StudeerSnel.nl

Robust 1+2 - robuust assignment

Robust and Multivariable Control Design (Technische Universiteit Delft)

StudeerSnel wordt niet gesponsord of ondersteund door een hogeschool of universiteit
Gedownload door Koen Ceton (koen604@gmail.com)

Robust Control

SC42145

Practical Assignment: Control Design for a Floating Wind Turbine

mr.X
mr.Y

December 18, 2020

TU Delft

Dit document is gratis te downloaden op StudeerSnel.nl

Gedownload door Koen Ceton (koen604@gmail.com)

Abstract

This report contains the solutions to the Practical Assignment for the course Robust Control (SC42145) and is written by mr.X and mr.Y. The report consists of an introduction, and one section for each of the exercises. To solve the exercises, Matlab scripts were used. These files can be found as an attachment to this report.

Gedownload door Koen Ceton (koen604@gmail.com)

Contents

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

1 Introduction

In this assignment, the pitch angle of the rotors of a floating wind turbine (FWT) is controlled. The figure below depicts a schematic representation of the movement of the FWT that must be controlled.

1.1 Model description

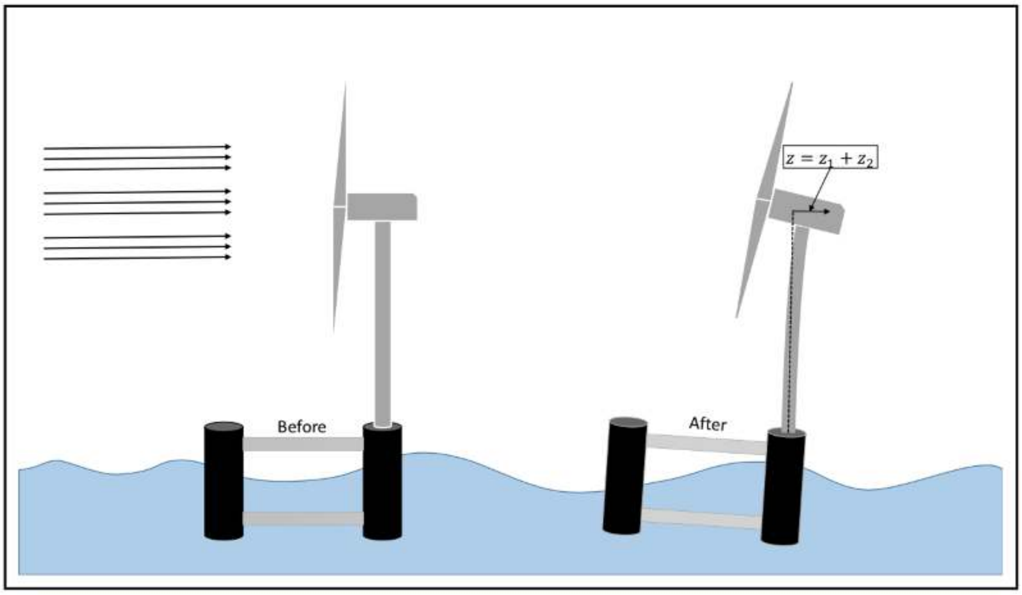

Figure 1: Schematic depiction of fore-aft movement.

This system can be described by the following state-space model:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where the system matrices A, B, C, D are given in the script and where

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \omega_r \\ \dot{z}_1 \\ z_1 \\ \dot{z}_2 \\ z_2 \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \beta \\ \tau_e \\ V \end{bmatrix} \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \omega_r \\ z \end{bmatrix} \tag{2}$$

The operational parameters of the FWT are given by:

Rated Power	1.8[MW]
Rated Torque	10^4[Nm]
Rated Speed	180[rad/s]

(3)

Gedownload door Koen Ceton (koen604@gmail.com)

2 Part 1: SISO Analysis and Control Design

In this part, the objective is to design a reference tracking SISO controller for the floating wind turbine that achieves the highest possible bandwidth frequency whilst meeting the following design requirements:

1. Stability and robustness
2. Small settling time
3. Overshoot $< 1\%$
4. No steady state error

2.1 Question 1: SISO system analysis

In Figure 2 and 3 the open-loop Bode plot and pole-zero map of the plant are depicted respectively.

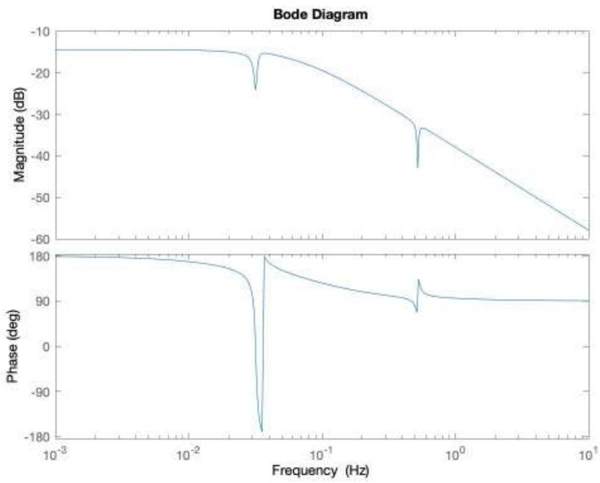

Figure 2: Open-loop bode plot of the SISO system

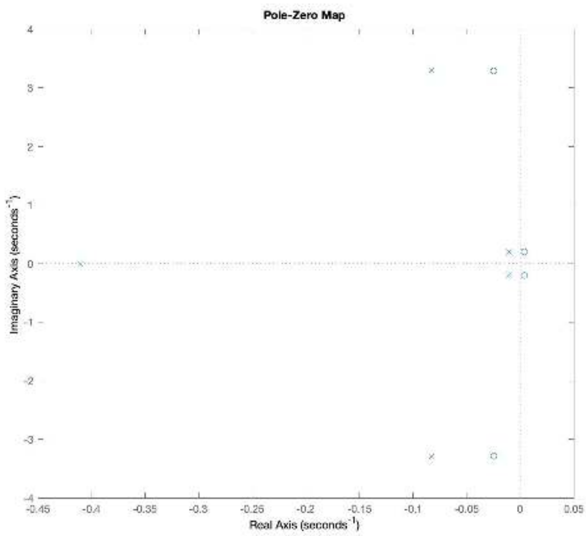

Figure 3: Pole-zero map of the SISO system

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

From the Bode plot in Figure 2 it can be observed that the magnitude is always below $-3dB$. Hence, the bandwidth is never reached. We can conclude logically that a higher gain is needed in our open-loop.

