EL2520
Control Theory and Practice

Welcome!

Elling W. Jacobsen
Div of Decision and Control Systems
School of Electrical Engineering and Computer Science
KTH

Todays Program

- Practical information, course content
- Introduction to MIMO control
- Signal norms, System gain
- The Small Gain Theorem

A. Practical information

Practical information

- Course information and schedule in Canvas
  https://kth.instructure.com/courses/7592

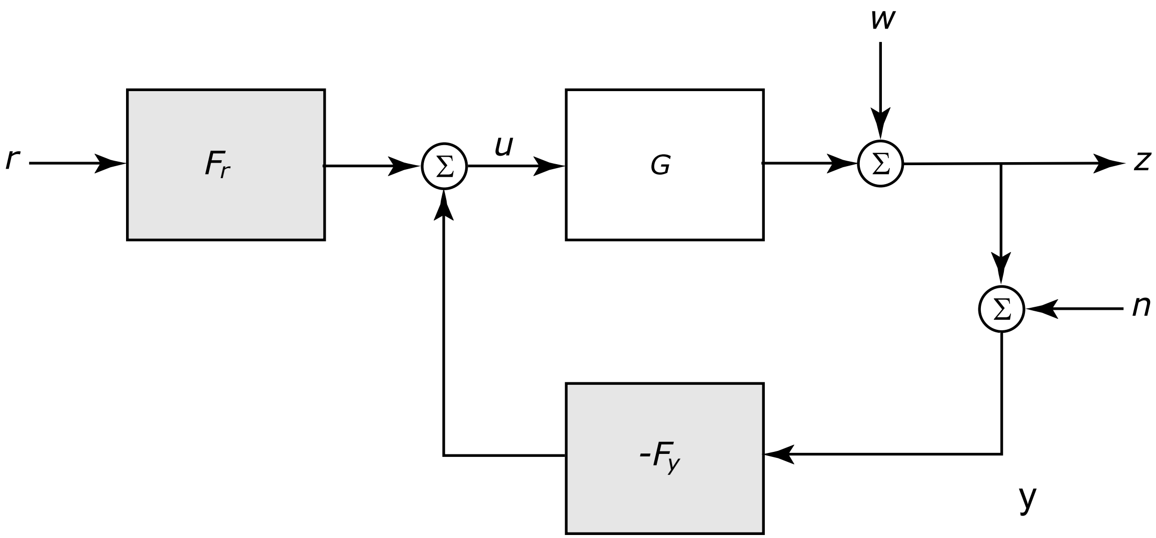
- Slides and lecture notes available on homepage before class
- Exercise notes available on homepage after exercise class
- Course book: English and Swedish versions; Kåren (SEK 590) or Internet
- Course material: course homepage
- Course administration: Service Center, Malvinas väg10 or https://www.kth.se/en/eecs/kontakt/studentexpedition-och-servicecenter
- Computer exercises: need kth.se account
- Hand in reports via Canvas
- Register for labs via home page (from April 17)
- Lab access: May 9-20 (weekdays only)

- Expectations and feedback: email to jacobsen@kth.se

Course elements

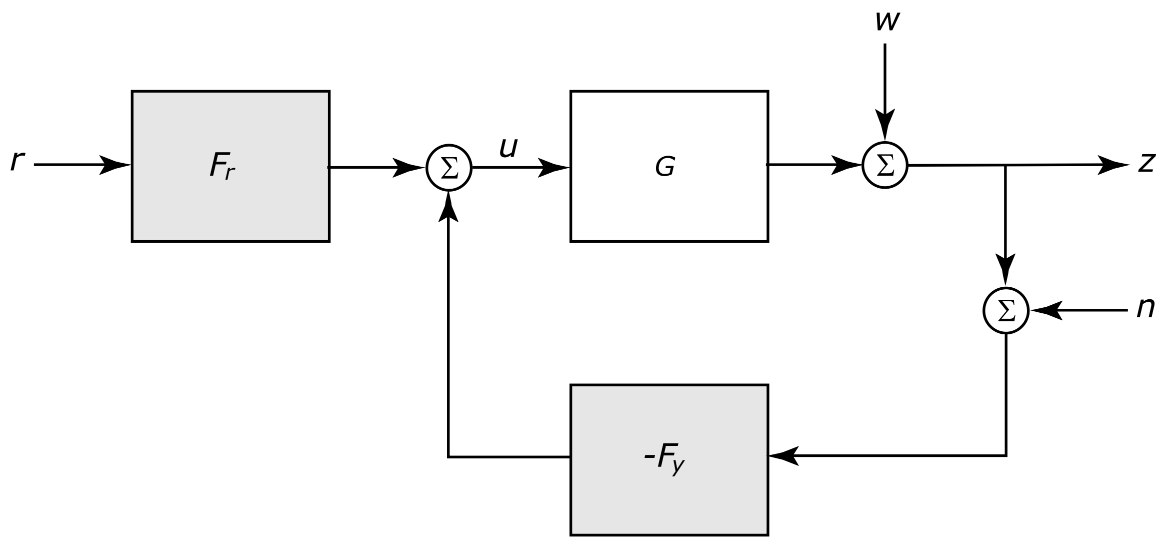
- 14 lectures, Elling W. Jacobsen
- 10 exercises, Lars Lindemann
- 4 computer exercises, Joakim Björk
  - –groups of 2 students
  - –two alternative times for each exercise
  - –report deadlines April 3, April 10, May 2, May 15
- 1 lab project, Joakim Björk
  - –groups of 4 students
  - –two sessions in laboratory
  - –book two times May 9-20 (not open for booking yet)
  - –report deadline: May 24
- Exam: 5 hours, written exam, Wed May 29, 08-13

Feedback Control



Given a system G, with measured signal y, determine a control input u so that the control variable z follows as closely as possible a reference signal r, despite disturbances w, and measurement noise n.

Why Feedback Control?



- Attenuate **unmeasured disturbances**
- Reduce impact of **uncertainty**
- Stabilize **unstable** systems

Powerful tool for tayloring system behavior!

Why EL2520?

- Fundamental limitations
- Sensitivity and robustness
- MIMO systems (Multi-Input-Multi-Output)
- Control problems cast as optimization problems
- Dealing with hard constraints
- Real-time optimal computer based control

- Applications

- Understand dynamic systems!

