

# Computer Exercise 1

## EL2520 Control Theory and Practice

Jiaqi Li  
jiaqli@kth.se  
960326-1711

Sifan Jiang  
sifanj@kth.se  
961220-8232

April 30, 2019

### Suppression of disturbances

The weight is

$$W_S(s) = \frac{1}{(s + \epsilon + i\sqrt{\omega^2 - \epsilon^2})(s + \epsilon - i\sqrt{\omega^2 - \epsilon^2})}$$
$$= \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$

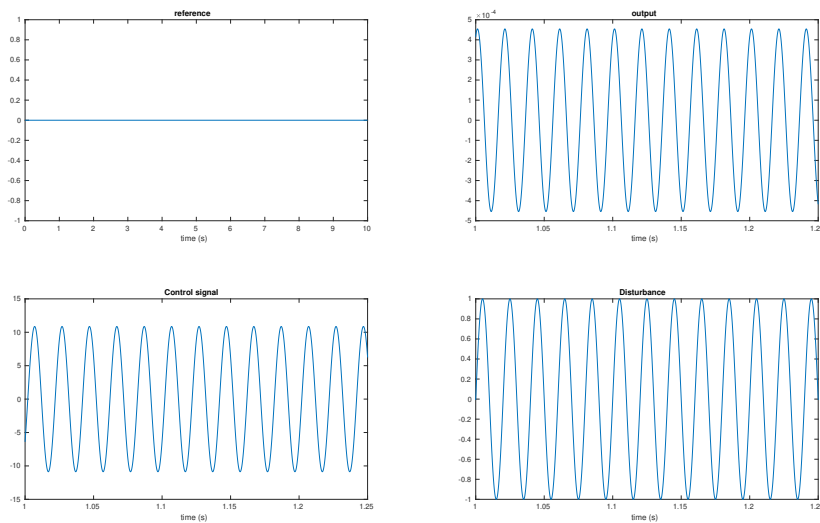


Figure 1: Simulation results with system  $G$ , using  $W_S$ .

- How much is the disturbance damped on the output?

The ratio between the disturbance amplitude and the output oscillations is  $4.543 \times 10^{-4}$ .

- What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?

Since  $|FG| \gg 1$ , we have:

$$\begin{aligned} |S| &\approx |FG|^{-1} \\ &= |FG(i100\pi)|^{-1} \\ &= (0.0920|F|)^{-1} \\ &= 4.543 \times 10^{-4} \end{aligned}$$

So,  $|F| = \frac{1}{0.0920 \times 4.543 \times 10^{-4}} = 2.3926 \times 10^4$ , which means the approximate amplification for a P-controller to get the same rate is  $2.3926 \times 10^4$ .

The disadvantage of such a P-controller is that it's not feasible to get controllers with such big amplification.

## Robustness

- What is the condition on  $T$  to guarantee stability according to the small gain theorem, and how can it be used to choose the weight  $W_T$ ?

Since  $G_0(s) = G(s)(1 + \Delta_G(s))$ ,  $\Delta_G(s) = -\frac{3}{s+2}$  is obtained. According to the small gain theorem,

$$|T(i\omega) \cdot \Delta_G(i\omega)| < 1 \Leftrightarrow |T(i\omega)| < |\Delta_G^{-1}(i\omega)|$$

Also, according to eq. (5), to ensure  $|T(i\omega)| \leq \gamma |W_T^{-1}(i\omega)|$  fulfilled, we let  $|\Delta_G^{-1}(i\omega)| \leq \gamma |W_T^{-1}(i\omega)|$ , thus obtain  $|W_T(i\omega)| \leq \gamma |\Delta_G(i\omega)|$ .

The weights are

$$\begin{aligned} W_S(s) &= \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})} \\ W_T(s) &= \frac{3}{s + 2} \end{aligned}$$

- Is the small gain theorem fulfilled?  
According to fig 2,
- Compare the results to the previous simulation

## Control signal

The weights are

$$\begin{aligned} W_S(s) &= \dots \\ W_T(s) &= \dots \\ W_U(s) &= \dots \end{aligned}$$

Compare the results to the previous simulations

.....  
.....

