Computer Exercise 2
EL2520 Control Theory and Practice

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3 Exercises

3.1 Poles, zeros and RGA

Minimum phase

3.1.1

The transfer function is

$$G(s) = \begin{bmatrix} \frac{0.0348}{s+0.05645} & \frac{0.0004649}{s^2+0.08217s+0.001452} \\ \frac{0.0004446}{s^3+0.07317s+0.001105} & \frac{0.03013}{s+0.05187} \end{bmatrix}$$

- Poles of $G(1,1)$ is -0.0564 and zeros of $G(1,1)$ is $\emptyset$.
- Poles of $G(1,2)$ are $-0.0564, -0.0257$ and zeros of $G(1,2)$ is $\emptyset$.
- Poles of $G(2,1)$ are $-0.0519, -0.0213$ and zeros of $G(2,1)$ is $\emptyset$.
- Poles of $G(2,2)$ is -0.0519 and zeros of $G(2,2)$ is $\emptyset$.

3.1.2

Poles of the system are

$$Poles = \begin{bmatrix} -0.0564 \\ -0.0519 \\ -0.0213 \\ -0.0564 \\ -0.0257 \\ -0.0519 \end{bmatrix}$$

Zeros of the system are

$$Zeros = \begin{bmatrix} -0.0093 \\ -0.0377 \\ -0.0564 \\ -0.0519 \end{bmatrix}$$

As we see, we have more zeros in the MIMO system, which means at frequency generalized by these zeros s_0, our system can have a non-zero input $u(t) = u_0 e^{s_0 t}$,we can have zero output $y(t) = 0 \; \forall t$ (See MIT Topic 6 and Topic 8).

3.1.3

The singular values of the system are shown in the Figure 1.

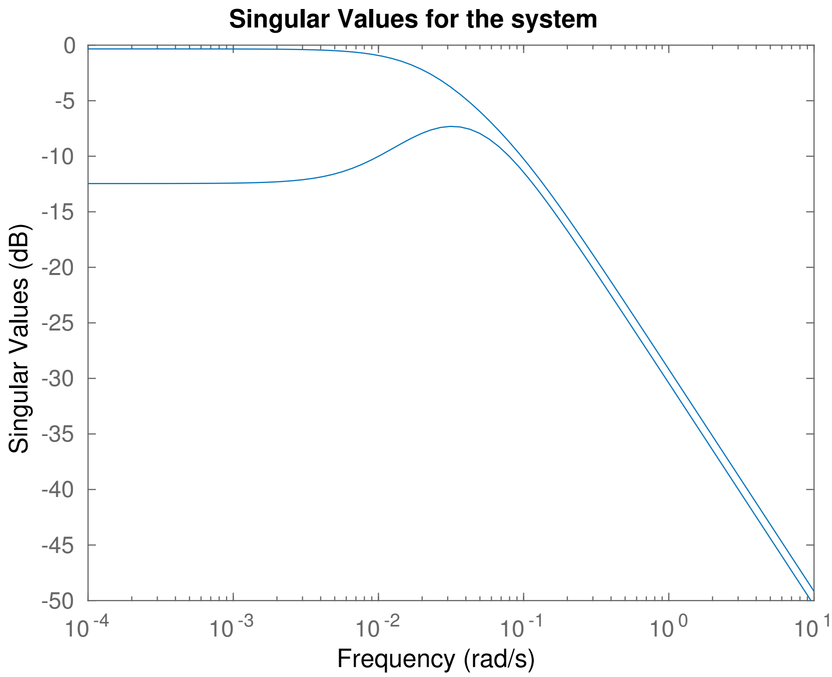

Figure 1: Singular values of the system.

The H_∞ norm of the system is 0.9619.

3.1.4

The RGA at frequency 0 is

$$RGA(G(0)) = \begin{bmatrix} 1.5625 & -0.5625 \\ -0.5625 & 1.5625 \end{bmatrix}$$

From RGA, we can conclude that we can using decentralized control and it is better to select λ_{11} and λ_{22} to pair.