Course structure

1.	Fundamentals of (modern) SISO control	(4 lectures)
2.	Modern control of multivariable linear systems	(7 lectures)
3.	Control of systems with constraints	(2 lectures)

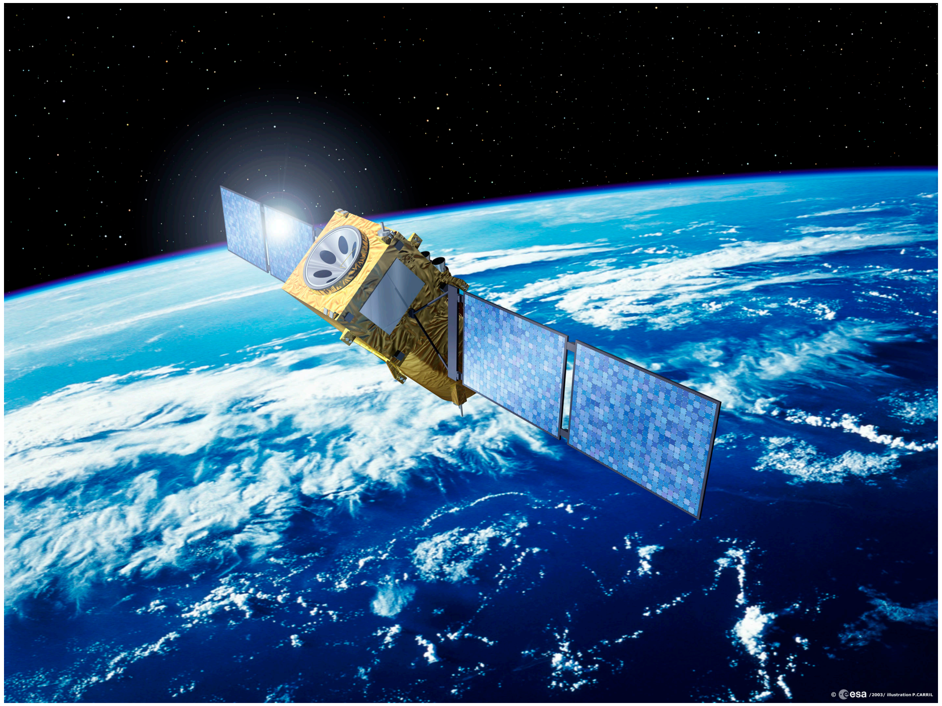
Applications

Use control theory to analyze and modify system properties!

Applications in most engineering domains:

- Aerospace
- Cars and heavy vehicles
- Autonomous systems and robots
- Process industry
- Consumer products
- Communication systems
- Economics
- Biology
- …

Aerospace

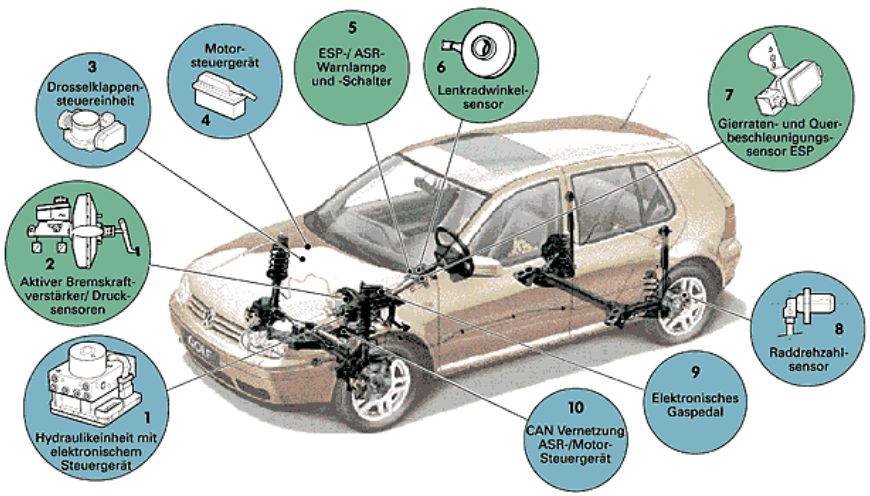


“Rocket Science”

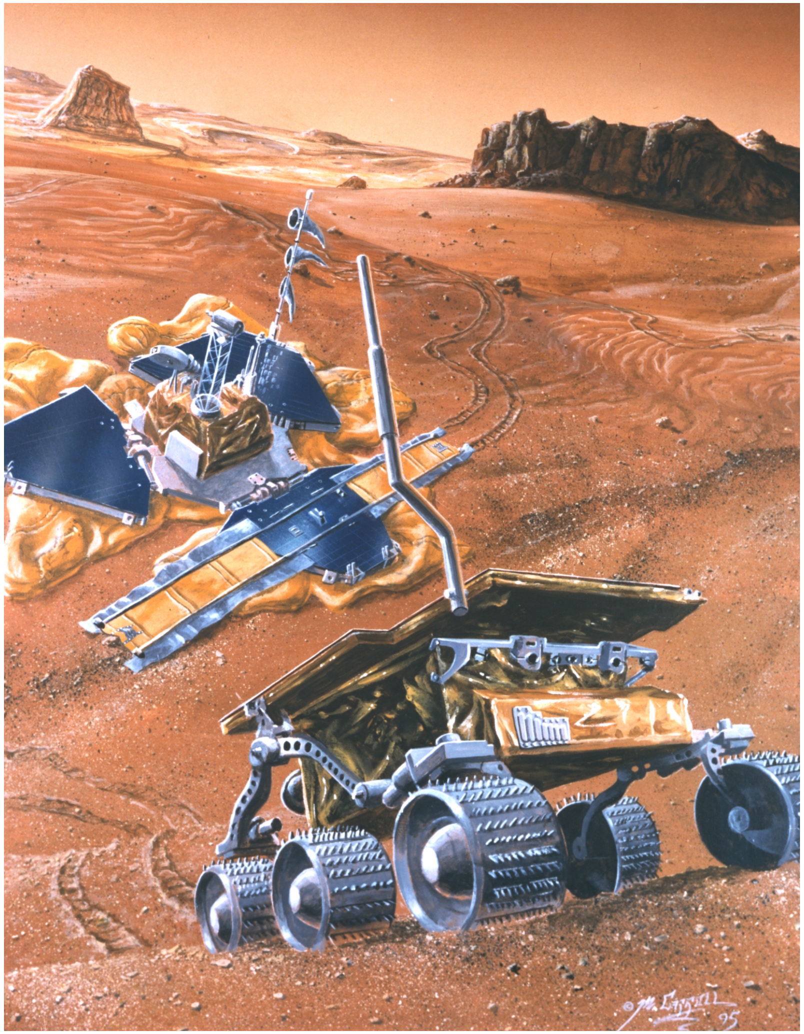
- SpaceX

Vehicle control

Elektronisches Stabilitätsprogramm (ESP)