From the pole-zero plot in Figure 3 we observe that there are no poles in the right-half-plane, hence there are no unstable poles. However, there are two zero's in the RHP that limit the bandwidth frequency. We can compute the maximum attainable bandwidth frequency using the following equation:

$$\omega_B^* < -\frac{x}{M} + \sqrt{x^2 + y^2\left(1 - \frac{1}{M^2}\right)} \tag{4}$$

where x and y denote the real and the imaginary part of the RHP zero's, respectively. For $M \to \infty$ and the zero pair of value $z = 0.0038 \pm 0.2i$ we get a maximum achievable bandwidth frequency of approximately 0.2 rad/s, which is about 0.032 Hz.

Lastly, there are 2 dips in the Bode plot that could hinder in achieving a high bandwidth frequency (> 0.1 Hz). This can be solved by using an anti-notch filter to correct for them.

2.2 Question 2: Open-loop design requirements

There are certain characteristics in the open-loop of a Bode plot that correspond to certain characteristics in the closed-loop step response, which we use to attain our design requirements. We decide to use a PID controller. The methodology we will use in order to select appropriate PID gains is called loop-shaping. In loop-shaping we select a certain controller structure and gains such that the desired characteristics of our open-loop are achieved. In this section we discuss what those characteristics are and how the PID gains influence them. We also discuss what the complementary sensitivity function means for our system.

1. Stability and robustness - The open-loop gain- and phase margin must be positive for stability, and the more positive they are, the more robust our system is. We can also use the complementary sensitivity function for stability and robustness. For the system to be stable, we want to see how much gain and phase we can add to our system before reaching the point -1 on the Nyquist plot. For robustness, we want these values to be as high as possible. The distance between the -1 point and our system (defined as C(s)*G(s)) is equal to $|-1 - C(s) * G(s)|$ which is the same as $|1 + C(s) * G(s)|$. Hence, to have maximal robustness, we want to maximize $|1 + C(s) * G(s)|$ over all frequencies. As our sensitivity function is $|\frac{1}{1+C(s)*G(s)}|$, it is clear that we want to minimize our Sensitivity function. To be more precise, we want to minimize our maximum sensitivity value, as it is at this frequency that the -1 point will be reached first. This is called the nominal sensitivity peak M_s. As our complementary sensitivity function is $|\frac{C(s)*G(s)}{1+C(s)*G(s)}|$, we could also formulate our goal as minimizing the complementary sensitivity function. There are some fundamental limitations to this, stated as $||T| - |S|| < 1$ and either $|T| > 1$ or $|S| > 1$.

2. Minimal settling time - We want our open-loop cross-over frequency (the frequency at which our gain is 0 dB) to be as high as possible. This is because the open-loop cross-over frequency directly influences the closed-loop bandwidth frequency (frequency at which the gain is -3 dB), which is related to the settling time. The relation is given in the following equation:

$$\omega_{\text{BW}} = \frac{4}{T_s\zeta}\sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} \tag{5}$$

   For our damping ratio we find:

$$\omega_{\text{BW}} = \frac{4.05}{T_{\text{s}}} \tag{6}$$

   The cross-over frequency can be increased with a Proportional gain. Still, we need to not have a P that is too high, as a high gain at low frequencies causes a high rise time (which logically causes a higher settling time). Hence, we need to find a certain gain and slope attenuation at low frequencies that is able to balance these two factors. A slope attenuation often used is - 20 dB /dec.

Gedownload door Koen Ceton (koen604@gmail.com)

3. Overshoot - We want a high phase margin in the open-loop, as this is inversely related to the closed-loop overshoot. This can be done with a Derivative action. To approximate the phase margin that we want to have in the open-loop we use a formula that computes overshoot as a function of the damping ratio (ζ), which is used for second-order systems. OS is given as a percentage (%).

$$OS = 100 \exp\left(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}\right) \tag{7}$$

We solve this for PO is 1, as we want a maximum overshoot of 1%, resulting in a ζ of approximately 0.83. Then, we compute the desired minimum phase margin for the open-loop with the following equation

$$\phi_M = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}} \tag{8}$$

Filling in our ζ we get a minimum phase margin of approximately 70.9°.

4. Steady state error - The steady state error can be corrected by making use of the Integrator. If done correctly, the gain in the controlled closed-loop Bode plot should be 0 dB at low frequencies which results in the final value of 1 for the step response.

