel Dekker, Inc.)

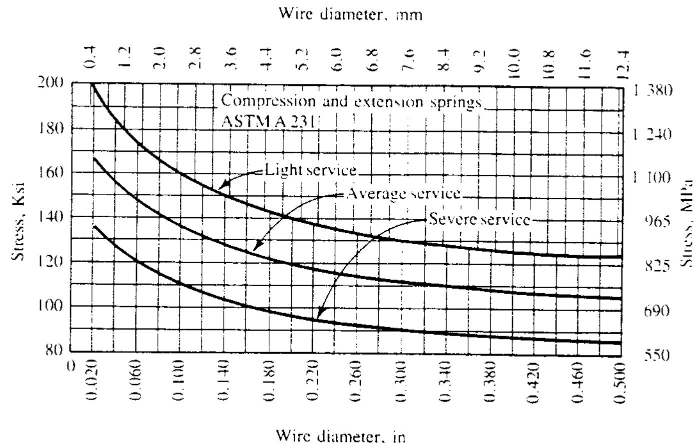


Figure 7-9 Design Stresses, ASTM A231 Steel Wire, Chromium-vanadium Alloy, Valve Spring Quality (Reprinted from Carlson, Harold. *Spring Designer's Handbook,* p. 147, by courtesy of Marcel Dekker, Inc.)

Recall that N_a is the number of *active* coils, as discussed in section 7-2. Table 7-4 lists the values for G for typical spring materials. Note again, in equation (7-6), that the wire diameter has a strong effect on the performance of the spring.

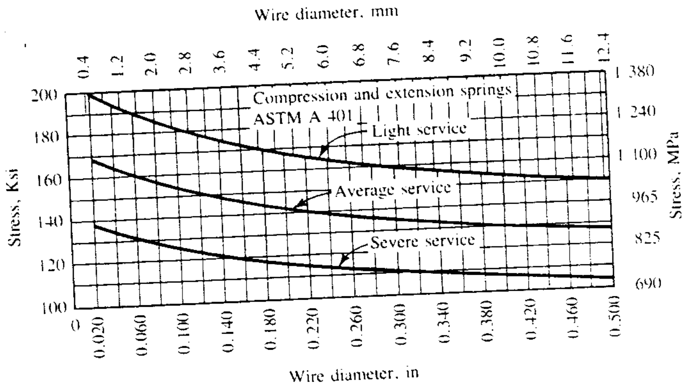


Figure 7-10 Design Stresses, ASTM A401 Steel Wire, Chromium-silicon Alloy, Oil-tempered (Reprinted from Carlson, Harold. *Spring Designer's Handbook*, p. 148, by courtesy of Marcel Dekker, Inc.)

Buckling

The tendency for a spring to buckle increases as the spring becomes tall and slender, much as for a column. Figure 7-13 shows plots of the critical ratio of deflection to the free length, versus the ratio of free length to the mean diameter for the spring. Three different end conditions are described in the figure. As an example of the use of this figure, consider a spring having squared and ground ends, a free length of 6.0 in, and a mean diameter of 0.75 in. We want to know what deflection would cause the spring to buckle. First compute

$$\frac{L_f}{D_m} = \frac{6.0}{0.75} = 8.0$$

Then, from Figure 7-13, the critical deflection ratio is 0.20. From this we can compute the critical deflection.

$$\frac{f_o}{L_f} = 0.20 \qquad \text{or} \qquad f_o = 0.20(L_f) = 0.20(6.0\text{ in}) = 1.20\text{ in}$$

That is, if the spring is deflected more than 1.20 in, the spring should buckle.

7-4 DESIGN OF HELICAL COMPRESSION SPRINGS

The objective of the design of helical compression springs is to specify the geometry for the spring to operate under specified limits of load and deflection, possibly with space limitations, also. The material and the type of service will be specified by considering the environment and the application.

A typical problem statement follows.