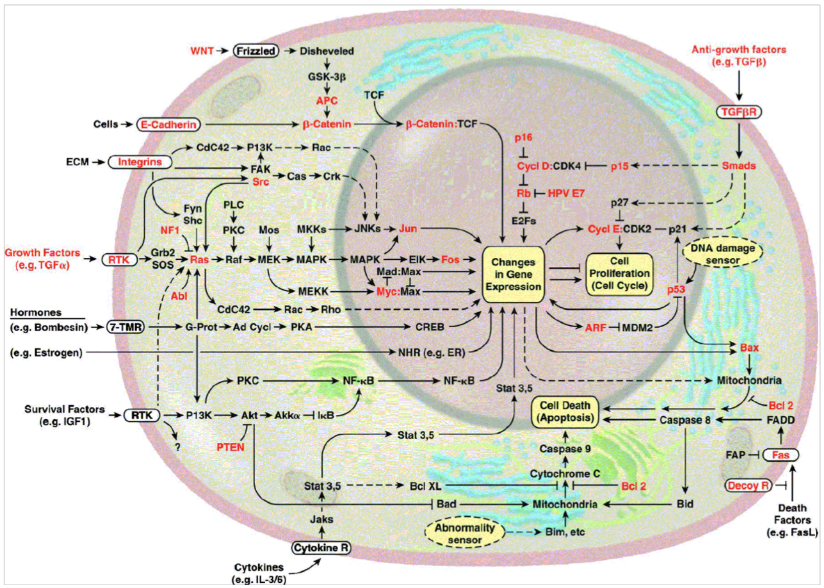
Autonomous systems and robotics

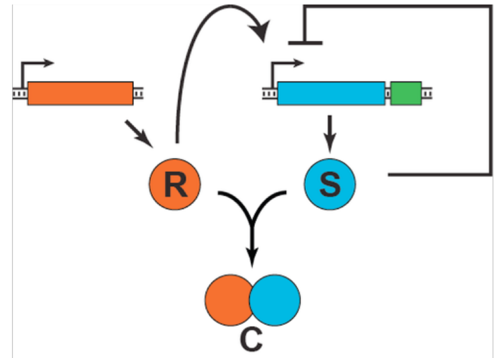


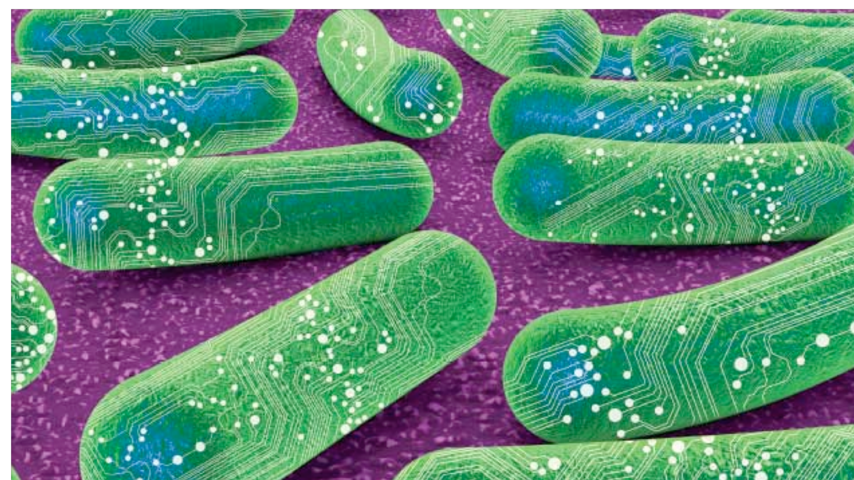
Process industry

Consumer products

Systems and Synthetic Biology







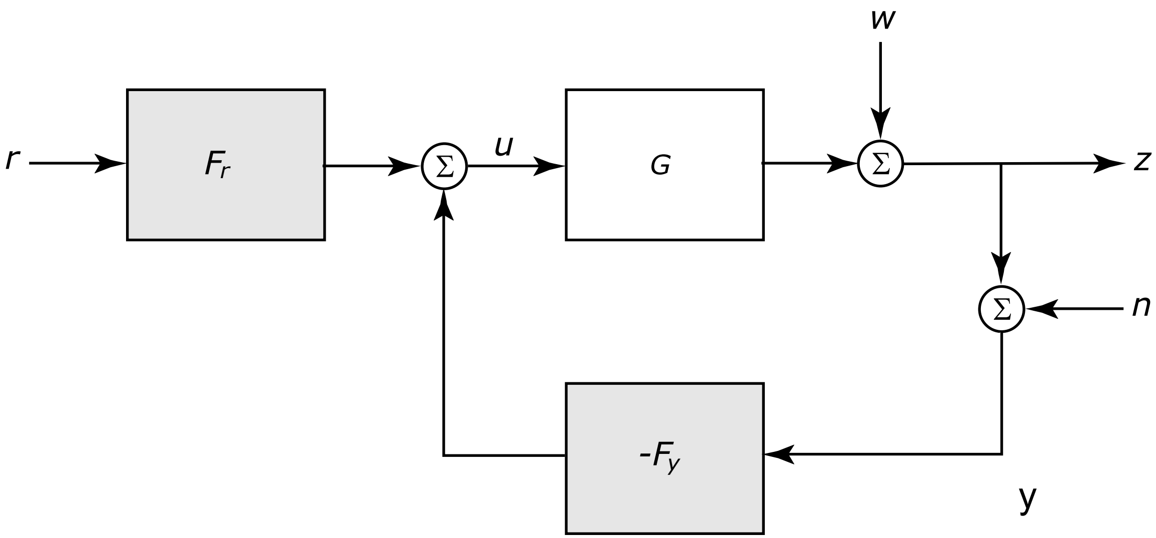
How to learn more?

