Computer Exercise 1
EL2520 Control Theory and Practice

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4 Exercises

4.1 Basics

4.1.1

Our system should fulfills two requirements.

1. Zero static control error
2. Phase margin of 30°at Cross frequency at 0.4 rad/s

To make the zero static control error, which means

$$\lim_{t\to\infty} e(t) = 0$$

Using Final Value Theorem, we get

$$\lim_{s\to 0} sE(s) = 0$$

where

$$E(s) = R(s) - Y(s) = R(s) - \frac{G(s)F(s)}{1+G(s)F(s)}R(s) = \frac{1}{1+G(s)F(s)}R(s)$$

Because $r(t)$ is a step response, so its Laplace transform is

$$R(s) = \frac{1}{s}$$

Finally, we get

$$\lim_{s\to 0}\frac{1}{1+F(s)G(s)} = \lim_{s\to 0}\frac{1}{1+\frac{3(-s+1)}{(5s+1)(10s+1)}K\frac{(\tau_D s+1)(\tau_I s+1)}{(\beta\tau_D s+1)(\tau_I s+\gamma)}} = \frac{\gamma}{3K+\gamma} = 0$$

So we can derive that

$$\gamma = 0$$

So we set $\gamma = 0.00001$

II

Then we design the lag controller first, to minimize the the phase lag caused by the lag compensator, we have

$$\tau_I = \frac{10}{w_c} = 25$$

Moreover, the phase lag caused by lag compensator (5∼12 for safety) is

$$\phi_{lag} = -5.71°$$

To satisfy the phase margin requirements, we get

$$\phi_m = 180° + \phi_{open-loop}$$

where

$$\phi_{open-loop} = \phi_G + \phi_{lead} + \phi_{lag}$$

we get

$$\phi_{lead} = 16.91°$$

Then, we can derive the coefficients using following equations

$$\sin \phi_{lead} = \frac{1-\beta}{1+\beta} \rightarrow \beta = 0.5493$$

$$\tau_D = \frac{1}{\sqrt{\beta} w_c} = 3.3731$$

$$|F(jw_c) * G(jw_c)| = 1 \rightarrow K = 2.1043$$

The parameter is listed in the Table 1 The bode plot for the lag-lead com-

K	β	τ_I	τ_D	γ
2.1043	0.5493	25	3.3731	0.00001

Table 1: Parameters for lag-lead compensator

pensator is shown in the Figure 1

4.1.2

The response behavior is listed in the Table 2 The step response for the

$Bandwidth$	$ResonancePeak$	$RiseTime$	$Overshoot$
0.76625	2.0825	2.3995	39.9626%