Dit document is gratis te downloaden op StudeerSnel.nl

Gedownload door Koen Ceton (koen604@gmail.com)

2.3 Question 3: Implementing SISO controller

To cancel out the notches in the Bode plot of the system, two anti-notch filters that are both of the following form:

$$C_{\text{notch}}(s) = \frac{s^2 + 2\omega\beta_1 s + \omega^2}{s^2 + 2\omega\beta_2 s + \omega^2} \tag{9}$$

where the values for ω are $0.0318 * 2\pi$ and $0.523 * 2\pi$ (the frequencies at which the notches occur in rad/s). The values for the amplification factor $\frac{\beta_1}{\beta_2}$ are found by trying different combinations in MATLAB. Choosing larger values for both betas will lead to a wider notch filter and the ratio between them translates to the magnitude of amplification. Next, a PID controller of the form

$$C_{\text{PID}} = K_p + \frac{K_i}{s} + \frac{K_d s}{T_f s + 1} \tag{10}$$

is used and tuned to meet the desired requirements of the system. The effects of the P, I and D actions were discussed in Section 2.2. To find an optimal combination of the P, I and D gains, we used the Ziegler-Nichols method, which works as follows:

1. Set all gains to zero.
2. Increase the P gain until the response to a disturbance is steady oscillation.
3. Increase the D gain until the the oscillations go away (i.e., it is critically damped).
4. Repeat steps 2 and 3 until increasing the D gain does not stop the oscillations.
5. Set P and D to the last stable values.
6. Increase the I gain until it brings you to the setpoint with the number of oscillations as desired.

The gains that are found this way are:

- $K_p = -0.07$
- $K_i = -0.45$
- $K_d = -9$
- $T_f = 1$

Combining the filters, the PID controller and our system leads to the following open-loop transfer function:

$$L = C_{\text{notch},1} C_{\text{notch},2} C_{\text{PID}} G_{\text{plant}} \tag{11}$$

In the following table our performance specifications are given. The resulting open-loop Bode, step response, sensitivity function and complementary sensitivity of the controlled system are given after the table in Figures 4-7. We can see that our all the requirements listed in Section 2.2 are met, as well as the design requirements from the task. The only problem is that our bandwidth frequency is still not very high at 0.0873 Hz. This is because of the two zero's in the RHP, as discussed in Section 2.1

Time-domain characteristics	
Rise time	24.3404 s
Settling time	30.5899 s
Overshoot	0.5971 %
Undershoot	0 %
Steady-state error	0
Frequency-domain characteristics	
Crossover frequency	0.197 rad/s
Bandwidth	0.5485 rad/s
Phase margin	79°
Gain margin	19.4 dB

(12)

Gedownload door Koen Ceton (koen604@gmail.com)

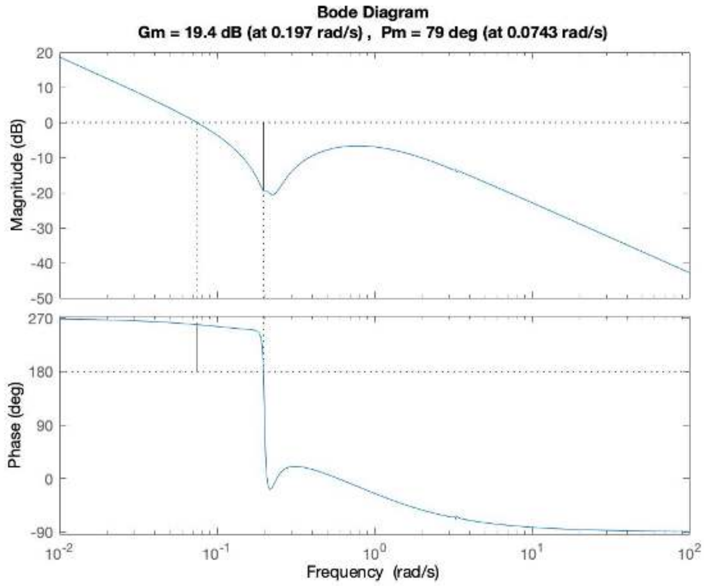

Figure 4: Closed-loop bode plot of the controlled system.

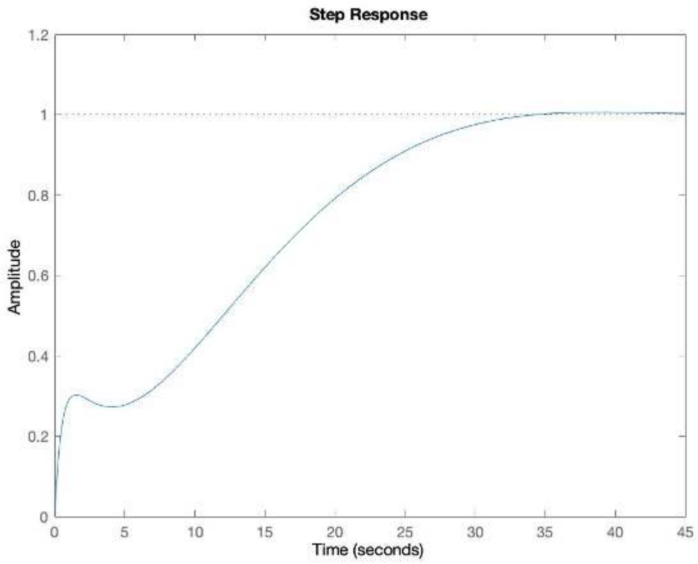

Figure 5: Step response of the controlled system.

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

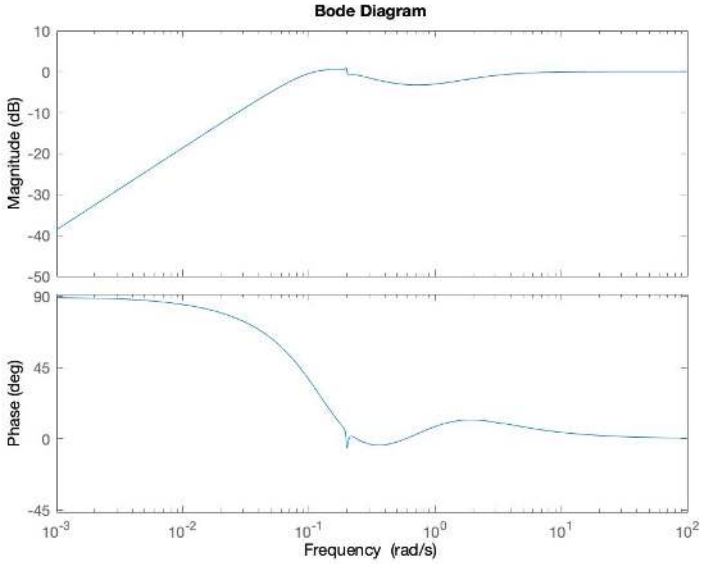

Figure 6: Sensitivity function.

