

Grading template for laboratory exercise 3

EL2820, Modeling of Dynamical Systems

September 2018

Chang, Hongsheng

951228-2451

Jiang, Sifan

961220-8232

	Pass	Fail	
The report is handed in on time?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Number of authors	$\leq 2^{st}$ <input type="checkbox"/>	> 2 <input type="checkbox"/>	
Author names and personal identity number filled out?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The report is well structured? The language is understandable?	yes <input type="checkbox"/>	often <input type="checkbox"/>	sometimes <input type="checkbox"/> no <input type="checkbox"/>
The figures are clear? (Captions, high resolution, etc.)	yes <input type="checkbox"/>	often <input type="checkbox"/>	sometimes <input type="checkbox"/> no <input type="checkbox"/>
The preparation task is solved and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The working region is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The sampling time is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A detailed description of the input signal is given and the choice is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The amount of data used for estimation and validation is specified?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Models of three different model structures have been estimated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The model order of each model is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A top 4 ranking of the estimated models have been made?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The best and worst models have been compared using Bode plots, and analyze through pole/zeros and correlation(residuals) analysis using (compare) and correlation analysis(resid)?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
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1 Preparation task

- Derivation of a physical model of the magnetic levitator in state-space form:

From figure in the lab description, we can get:

$$m\ddot{z} = \gamma\dot{z} + E_r - F_{ul} - mg \quad (1)$$

$$m\ddot{y} = \gamma\dot{y} - E_a + F_{lu} - mg \quad (2)$$

Repulsive magnet force is proportional to $m|y - z|^{-4}$, and here, constant T is given to represent the proportion:

$$F_{ul} = F_{lu} = Tm \frac{1}{(y - z)^4} \quad (3)$$

From exercise 2.5, the electromagnetic force can be obtained, where K is a constant and A is the area of the coil:

$$B = \mu_0 \frac{N\gamma^2 I(t)}{2y^3} \quad (4)$$

$$E = \frac{1}{2} A \frac{B^2(t)}{\mu_0} \quad (5)$$

$$E = \frac{AN^2\mu_0\gamma^4}{8} \frac{I^2(t)}{y^6(t)} = Km \frac{I^2(t)}{y^6(t)} \quad (6)$$

$$\text{So: } E_a = Km \frac{I^2(t)}{y^6(t)} \quad E_r = Km \frac{I^2(t)}{z^6(t)} \quad (7)$$

Then we can obtain state space equations:

$$\dot{a} = \frac{\gamma}{m} \dot{z} + K \frac{I^2}{z^6} - T \frac{1}{(y - z)^4} - g \quad (8)$$

$$\dot{b} = \frac{\gamma}{m} \dot{y} + K \frac{I^2}{y^6} - T \frac{1}{(y - z)^4} - g \quad (9)$$

$$\dot{z} = a \quad (10)$$

$$\dot{y} = b \quad (11)$$

- Suggestion for a suitable model order for a linear model:

In stationarity, we have:

$$x = \begin{bmatrix} a \\ b \\ z \\ y \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{z} \\ \dot{y} \end{bmatrix} = 0 = \begin{bmatrix} f_1(a, b, z, y, I_0) \\ \vdots \\ f_4(a, b, z, y, I_0) \end{bmatrix} \quad (12)$$

The linearized system is given by:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}I(t) \quad (13)$$

$$\mathbf{Y} = \mathbf{C}\mathbf{X} + D \quad (14)$$

$$\mathbf{A} = \left[\begin{array}{ccc|c} \frac{\partial f_1}{\partial a} & \cdots & \frac{\partial f_1}{\partial y} & \\ \vdots & \ddots & \vdots & \\ \frac{\partial f_4}{\partial a} & \cdots & \frac{\partial f_4}{\partial y} & \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \left[\begin{array}{ccc|c} \frac{\gamma}{m} & 0 & -\frac{4T}{(y^0-z^0)^5} - \frac{6KI^2}{z^{07}} & -\frac{4T}{(y^0-z^0)^5} \\ 0 & \frac{\gamma}{m} & \frac{4T}{(y^0-z^0)^5} & -\frac{4T}{(y^0-z^0)^5} + \frac{6KI^2}{y^{07}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right] \quad (15)$$

$$\mathbf{B} = \left[\begin{array}{c} \frac{\partial f_1}{\partial I} \\ \vdots \\ \frac{\partial f_4}{\partial I} \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \left[\begin{array}{c} \frac{2KI}{z^{06}} \\ -\frac{2KI}{z^{06}} \\ 0 \\ 0 \end{array} \right] \quad (16)$$

$$\mathbf{C} = [0 \ 0 \ 0 \ 1]^T \quad \mathbf{D} = [0 \ 0 \ 0 \ 0]^T \quad (17)$$