3.1.5

The step responses of each subplant are shown in the Figure 2.

As we see, there is some coupling among these subplants, so it agrees with the RGA results.

Non-minimum phase

3.1.1

The transfer function is

$$G(s) = \begin{bmatrix} \dfrac{0.02088}{s + 0.05106} & \dfrac{0.002586}{s^2 + 0.1369s + 0.004382} \\ \dfrac{0.003163}{s^2 + 0.1378s + 0.004265} & \dfrac{0.01808}{s + 0.04692} \end{bmatrix}$$

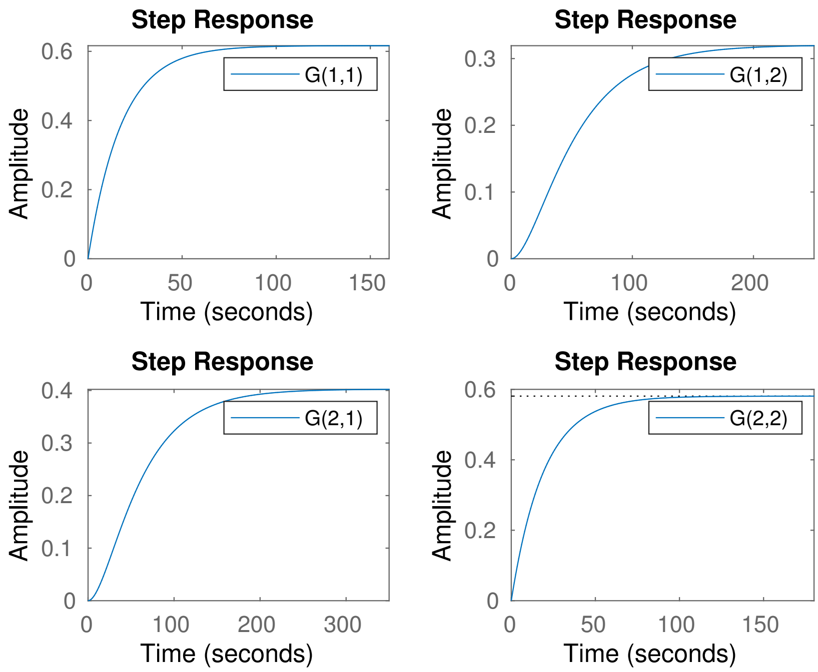

Figure 2: Step responses of each subplant.

- Poles of $G(1,1)$ is -0.0511 and zeros of $G(1,1)$ is $\emptyset$.
- Poles of $G(1,2)$ are $-0.0858, -0.0511$ and zeros of $G(1,2)$ is $\emptyset$.
- Poles of $G(2,1)$ are $-0.0909, -0.0469$ and zeros of $G(2,1)$ is $\emptyset$.
- Poles of $G(2,2)$ is -0.0469 and zeros of $G(2,2)$ is $\emptyset$.

3.1.2

Poles of the system are

$$Poles = \begin{bmatrix} -0.0511 \\ -0.0469 \\ -0.0858 \\ -0.0909 \end{bmatrix}$$

Zeros of the system are

$$Zeros = \begin{bmatrix} 0.0589 \\ -0.2356 \end{bmatrix}$$

As we see, we have more zeros in the MIMO system, which means at frequency generalized by these zeros s_0, our system can have a non-zero input $u(t) = u_0 e^{s_0 t}$,we can have zero output $y(t) = 0 \; \forall t$ (See MIT Topic 6 and Topic 8).

3.1.3

The singular values of the system are shown in the Figure 3.

The H_∞ norm of the system is 1.0702.