- Internet
- Books (see course information, home page)
- Journals (IEEE Trans Automatic Control, Automatica, Control Systems Magazine …
- Courses (Hybrid Systems, MPC, Reinforcement Learning, Nonlinear Control, Modeling of Dynamical Systems, …)
- Software (Matlab, …)

- Interact with us!

B. Introduction to Multivariable Systems and Control

Multivariable feedback control



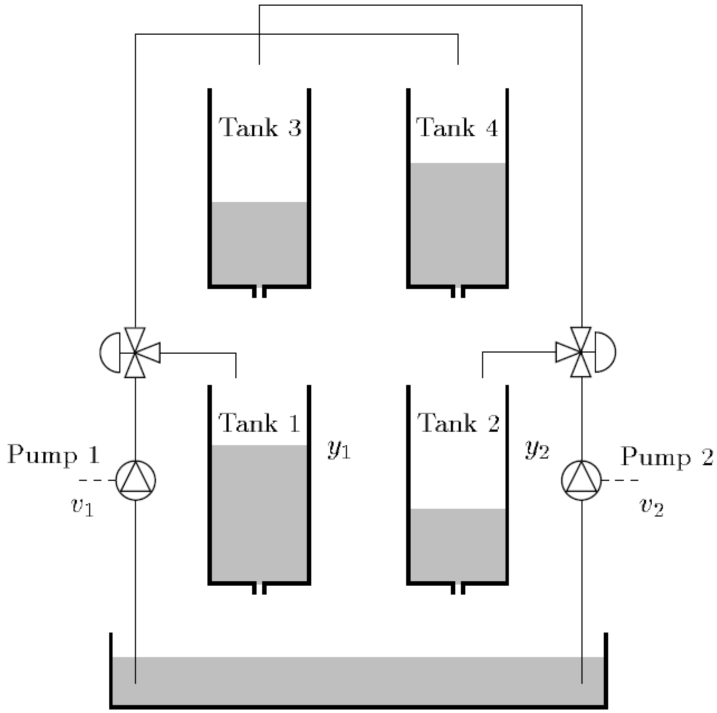
Multi-Input-Multi-Output (MIMO): all signals are vectors, all transfer-functions are matrices

Multivariable systems

Key aspects:

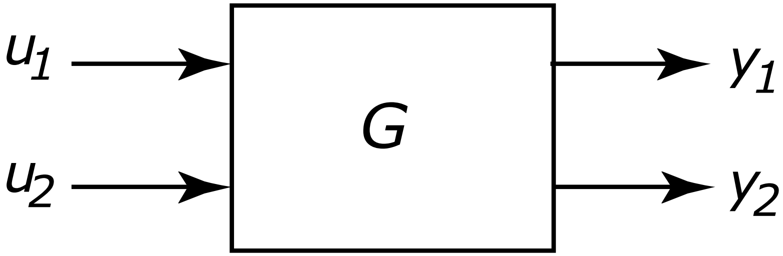
- several inputs and outputs,
- dynamics coupled (each input affects several outputs)

Laboratory project example: quadruple tank system



The need for multivariable control

Example. Consider a linear system with two inputs and two outputs

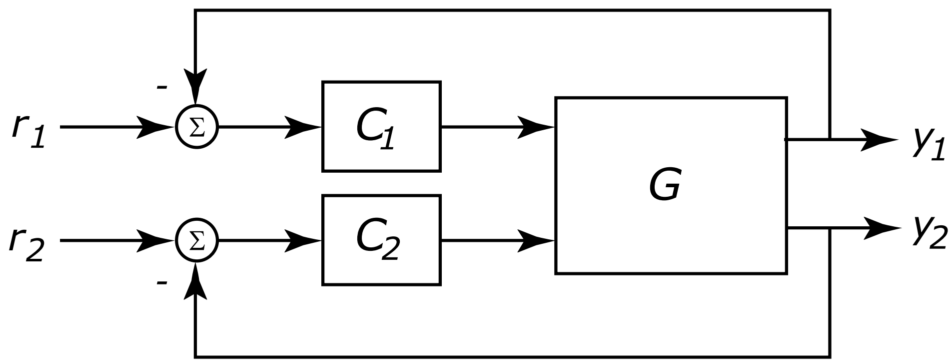


$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Note: both inputs influence both outputs!

Decentralized control

Simple approach: pair inputs and outputs, use SISO control



Use u_1 to control y_1 and u_2 to control y_2. Use PI-control in each loop

$$U_i(s) = \frac{K_i(s+1)}{s}(R_i(s) - Y_i(s)) \quad i = 1, 2$$

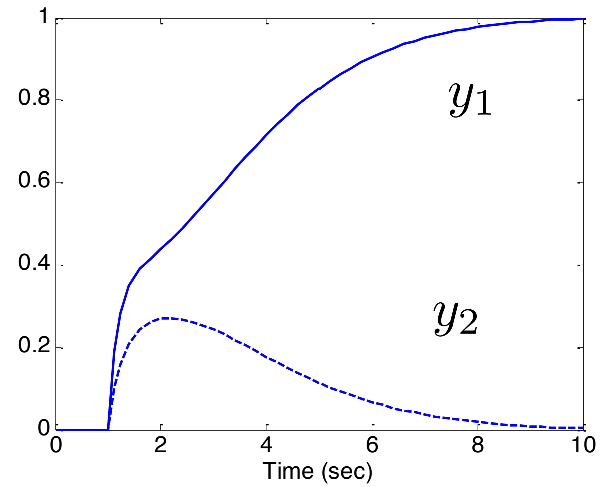
gives transfer functions for individual loops

$$G_{r_1 \to y_1} = \frac{2K_1}{s + 2K_1} \;; \quad G_{r_2 \to y_2} = \frac{K_2}{s + K_2}$$

