Computer Exercise 3
EL2520 Control Theory and Practice

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Suppression of disturbances

The process to be controlled is identified by the transfer function G:

$$G(s) = 10^4 \frac{s+2}{(s+3)(s+100)^2}$$

The weight W_S is

$$W_S(s) = \frac{10^4}{(s+p_1)(s+p_2)}$$
$$p_k = -0.1 \pm i\sqrt{(100\pi)^2 - 0.1^2}, k = \{1, 2\}$$

Figure 1 illustrates the amplitudes of the reference signal (0.0), the output, the control signal (10.0) and the disturbance (1.0). The amplitude of the output is at most $3 \cdot 10^{-5}$ in steady-state, hence the rate between the disturbance and the output oscillations is approximately $3 \cdot 10^5$.

If the controller was a proportional one, it would need to have a value around 10^8 for the disturbances to be damped equally well as in the above case. However, in this case the amplitude of the control signal would increase by 2 orders of magnitude. At the same time, a P-controller would treat all frequencies in the same manner, i.e. it would attenuate all frequencies equally; not just the 50 Hz one.

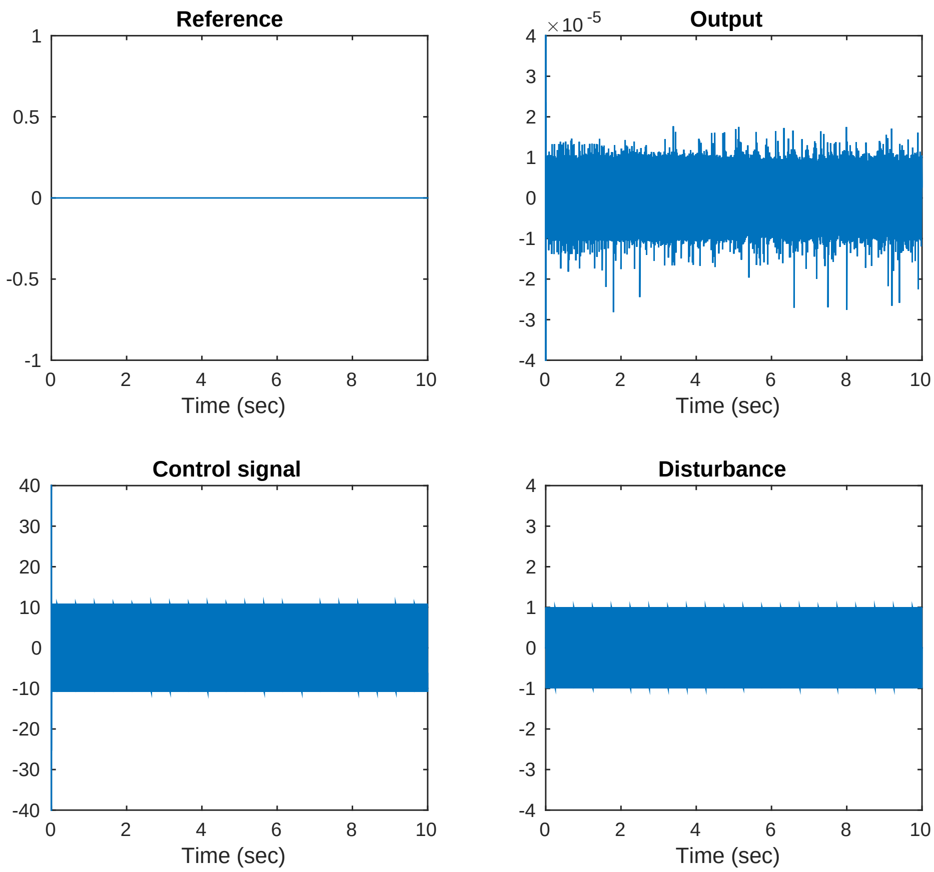

Figure 1: Simulation results with system G, using W_S.

