Computer Exercise 1
EL2520 Control Theory and Practice

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1 Exercises

1.1 Basics

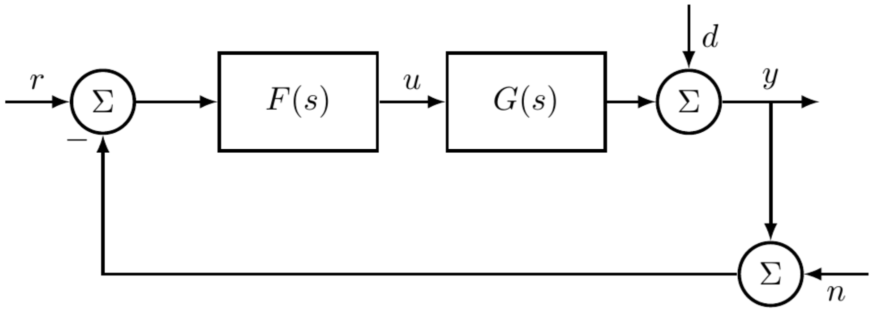

Figure 1: F-controller, G-system, r-reference signal, u-control signal, d-disturbance signal, y-output signal, n-measurement noise.

Consider a system which can be modeled by the transfer function

$$G(s) = \frac{3(-s+1)}{(5s+1)(10s+1)}$$

1. **Question:** Use the procedure introduced in the basic course to construct a lead-lag controller which eliminates the static control error for a step response in the reference signal.

$$F(s) = K \underbrace{\frac{\tau_D s + 1}{\beta \tau_D s + 1}}_{\text{Lead}} \underbrace{\frac{\tau_I s + 1}{\tau_I s + \gamma}}_{\text{Lag}}$$

The phase margin should be 30° at the cross-over frequency $\omega_c = 0.4$ rad/s.

Answer: For the closed-loop system, in order to have a phase margin $\phi_m = 30°$, a cross-over frequency $\omega_c = 0.4$ rad/s, and zero steady-state error for a step response in the reference signal, we consider a lead-lag controller F shown above, where K, τ_D, τ_I, β, and γ are parameters should be configured so that the closed-loop system satisfies the requirements.

For a step reference, the error is given by

$$E(s) = R(s) - Y(s) = \frac{1}{1+F(s)G(s)}R(s) = \frac{1}{1+F(s)G(s)}\frac{1}{s}$$

The steady-state error then can be obtained using Final Value Theorem:

$$e(\infty) = \lim_{t\to 0} e(t) = \lim_{s\to 0} sE(s) = \frac{1}{1+F(0)G(0)} = \frac{\gamma}{\gamma+3K}$$

To get zero steady-state error, $e(\infty)=0$, either $\gamma=0$ or $K\to\infty$. So, $\gamma=0$ is chosen here. Then we can design the lag controller. To minimize the phase lag caused by the lag compensator, we would obtain:

$$\tau_I = \frac{10}{\omega_c} = \frac{10}{0.4} = 25$$

then the lag component becomes to $F_I(s) = \frac{25s+1}{25s}$, and the phase margin of $F_I(s)G(s)$ is equal to

In all, we have our controller:

$$F(s) =$$

2. **Question:**

   Answer:

3. **Question:**

   Answer:

1.2 Disturbance attenuation

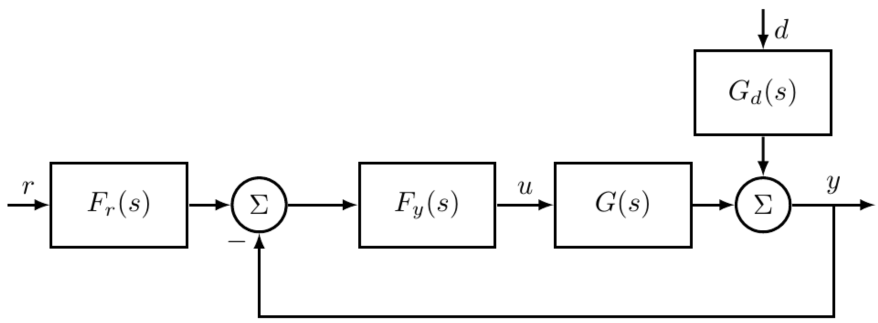

Figure 2: F_r-prefilter, F_y-feedback controller, G-system, G_d-disturbance dynamics, r-reference signal, u-control signal, d-disturbance signal, y-measurement signal.

The block diagram of the control system is given in fig 2, where the transfer functions have been estimated to

$$G(s) = \frac{20}{(s+1)((\frac{s}{20})^2+\frac{s}{20}+1)}$$

$$G_d(s) = \frac{10}{s+1}$$

1. **Question:** For which frequencies is control action needed? Control is needed at least at frequencies where $|G_d(j\omega)| > 1$ in order for disturbances to be attenuated. Therefore the cross-over frequency must be large enough. First, try to design F_y such that $L(s) \approx \omega_c/s$ and plot the closed-loop transfer function from d to y and the corresponding step response. (A simple way to find $L(s) = \omega_c/s$ is to let $F_y = G^{-1}\omega_c/s$. However, this controller is not proper. A procedure to fix this is to "add" a number of poles in the controller such that it becomes proper. How should these poles be chosen?)

   Answer: $\omega_c = 9.9473$ rad/s is the cross-over frequency of G_d so that $G_d(j\omega_c) = 1$. And since control is needed at least at frequencies where $|G_d(j\omega)| > 1$, $\omega \in [0, 9.9473]$ rad/s is the frequencies which control action needed.

   To design F_y such that $L_s \approx \omega_c/s$, a naive approach can be first considered:

$$\begin{aligned} F_y^*(s) &= G^{-1}(s)\omega_c/s \\ &= \frac{(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)\omega_c}{20s} \end{aligned}$$

   so that $L(s) = F_y^*(s)G(s) = \omega_c/s$. However, the relative degree of F_y^* is -2, meaning future information in time domain would be needed, which is impossible. So poles such be "added" to make the controller proper.

The feedback controller in exercise 4.2.2 is

$$F_y(s) = \ldots$$

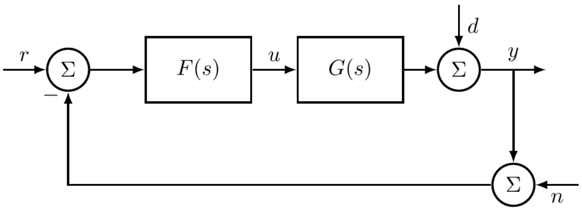

Figure 3: Step disturbance, exercise 4.2.2

The feedback controller and prefilter in exercise 4.2.3 is

$$F_y(s) = \ldots$$

$$F_r(s) = \ldots$$

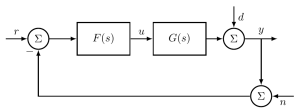

Figure 4: Reference step, exercise 4.2.3

Did you manage to fulfill all the specifications? If not, what do you think makes the specifications difficult to achieve?

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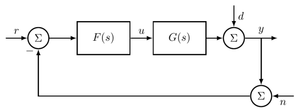

Figure 5: Control signal for a disturbance or a reference step (plus a combination of these)

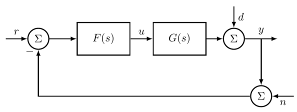

Figure 6: Bode diagram of sensitivity and complementary sensitivity functions, exercise 4.2.4