# Computer Exercise 2
# EL2520 Control Theory and Practice

Alexandros Filotheou
alefil@kth.se
$871108 - 5590$

April 14, 2016

## Minimum phase case

For the minimum phase case, the Relative Gain Array suggests that inputs $u_1$ and $u_2$ should be paired to outputs $y_1$ and $y_2$ respectively. The controller $F(s)$ will thus be of the form:

$$F(s) = \begin{bmatrix} f_1(s) & 0 \\ 0 & f_2(s) \end{bmatrix}$$

where $f_i = K_i(1 + \frac{1}{T_i s})$, $i \in \{1, 2\}$. Here,

$$T_i = \frac{1}{\omega_c^{mp}} tan(\phi_m - \pi/2 - arg(G_{ii}(j\omega_c^{mp})))$$

and

$$K_i = \frac{1}{\left| G_{ii}(j\omega_c^{mp})(1 + \frac{1}{j\omega_c^{mp} T_i}) \right|}$$

where $\omega_c^{mp} = 0.1$ rad/s and $\phi_m = 60°$. From the above two relations we conclude that

$$f_1(s) = 1.6776(1 + \frac{1}{5.9037s})$$
$$f_2(s) = 2.0137(1 + \frac{1}{6.3911s})$$

Figure 1 shows the step response of the system for an input step signal applied at input $u_1$ at $t = 100$s and at input $u_2$ at $t = 500$s.

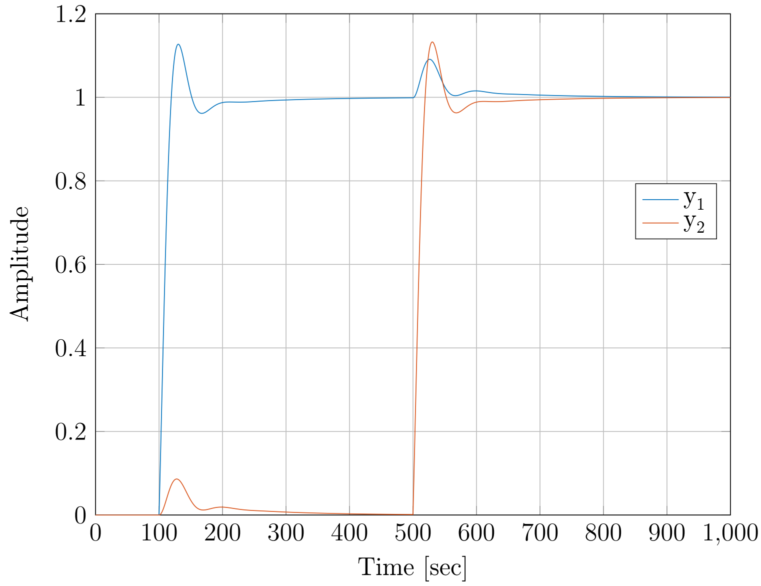


Figure 1: Step responses in outputs $y_1$ (`blue`) and $y_2$ (`red`).

Tables 1 and 2 feature the overshoot and rise time performance metrics with regard to the step responses of the system. It is evident that all inputs are coupled to all outputs, but the applied control actions result in swift and satisfactory performance.

Figure 2 shows the frequency response of $L_{11}(j\omega) = G_{11}(j\omega)f_1(j\omega)$ (`blue`) and $L_{22} = G_{22}(j\omega)f_2(j\omega)$ (`red`), where $L = GF$. The phase margin is 60° at the required gain crossover frequency $\omega_c^{mp} = 0.1$ rad/s.

| Input/Output | $y_1$ | $y_2$ |
|---|---|---|
| $u_1$ | <u>12.7%</u> | 8.6% |
| $u_2$ | 9.1% | <u>13.2%</u> |

Table 1: Overshoot figures regarding the step responses depicted in figure 1. Underlined percentages reference control-coupled inputs/outputs.

| Input/Output | $y_1$ | $y_2$ |
|---|---|---|
| $u_1$ | 14.2 | |
| $u_2$ | | 14.3 |

Table 2: Rise times in seconds regarding the step responses depicted in figure 1.

