Computer Exercise 1
EL2520 Control Theory and Practice

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1 Exercises

1.1 Basics

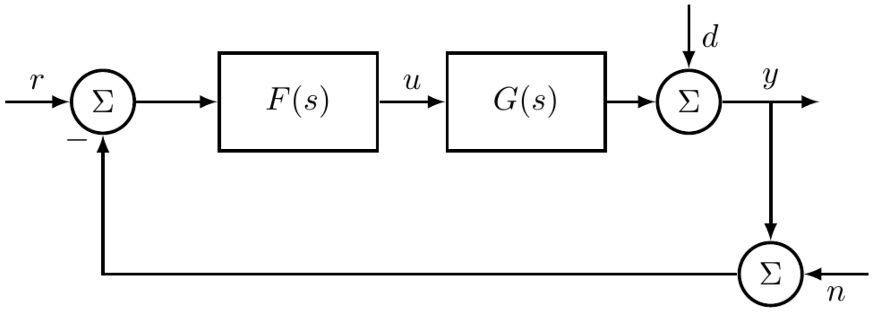

Figure 1: F-controller, G-system, r-reference signal, u-control signal, d-disturbance signal, y-output signal, n-measurement noise.

Consider a system which can be modeled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}$$

1. **Question:** Use the procedure introduced in the basic course to construct a lead-lag controller which eliminates the static control error for a step response in the reference signal.

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{\text{Lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{\text{Lag}}$$

The phase margin should be 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.

Answer: For the closed-loop system, in order to have a phase margin $\phi_m = 30°$, a cross-over frequency $\omega_c = 0.4$ rad/s, and zero steady-state error for a step response in the reference signal, we consider a lead-lag controller F shown above, where K, τ_D, τ_I, β, and γ are parameters should be configured so that the closed-loop system satisfies the requirements.

For a step reference, the error is given by

$$E(s) = R(s) - Y(s) = \frac{1}{1+F(s)G(s)}R(s) = \frac{1}{1+F(s)G(s)}\frac{1}{s}$$

The steady-state error then can be obtained using Final Value Theorem:

$$e(\infty) = \lim_{t\to 0} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1+F(0)G(0)} = \frac{\gamma}{\gamma+3K}$$

To get zero steady-state error, $e(\infty) = 0$, either $\gamma = 0$ or $K \to \infty$. So, $\gamma = 0$ is chosen here. Then we can design the lag controller. To minimize the phase lag caused by the lag compensator, we would obtain:

$$\tau_I = \frac{10}{\omega_c} = \frac{10}{0.4} = 25$$

then the lag component becomes to $F_I(s) = \frac{25s+1}{25s}$, and the phase margin of $F_I(s)G(s)$ is equal to

In all, we have our controller:

$$F(s) =$$

2. **Question:**

   Answer:

3. **Question:**

   Answer:

1.2 Disturbance attenuation

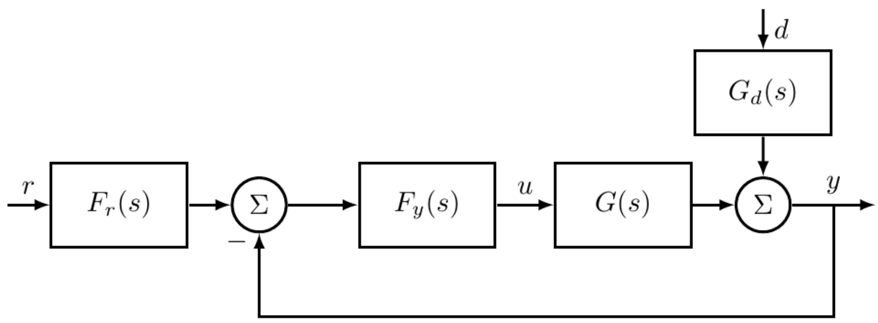

Figure 2: F_r-prefilter, F_y-feedback controller, G-system, G_d-disturbance dynamics, r-reference signal, u-control signal, d-disturbance signal, y-measurement signal.