Example Problem 7-1. A helical compression spring is to exert a force of 8.0 lb when compressed to a length of 1.75 in. At a length of 1.25 in, the force must be 12.0 lb. The

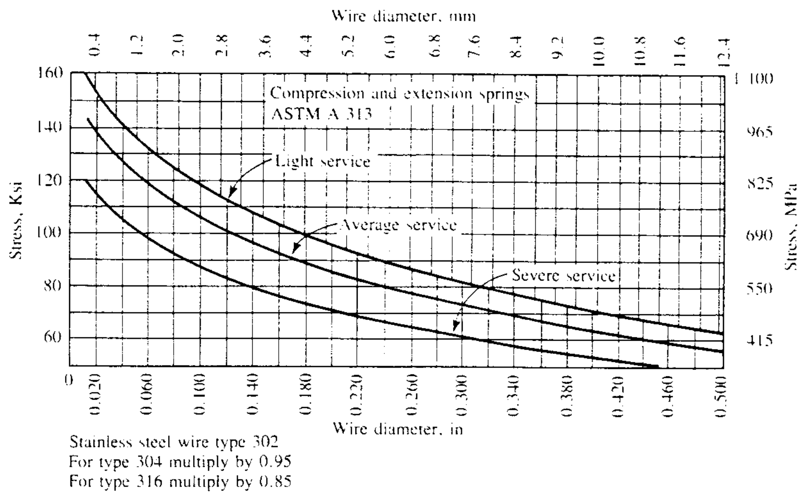


Figure 7-11 Design Stresses, ASTM A313 Corrosion-resistant Stainless Steel Wire (Reprinted from Carlson, Harold. *Spring Designer's Handbook*, p. 150, by courtesy of Marcel Dekker, Inc.)

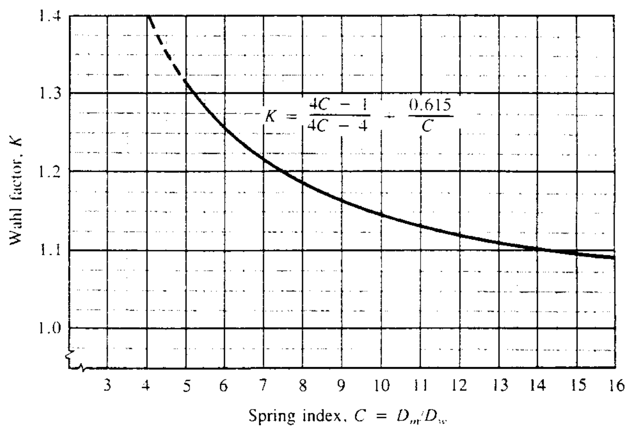


Figure 7-12 Wahl Factor vs. Spring Index for Round Wire

spring will be installed in a machine that cycles slowly, and approximately 200 000 cycles total are expected. The temperature will not exceed 200°F. It is planned to install the spring in a hole having a diameter of 0.75 in.

For this application, specify a suitable material, wire diameter, mean diameter, OD, ID, free length, solid length, number of coils, and type of end condition. Check the stress at the maximum operating load and at the solid length condition.

Table 7-4 Modulus of Elasticity: *G* (Shear); *E* (Tension)

MATERIAL AND ASTM NO.	SHEAR MODULUS, *G* (psi)	SHEAR MODULUS, *G* (GPa)	TENSION MODULUS, *E* (psi)	TENSION MODULUS, *E* (GPa)
Hard drawn steel: A227	11.5×10^6	79.3	28.6×10^6	197
Music wire: A228	11.85×10^6	81.7	29.0×10^6	200
Oil-tempered: A229	11.2×10^6	77.2	28.5×10^6	196
Chromium-vanadium: A231	11.2×10^6	77.2	28.5×10^6	196
Chromium-silicon: A401	11.2×10^6	77.2	29.5×10^6	203
Stainless steels: A313				
Types 302, 304, 316	10.0×10^6	69.0	28.0×10^6	193
Type 17-7 PH	10.5×10^6	72.4	29.5×10^6	203
Spring brass: B134	5.0×10^6	34.5	15.0×10^6	103
Phosphor bronze: B159	6.0×10^6	41.4	15.0×10^6	103
Beryllium copper: B197	7.0×10^6	48.3	17.0×10^6	117
Monel and K-Monel	9.5×10^6	65.5	26.0×10^6	179
Inconel and Inconel-X	10.5×10^6	72.4	31.0×10^6	214

Note: Data are average values. Slight variation with wire size and treatment may occur.

Two solution procedures will be shown, and each will be implemented in a computer program and worked out in the solution. The numbered steps can be used as a guide for future problems and as a kind of algorithm for the computer programs.