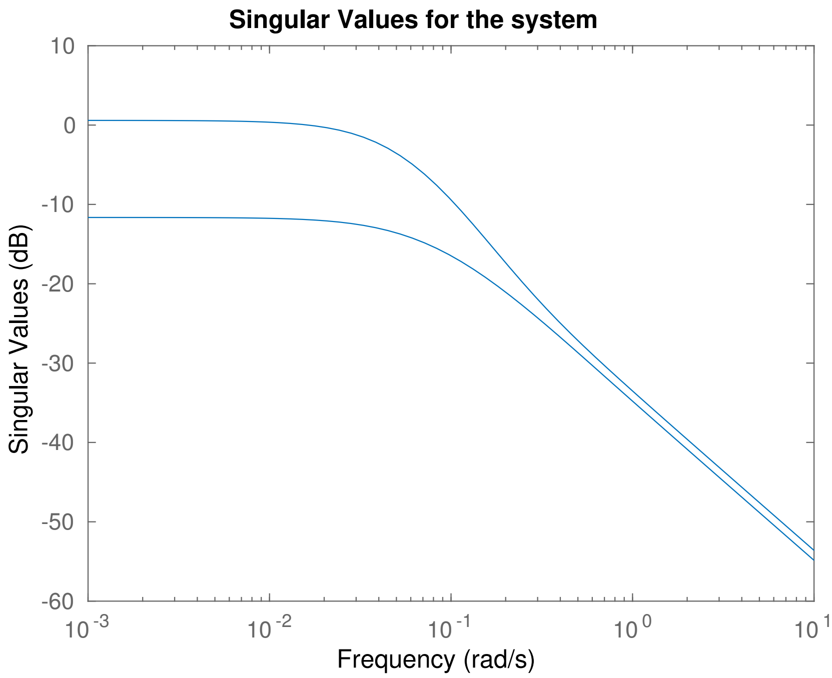

Figure 3: Singular values of the system.

3.1.4

The RGA at frequency 0 is

$$RGA(G(0)) = \begin{bmatrix} -0.5625 & 1.5625 \\ 1.5625 & -0.5625 \end{bmatrix}$$

From RGA, we can conclude that we can using decentralized control and it is better to select λ_{12} and λ_{21} to pair.

3.1.5

The step responses of each subplant are shown in the Figure 4.
As we see, there is some coupling among these subplants, so it agrees with the RGA results.

3.1.6

The main difference between these two cases is

3.2 Decentralized Control

Minimum phase

3.2.1

The Decentralized controller is

$$F = \begin{bmatrix} 1.6776(1 + \frac{1}{0.5904s}) & 0 \\ 0 & 2.0137(1 + \frac{1}{0.6391s}) \end{bmatrix}$$