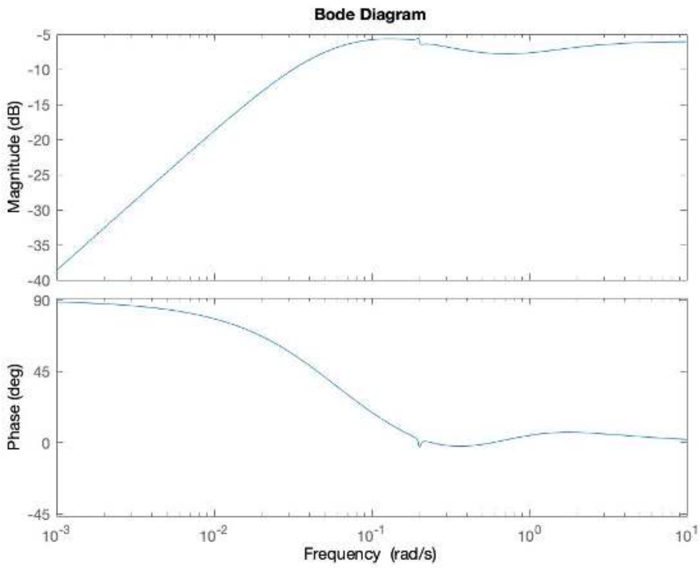

Figure 7: Complementary sensitivity function.

Gedownload door Koen Ceton (koen604@gmail.com)

2.4 Question 4: Disturbance rejection

Now we take a look at the disturbance rejection of the controller. For this the following block diagram and output relation from the lecture slides are considered:

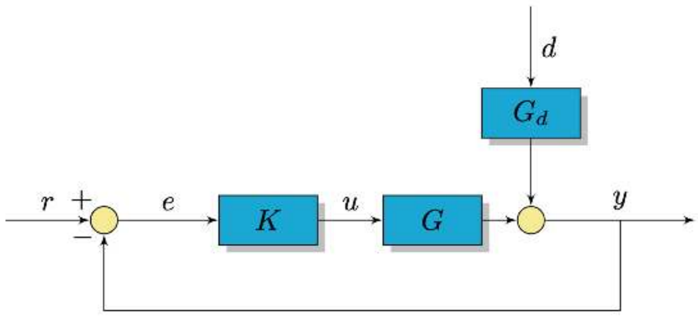

Figure 8: Closed loop system including disturbance.

$$y = \underbrace{(I + GK)^{-1}GK}_{T}\, r + \underbrace{(I + GK)^{-1}}_{S}\, G_d d \tag{13}$$

Since we want to use the controller to drive the disturbance on the output to 0, the reference is set to 0. This means that the first term containing the complementary sensitivity function is eliminated. We can achieve this by multiplying with the sensitivity function. We multiplied the sensitivity function with the third column of the transfer function, as asked in the assignment. We are left with:

$$y = SG_d d \tag{14}$$

After implementing the SISO controller for the disturbance rejection on the output of the system, the following step response is achieved:

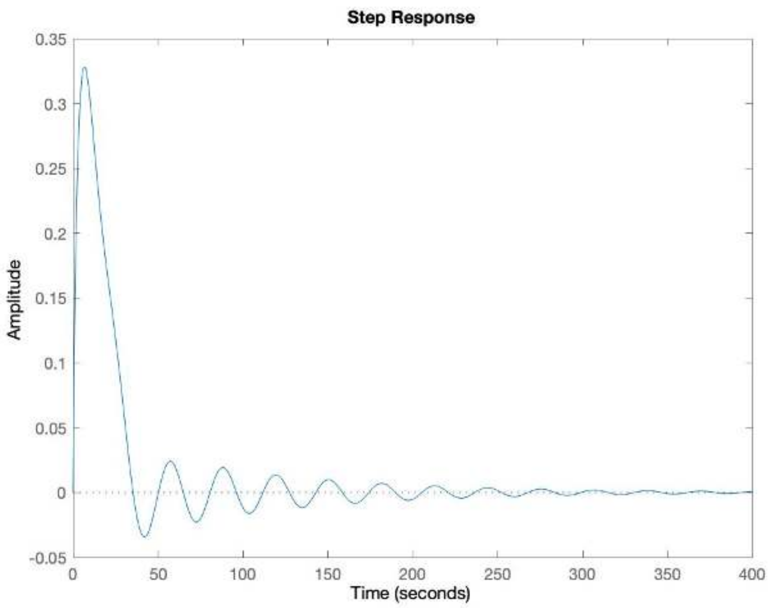

Figure 9: The step response of the system including disturbance.

It is clear from figure 9 that this is not an ideal step response. The overshoot and settling time are too big, so the controller needs to be changed. A Derivative action could, for example, yield better results.

