Applied Estimation Lab 1

John Folkesson

Prepartion Questions (Chap 1-3,7 in Thrun)

VETENSKAP OCH KONST

KTH

Centre for Autonomous Systems

Question 1

- What is the difference between a 'control' $\mathbf{u}_t$, a 'measurement' $\mathbf{z}_t$ and the state $\mathbf{x}_t$? Give examples of each?

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- What is the difference between a 'control' $\mathbf{u}_t$, a 'measurement' $\mathbf{z}_t$ and the state $\mathbf{x}_t$? Give examples of each?
- The control is part of the dynamic model beween $\mathbf{x}_{t-1}$ and $\mathbf{x}_t$.
- $\mathbf{x}_t$ is modeled as depending on both $\mathbf{u}_t$ and $\mathbf{x}_{t-1}$ by $\mathbf{g}(\mathbf{u}_t, \mathbf{x}_{t-1})$.
- Measurement is modeled as depending on the state at time t, $\mathbf{x}_t$.
- $\mathbf{z}_t$ gives undirect information on the state at time t, $\mathbf{x}_t$ thru the function $\mathbf{h}(\mathbf{x}_t)$.
- difference is that $\mathbf{u}_t$ is used in predict while $\mathbf{z}_t$ is used in update.

Question 2

- Can the uncertainty in the belief increase during an update? Why (or not)?

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- No
- $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
- $\Sigma_t = \bar{\Sigma}_t - K_t C_t \bar{\Sigma}_t$
- change is $-\bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1} C_t \bar{\Sigma}_t = -MM^T$
- and $\mathbf{x}^T MM^T \mathbf{x} = \mathbf{y}^T \mathbf{y} \geq 0$ for any x.
- Alternatively: $\Sigma_t = (\bar{\Sigma}_t^{-1} + C_t^T Q_t^{-1} C_t)^{-1}$
- $C_t^T Q_t^{-1} C_t$ is positive semidefinate.

Question 3

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- $\mu_t = (I - W)\bar{\mu}_{\mathbf{t}} + W\mu_{\mathbf{z}}$
  $W = \Sigma_t C_t^T Q_t^{-1} C_t$
  and $C_t\mu_{\mathbf{z}} = (\mathbf{z}_t - \bar{\mathbf{z}}_{\mathbf{t}} + C_t\bar{\mu}_{\mathbf{t}})$ shown in Lecture 5.
- So an answer is the relative size of Q and Σ, measurement error and uncertainty after the update is done.
- Just the measurement covariance, Q_t is an acceptable answer even if it is less precise.

Question 4

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- What would be the result of using a too large a covaraince (Q matrix) for the measurement model?
- Estimates would be pessimistic (conservative) and convergence would be slower.

Question 5

- What would give the measurements an increased effect on the updated state estimate?

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- Decreasing the measurement model uncertainty, Q, would increase the Kalman gain.

Question 6

- What happens to the belief uncertainty during prediction? How can you show that?

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- What happens to the belief uncertainty during prediction? How can you show that?
- It usually increases.
- Normally the dynamic model adds noise to the state.
- $\bar{\Sigma}_t = R_t + A_t \Sigma_{t-1} A_t^T$
- So if $A_t \geq I$ then this is adding a positive definate matrix to something bigger or equal to Σ.
- In general uncertianty could go down during predict but most real systems it increases. This is because A_t is often about I and R is significantly large.

Question 7

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- How can we say that the Kalman filter is the optimal and minimum least square error estimator?
- We know that the Kalman filter gives us the true posterior distrubution for a linear Gaussain system. Thus the question is really asking is there any other mean μ_{better} for the state that has lower expected square error that the Gaussian mean μ.
- $\int_{-\infty}^{\infty}(\mathbf{x}-\mu_{better})^2 G(\mathbf{x},\mu,\Sigma)d\mathbf{x}$.
- $\int_{-\infty}^{\infty}(\mathbf{x}^2-2\mathbf{x}\mu_{better}+\mu_{better}^2)G(\mathbf{x},\mu,\Sigma)d\mathbf{x}$.
- $\Sigma+\mu^T\mu-\mu^T\mu_{better}-\mu_{better}^T\mu+\mu_{better}^T\mu_{better})$ minimize by differentiating.
- $0=-2\mu+2\mu_{better}$
- $\mu_{better}=\mu$

Question 8

- Is it (KF) a MLE estimator? MAP?

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- MLE yes if we start with an update from no prior then the mean is the MLE of the Gaussian posterior (the top of the bell curve)
- MAP yes if we start from some Gaussian prior on the state.

Question 9

- How does the extended Kalman filter relate to the Kalman filter EKF?

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- How does the extended Kalman filter relate to the Kalman filter EKF?
- The EKF applies the KF to the linearized nonlinear system.
- $G_t \iff A_t$
- $\mathbf{g}(\mathbf{u}_t, \mu_{t-1}) \iff A_t\mu_{t-1} + B_t\mathbf{u}_t$
- $H_t \iff C_t$
- $\mathbf{y}_t$ is defined non-linearly wrt. $\bar{\mu}_t$

Question 10

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- No. Consistency depends on the significance of the nonlinearity.

Question 11

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- If our filter seems to diverge often can we change any parameter to try and reduce this?
- Yes we might change our modeled uncertianties Q and R. Typically divergence occures on update and increasing the relative size of the measurement covaraince, Q, will help.
- If the divergence was due to a poor data association then we might try changing our matching threshold.

Question 12

- If a robot is completely unsure of its location and measures the range r to a know landmark with Gaussain noise what does its posterior belief of its location $p(x, y, \theta | r)$ look like?

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- If a robot is completely unsure of its location and measures the range r to a know landmark with Gaussain noise what does its posterior belief of its location $p(x, y, \theta | r)$ look like?
- It will have a uniform distrubution over the heading θ between $-\pi$ and π. The position will be a sort of donut/ring. So a Gaussian on ρ in radial coordinate with uniform distribution on the angle ϕ.
- $x = \rho \cos \phi$
- $y = \rho \sin \phi$
- $\exp -(\rho - r)^2/(2\sigma_r^2)$

Question 13

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- If the above measurement also included a bearing how would the posterior look?
- The same except that the heading and angle around the ring would be Gaussian with a completely correlated covariance.
- $\exp -[(\rho - r)^2/(2\sigma_r^2) + (b - \phi + \theta)^2/(2\sigma_b^2)]$

Question 14

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- If the robot moves with relatively good motion estimation (prediction error is small) but a large initial uncertainty in heading θ how will the posterior look after traveling a long distance without seeing any features?
- It will look like a cresent/arc/C shape.
- The heading θ will be correlated with position along the arc.

Question 15

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- If the above robot then sees a point feature and measures range and bearing to it how might the EKF update go wrong?
- The linearization (Jacoboian) will produce an update 'direction' that is a straight line which can not move the estimate along the curved cresent.
- The Gaussian will not be able to represent the cresent shape and thus the update might diverge.