The bode diagrams of the open-loop controller L is shown in the Figure 5

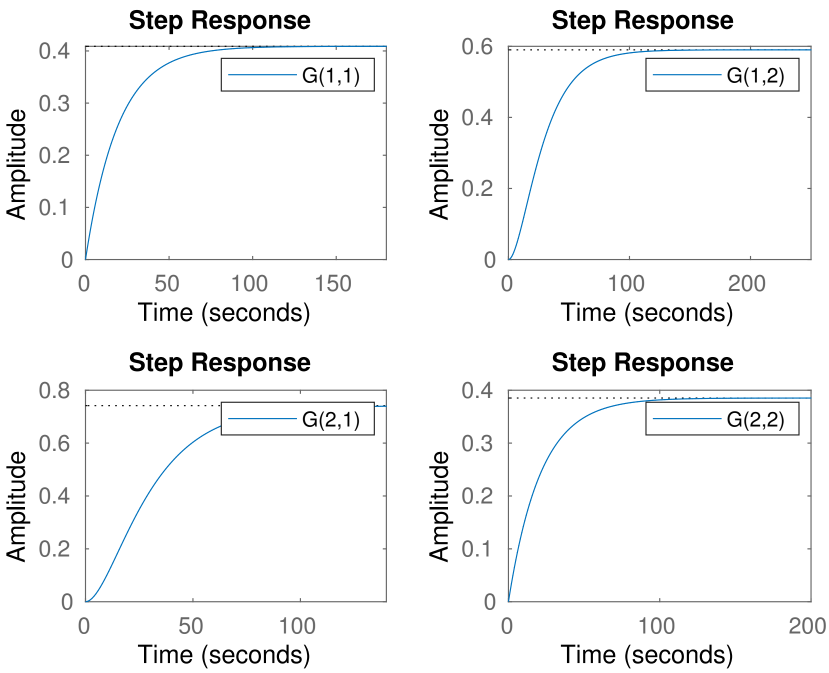

Figure 4: step responses of each subplant

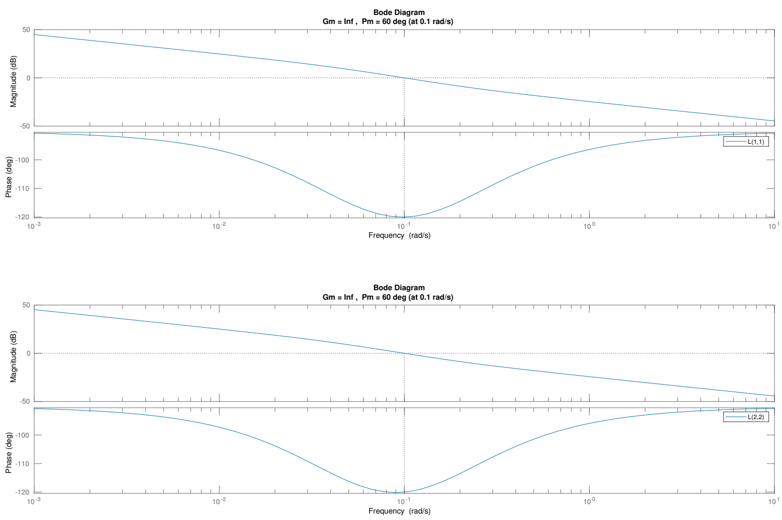

Figure 5: Bode diagrams of the open-loop controller L

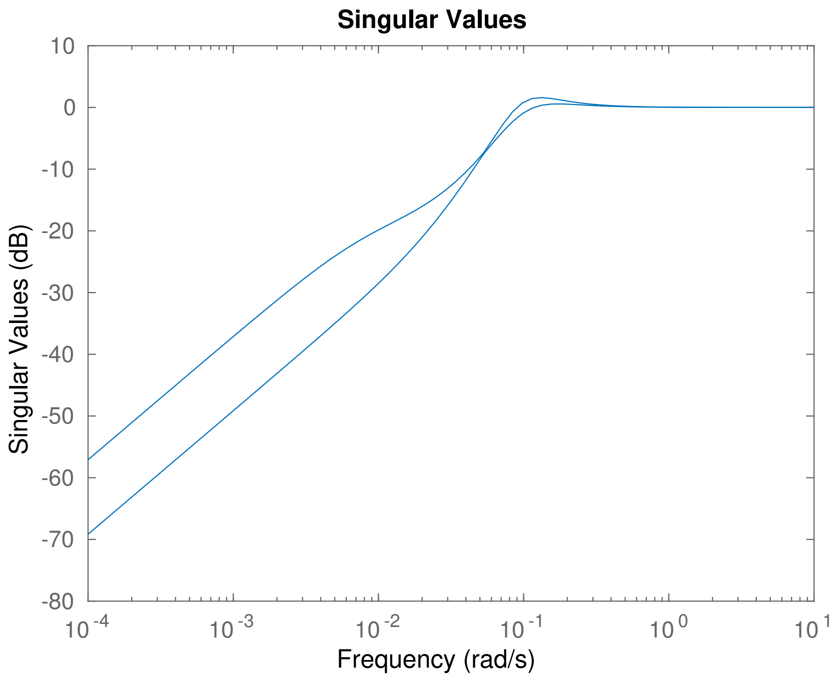

Figure 6: The singular values of the sensitivity S

3.2.2

The singular values of the sensitivity and complementary sensentivity are shown in the Figure 6 and Figure 7
As we see, the controller will be more sensitive and less robust to the disturbance at low frequency, and less sensitive and more robust to the disturbance at high frequency.