The block diagram of the control system is given in fig 2, where the transfer functions have been estimated to

$$G(s) = \frac{20}{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)}$$
$$G_d(s) = \frac{10}{s+1}$$

1. **Question:** For which frequencies is control action needed? Control is needed at least at frequencies where $|G_d(j\omega)| > 1$ in order for disturbances to be attenuated. Therefore the cross-over frequency must be large enough. First, try to design F_y such that $L(s) \approx \omega_c/s$ and plot the closed-loop transfer function from d to y and the corresponding step response. (A simple way to find $L(s) = \omega_c/s$ is to let $F_y = G^{-1}\omega_c/s$. However, this controller is not proper. A procedure to fix this is to "add" a number of poles in the controller such that it becomes proper. How should these poles be chosen?)

   Answer: $\omega_c = 9.9473$ rad/s is the cross-over frequency of G_d so that $G_d(j\omega_c) = 1$. And since control is needed at least at frequencies where $|G_d(j\omega)| > 1, \omega \in [0, 9.9473]$ rad/s is the frequencies which control action needed.

   To design F_y such that $L_s \approx \omega_c/s$, a naive approach can be first considered:

$$\begin{aligned} F_y^*(s) &= G^{-1}(s)\omega_c/s \\ &= \frac{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)\omega_c}{20s} \end{aligned}$$

   so that $L^*(s) = F_y^*(s)G(s) = \omega_c/s$. However, the relative degree of F_y^* is -2, meaning future information in time domain would be needed, which is impossible. So poles such be "added" to make the controller proper.

   To make the relative degree to be at least 0, two poles, p_1 and p_2, are added such that

$$\begin{aligned} F_y(s) &= \frac{G^{-1}(s)\omega_c p_1 p_2}{s(s+p_1)(s+p_2)} \\ &= \frac{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)\omega_c}{20s} \cdot \frac{p_1 p_2}{(s+p_1)(s+p_2)} \end{aligned}$$

   To simplify the addition of poles, poles are set such that $p_1 = p_2 = p$.
   The open-loop transfer function becomes to $L(s) = \frac{p_1 p_2 \omega_c}{s(s+p_1)(s+p_2)}$. So the closed-loop transfer function from d to y is $G_{d_{cl}} = \frac{G_d}{1+L}$. The characteristics equation of $G_{d_{cl}}$ is

$$s^4 + (2p+1)s^3 + (2p+p^2)s^2 + p^2(w_c+1)s + p^2 w_c = 0$$

   To make the system stable, the poles in the characteristics equation should be in the LHP. To satisfy such requirement, Routh-Hurwitz Stability Criterion is used to test the stability and find possible value for p. Thus, a sufficient condition is that p should be positive and large enough. However if the value of poles added is too large, the step response would be too rapid and the amplitude of step response would also be too large. $p = 100$ is finally chosen as a decision of the trade-off. The bode plot of the open-loop

The feedback controller in exercise 4.2.2 is

$$F_y(s) = \dots$$

The feedback controller and prefilter in exercise 4.2.3 is

$$F_y(s) = \dots$$

$$F_r(s) = \dots$$

Did you manage to fulfill all the specifications? If not, what do you think makes the specifications difficult to achieve?

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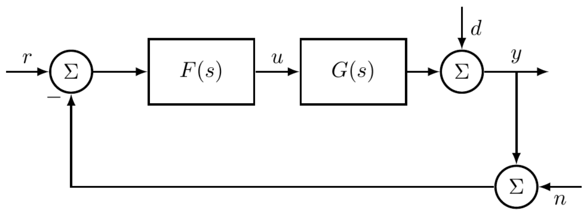

Figure 3: Step disturbance, exercise 4.2.2

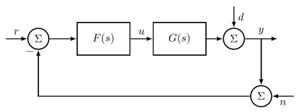

Figure 4: Reference step, exercise 4.2.3

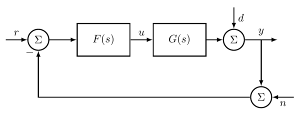

Figure 5: Control signal for a disturbance or a reference step (plus a combination of these)

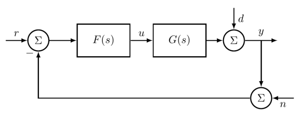

Figure 6: Bode diagram of sensitivity and complementary sensitivity functions, exercise 4.2.4