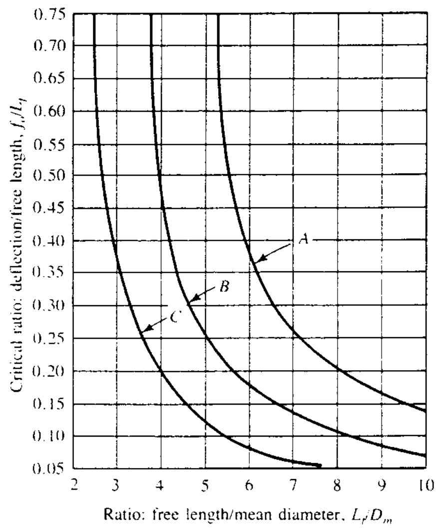
Solution Method 1. The procedure works directly toward the overall geometry of the spring by specifying the mean diameter to meet the space limitations. The process requires that the designer have tables of data available for wire diameters (such as Table 7-2) and design stresses for the material from which the spring will be made (such as Figures 7-6 to 7-11). An initial estimate for the design stress for the material must be made by consulting the charts of design stress versus wire diameter to make a reasonable choice. In general, more than one trial must be made, but the results of early trials will help you to decide the values to use for later trials.

Step 1. Specify a material and its shear modulus of elasticity, G.
For this problem, several standard spring materials can be used. Let's select ASTM A231 chromium-vanadium steel wire, having a value of $G = 11\,200\,000$ psi.

Step 2. From the problem statement, identify the operating force, F_o; the operating length at which that force must be exerted, L_o; the force at some other length, called the *installed force*, F_i; and the installed length, L_i.
Remember, F_o is the maximum force the spring experiences under normal operating conditions. Many times, the second force level is not specified. In that case, let F_i = zero and specify a design value for the free length, L_f, in place of L_i.
For this problem, $F_o = 12.0$ lb; $L_o = 1.25$ in; $F_i = 8.0$ lb; and $L_i = 1.75$ in.

Step 3. Compute the spring rate, k.

$$k = \frac{F_o - F_i}{L_i - L_o} = \frac{12.0 - 8.0}{1.75 - 1.25} = 8.00 \text{ lb/in}$$



Curve *A*: Fixed ends (e.g., squared and ground ends on guided, flat, parallel surfaces)
Curve *B*: One fixed end; one pinned end (e.g., one end on flat surface, one in contact with a spherical ball)
Curve *C*: Both ends pinned (e.g., ends in contact with surfaces which are pinned to the structure and permitted to rotate)

Figure 7-13 Spring Buckling Criteria. If actual ratio of L_O/L_f is greater than critical ratio, spring will buckle at operating deflection.

Step 4. Compute the free length, L_f.

$$L_f = L_i + F_i/k = 1.75 \text{ in} + [(8.00 \text{ lb})/(8.00 \text{ lb/in})] = 2.75 \text{ in}$$

The second term in the preceding equation is the amount of deflection from free length to the installed length in order to develop the installed force, F_i. Of course, this step becomes unnecessary if the free length is specified in the original data.

Step 5. Specify an initial estimate for the mean diameter, D_m.
Keeping in mind that the mean diameter will be smaller than the outside diameter and larger than the inside diameter, judgment is necessary to get started. For this problem, let's specify $D_m = 0.60$ in. This should permit the installation into the 0.75-in diameter hole.

Step 6. Specify an initial design stress.
The charts for the design stresses for the selected material can be consulted, considering also the service. In this problem, we should use average service. Then, for the ASTM A231 steel, as shown in Figure 7-9, a nominal design stress would be 130 000 psi. This is strictly an estimate based on the strength of the material. The process includes a check on stress later.

Step 7. Compute the trial wire diameter by solving equation (7-4) for D_w. Notice that everything else in the equation is known except the Wahl factor, K, because it depends on the wire diameter itself. But K varies only little over the normal range of spring indexes, C. From Figure 7-12, it can be seen that $K = 1.2$ is a nominal value. This, too, will be checked later. With the assumed value of K, some simplification can be done.

$$D_w = \left[\frac{8KF_oD_m}{\pi\tau_d}\right]^{1/3} = \left[\frac{(8)(1.2)(F_o)(D_m)}{(\pi)(\tau_d)}\right]^{1/3}$$

Combining constants gives

$$D_w = \left[\frac{8KF_oD_m}{\pi\tau_d}\right]^{1/3} = \left[\frac{(3.06)(F_o)(D_m)}{\tau_d}\right]^{1/3} \qquad (7\text{-}7)$$

For this problem,

$$D_w = \left[\frac{(3.06)(F_o)(D_m)}{\tau_d}\right]^{1/3} = \left[\frac{(3.06)(12)(0.6)}{130\,000}\right]^{0.333}$$

$$D_w = 0.055\,3 \text{ in}$$

Step 8. Select a standard wire diameter from the tables and then determine the design stress and maximum allowable stress for the material at that diameter. The design stress will normally be for average service, unless high cycling rates or shock indicate that severe service is warranted. The light service curve should be used with care because it is very near to the yield strength. In fact, we will use the light service curve as an estimate of the maximum allowable stress.