3.2.3

The responses of the control signals and outputs are shown in the Figure 8 and Figure 9
As shown in the figure, the outputs are coupled but it is not strongly coupled. When given a step response, the unrelated output will gain some impulse, but it will soon be steady. So we can conclude that the controller is relatively good.

Nonminimum phase

3.2.1

The Decentralized controller is

$$F = \begin{bmatrix} 0 & 0.1469(1 + \frac{1}{4.8107s}) \\ 0.1437(1 + \frac{1}{3.9426s}) & 0 \end{bmatrix}$$

The bode diagrams of the open-loop controller L is shown in the Figure 10

3.2.2

The singular values of the sensitivity and complementary sensentivity are shown in the Figure 11 and Figure 12
As we see, the controller will be more sensitive and less robust to the disturbance at low frequency, and less sensitive and more robust to the disturbance at high frequency.

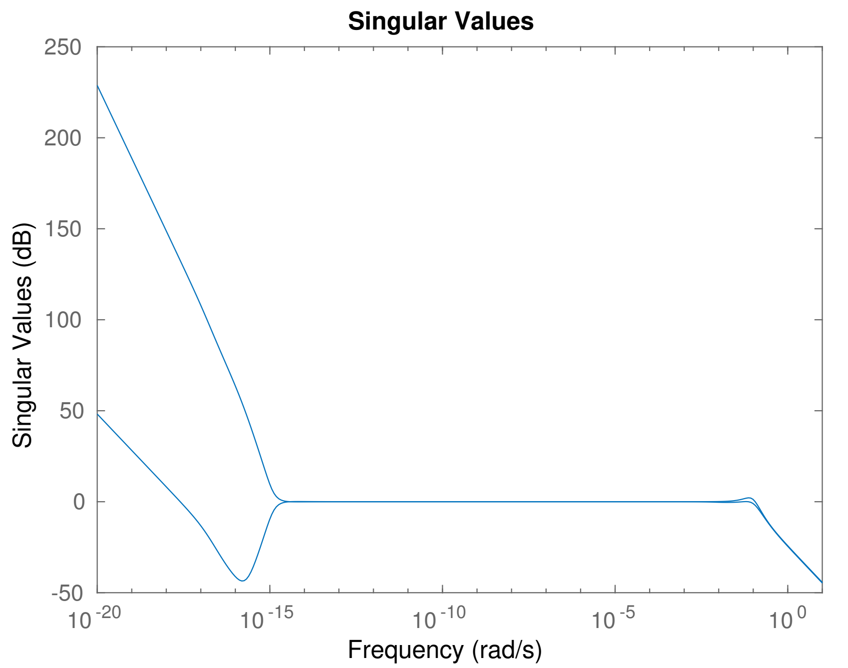

Figure 7: The singular values of the complementary sensitivity T

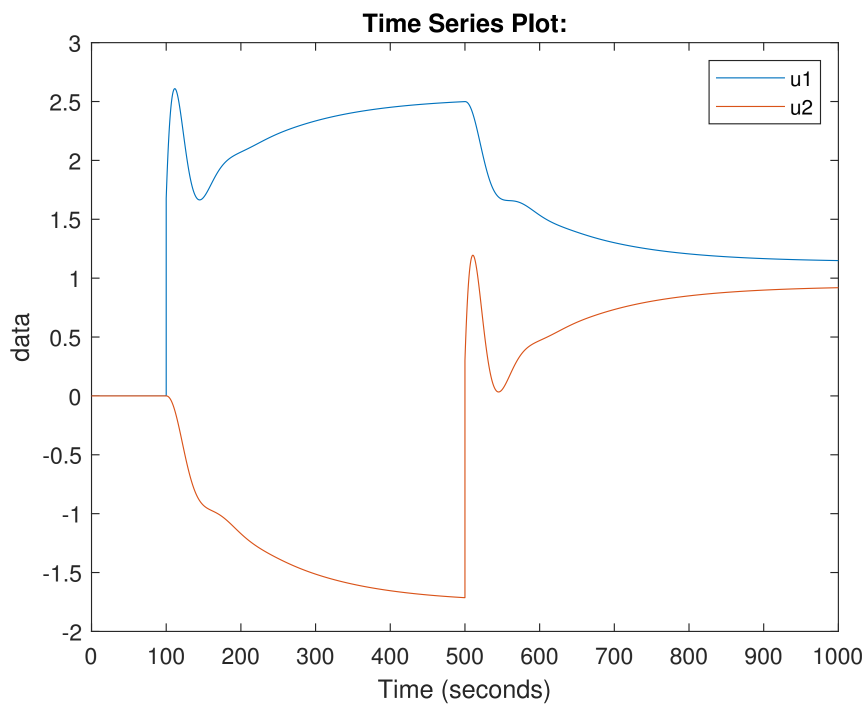

Figure 8: Responses of the control signal u

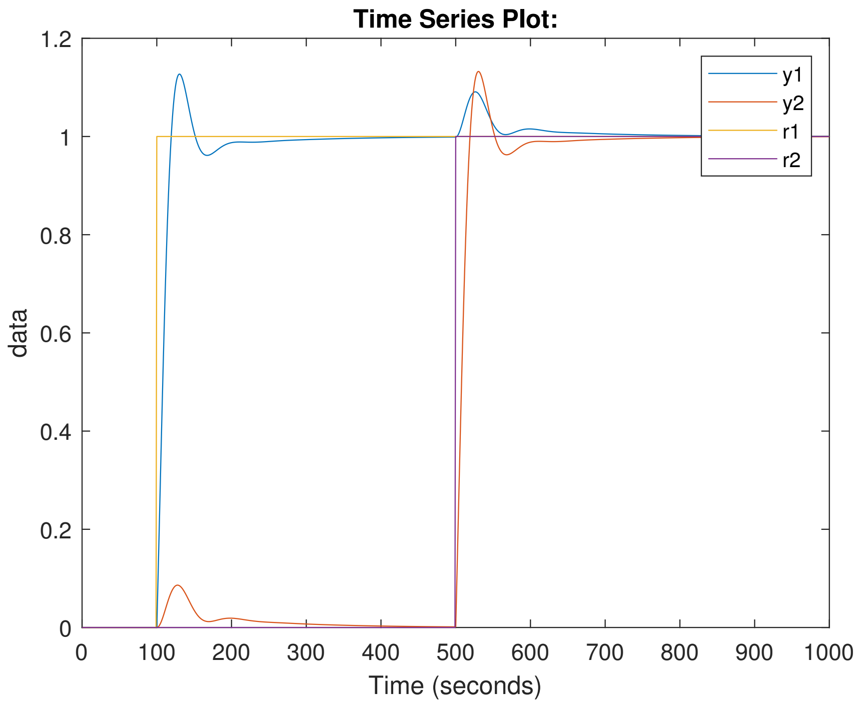

Figure 9: Responses of the output y with references r

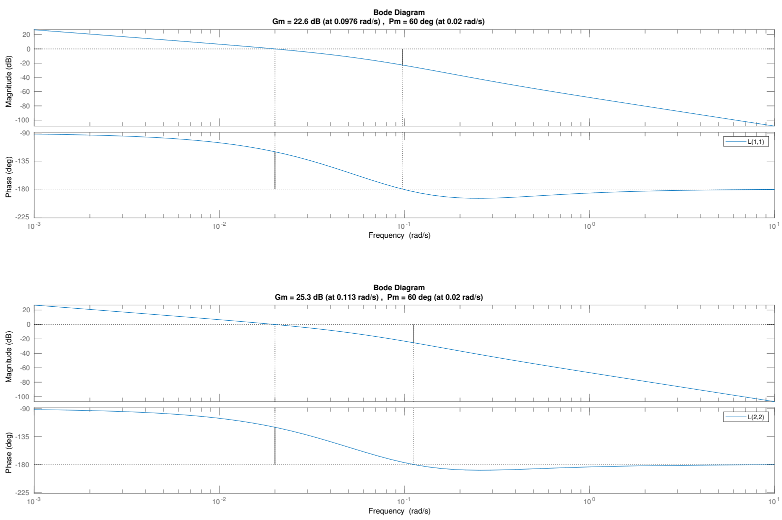

Figure 10: Bode diagrams of the open-loop controller L