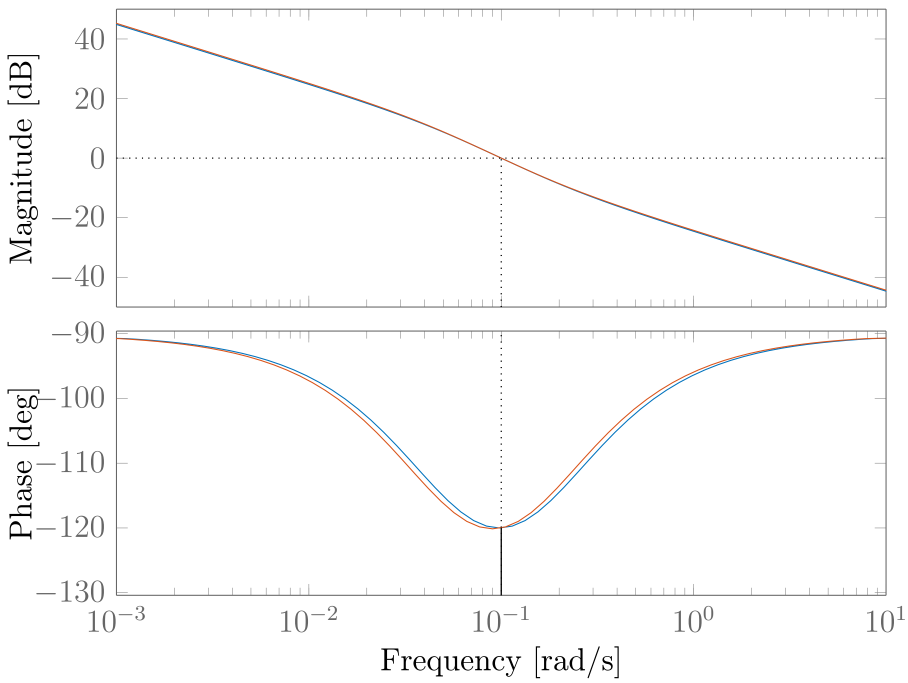


Figure 2: The frequency response of $L_{11}(j\omega) = G_{11}(j\omega)f_1(j\omega)$ (`blue`) and $L_{22} = G_{22}(j\omega)f_2(j\omega)$ (`red`), where $L = GF$. The phase margin is 60° and the gain crossover frequency is $\omega_c^{mp} = 0.1$ rad/s.

## Non-minimum phase case

For the non-minimum phase case, the Relative Gain Array suggests that inputs $u_1$ and $u_2$ should be paired to outputs $y_2$ and $y_1$ respectively. The controller $F(s)$ will thus be of the form:

$$F(s) = \begin{bmatrix} 0 & f_1(s) \\ f_2(s) & 0 \end{bmatrix}$$

where $f_i = K_i(1 + \frac{1}{T_i s}), i \in \{1, 2\}$. Here,

$$T_i = \frac{1}{\omega_c^{nmp}} tan(\phi_m - \pi/2 - arg(G_{ji}(j\omega_c^{nmp})))$$

and

$$K_i = \frac{1}{\left|G_{ji}(j\omega_c^{nmp})(1 + \frac{1}{j\omega_c^{nmp} T_i})\right|}$$

where $i \neq j$, $\omega_c^{nmp} = 0.02$ rad/s and $\phi_m = 60°$. From the above two relations we conclude that

$$f_1(s) = 0.1437(1 + \frac{1}{4.8107s})$$
$$f_2(s) = 0.1469(1 + \frac{1}{3.9426s})$$

Figure 3 shows the step responses of the system for an input step signal applied at input $u_1$ at $t = 100$s and at input $u_2$ at $t = 500$s.

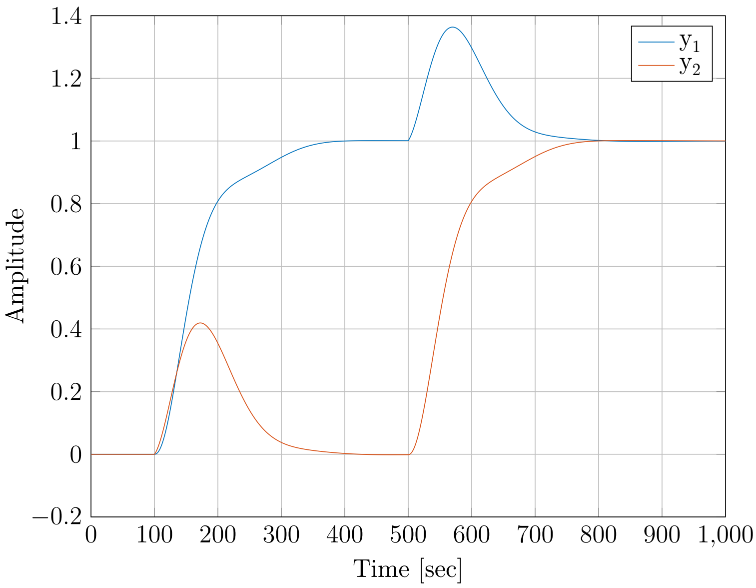


Figure 3: Step responses in outputs $y_1$ (`blue`) and $y_2$ `red`.