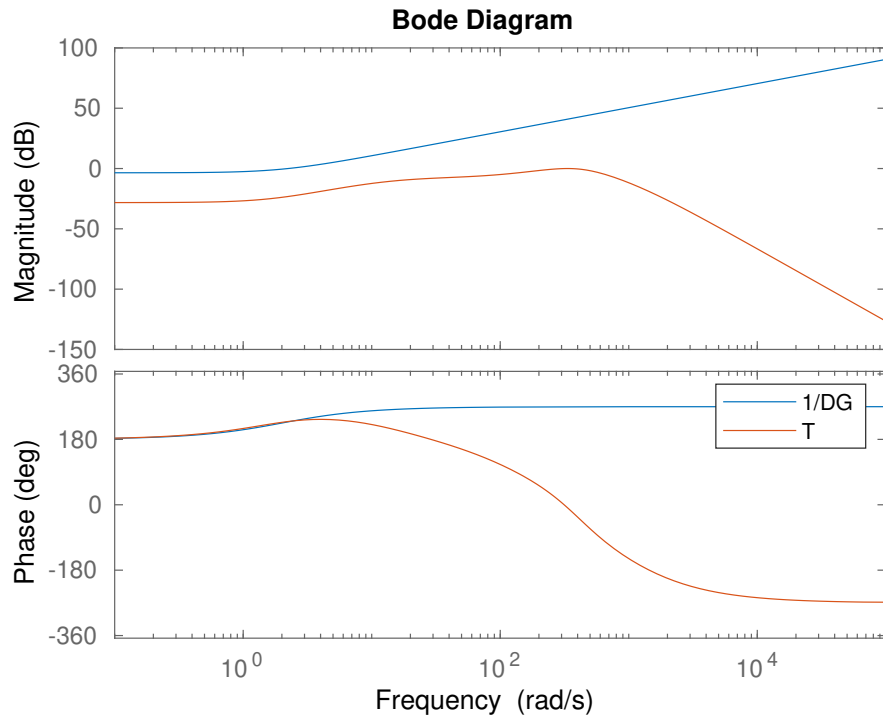


Figure 2: Bode diagram showing that the small gain theorem is satisfied.

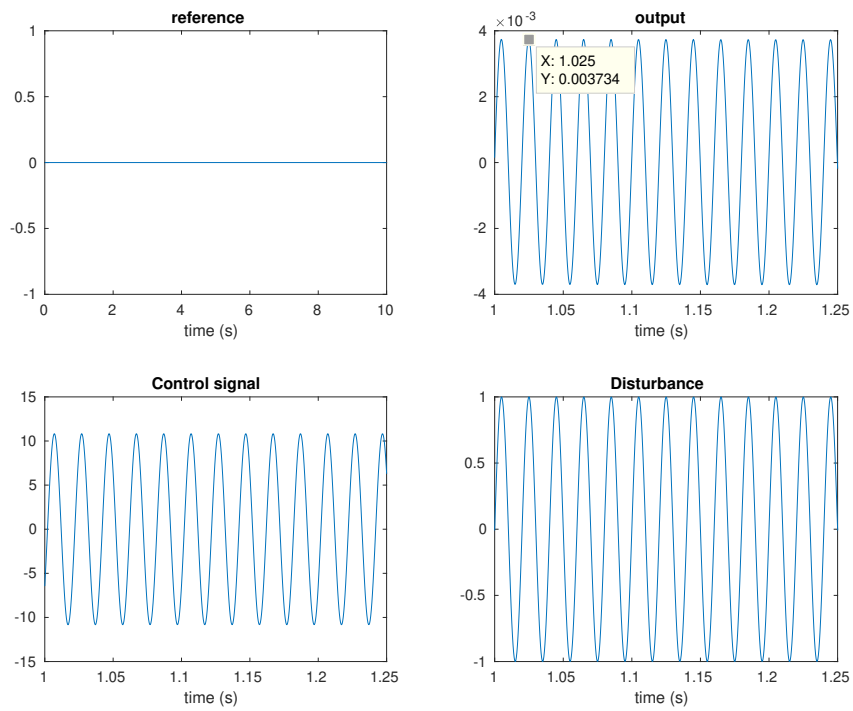


Figure 3: Simulation results with system  $G_0$ , using  $W_S$  and  $W_T$ .

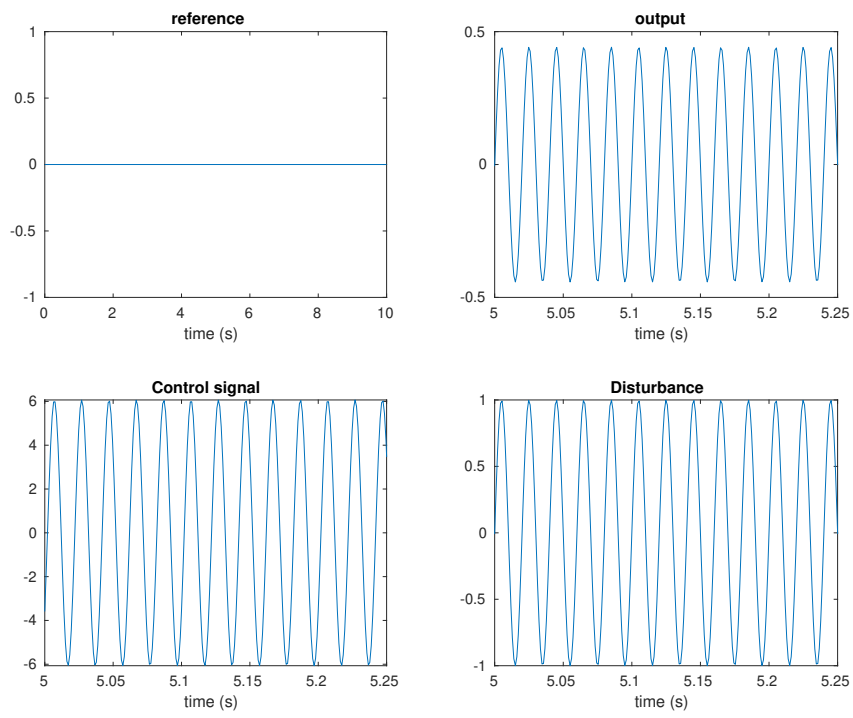


Figure 4: Simulation results with system  $G_0$ , using  $W_S$ ,  $W_T$  and  $W_U$ .