For this problem, the next larger standard wire size is 0.062 5 in, no. 16 on the U.S. Steel Wire Gage chart. For this size, the curves for ASTM A231 steel wire show the design stress to be approximately 145 000 psi for average service, and the maximum allowable stress to be 170 000 psi from the light service curve.

Step 9. Compute the actual values of C and K, the spring index, and the Wahl factor.

$$C = D_m/D_w = 0.60/0.062\,5 = 9.60$$

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = \frac{4(9.60)-1}{4(9.60)-4} + \frac{0.615}{9.60} = 1.15$$

Step 10. Compute the actual expected stress due to the operating force, F_o, from equation (7-4).

$$\tau_o = \frac{8KF_oD_m}{\pi D_w^3} = \frac{(8)(1.15)(12.0)(0.60)}{(\pi)(0.062\,5)^3} = 86\,450 \text{ psi}$$

Comparing this with the design stress of 145 000 psi, it is safe.

Step 11. Compute the number of active coils required to give the proper deflection characteristics for the spring. Using equation (7-6) and solving for N_a,

$$f = \frac{8FC^3N_a}{GD_w}$$

$$N_a = \frac{fGD_w}{8FC^3} = \frac{GD_w}{8kC^3} \quad \text{(Note: } F/f = k\text{, the spring rate)} \qquad (7\text{-}8)$$

Then, for this problem,

$$N_a = \frac{GD_w}{8kC^3} = \frac{(11\,200\,000)(0.062\,5)}{(8)(8.0)(9.60)^3} = 12.36 \text{ coils}$$

Notice that $k = 8.0$ lb/in is the spring rate. Do not confuse this with K, the Wahl factor.

Step 12. Compute the solid length, L_s; the force on the spring at solid length, F_s; and the stress in the spring at solid length, τ_s. This computation will give the maximum stress the spring will receive.

The solid length occurs when all the coils are touching, but recall that there are two inactive coils.

$$L_s = D_w(N_a + 2) = 0.062\,5(14.36) = 0.898 \text{ in}$$

The force at solid length is the product of the spring rate times the deflection to solid length $(L_f - L_s)$.

$$F_s = k(L_f - L_s) = (8.0 \text{ lb/in})(2.75 - 0.898) \text{ in} = 14.8 \text{ lb}$$

Because the stress in the spring is directly proportional to the force, a simple method of computing the solid length stress is

$$\tau_s = (\tau_o)(F_s/F_o) = (86\,450 \text{ psi})(14.8/12.0) = 106\,750 \text{ psi}$$

When this is compared with the maximum allowable stress of 170 000 psi, it is safe and the spring will not yield when compressed to solid length.

Step 13. Complete the computations of geometric features and compare them with space and operational limitations.

$$\text{OD} = D_m + D_w = 0.60 + 0.062\,5 = 0.663 \text{ in}$$

$$\text{ID} = D_m - D_w = 0.60 - 0.062\,5 = 0.538 \text{ in}$$

These dimensions are satisfactory for installation in a hole having a diameter of 0.75 in.

This procedure completes the design of one satisfactory spring for this application. It may be desirable to make other trials to attempt to find a more nearly optimum spring. Also, the tendency to buckle should be checked along with the coil clearance. These items will be discussed within the development of method 2.

Computer Program for Spring Design Method 1

In the following discussion, the listing of the code for a computer program in the BASIC language performs the 13-step design procedure developed previously. Figure 7-14 is a flowchart for the program. The *REM*, or remark, statements in the program listing should help you follow the program. Notice that several checks are made and messages are printed if (1) the stresses are too high, (2) the spring index is less than 5.0, or (3) the computed solid length happens to exceed the specified operating length, a clearly impossible situation. Statements 890–940 allow the operator to elect several options on how to proceed after completing any given design. Many designs can be completed in a short time with such a program.