AUTOMATIC CONTROL

KTH

EL2520 Control Theory and Practice – Advanced Course

Exam (*Tentamen*) 2011–05–23, kl 08.00–13.00

Aids:

The course book for EL2520 (advanced course) and EL1000/EL1110 (basic course) or equivalent if approved beforehand, copies of slides from this years lectures, mathematical tables and pocket calculator
Note that exercise material (*övningsuppgifter, ex-tentor och lösningar*) are NOT allowed.

Observe:

Do not treat more than one problem on each page

Each step in the solution must be motivated

Lacking motivation can result in point deductions

Write answer (with unit in relevant cases)

Write name and personal number on each page

Do only write on one side of each page

Mark the total number of pages on the cover

The exam consists of five problems of which each can give up to 10 points. The points for subproblems have been marked

Grading:

Grade A: ≥ 43, Grade B: ≥ 38

Grade C: ≥ 33, Grade D: ≥ 28

Grade E: ≥ 23, Grade Fx: ≥ 21

Results:

The results will be available 2011-06-14 at STEX, Studerandeexpeditionen, Osquldasv. 10.

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Good Luck!

1. (a) A system is described by the transfer-matrix

$$G(s) = \frac{1}{s+1}\begin{pmatrix} 1 & -s+1 \\ 2 & 1 \end{pmatrix}$$

i. Determine the poles and zeros (with multiplicities) of the system. How many states are needed in a state-space realization of the system? (3p)

ii. The system is to be controlled with a diagonal controller. Use the RGA to determine a proper pairing of inputs and outputs when the bandwidth should be around $\omega_B = 1\ rad/s$. (3p)

(b) Given the linear system

$$\dot{y} = y(t) + u(t)$$

Determine a controller such that the criterion

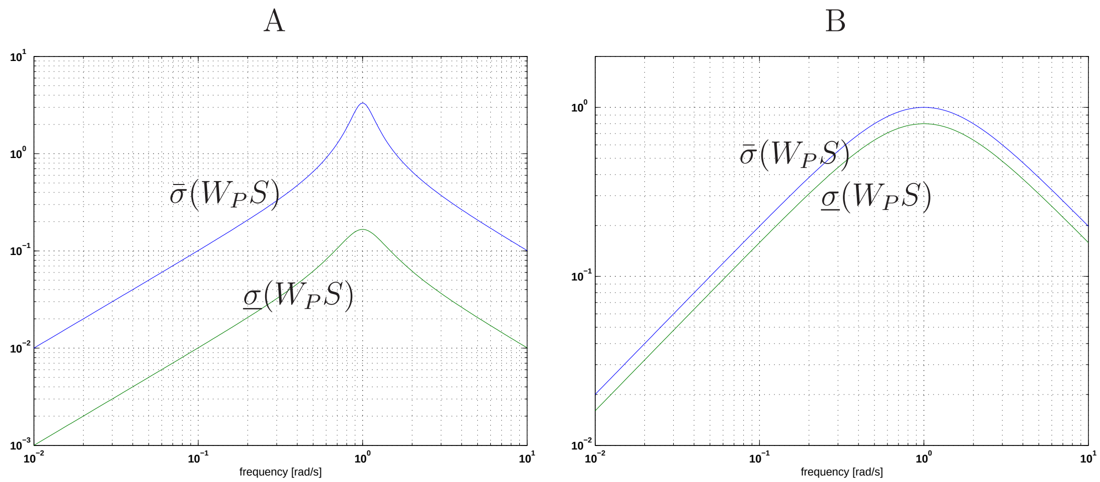
$$J = \int_0^\infty y^2(t) + \beta u^2(t) dt$$

is minimized for any arbitrary initial condition $y(0)$. What happens with the closed-loop pole as $\beta \to 0$ and $\beta \to \infty$, respectively? (4p)

2. (a) Two alternative controllers have been designed for a linear system based on minimizing the norm of the weighted sensitivity

$$\|W_P S\|_m$$

using H_2-optimal control ($m = 2$) and H_∞-optimal control ($m = \infty$), respectively. The resulting singular values of the weighted sensitivity functions are shown below.



Which of the two controllers, A and B, correspond to H_2- and H_∞-optimization respectively? Motivate! (3p)

(b) The system

$$G(s) = \frac{1}{10s+1}\begin{pmatrix} 1 & s \\ 2s+1 & 1 \end{pmatrix}$$

is to be controlled such that the response in the two outputs are completely decoupled.

i. A first suggestion is design a one-degree-of-freedom controller $F_y(s)$ such that the closed-loop transfer-function from setpoint to output becomes

$$Y(s) = \frac{1}{\lambda s + 1}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} R(s)$$

Is the resulting controller an appropriate choice? Motivate! (3p)

ii. Design a controller so that the closed-loop system is completely decoupled and has the same singular values from R to Y as the transfer-matrix in problem (i) and, furthermore, is internally stable. (4p)

