Computer Exercise 1
EL2520 Control Theory and Practice

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Suppression of disturbances

The weight is

$$W_S(s) = \frac{1}{(s + \epsilon + i\sqrt{\omega^2 - \epsilon^2})(s + \epsilon - i\sqrt{\omega^2 - \epsilon^2})}$$

$$= \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$
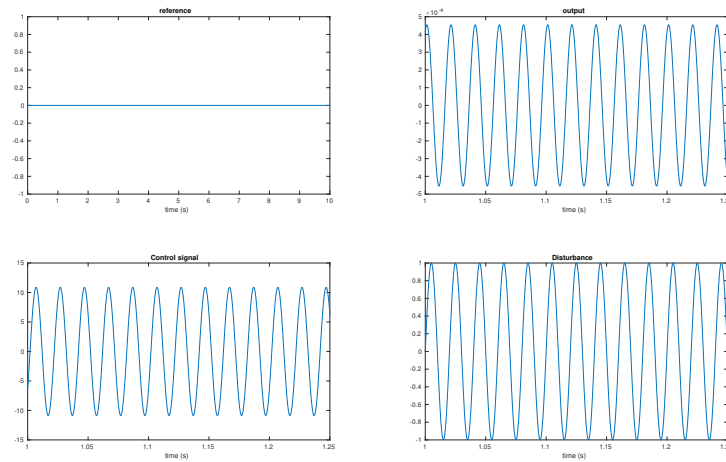

Figure 1: Simulation results with system G, using W_S.

- How much is the disturbance damped on the output?

  The rate between the output oscillations and the disturbance amplitude is 4.543×10^{-4}.

- What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?

  Since $|FG| \gg 1$, we have:

$$|S| \approx |FG|^{-1}$$
$$= |FG(i100\pi)|^{-1}$$
$$= (0.0920|F|)^{-1}$$
$$= 4.543 \times 10^{-4}$$

So, $|F| = \frac{1}{0.0920 \times 4.543 \times 10^{-4}} = 2.3926 \times 10^4$, which means the approximate amplification for a P-controller to get the same rate is 2.3926×10^4.

The disadvantage of such a P-controller is that it's not feasible to get controllers with such big amplification.

Robustness

- What is the condition on T to guarantee stability according to the small gain theorem, and how can it be used to choose the weight W_T?

  Since $G_0(s) = G(s)(1 + \Delta_G(s))$, $\Delta_G(s) = -\frac{3}{s+2}$ is obtained. According to the small gain theorem,

  $$|T(i\omega) \cdot \Delta_G(i\omega)| < 1 \Leftrightarrow |T(i\omega)| < |\Delta_G^{-1}(i\omega)|$$

  Also, according to eq. (5), to ensure $|T(i\omega)| \leq \gamma|W_T^{-1}(i\omega)|$ fulfilled, we let $|\Delta_G^{-1}(i\omega)| \leq \gamma|W_T^{-1}(i\omega)|$, thus obtain $|W_T(i\omega)| \leq \gamma|\Delta_G(i\omega)|$. So, we can choose $|W_T(i\omega)| = \gamma|\Delta_G(i\omega)|$. Setting γ to 10^{-4}, the weights are

  $$W_S(s) = \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$
  $$W_T(s) = 10^{-4}\frac{3}{s + 2}$$

- Is the small gain theorem fulfilled?

  According to fig 2, small gain theorem $|T(i\omega)| < |\Delta_G^{-1}(i\omega)|$ is always fulfilled.
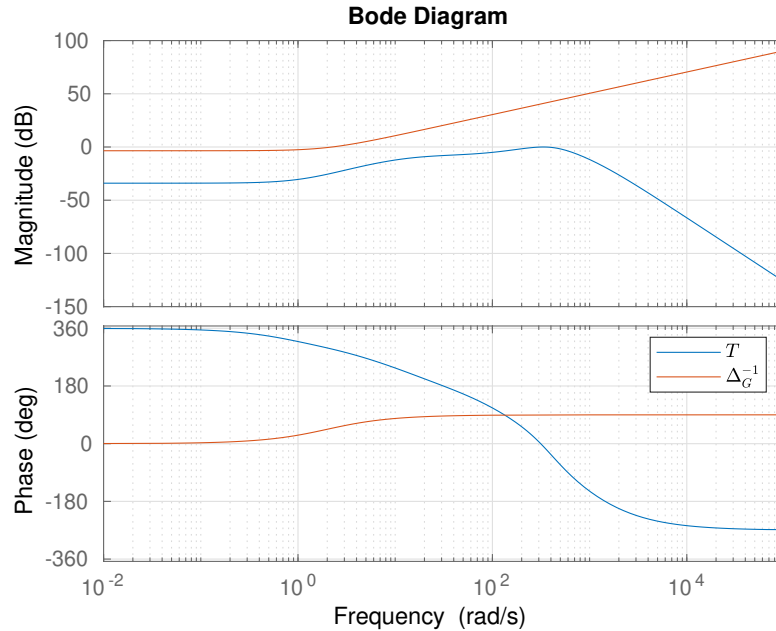