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

3 Part 2: Multi variable Mixed-Sensitivity

3.1 Question 1: RGA

The Relative Gain Array (RGA), which is depicted in (15), is a method for determining the best input-ouput pairings of a MIMO system.

$$\mathrm{RGA(G)} = \Lambda(\mathrm{G}) \triangleq \mathrm{G} \times \left(\mathrm{G}^{-1}\right)^{\mathrm{T}} \tag{15}$$

Here, $\times$ is the elementwise Hadamard product of the system matrix G and its inverse transposed. After computing the RGA for both frequencies $\omega = 0$ and $\omega = 0.4 * 2\pi$, the following results are obtained

$$RGA_{\omega=0} = \begin{bmatrix} -0.6554 & 1.6554 \\ 1.6554 & -0.6554 \end{bmatrix} \quad RGA_{\omega=0.4*2\pi} = \begin{bmatrix} -0.0918 & 1.0918 \\ 1.0918 & -0.0918 \end{bmatrix} \tag{16}$$

To avoid instability caused by interactions at low frequencies, pairings with negative RGA elements should be avoided. We see that at a frequency of $\omega = 0$ the diagonal elements are negative. To avoid instability caused by interactions in the crossover region, pairings for which the RGA elements are close to identity are preferred. Since the diagonal elements of the RGA matrix are negative for low frequencies ($\omega = 0$), and the off diagonal elements are close to identity at the crossover frequency ($\omega = 0.4 * 2\pi$), the input-output pairs that avoid instability are u_2, y_1 and u_1, y_2.

3.2 Question 2: MIMO poles and zeros

Using functions in MATLAB, the poles and zeros for the plant G are computed, which yields

$$p_1 = -0.4104 \quad p_2 = -0.0832 \pm 3.2936j \quad p_3 = -0.0106 \pm 0.2022j \tag{17}$$

$$z = -0.0078 \pm 1.2358j \tag{18}$$

As there are no right-half-plane poles or zeros, we can conclude that there are no limitations to this system. Because all poles have a real negative value, the system is asymptotically stable.

3.3 Question 3: Performance weight

In order to design an appropriate W_{p11} we use the following formula from slide 2-22:

$$w_{p,i} = \frac{s/M_i + \omega_{Bi}}{s + \omega_{Bi} A_i} \tag{19}$$

where ω_{Bi} is the cut-off frequency, A_i the attenuation of low-frequency disturbances and M_i the desired bound on the H_∞ norm. These are 0.4 Hz ($0.4 * 2\pi$ in rad/s), 10^{-4} and 1.8, respectively. Hence, we get:

$$W_{p11} = \frac{\frac{s}{1.8} + 0.4 \cdot 2 \cdot \pi}{s + 0.4 \cdot 2 \cdot \pi \cdot 10^{-4}} \tag{20}$$

We can define the performance weights in the following way:

W_p	$\begin{bmatrix} W_{p11} & 0 \\ 0 & 0.2 \end{bmatrix}$
W_u	$\begin{bmatrix} 0.01 & 0 \\ 0 & \frac{5\cdot10^{-3}s^2+7\cdot10^{-4}s+5\cdot10^{-5}}{s^2+14\cdot10^{-4}s+10^{-6}} \end{bmatrix}$

Figure 10: Performance weights

Gedownload door Koen Ceton (koen604@gmail.com)

3.4 Question 4: Block diagram

The block diagram with the model of the floating wind turbine and controller is drawn below, where w denotes $\begin{bmatrix} \beta & \tau_\epsilon \end{bmatrix}^T$

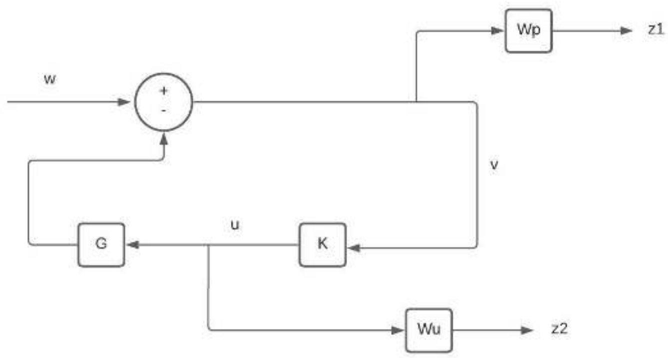

Figure 11: Block diagram of the FWT and controller including signals and performance weights W_u and W_p

3.5 Question 5: Generalized plant

Generalized plant:

$$z_1 = W_p w - W_p G u \tag{21}$$
$$z_2 = W_u u \tag{22}$$
$$v = w - Gu \tag{23}$$

This can be written as:

$$\begin{bmatrix} z_1 \\ z_2 \\ v \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} W_P & -W_P G \\ 0 & W_u \\ \hline I & -G \end{array}\right]}_{P} \begin{bmatrix} w \\ u \end{bmatrix} \tag{24}$$

where

$$P = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right] \tag{25}$$

3.6 Question 6: Performance weight interpretation

The control design target here is that the torque should not have a steady-state contribution, but only be used to correct for disturbances. This is where we can make use of our give performance weights. Adding a disturbance to our system would results in the following output equation:

$$y = \underbrace{(I + GK)^{-1} K r}_{\text{T}} + \underbrace{(I + GK)^{-1}}_{\text{S}} G_D d \Leftrightarrow y = Tr + SG_D d \tag{26}$$

where S denotes the sensitivity function and T the complementary sensitivity function. From this we notice that the sensitivity function relates the disturbance to the output of the system. We have two limitations that are relevant here, the first one being that the H_∞ norm , $|W_p S|$, must be smaller than 1 and the second one that for disturbance rejection we must satisfy $|SG_d(j\omega)| < 1$ for all frequencies. From this it follows that a suitable W_p is similar to $|G_d(j\omega)|$ at frequencies where $|G_d| > 1$ and that our maximum peak magnitude of S must not exceed $\frac{1}{|W_P(s)|}$.

From equation (19) we see that $\frac{1}{|W_P(s)|}$ is equal to A_i at low frequencies, equal to M at high frequencies

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

and that the asymptote crosses 1 at the frequency ω_B^*, which is approximately the bandwidth requirement. The top-left entry of W_p denotes that we want a varying output response for rotational velocity under influence of disturbances at certain frequencies, and that this output response should go to 0 after some time. The bottom-right entry denotes that under disturbance the tower placement should undergo a steady-state displacement.