Table 2: response behavior for lag-lead compensator with step input

lag-lead compensator is shown in the Figure 2

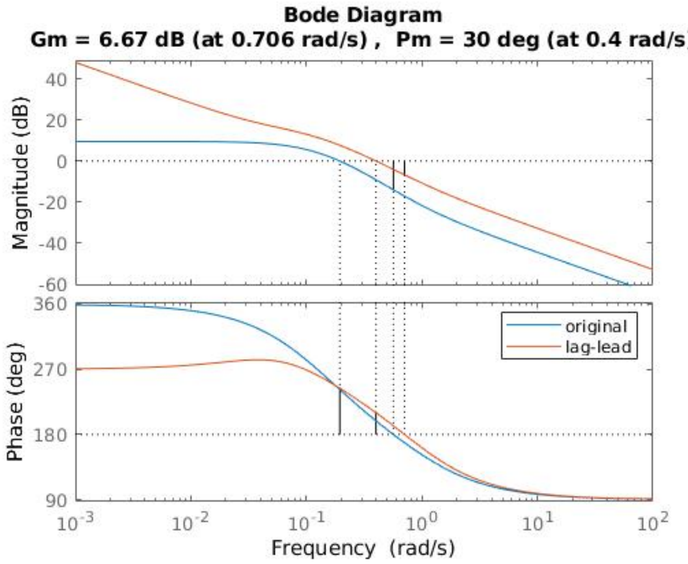

Figure 1: Bode Plot for the lag-lead compensator

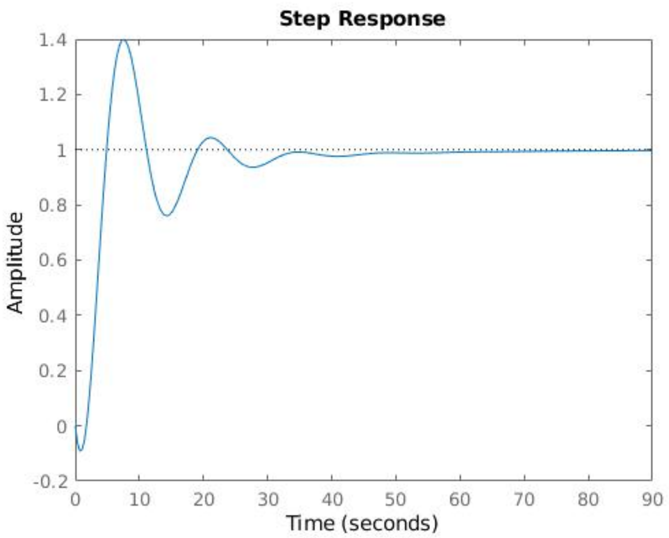

Figure 2: The step response for the lag-lead compensator $\phi_m = 30$

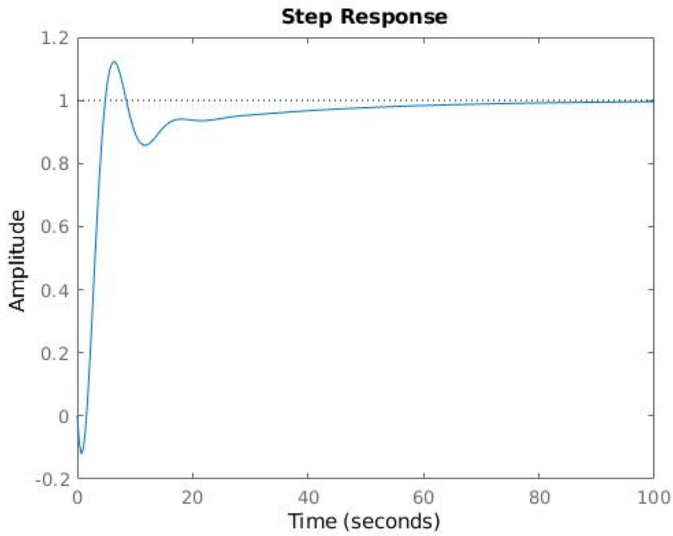

Figure 3: The step response for the lag-lead compensator with $\phi_m = 50$

4.1.3

The phase angle for lead controller is then

$$\phi_{lead} = 36.91° \leq 65°$$

which means it passes safety check. The response behavior is listed in the Table 3

$Bandwidth$	$ResonancePeak$	$RiseTime$	$Overshoot$
0.91213	1.2931	2.4257	12.3471%

Table 3: response behavior for lag-lead compensator with step input

The step response for the lag-lead compensator is shown in the Figure 3

4.2 Disturbance attenuation

4.2.1

The crossover frequency can be obtained from equation

$$|G(jw)| = 1 \rightarrow w_c = 9.95rad/s$$

If we design $F_y = G^{-1}\frac{w_c}{s} = \frac{w_c(s+1)((\frac{s}{20})^2+\frac{s}{20}+1)}{20s}$, it has higher number of zeros than higher number of poles. It means that we need information from

future time domain, which cannot be realized in the real world.
Simple Proof

$$\frac{U(s)}{E(s)} = F_y(s) = \frac{w_c(s+1)((\frac{s}{20})^2 + \frac{s}{20} + 1)}{20s} = O(s^2) + O(s) = O(s^2)$$

$$\xrightarrow{InverseLaplace} u(t) = e\,''(t) = \lim_{h \to 0} \frac{e(x+h) - 2e(x) + e(x-h)}{h^2}$$