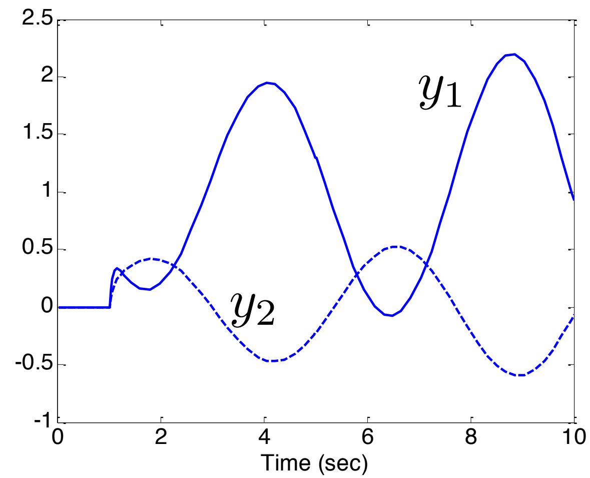
Stable for all positive values of K_1, K_2!

Decentralized control

Response to step-change in r_1, with K_1=1, K_2=2
Seems OK (but slow)

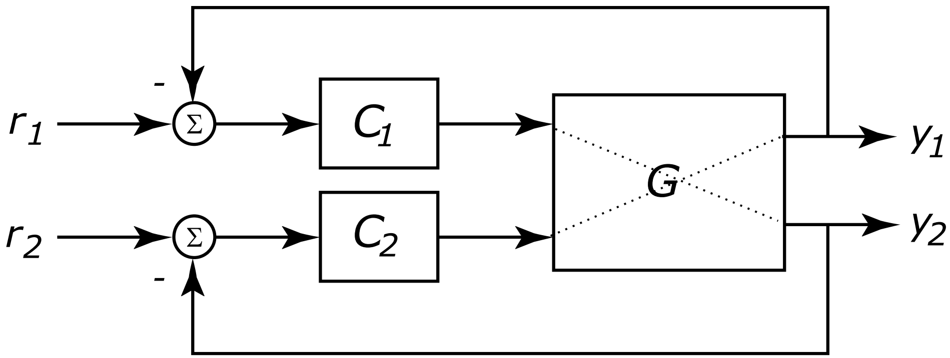


and with K_1=4, K_2=8



Multivariable system unstable, even if SISO analysis indicates stability!

What is happening?



$$Y_1(s) = \frac{2}{s+1}U_1(s) + \frac{3}{s+2}U_2(s)$$
$$Y_2(s) = \frac{1}{s+1}U_1(s) + \frac{1}{s+1}U_2(s)$$

Interactions in the system make the control loops coupled!

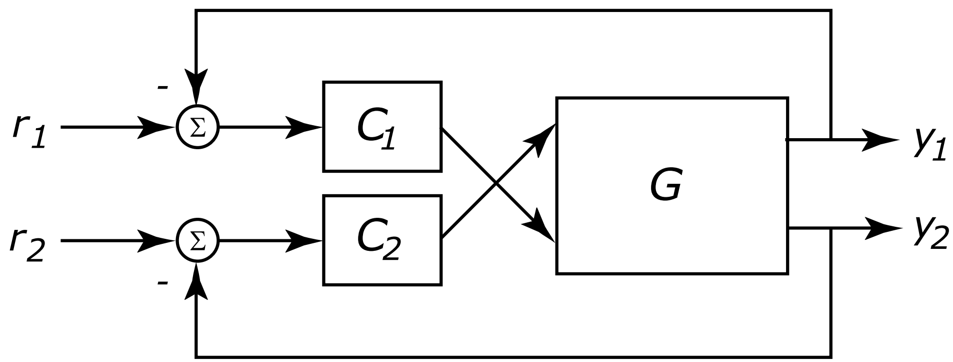
Multivariable analysis

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix} \left(\begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} \right) \Rightarrow Y = (I + GC)^{-1} GC\, R$$

Elements of closed-loop transfer matrix very different from SISO analysis!

Decentralized control

What if we pair signals “the other way around”? (with new controllers)