From equation (26) it can also be seen that the complementary sensitivity function relates the disturbance to the input, which can solve our design target stated above. From the block diagram we see that we can influence this complementary sensitivity function ($K * s$) with our second performance weight, W_u. First, it is important to note some fundamental limitation to sensitivity function S and complementary sensitivity function KS. These are states as $|T| - |S| < 1$ and either $|T| > 1$ or $|S| > 1$. KS also has the limitation that its maximum should not exceed $\frac{1}{|W_u(s)|}$. Looking at our W_u, we notice a constant in the top-left diagonal, which is because we want to have a steady-state contribution of the blade pitch angle β under influence of disturbances. The bottom-right entry of the weight influences the torque, which gives certain responses at certain frequencies of the disturbance, and goes to 0 as time passes as required.

3.7 Question 7: MIMO controller

Using the *hinfsyn* command in MATLAB, the mixed-sensitivity generalized H_∞-controller K is computed by using the predefined generalized plant. In order to check the internal stability of our closed-loop system we use the MIMO Nyquist criterion, which states that the closed-loop system is stable if and only if the contour of the open-loop KP (which traverses the right-half-plane in clockwise direction) encircles the point -1 as many times as there are open-loop poles in the right-half-plane in counter-clockwise direction. Using the MATLAB function 'pole', we notice two poles in the right-half-plane, at $0.0083 + 3.4260i$ and $0.0083 - 3.4260i$ (a complex pair). This can also be seen with a pole-zero plot.
Looking at the Nyquist plot, we see that indeed the contour traverses the right-half-plane in clockwise direction, and that there are two counter-clockwise encirclements of the point -1. Hence, we can conclude our system to be stable. Looking at the controller and the generalized plant in the command window of MATLAB, we notice our A matrix to be of size 8×8, hence they both have 8 states.

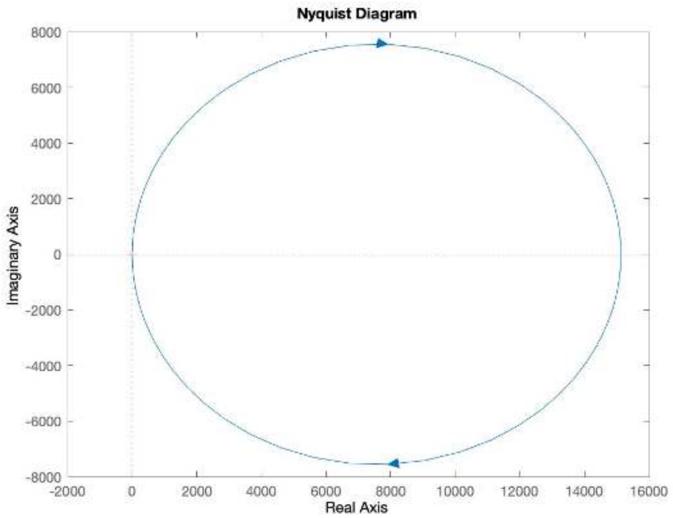

Figure 12: Full Nyquist plot of the open loop (KP)

Gedownload door Koen Ceton (koen604@gmail.com)

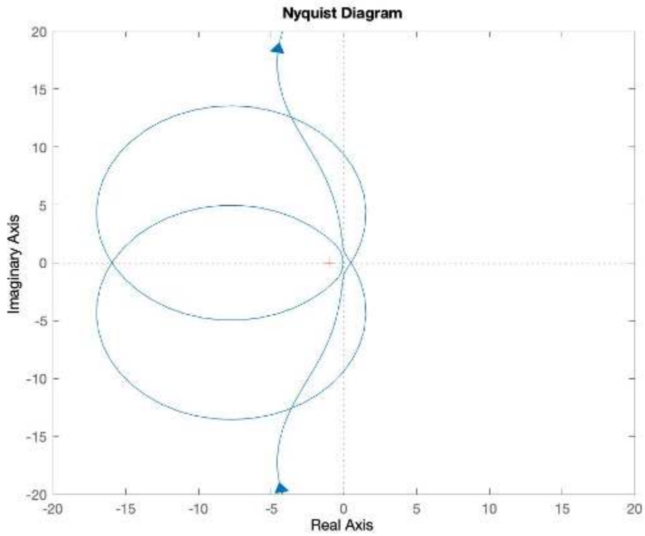

Figure 13: Zoom of the origin of open loop Nyquist plot, where the red cross denotes the -1 point on the real axis

3.8 Question 8: Time-domain simulation

As explained earlier, we have a steady-state contribution of the blade pitch angle β as well as a steady-state displacement of the tower placement from the given weights W_p and W_u. This is why we will always have a (small) error in our system and why the step response of the disturbance rejection does not go to 0 over time. Still, the output converges to amplitudes (0.0341 and 0.0203) that are reasonably close to 0. The same limitations to the step response for reference tracking.