Then we need to add poles to make the controller proper, the least poles we should add is 2. Moreover, we should consider the addition of poles effects on the response behavior.
To simplify the addition of poles, we add two poles and set $p_1 = p_2 = p$ and now open-loop transfer equation becomes $L(s) = \frac{p_1 p_2 w_c}{s(s+p_1)(s+p_2)}$. So the closed-loop transfer function $G_{cl} = \frac{G_d}{1+L}$ have characteristics equation

$$s^4 + (2p+1)s^3 + (2p+p^2)s^2 + p^2(w_c+1)s + p^2 w_c = 0$$

The system should be stable, so the poles of characteristic equation should be in the LHP. To satisfy such condition, we use Routh-Hurwitz Stability Criterion. Thus we get a sufficient condition that p should be positive and large. So we set $p = 100$ However, if the poles added is too large, the step response will be so fast so that the amplitude of step response will also be large. It is a trade-off. The bode plot of the open-loop transfer function and step response are shown in the Figure 4 and Figure 5

4.2.2

By testing several values (1 5 10 20) of w_I , the step response starts to satisfy the specification that "for a step in the disturbance, we have $|y(t)| \leq 1\ \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5s$" when $5 < w_I < 10$. So we choose $w_I = 8$ Thus

$$F_y = \frac{(s+10)((\frac{s}{20})^2 + \frac{s}{20} + 1)}{2s}$$

which is not proper because it needs information in the future time domain. To solve this, we add two poles so the new F_y is

$$F_y = \frac{p_1 p_2 (s+10)((\frac{s}{20})^2 + \frac{s}{20} + 1)}{2s(s+p_1)(s+p_2)}$$

The poles should also fulfills Routh-Hurwitz Stability criterion for $1+F_y G = 0$, so we use $p_1 = p_2 = 100$. The step response for the closed-loop transfer function is shown in the Figure 6

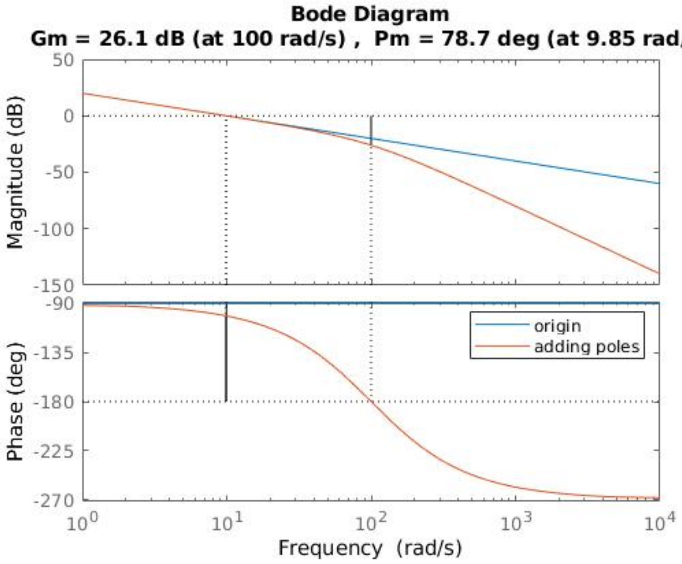

Figure 4: Bode Plot for open-loop transfer function with $F_y = G^{-1}\frac{w_c}{s}$ and $F_y = G^{-1}\frac{p_1 p_2 w_c}{s(s+p_1)(s+p_2)}$