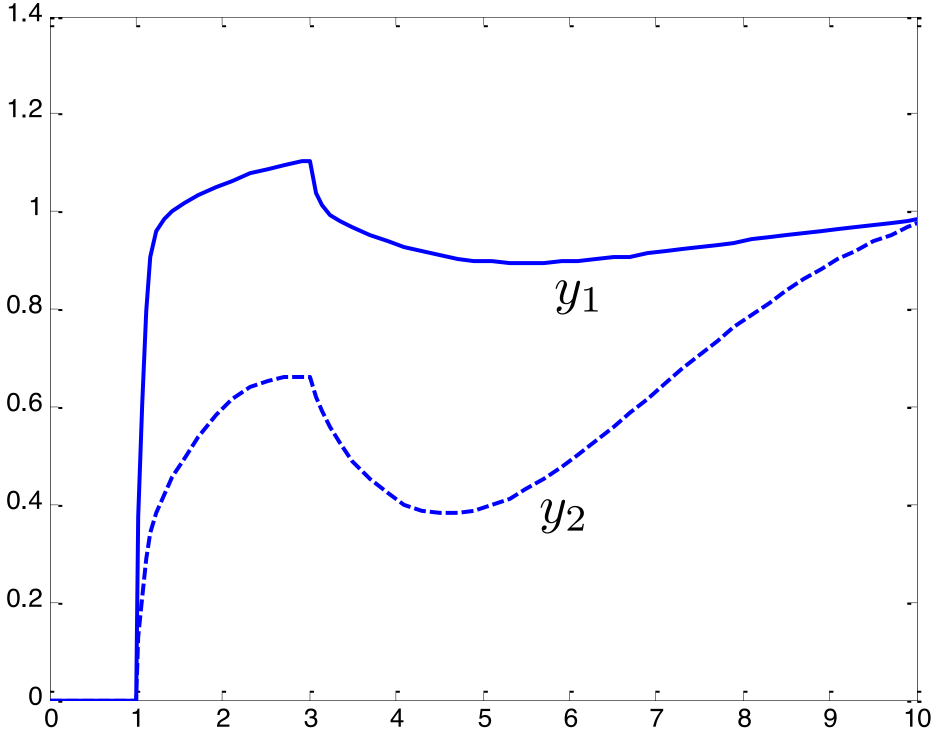


Turns out we can no longer stabilize system with positive gains

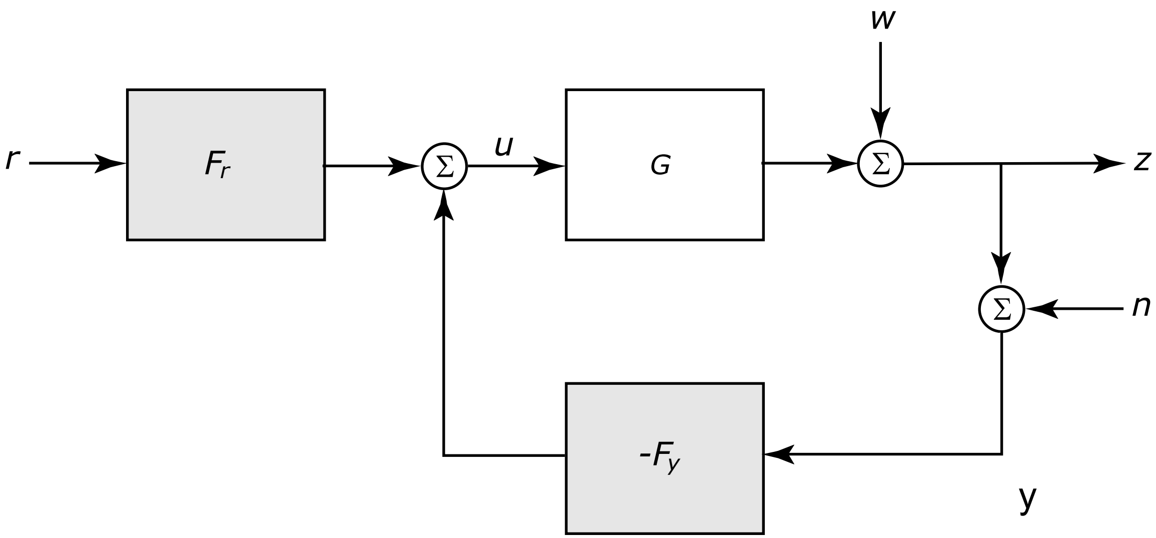
- need positive and negative controller gains
- system becomes unstable if one controller in open-loop

Control with reversed pairing

Response to step-changes: $r_1(t)=1(t>1)$, and $r_2(t)=1(t>3)$,
With controller gains $K_1=-0.75$, $K_2=5$



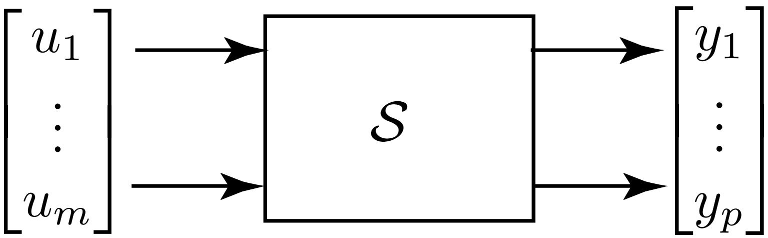
What to do?

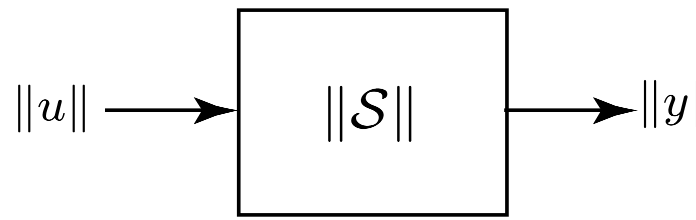


- Design MIMO controllers, i.e., for vector signals / transfer-matrices
- Formulate it as an optimization problem, e.g., minimize control error and input usage
- Need to quantify size of signals (and systems): norms

C. Signal norms and System gain

Systems as ”mapping of signals”





Key components:

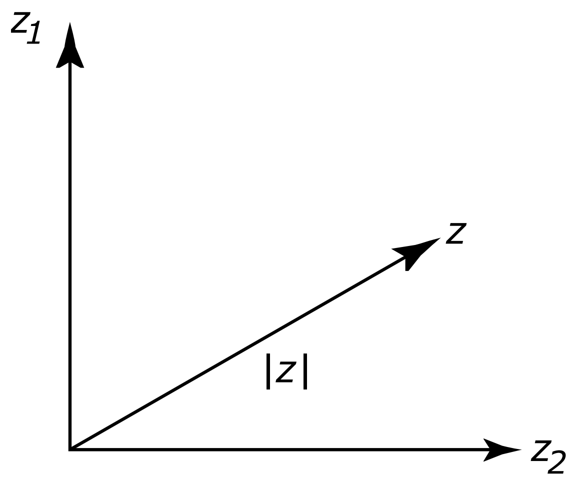
- Signal norm: quantifies ”size” of signals
- System gain: quantifies system’s (maximum) amplification
- Frequency responses

Admits natural extensions from scalar to multivariable systems!

Vector norms

Vector (spatial) norms measure the "size" of vectors.