Robustness

The system's transfer function is now given by G_0:

$$G_0(s) = G(s)(1 + \Delta_G(s)) = 10^4 \frac{s+2}{(s+3)(s+100)^2} \cdot \frac{s-1}{s+2}$$

Here, $\Delta_G(s) = -\dfrac{3}{s+2}$, which is stable. According to the small gain theorem, the closed loop is stable if

$$\begin{aligned} |T(i\omega) \cdot \Delta_G(i\omega)| < 1 \Leftrightarrow \\ |T(i\omega)| < \left|\frac{i\omega + 2}{3}\right| \end{aligned}$$

Hence the weight W_T can be chosen as $-\Delta_G$. The weights are then:

$$\begin{aligned} W_S(s) &= \frac{10^4}{(s+p_1)(s+p_2)} \\ W_T(s) &= \frac{3}{s+2} \end{aligned}$$

where $p_k = -0.1 \pm i\sqrt{(100\pi)^2 - 0.1^2}, k = \{1, 2\}$. Figure 2 illustrates that the small gain theorem holds: the amplitude of $T \cdot \Delta_G$ is less than one for all frequencies. Figure 3 illustrates the amplitudes of the reference signal (0.0), the output, the control signal ($\approx$ 10.0) and the disturbance (1.0). The amplitude of the output is less than $5 \cdot 10^{-4}$ in steady-state, hence the rate between the disturbance and the output oscillations is approximately 2000, which is less than in the previous case.

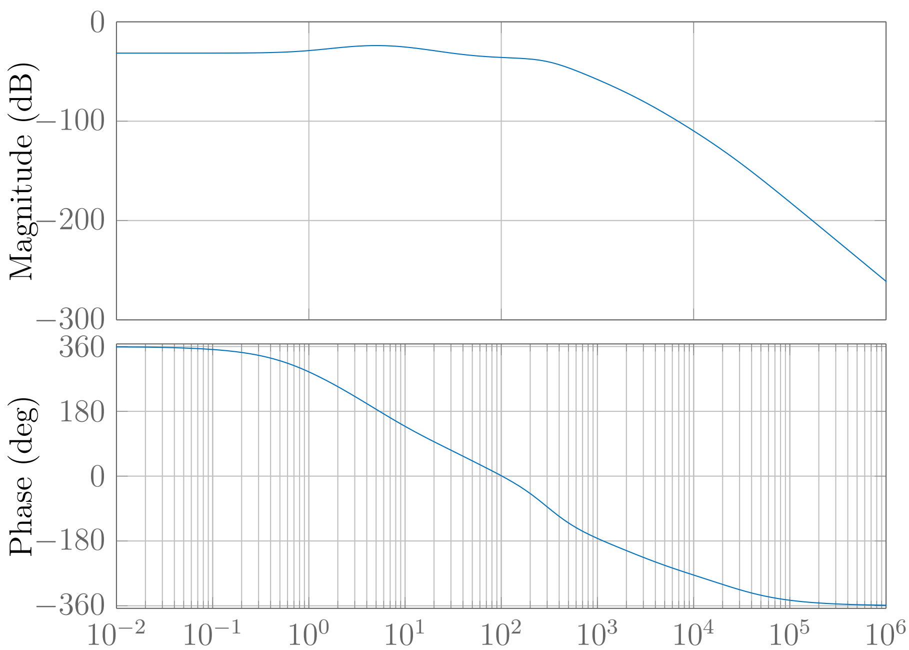

Figure 2: Bode diagram of $T(i\omega) \cdot \Delta_G(i\omega)$. The amplitude is always less than 1, hence the small gain theorem is fulfilled.