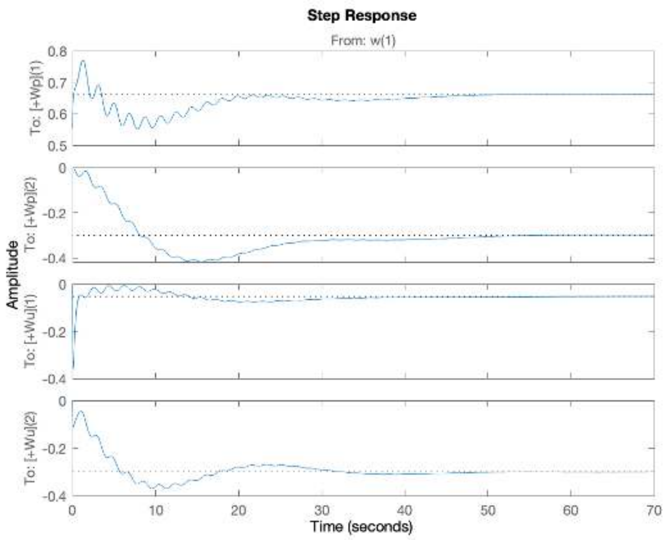

Figure 14: Step response of controlled closed loop system

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

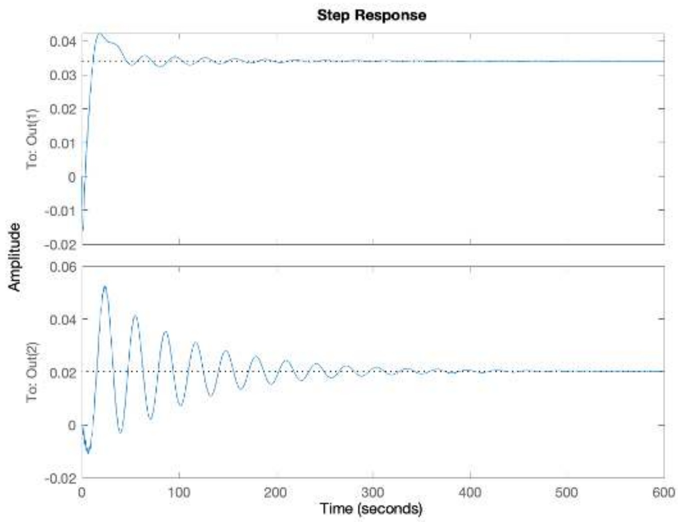

Figure 15: Disturbance rejection

Gedownload door Koen Ceton (koen604@gmail.com)

4 2.1: MIMO weighting design

4.1 Question 1: Generalized plant

For this system we have only one output, which is the rotational velocity ω_r and we have three inputs - blade pitch angle β, torque τ_ϵ and the disturbance $G_d d$.

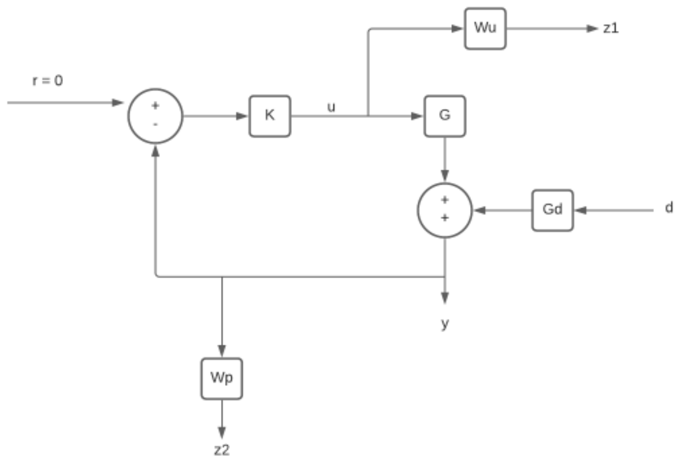

Figure 16: The full block diagram of the generalized plant

4.2 Question 2: Weight on the error

Looking at our generalized plant, we see that the output of the model is equal to the error. The weight on the error is defined as W_p, and can be determined using equation (27)

$$w_{p,i} = \frac{s/M_i + \omega_{Bi}}{s + \omega_{Bi} A_i} \tag{27}$$

The error weight denotes how 'important' it is, relatively speaking, to correct for any error. This is why we want to have a high error weight for high frequencies, to cause the torque to do its work, and low error weight at low frequencies, as we do not want the torque to start correcting there for errors. We will try some values in MATLAB later on to find an error weight that satitisfies our design goals.

4.3 Question 3: Controller weights

The controller weights, denoted as W_u, are designed in such a way that the blade pitch angle β is used for low frequency inputs and the torque control τ_e is used for high frequency inputs. This can be done by using a low-pass filter (LPF) for β and a high-pass filter (HPF) for τ_e. This is because according to the assignment we want to have blade pitch control for low frequencies and torque to counteract high frequency changes in wind speed.
The filters have the following form

$$W_{LPF} = \frac{a}{s+a} \quad W_{HPF} = \frac{s}{s+b} \tag{28}$$

where parameters a and b denote high and low frequencies, respectively, that can be tuned to improve performance. The controller weight matrix will then become

$$W_u = \begin{bmatrix} W_{LPF} & 0 \\ 0 & W_{HPF} \end{bmatrix} \tag{29}$$

Dit document is gratis te downloaden op StudeerSnel.nl

Gedownload door Koen Ceton (koen604@gmail.com)

4.4 Question 4: Controller synthesis

Again the controller is synthesized using the command $hinfsyn$, where the following generalized system is implemented

$$\begin{aligned} z_1 &= W_p G_d d + W_p G u \\ z_2 &= W_u u \\ v &= -G_d d - G u \end{aligned} \tag{30}$$

Which can be written as

$$\begin{bmatrix} z_1 \\ z_2 \\ v \end{bmatrix} = \underbrace{\left[\begin{array}{c|c} W_p G_d & W_p G \\ 0 & W_u \\ \hline -G_d & -G \end{array}\right]}_{P} \begin{bmatrix} d \\ u \end{bmatrix} \tag{31}$$

where

$$P = \left[\begin{array}{c|c} P_{11} & P_{12} \\ \hline P_{21} & P_{22} \end{array}\right] \tag{32}$$

4.5 Question 5: Simulation

In Figure 17, the wind speed data is plotted with the simulation of the control inputs. It is observed that the controller works as intended, as the first input (β) varies over time, but the value of the second control input (τ_e) stays around zero. This is desired as torque control affects the power production of the wind turbine. In the Figures 18, 19 and 20, the frequency response of the weights are plotted against the error, β and τ, respectively. It is observed that the error drops to zero as the frequency increases. From Figure 19 it can be concluded that for high frequencies, the controller weight pulls down the control input for the blade pitch angle β as desired. Finally, in Figure 20 it is observed that for low frequencies the control input for the torque τ_e is zero, where for higher frequencies its value increases, again exactly as desired. This confirms that the controller works as intended.