- common choice: Euclidian norm (also known as 2-norm)

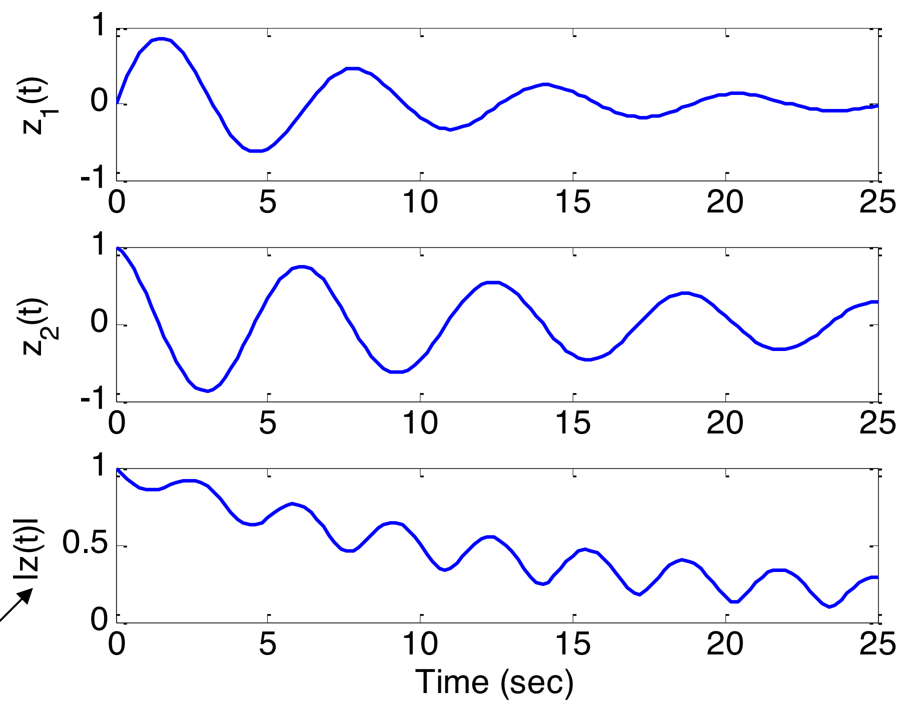


$$|z|^2 = \sum_i z_i^2 = z^T z$$

Signal norms

Signals are functions of time (or frequency)

- signal norms measure size across both space and time.



$\Rightarrow \|z\|$

summing up over channels

summing up over time

Signal norms

The *peak-norm*, or L_∞ -norm, of a signal is defined as

$$\|z\|_\infty = \sup_{t \geq 0} |z(t)|$$

A signal is *bounded* if its peak-norm is bounded ($\|z\|_\infty < \infty$)

The *energy-norm*, or L_2-norm, of a signal is defined as

$$\|z\|_2 = \sqrt{\int_{-\infty}^{\infty} |z(t)|^2 \, dt}$$

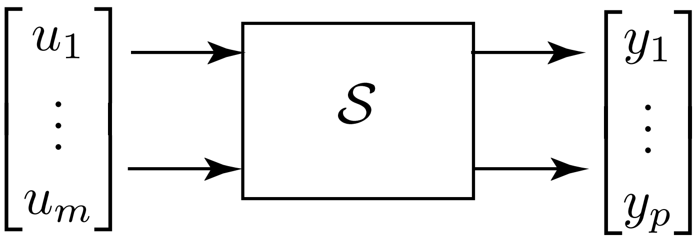
A signal is *finite-energy* if $\|z\|_2 < \infty$

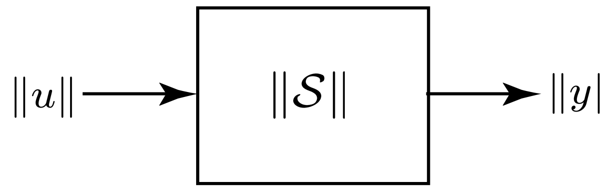
Note: - bounded signals may have infinite energy (and vice versa)

- we will only work with the 2-norm in this course

The energy-gain of a system

Measures maximum “energy amplification” of system





The amplification for a specific signal $u \neq 0$ is given by

$$\frac{\|y\|_2}{\|u\|_2} = \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

The system gain is the maximum amplification (over all finite-energy signals)

$$\|\mathcal{S}\| = \sup_{0<\|u\|_2<\infty} \frac{\|\mathcal{S}u\|_2}{\|u\|_2}$$

Energy gain of scalar linear systems

Stable scalar linear time-invariant system $\mathcal{S} : Y(s) = G(s)U(s)$

Assume that $|G(i\omega)| \leq K$ with equality for $\omega = \omega^\star$

Then, Parseval's theorem yields

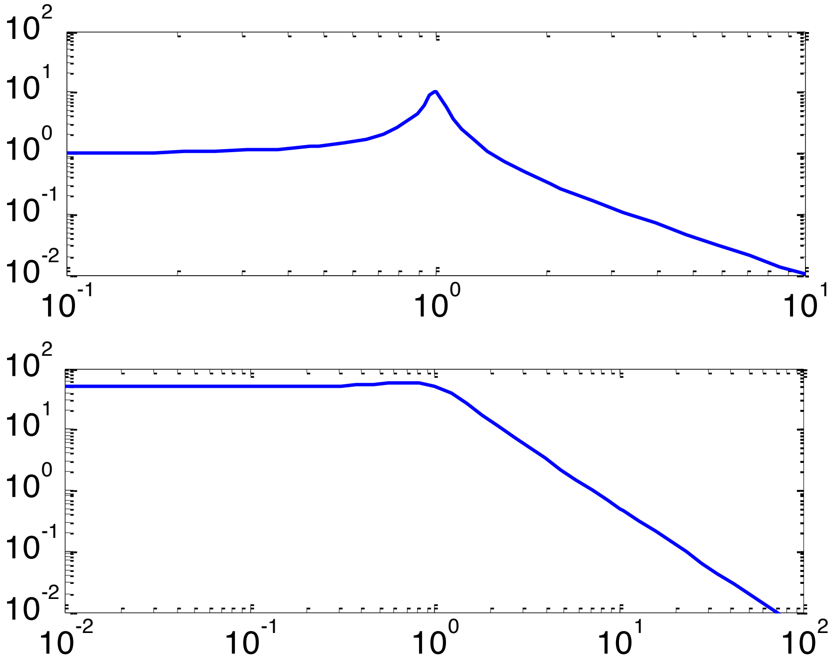
$$\|y\|_2^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |Y(i\omega)|^2 d\omega =$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 |U(i\omega)|^2 d\omega \leq K^2 \|u\|_2^2$$

Since equality holds for $u(t) = \sin(\omega^\star t)$, we have

$$\|\mathcal{S}\| = \sup_{\omega} |G(i\omega)| = \|G\|_\infty$$

Quiz: energy gains and Bode diagrams

Quiz: the Bode amplitude diagrams below represent two different linear time-invariant systems. Which one has the largest energy-gain?



