Computer Exercise 1
EL2520 Control Theory and Practice

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1 Exercises

1.1 Basics

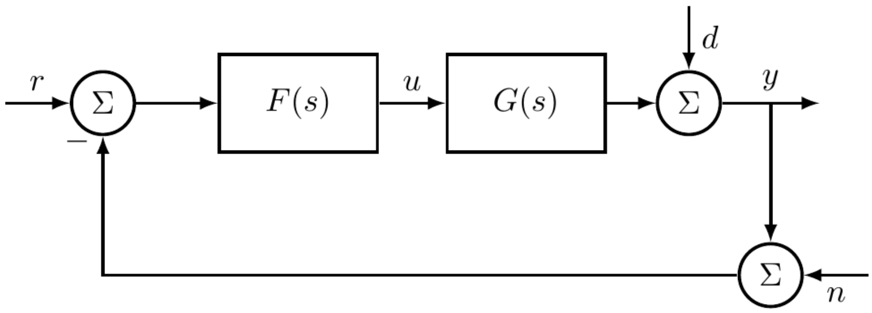

Figure 1: F-controller, G-system, r-reference signal, u-control signal, d-disturbance signal, y-output signal, n-measurement noise.

Consider a system which can be modeled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}$$

1. **Question:** Use the procedure introduced in the basic course to construct a lead-lag controller which eliminates the static control error for a step response in the reference signal.

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{\text{Lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{\text{Lag}}$$

The phase margin should be 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.

Answer: For the closed-loop system, in order to have a phase margin $\phi_m = 30^\circ$, a cross-over frequency $\omega_c = 0.4$ rad/s, and zero steady-state error for a step response in the reference signal, we consider a lead-lag controller F shown above, where K, τ_D, τ_I, β, and γ are parameters should be configured so that the closed-loop system satisfies the requirements.

For a step reference, the error is given by

$$E(s) = R(s) - Y(s) = \frac{1}{1 + F(s)G(s)} R(s) = \frac{1}{1 + F(s)G(s)} \frac{1}{s}$$

The steady-state error then can be obtained using Final Value Theorem:

$$e(\infty) = \lim_{t\to 0} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1+F(0)G(0)} = \frac{\gamma}{\gamma+3K}$$

To get zero steady-state error, $e(\infty)=0$, either $\gamma=0$ or $K\to\infty$. So, $\gamma=0$ is chosen here. Then we can design the lag controller. To minimize the phase lag caused by the lag compensator, we would obtain:

$$\tau_I = \frac{10}{\omega_c} = \frac{10}{0.4} = 25$$

then the lag component becomes to $F_I(s) = \frac{25s+1}{25s}$
In all, we have our controller:

$$F(s) =$$

2. **Question:**

   Answer:

1.2 Disturbance attenuation

How should the extra poles be chosen in exercise 4.2.1? Motivate!

. . . .
. . . .

The feedback controller in exercise 4.2.2 is

$$F_y(s) = \ldots$$

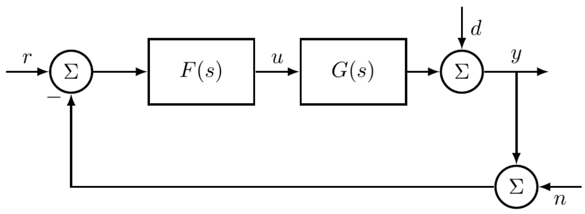

Figure 2: Step disturbance, exercise 4.2.2

The feedback controller and prefilter in exercise 4.2.3 is

$$F_y(s) = \ldots$$

$$F_r(s) = \ldots$$

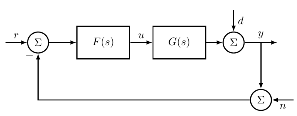

Figure 3: Reference step, exercise 4.2.3

Did you manage to fulfill all the specifications? If not, what do you think makes the specifications difficult to achieve?

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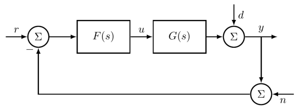

Figure 4: Control signal for a disturbance or a reference step (plus a combination of these)

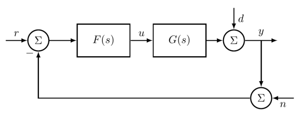

Figure 5: Bode diagram of sensitivity and complementary sensitivity functions, exercise 4.2.4