Exercise session 1:

2: 1.1 4.3 7.1 8.1

7.1) $G(s) = S-1 / _{S+1}$ and the controller $F(s) = S+2 / _{S+1}$

$G_c(r \to z)$, $S(w \to z)$ and $T(n \to z)$

G_c: $z = \underbrace{G / _{1+FG}}_{G_c} \cdot r = \frac{1}{2} \frac{s-1}{s+3/2} \cdot r$ stable.

$$S = \frac{1}{1+FG} = \frac{1}{2} \frac{s+1}{s+\frac{3}{2}}$$ stable.

$$T = \frac{GF}{1+GF} = \frac{1}{2} \frac{s+2}{s+\frac{3}{2}}$$ stable.

SF: $u = SF \cdot n$

$$SF = S \cdot F = \frac{s+1}{2s+3} \cdot \frac{s+2}{s-1}$$ unstable.

Alternatively to checking "the gang of four",

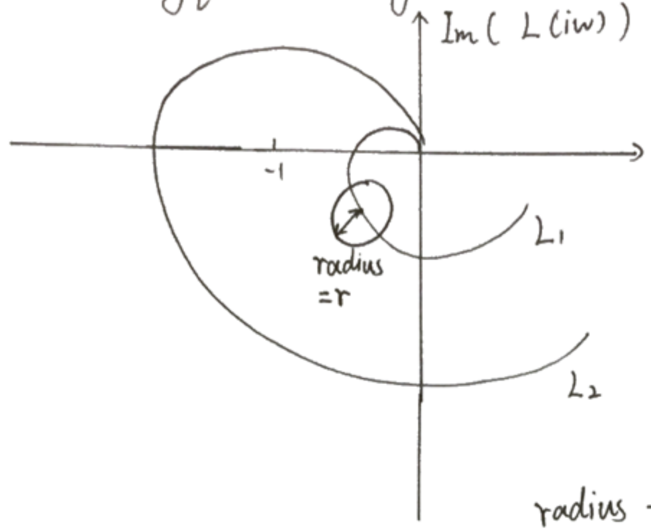
We know that RHP (right-half plane) pole/zero cancellations between F and G lead to internal ~~un~~stability.

Rule: ① A RHP pole of G at $s=p$ leads to a RHP zero of S at $s=p$
if not: internal unstable.

② A RHP zero of G at $s=z$ leads to a RHP zero of T at $s=z$.
if not: internal unstable.

8.1) Assume a given F and G such that the open loop and closed loop systems are stable.

Recall the Nyquist stability criterion



L_1 : the corresponding closed loop system is stable.

L_2 : not stable

Nyquist criterion: The closed loop system is stable if (L(iw)) does not encircle the critical point -1.

Robustness w.r.t modelling errors.

$$L_p(iw) = (1 + \Delta_L(iw))\, L(iw) = L(iw) + \Delta_L(iw)\, L(iw)$$

L_p : real physical system

L : modelled system $F(iw)\, G(iw)$

Δ_L : unknown error.

We want: $|\Delta_L| <$ radius, then L_p will be contained in a circle of radius r and center of L.

If this circle never ~~encircle~~ contains -1, we got robust stability.

$|1 + L(iw)|$ distance from -1 to $L(iw)$

Therefore, we need $|L(i\omega)+1| > r > |\Delta_L(i\omega)\, L(i\omega)| \;\; \forall \omega$

$$\iff \frac{|L(i\omega)+1|}{|L(i\omega)|} > |\Delta_L(i\omega)| \quad \forall \omega$$

$$\iff \frac{1}{|T(i\omega)|} > |\Delta_L(i\omega)| \quad \forall \omega$$

$\iff$ $\Delta_L(i\omega)$ has lie under $\frac{1}{|T(i\omega)|}$ to obtain stability.