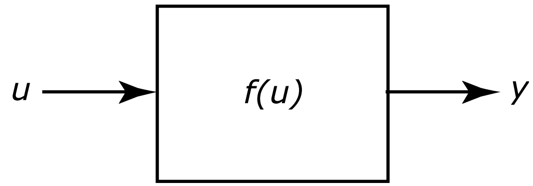
Example: gain of nonlinear system

Static nonlinear system $\mathcal{S} : y(t) = f(u(t))$

where

$$|f(x)| \leq K|x|$$

with equality for $x = x^\star$



Since

$$\|y\|_2^2 = \int_{-\infty}^{\infty} |f(u(t))|^2 \, dt \leq \int_{-\infty}^{\infty} K^2 |u(t)|^2 \, dt = K^2 \|u\|_2^2$$

the energy gain is

$$\|\mathcal{S}\| = \sup_u \frac{\|y\|_2}{\|u\|_2} = K$$

Example: gain of static linear systems

Consider the static linear system $y = Au$ with gain

$$\|A\| = \sup_{u \neq 0} \frac{|Au|}{|u|}$$

Since

$$\|A\|_2^2 = \sup_{u \neq 0} \frac{|Au|^2}{|u|^2} = \sup_{u \neq 0} \frac{u^T A^T A u}{u^T u} = \lambda_{\max}(A^T A)$$

Thus, the gain is the square root of the maximum eigenvalue of A^TA.

(the square roots of eig(A^TA) are called the *singular values* of A; more in Lec 5)

Quiz: a flavour of multivariable systems

Quiz: What is the gain of the following (static) systems?

a) $$y = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} u$$

b) $$y = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} u$$

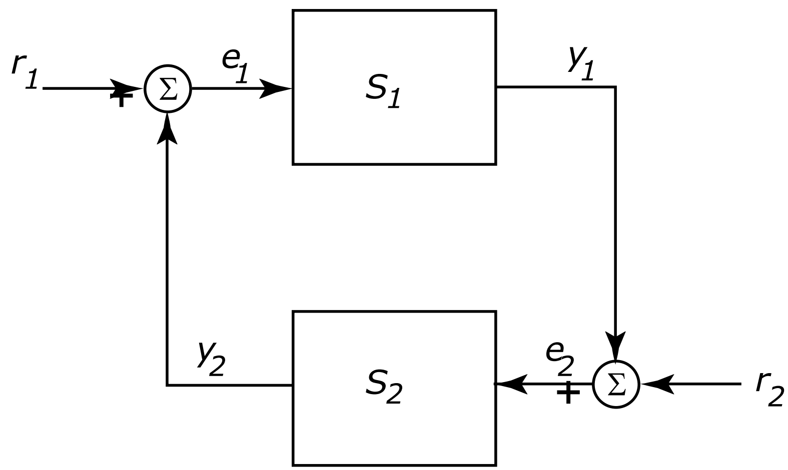
What are the corresponding "worst-case" input vectors?
(vectors u with |u|=1 that give the maximum value of |y|)

Input-output stability

Definition. A system $\mathcal{S}$ is *input-output stable* if $\|\mathcal{S}\| < \infty$

The Small Gain Theorem

Theorem. Consider the interconnection



If $\mathcal{S}_1$ and $\mathcal{S}_2$ are input-output stable and the loop gain

$$\|\mathcal{S}_1\| \cdot \|\mathcal{S}_2\| < 1$$

Then, the closed-loop system with r_1, r_2 as inputs and e_1, e_2, y_1, y_2 as outputs is input-output stable.

Proof sketch

$$e_1 = r_1 + \mathcal{S}_2(r_2 + y_1)$$

$$y_1 = \mathcal{S}_1 e_1$$

$$\|e_1\| \leq \|r_1\| + \|\mathcal{S}_2\|(\|r_2\| + \|\mathcal{S}_1\| \cdot \|e_1\|)$$

$$\|e_1\| \leq \frac{\|r_1\| + \|\mathcal{S}_2\| \cdot \|r_2\|}{1 - \|\mathcal{S}_2\| \cdot \|\mathcal{S}_1\|}$$

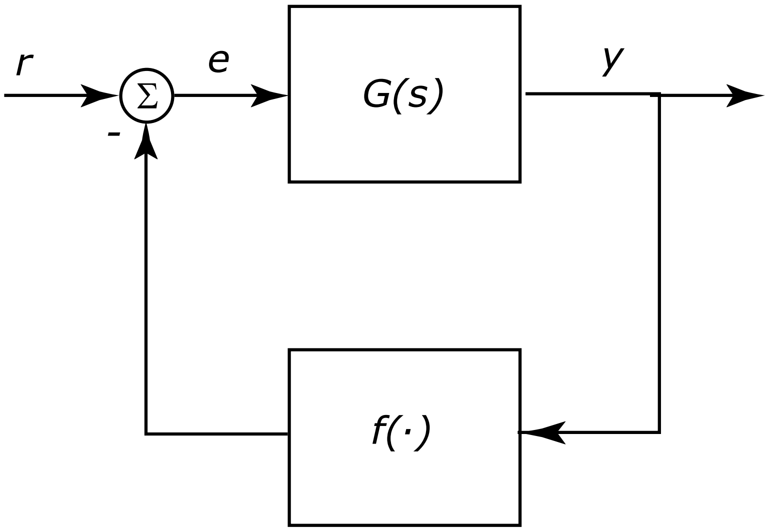
Hence, the gain from r_1, r_2 to e_1 is finite.

A similar argument proves that the gain from r_1, r_2 to e_2 is finite

Note: for linear system, it is sufficient that $\|\mathcal{S}_1 \mathcal{S}_2\| \leq 1$

Quiz: a nonlinear interconnection

Is the feedback interconnection



with $G(s) = \dfrac{0.4}{s+1}$ and $|f(y)| \leq 2|y|$ input-output stable?

Summary

- Systems as mappings of signals
- Norm
  - vector norms: measure size of vector “across channels”
  - signal norms: measure size of signal across time and space
- Gain
  - the maximum amplification of bounded energy signals
  - for stable linear systems, gain is infinity norm of frequency response
- Input-output stability and the small gain theorem

Next lecture: The closed-loop system (Chapter 6, Lecture notes 2)