# Computer Exercise 1 EL2520 Control Theory and Practice

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March 24, 2019

## 1 Exercises

#### 1.1 Basics

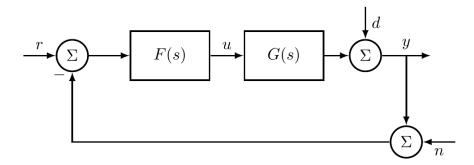


Figure 1: F-controller, G-system, r-reference signal, u-control signal, d-disturbance signal, y-output signal, n-measurement noise.

Consider a system which can be modeled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}$$

1. **Question:** Use the procedure introduced in the basic course to construct a lead-lag controller which eliminates the static control error for a step response in the reference signal.

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{\text{Lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{\text{Lag}}$$

The phase margin should be  $30^{\circ}$  at the cross-over frequency  $\omega_c = 0.4 \text{ rad/s}$ .

Answer: For the closed-loop system, in order to have a phase margin  $\phi_m = 30^\circ$ , a cross-over frequency  $\omega_c = 0.4$  rad/s, and zero steady-state error for a step response in the reference signal, we consider a lead-lag controller F shown above, where K,  $\tau_D$ ,  $\tau_I$ ,  $\beta$ , and  $\gamma$  are parameters should be configured so that the closed-loop system satisfies the requirements.

For a step reference, the error is given by

$$E(s) = R(s) - Y(s) = \frac{1}{1 + F(s)G(s)}R(s) = \frac{1}{1 + F(s)G(s)}\frac{1}{s}$$

The steady-state error then can be obtained using Final Value Theorem:

$$e(\infty) = \lim_{t \to 0} e(t) = \lim_{s \to 0} sE(s) = \frac{1}{1 + F(0)G(0)} = \frac{\gamma}{\gamma + 3K}$$

To get zero steady-state error,  $e(\infty)=0$ , either  $\gamma=0$  or  $K\to\infty$ . So,  $\gamma=0$  is chosen here.

Then

In all, we have our controller:

$$F(s) =$$

## 2. Question:

**Answer:** 

### 1.2 Disturbance attenuation

How should the extra poles be chosen in exercise 4.2.1? Motivate!

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The feedback controller in exercise 4.2.2 is

$$F_{u}(s) = \dots$$

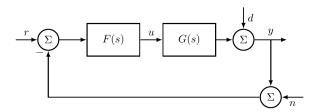


Figure 2: Step disturbance, exercise 4.2.2

The feedback controller and prefilter in exercise 4.2.3 is

$$F_y(s) = \dots$$

$$F_r(s) = \dots$$

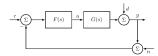


Figure 3: Reference step, exercise 4.2.3

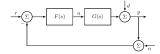


Figure 4: Control signal for a disturbance or a reference step (plus a combination of these)

Did you manage to fulfill all the specifications? If not, what do you think makes the specifications difficult to achieve?

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$$\xrightarrow{F} \underbrace{\Sigma} \qquad F(s) \qquad \stackrel{u}{\longrightarrow} \underbrace{G(s)} \qquad \stackrel{d}{\longrightarrow} \underbrace{U} \qquad \stackrel{d}{\longrightarrow} \underbrace{U$$

Figure 5: Bode diagram of sensitivity and complementary sensitivity functions, exercise 4.2.4