3. (a) Consider the two systems below

i)

$$y = 2.5\frac{5s+1}{10s+1}u + \frac{3}{10s+1}d$$

ii)

$$y = 2.5\frac{-5s+1}{10s+1}u + \frac{3}{10s+1}d$$

Both systems have been scaled so that the aim of the control is to keep $|y| < 1$, the input is constrained so that $|u| < 1$ and the expected disturbance magnitude $|d| < 1$. Assume all signals to be sinusoidal and determine if acceptable control is possible in any of the two systems. If acceptable control is feasible, then determine the maximum measurement delay that can be tolerated while still maintaining acceptable control. (4p)

(b) The following specifications are given for a scalar closed-loop control system: disturbances d on the output y should be attenuated at least by a factor 10 for all frequencies up to $\omega = 0.5\ rad/s$ and at least by a factor 100 at steady-state. Measurement noise should be attenuated at the output by at least a factor 10 for frequencies above 1 rad/s.

i) Formulate requirements on the sensitivity S and complementary sensitivity T that satisfies the above requirements.

ii) Translate the requirements from i) into (approximate) requirements on the loop gain L. Draw a simple figure showing the constraints imposed on L as a function of frequency.

iii) Can it be difficult to achieve the specifications? Motivate.

iv) Determine a weight function w_T so that $\|w_T T\|_\infty < 1$ implies that the specifications on T are satisfied.

(6p)

4. (a) We shall consider Model Predictive Control of a system with a constrained input. The model is

$$\begin{aligned} x_{k+1} &= -2x_k + u_k \\ y_k &= 2x_k \end{aligned}$$

where $|u_k| < 1 \ \forall k$. The optimization problem solved in the MPC is

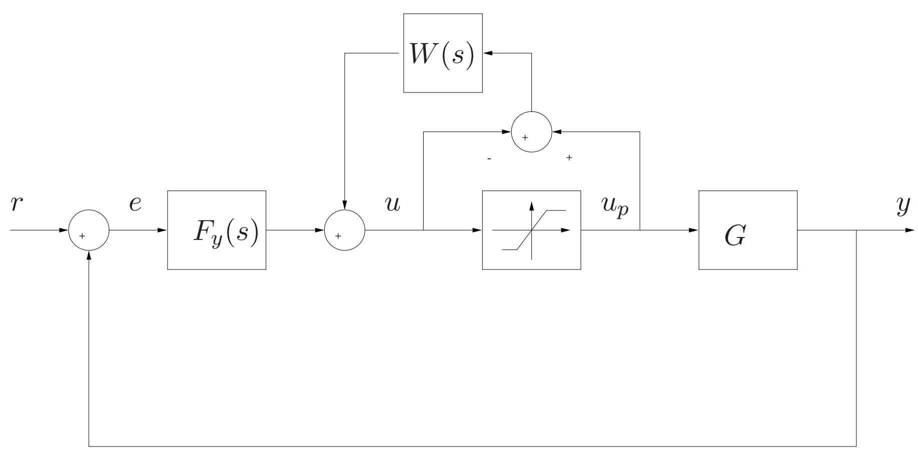
$$\min_u \left[\sum_{t=k}^{t=k+N_P} Q_y y_t^2 + \sum_{t=k}^{t=k+N_P} u_t^2 \right]$$

subject to $|u| < 1$. For the prediction and control horizon $N_P = 1$, translate the MPC problem into a Quadratic Programming (QP) problem

$$\min_u u^T H u + h^t u \ : \quad Lu \leq b$$

That is, determine H, h, L and b. (4p)

(b) Consider the block-diagram below for a feedback system with a constrained input and anti reset windup.



(The signs in the summations are such that $u = W(s)(u_p - u) + F_y(s)(r - y)$)

The transfer-functions are

$$G(s) = \frac{1}{s+1}; \quad F_y(s) = K \ ; \quad W(s) = \frac{1}{10s}$$

The constraint is such that $|u_p| < 1$.

Show that the gain of the saturation is 1 and derive a sufficient condition for stability of the closed-loop system. Motivate why the derived condition is not necessary. (6p)

5. A system with two inputs and two outputs has the transfer-function

$$G_p(s) = \frac{1}{100s+1}\begin{pmatrix} 1 & \epsilon \\ \epsilon & 3 \end{pmatrix}$$

(a) In the control design one decides to completely neglect the off-diagonal terms in $G_p(s)$, that is, the controller is designed based on the simplified model

$$G(s) = \frac{1}{100s+1}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

Determine the corresponding relative model uncertainty Δ_G when $G(s)$ is used as the nominal model and $G_p(s)$ should be within the uncertainty set. (2p)

(b) One decides to design a diagonal PI-controller

$$F(s) = \begin{pmatrix} K_1\frac{T_1 s+1}{T_1 s} & 0 \\ 0 & K_2\frac{T_2 s+1}{T_2 s} \end{pmatrix}$$

Determine the controller parameters so that the closed-loop system based on the simplified model gets two poles at $s=-1$. (2p)

(c) Derive a robustness criterion based on the small gain theorem and apply this to determine for what values of ϵ the closed-loop system with the controller from (b) can be guaranteed to be stable. (4p)

(d) For what values of ϵ will the closed-loop system be stable? Compare with the results from (c). (2p)