Figure 2: Bode diagram showing that the small gain theorem is satisfied.

- Compare the results to the previous simulation, which is illustrated in fig 1: the rate between the output oscillations and the disturbance amplitude increases to 3.743×10^{-3}.
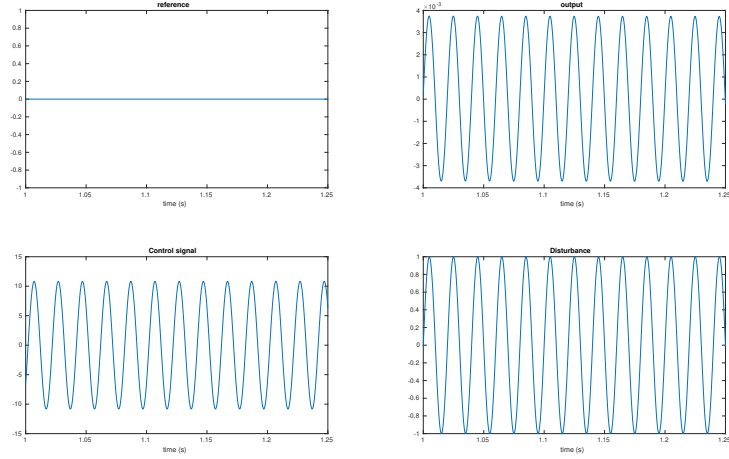
Figure 3: Simulation results with system G_0, using W_S and W_T.

Control signal

The weights are

$$W_S(s) = \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$

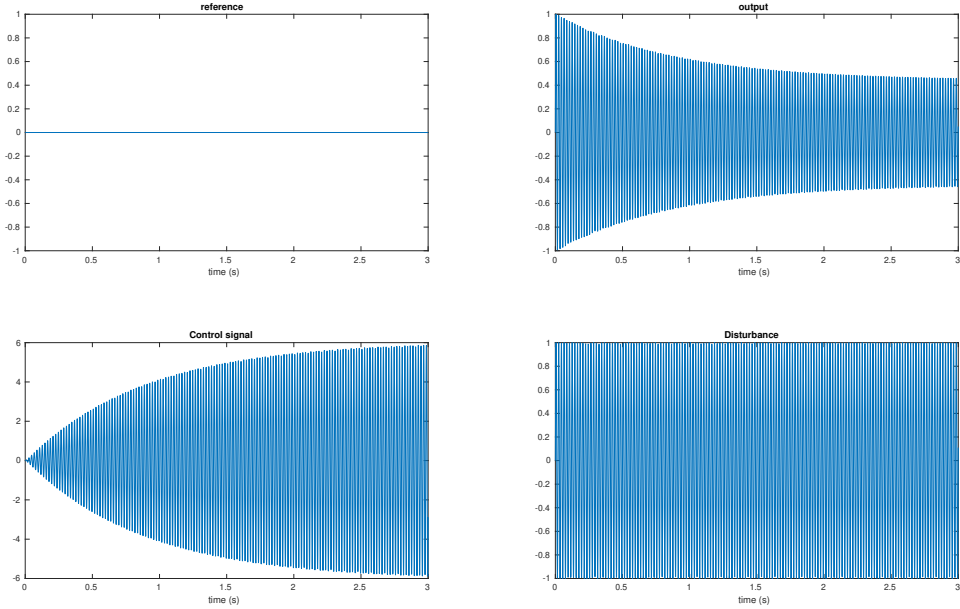$$W_T(s) = 10^{-4}\frac{3}{s+2}$$

$$W_U(s) = \frac{0.5}{s+2}$$

Figure 4: Simulation results with system G_0, using W_S, W_T and W_U.

Compare the results to the previous simulations: the amplitude of control signal is roughly half of the previous case. However, the output is increased to 0.5.

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