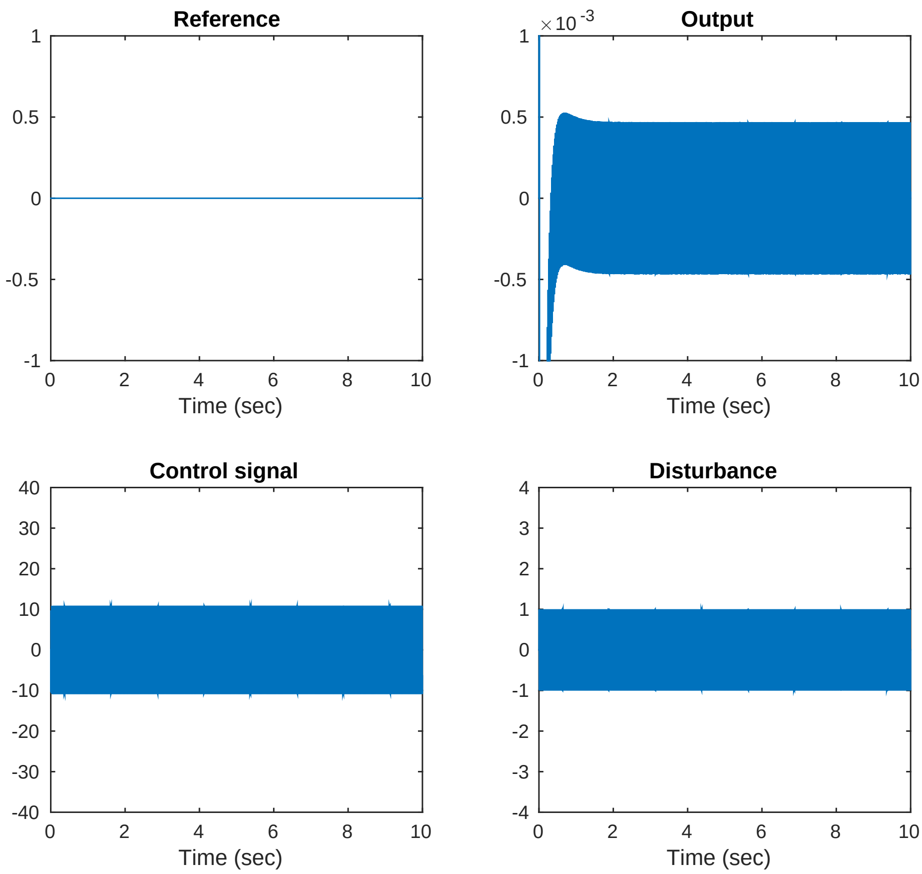

Figure 3: Simulation results with system G_0, using W_S and W_T.

Control signal

The weights are

$$W_S(s) = \frac{100}{(s+p_1)(s+p_2)}$$
$$W_T(s) = \frac{3}{s+2}$$
$$W_U(s) = \frac{100}{(s+p_1)(s+p_2)}$$

where $p_k = -0.1 \pm i\sqrt{(100\pi)^2 - 0.1^2}, k = \{1, 2\}$.

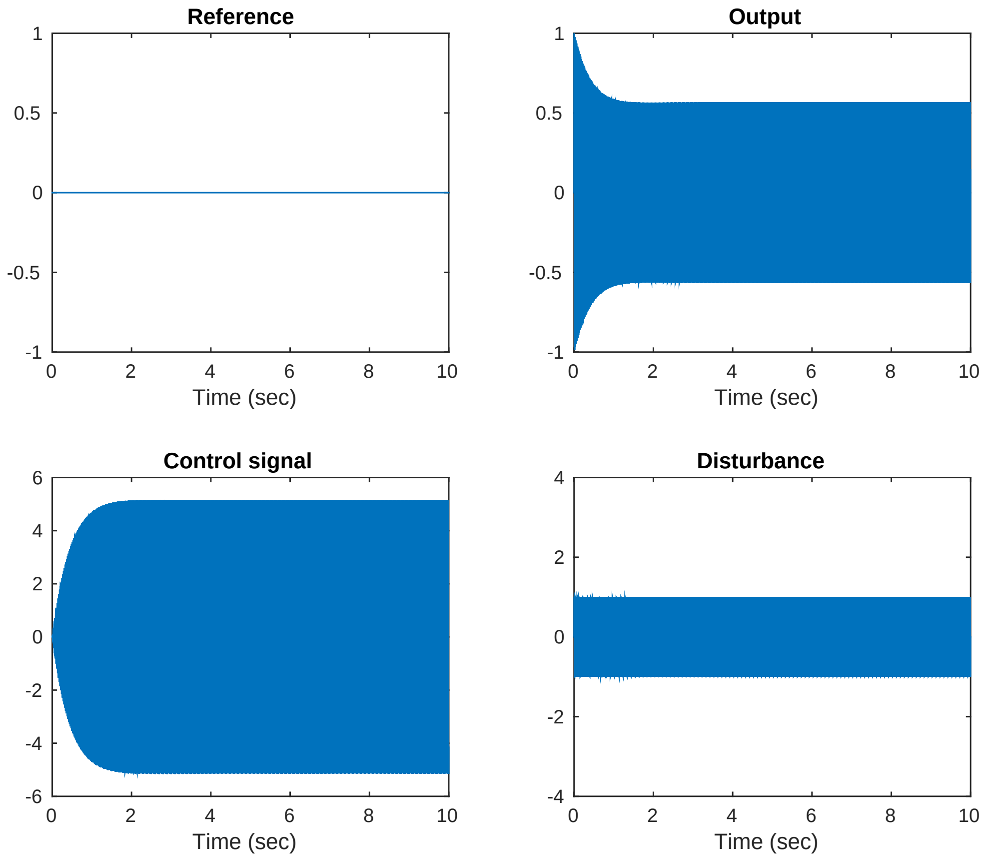

Figure 4: Simulation results with system G_0, using W_S, W_T and W_U.

We can see that a specification on the control signal u to be halved results in inadmissible output signal: its value instead of following the reference signal shoots up to an amplitude of 0.5.