Tables 1 and 2 feature the overshoot and rise time performance metrics with regard to the step responses of the system. The large overshoot figures are in line with the high sensitivity magnitude. It is evident that all inputs are coupled to all outputs, but the applied control actions result in adequate, but slow, performance, and certainly worse than in the minimum phase case.

Figure 4 shows the frequency response of $L_{11}(j\omega) = G_{12}(j\omega)f_2(j\omega)$ (`blue`) and $L_{22}(j\omega) = G_{21}(j\omega)f_1(j\omega)$ (`red`), where $L = GF$. The phase margin is $60°$ at the required gain crossover frequency $\omega_c^{nmp} = 0.02$ rad/s.

| Input/Output | $y_1$ | $y_2$ |
|---|---|---|
| $u_1$ | 0% | 41.9% |
| $u_2$ | 36.3% | 0% |

Table 3: Overshoot figures regarding the step response depicted in figure 3. Underlined percentages reference control-coupled inputs/outputs.

| Input/Output | $y_1$ | $y_2$ |
|---|---|---|
| $u_1$ | 137.1 | |
| $u_2$ | | 135 |

Table 4: Rise times in seconds regarding the step responses depicted in figure 3.

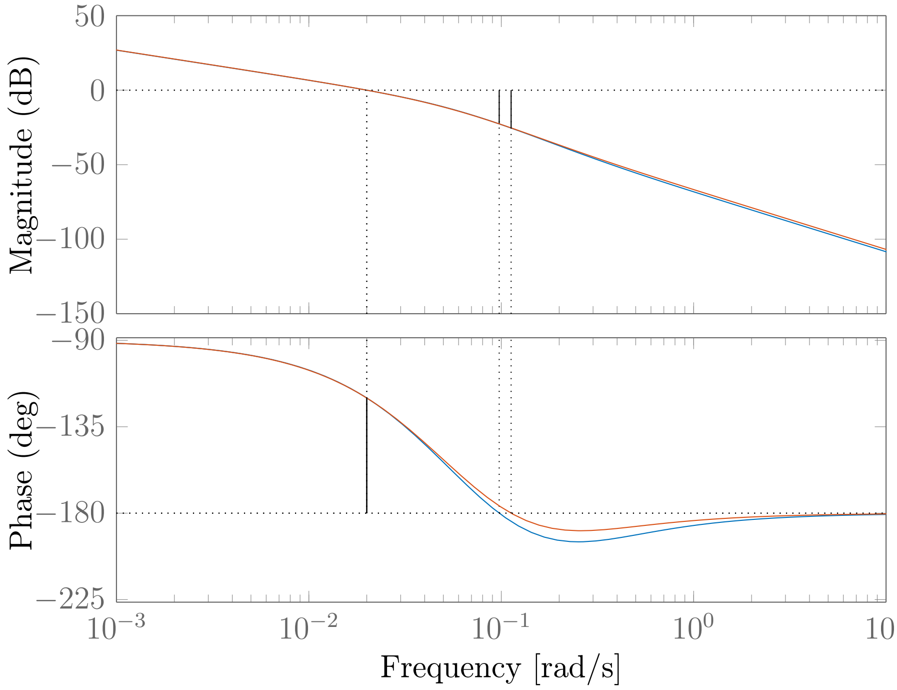


Figure 4: $L_{11}(j\omega) = G_{12}(j\omega)f_2(j\omega)$ (`blue`) and $L_{22}(j\omega) = G_{21}(j\omega)f_1(j\omega)$ (`red`), where $L = GF$. The phase margin is $60°$ and the gain crossover frequency is $\omega_c^{nmp} = 0.02$ rad/s.

# References

[1] T. Glad and L. Ljung, Control Theory - Multivariable and Nonlinear Methods

[2] S. Skogestad and I. Postlethwaite, Multivariable Feedback Control - Analysis and Design