# Computer Exercise 1
# EL2520 Control Theory and Practice

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April 30, 2019

## Suppression of disturbances

The weight is

$$\begin{aligned} W_S(s) &= \frac{1}{(s+\epsilon+i\sqrt{\omega^2-\epsilon^2})(s+\epsilon-i\sqrt{\omega^2-\epsilon^2})} \\ &= \frac{1}{(s+0.5+i\sqrt{(100\pi)^2-0.5^2})(s+0.5-i\sqrt{(100\pi)^2-0.5^2})} \end{aligned}$$

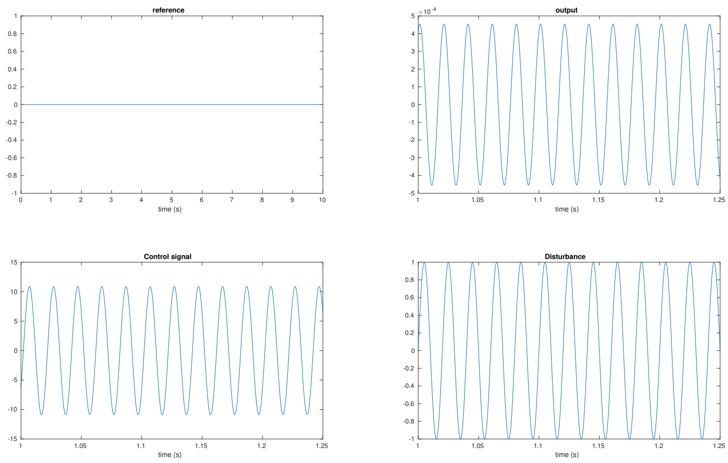

Figure 1: Simulation results with system $G$, using $W_S$.

- How much is the disturbance damped on the output?
  The rate between the output oscillations and the disturbance amplitude is $4.543 \times 10^{-4}$.
- What amplification is required for a P-controller to get the same performance, and what are the disadvantages of such a controller?
  Since $|FG| \gg 1$, we have:

$$\begin{aligned} |S| &\approx |FG|^{-1} \\ &= |FG(i100\pi)|^{-1} \\ &= (0.0920|F|)^{-1} \\ &= 4.543 \times 10^{-4} \end{aligned}$$

So, $|F| = \frac{1}{0.0920 \times 4.543 \times 10^{-4}} = 2.3926 \times 10^4$, which means the approximate amplification for a P-controller to get the same rate is $2.3926 \times 10^4$.

The disadvantage of such a P-controller is that it's not feasible to get controllers with such big amplification.

## Robustness

- What is the condition on $T$ to guarantee stability according to the small gain theorem, and how can it be used to choose the weight $W_T$?

  Since $G_0(s) = G(s)(1 + \Delta_G(s))$, $\Delta_G(s) = -\frac{3}{s+2}$ is obtained. According to the small gain theorem,

$$|T(i\omega) \cdot \Delta_G(i\omega)| < 1 \Leftrightarrow |T(i\omega)| < |\Delta_G^{-1}(i\omega)|$$

  Also, according to eq. (5), to ensure $|T(i\omega)| \leq \gamma |W_T^{-1}(i\omega)|$ fulfilled, we let $|\Delta_G^{-1}(i\omega)| \leq \gamma |W_T^{-1}(i\omega)|$, thus obtain $|W_T(i\omega)| \leq \gamma |\Delta_G(i\omega)|$. So, we can choose $|W_T(i\omega)| = \gamma |\Delta_G(i\omega)|$. Setting $\gamma$ to $10^{-4}$, the weights are

$$W_S(s) = \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$
$$W_T(s) = 10^{-4} \frac{3}{s+2}$$

- Is the small gain theorem fulfilled?

  According to fig 2, small gain theorem $|T(i\omega)| < |\Delta_G^{-1}(i\omega)|$ is always fulfilled.