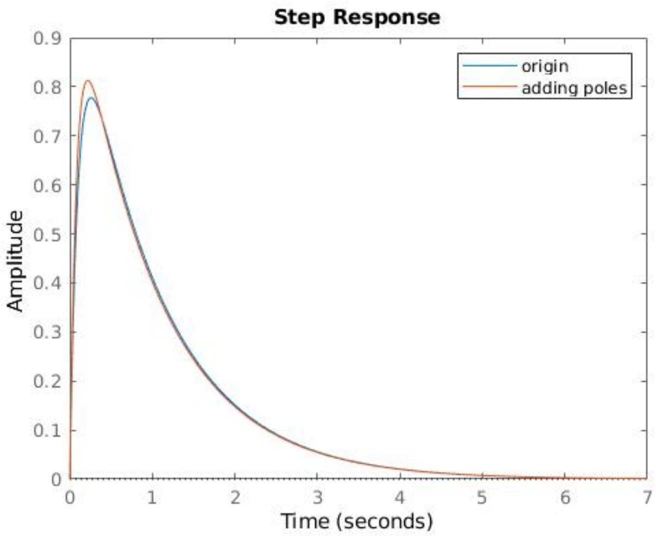

Figure 5: The step response for closed-loop transfer function with $F_y = G^{-1}\frac{w_c}{s}$ and $F_y = G^{-1}\frac{p_1 p_2 w_c}{s(s+p_1)(s+p_2)}$

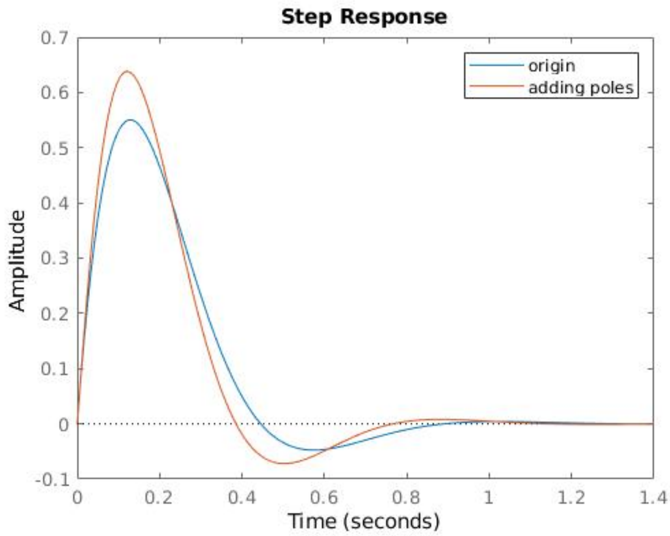

Figure 6: The step response for closed-loop transfer function with $F_y = \frac{s+w_I}{s} G^{-1} G_d$ and $F_y = \frac{p_1 p_2 w_c}{s(s+p_1)(s+p_2)} G^{-1} G_d$

4.2.3

Because there is no specification for lead compensator, we can design it by our own. We assume the lead compensator add 15° phase margin at crossover frequency $w_c = 15rad/s$(should be larger than 9.95 rad/s)

$$\sin \phi_{lead} = \frac{1-\beta}{1+\beta} \rightarrow \beta = 0.5888$$

$$\tau_D = \frac{1}{\sqrt{\beta} w_c} = 0.0869$$

$$|F_{lead}(jw_c) * F_y(jw_c) * G(jw_c)| = 1 \rightarrow K = 1.0407$$

If the prefilter is too complex, it could have some kind of effects on system behavior such as delay, change of rising time and so on. Because the crossover frequency $w_c = 9.95rad/s$ Then we assume the prefilter $F_r = \frac{1}{1+0.1s}$, which has a low pass frequency at $w = 10rad/s$
The step response of the reference is shown in the Figure 7
Then, we check the size of control signals. The step response of control signal of r and d is shown in the Figure 8

4.2.4

Check design specification:

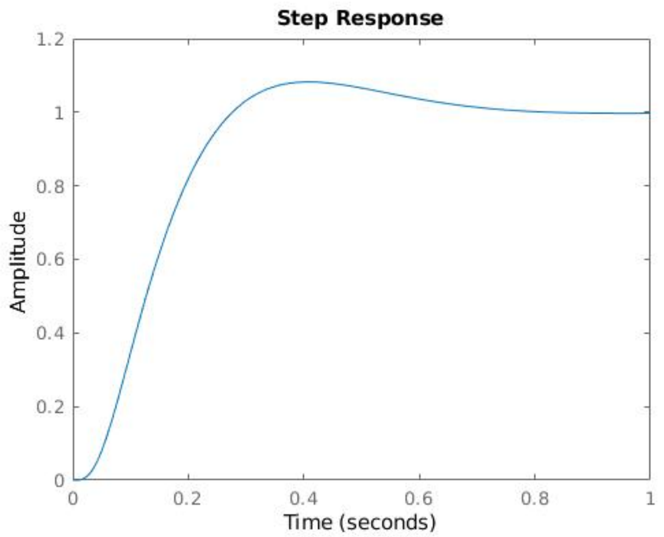

Figure 7: The step response of reference

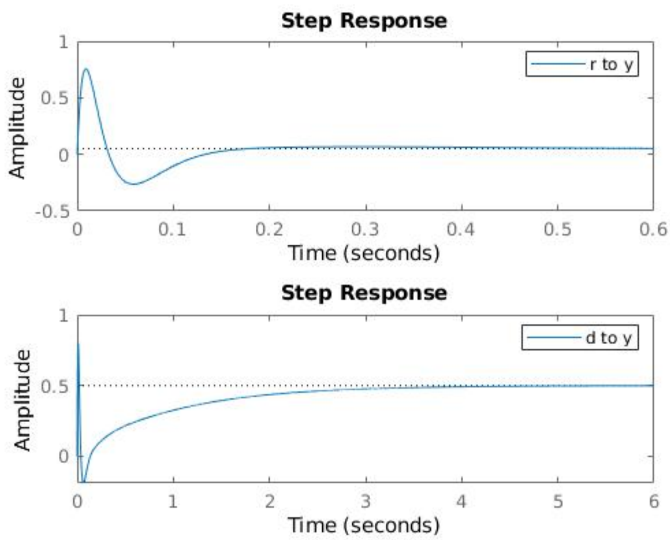

Figure 8: The step response of r and d

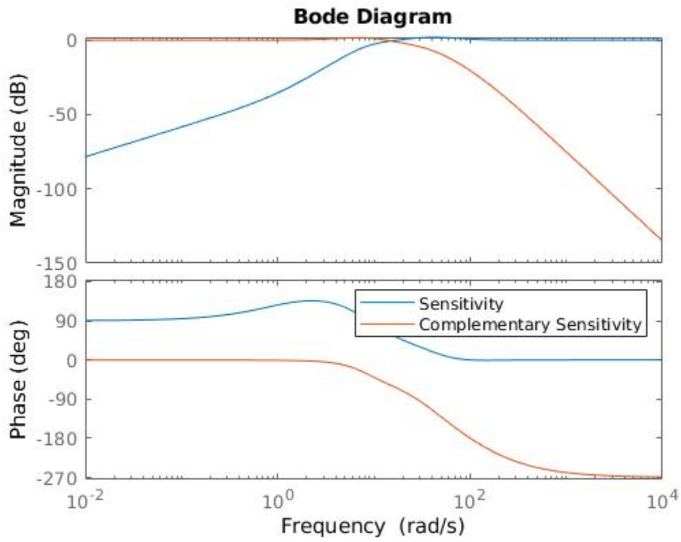

Figure 9: The bode diagrams of sensitivity and complementary sensitivity

1. The rise time for a step change in the reference signal is $0.17255s < 0.2s$ and the overshoot is $8.246\% < 10\%$.
2. For a step in the disturbance, we have $|y(t)| \leq 1 \forall t$ and $|y(t)| \leq 0.1$ for $t > 0.5s$
3. Control signals are scaled the control signal obeys $|u(t)| \leq 1 \forall t$

The bode diagrams of sensitivity and complementary sensitivity is shown in the Figure 9