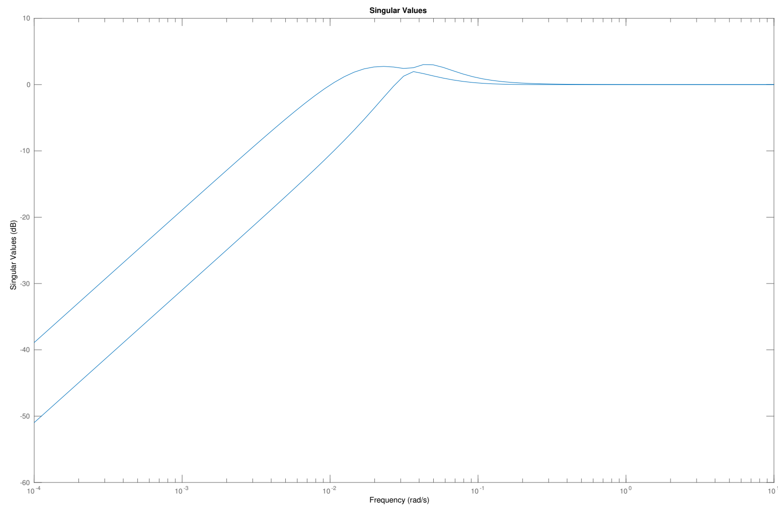

Figure 11: The singular values of the sensitivity S

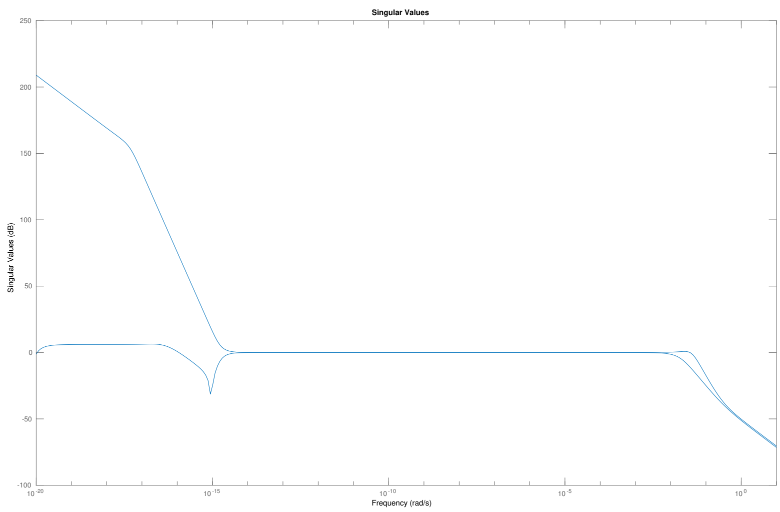

Figure 12: The singular values of the complementary sensitivity T

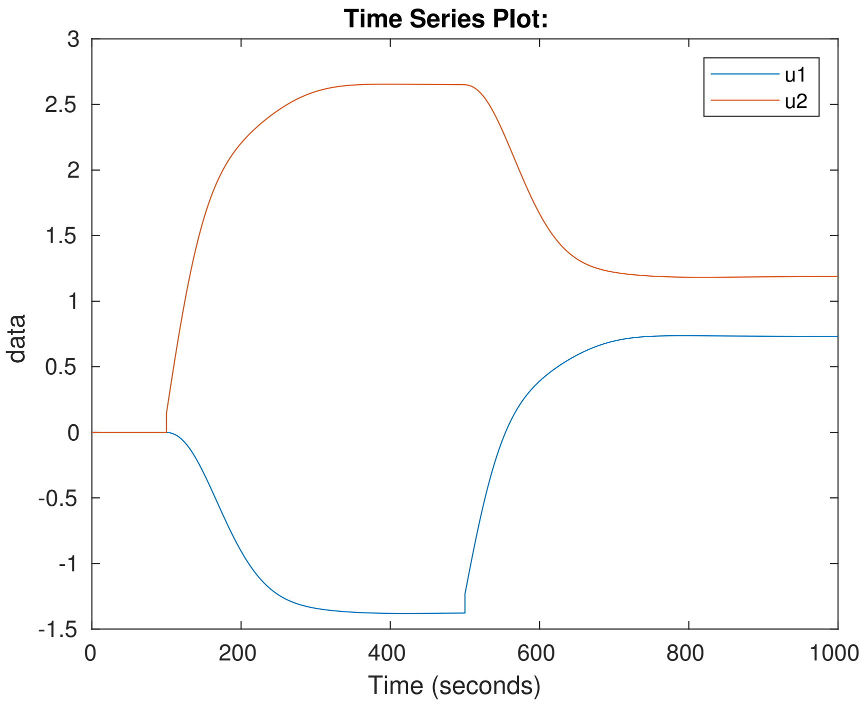

Figure 13: Responses of the control signal u

3.2.3

The responses of the control signals and outputs are shown in the Figure 13 and Figure 14
As shown in the figure, the outputs are coupled and it is still strongly coupled. When given a step response, the unrelated output will gain more impulse than minimum phase case, but it needs more time to be steady again. So we can conclude that the controller is not good enough.

