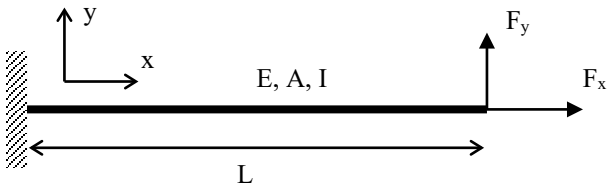
Exercise 8: Sensitivity Analysis

Discrete sensitivity analysis of a beam structure modeled with finite elements

Consider a clamped-free beam, length L, Young's modulus E, cross-sectional area A, moment of inertia I, and a load with components (F_x, F_y) at the tip. The beam length L is considered as a design variable. The tip displacements (u_x, u_y) are the response functions. Geometrical nonlinearities are not considered here.



Analytical responses:

$$u_x = \frac{F_x L}{EA} \qquad u_y = \frac{F_y L^3}{3EI}$$

Assignment

Compute the exact sensitivities using the analytical responses. These can be used as a reference. Use the provided Matlab finite element scripts to study:

1. Global Finite Difference (GFD) sensitivities:
   A script named `gfd_beam.m` is provided where forward and central finite difference sensitivity analyses are implemented. You can modify the load vector P at the beginning of the script.
   a) Have a look at the script, in particular the subroutines for forward and central finite difference. Verify by inspection that these compute the GFD sensitivities.
   b) Run the script for each load separately (Fx, Fy). It produces plots that show the accuracy of both types of finite difference sensitivities as a function of the relative design perturbation (ΔL/L). Comment on the findings, are they as expected? Indicate for each load which perturbations give sufficiently accurate results.
   c) The input argument `gfd_beam` of sets the number of elements for this beam. Increase it to 100 and repeat the study. Comment on the findings.
   d) Increase the number of elements until the elapsed time is about 3 seconds for each method. This allows for reliable time comparisons. Write down the elapsed times and the number of elements used.

2. Semi-analytical (SA) sensitivities:
   A script named `sa_beam.m` is provided where the calculation of sensitivities using the semi-analytical approach is implemented. You can modify the load vector P at the beginning of the script.
   a) Have a look at the script, in particular the subroutines. Verify by inspection that these compute the SA sensitivities using forward and central finite difference.
   b) Run also this script for each load separately (Fx, Fy). It produces plots that show the accuracy of both types of SA sensitivities as a function of the relative design perturbation (ΔL/L). Comment on the findings, are they as expected? Indicate for each load which perturbations give sufficiently accurate results.
   c) The input argument `sa_beam` sets the number of elements for this beam. Increase it to 100 and repeat the study. Comment on the findings.
   d) Run the script using the number of elements determined at 1(d), and write down the elapsed times.

3. Final comparison:
   To conclude, compare your findings from part 1 and 2 of this assignment. Discuss the results and draw conclusions with respect to:
   a) the usable range of design perturbations, and
   b) the computation time.