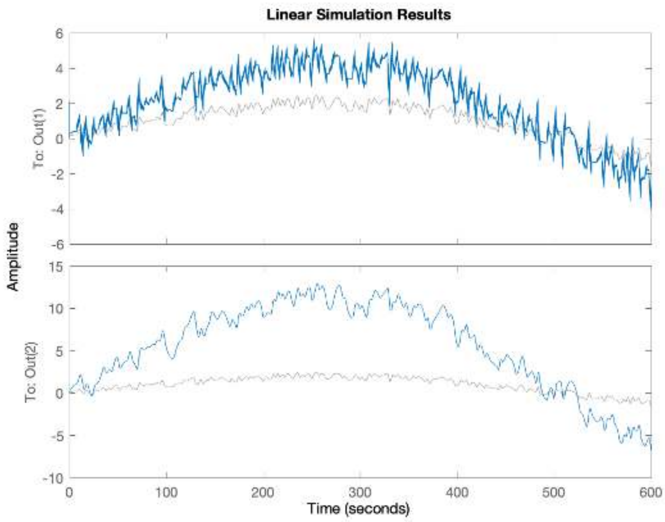

Figure 17: Simulation of wind speed input versus both control inputs

Gedownload door Koen Ceton (koen604@gmail.com)

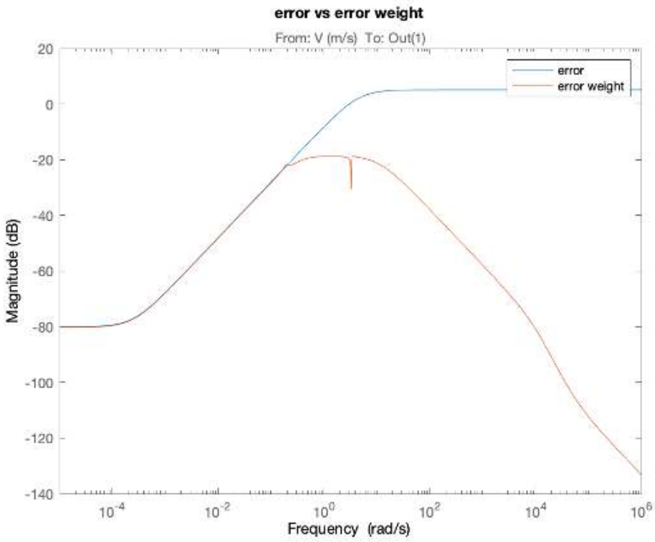

Figure 18: Frequency response of error versus error weight

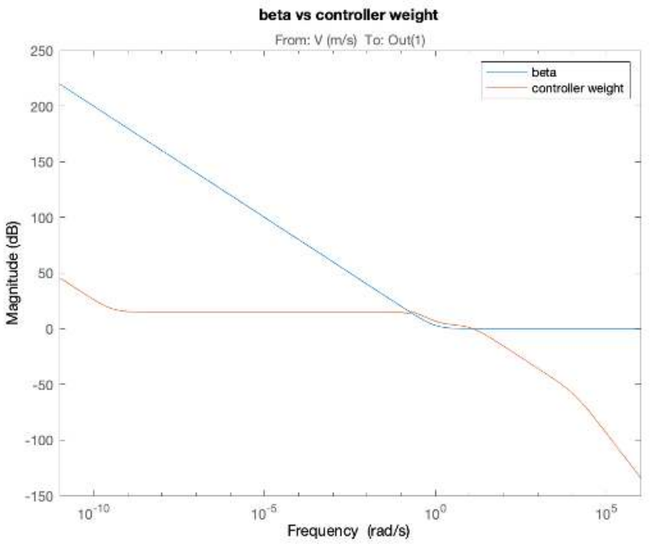

Figure 19: Frequency response of beta versus controller weight

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)

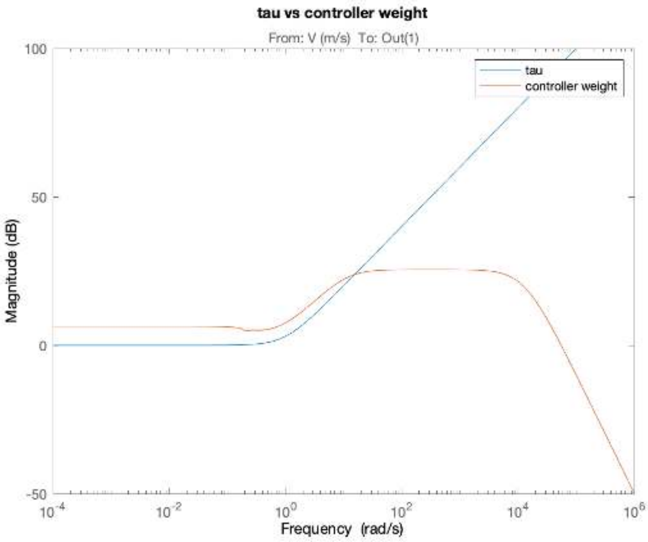

Figure 20: Frequency response of tau versus controller weight

Gedownload door Koen Ceton (koen604@gmail.com)

5 Part 3: MIMO Weighting Design

Dit document is gratis te downloaden op StudeerSnel.nl
Gedownload door Koen Ceton (koen604@gmail.com)