3.2.4

The main difference between these two case is that one is minimum phase and one is non minimum phase, which means one is stable and another is unstable. The RGA for the minimum phase is 1.0131 at desired crossover frequency $w_c = 0.1rad/s$. It means the outputs are not strongly coupled. But the RGA for the non minimum phase is 2.1813 at desired crossover frequency $w_c = 0.02rad/s$. It means the outputs are more strongly coupled. So it is more difficult to build a good controller to decouple it.

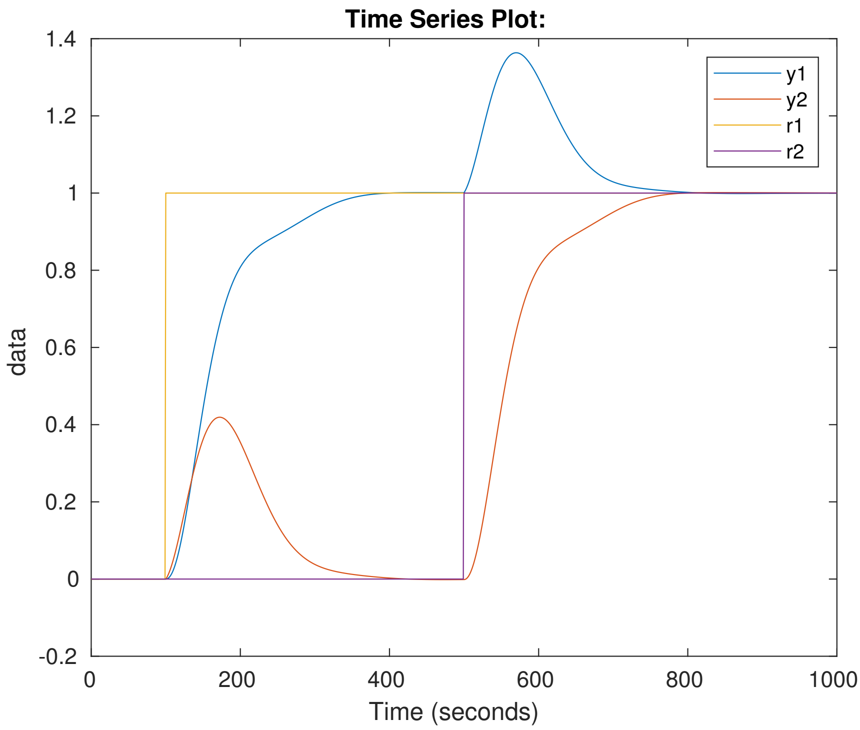

Figure 14: Responses of the output y with references r