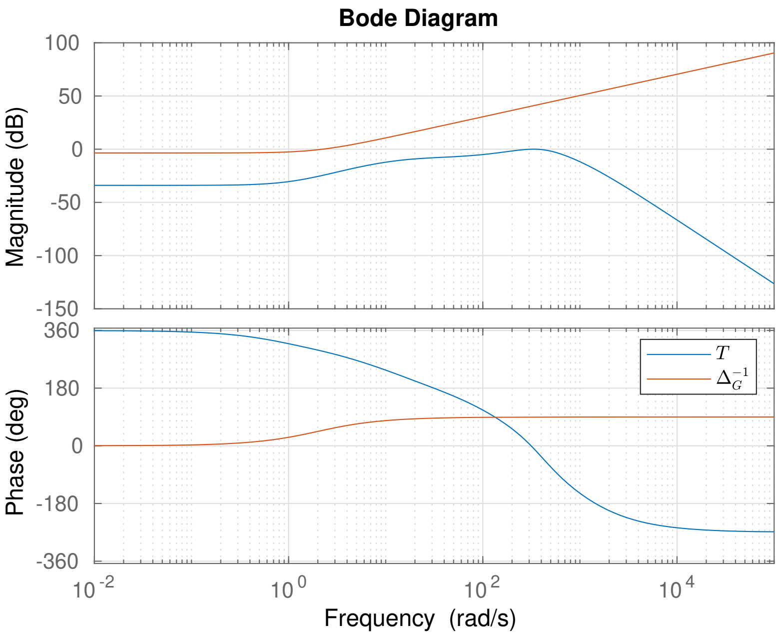


Figure 2: Bode diagram showing that the small gain theorem is satisfied.

- Compare the results to the previous simulation, which is illustrated in fig 1, the rate between the output oscillations and the disturbance amplitude increases to $3.743 \times 10^{-3}$.

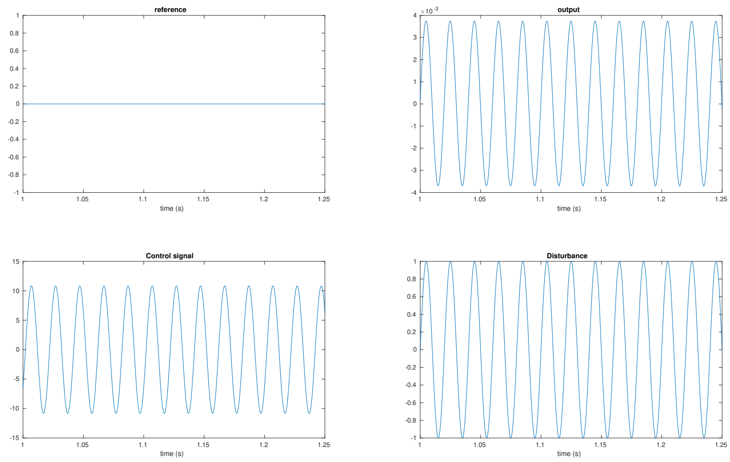


Figure 3: Simulation results with system $G_0$, using $W_S$ and $W_T$.

# Control signal

The weights are

$$W_S(s) = \frac{1}{(s + 0.5 + i\sqrt{(100\pi)^2 - 0.5^2})(s + 0.5 - i\sqrt{(100\pi)^2 - 0.5^2})}$$
$$W_T(s) = 10^{-4}\frac{3}{s+2}$$
$$W_U(s) = \frac{0.5}{s+2}$$

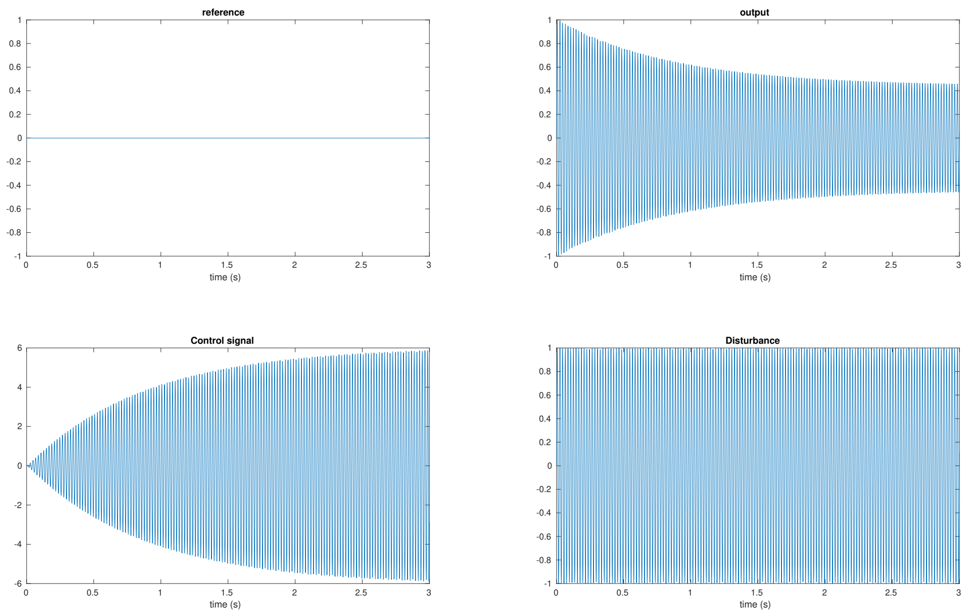


Figure 4: Simulation results with system $G_0$, using $W_S$, $W_T$ and $W_U$.

Compare the results to the previous simulations, ????