Then the transform function is given as, where \mathbf{I} is the identity matrix and N_i^a is a polynomial of i degree with label a on nominator:

$$G(q) = C^T(q\mathbf{I} - A)^{-1}B + D \quad (18)$$

$$= \frac{N_3^a(q)}{D_4^a(q)} + \frac{N_0^b(q)}{D_4^b(q)} \quad (19)$$

Approximating $G(q)$ with $D_4^a(q) = D_4^b(q)$, the transfer function would be $G(q) = \frac{N_3(q)}{D_4(q)}$. Plus, since our sampling time would be $2ms$ and the delay time would be $9ms$ in our experiment, base on the equation $n_k = \lfloor \frac{\tau}{h} \rfloor + 1$ from the lecture slides, we get $n_k = 5$. In all, our suggestion of the model order for ARX model could be ARX(4 3 5). Then the ARMAX and BJ models can be modified accordingly.

- MATLAB codes for the required functions are attached in the page 6.
- Plot for the spectrum of the binary random signal for the required values of α is shown in Fig. 1.

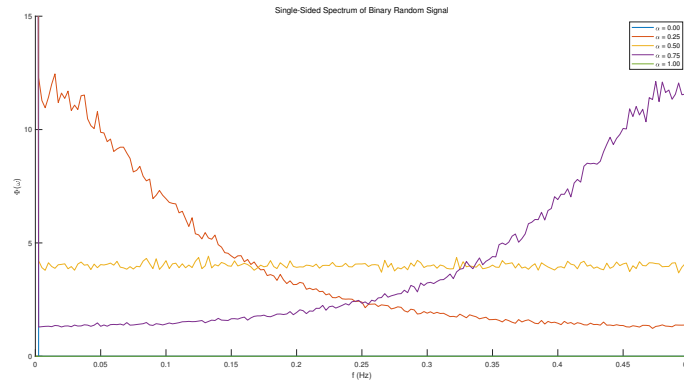


Figure 1: Spectrum of binary random signal.

2 Working region

- After the experiment, we choose working region as $1.5A \sim 3.6A$.
- Motivation for working region:
From Fig. 2, we can see that when $u \in [1.5A, 3.6A]$, the plot have linearity.
- Plot for illustrating working region is shown in Fig. 2.

3 Sampling time

- In the experiment, we chose $T_s = 2ms$ as sampling time.
- Motivation for sampling time:
Fig. 3 is obtained with the sampling time of $1ms$, and we can see the rise time is about $13ms$, which means there are about 13 points in one rise time. According to the requirement of “the sampling time is set to give around four to ten samples per rise time” in the lab description, we choose $2ms$ as sampling time so that there will be 6 to 7 samples per rise time.
- Plot of step responses yielding sampling time is shown in Fig.3.

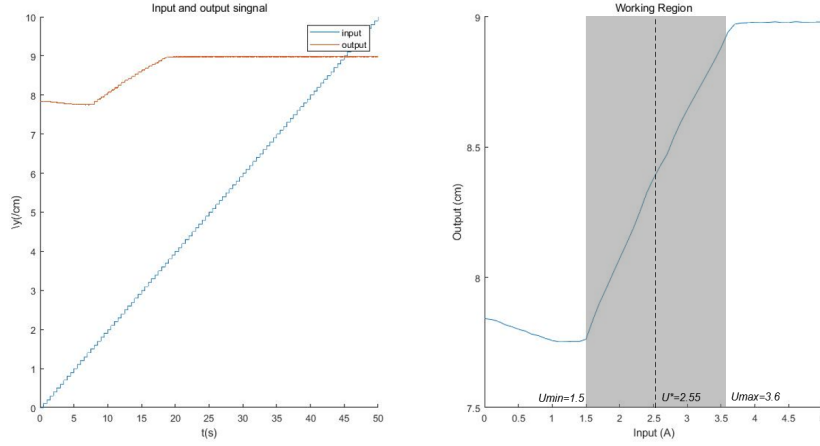


Figure 2: Working region.

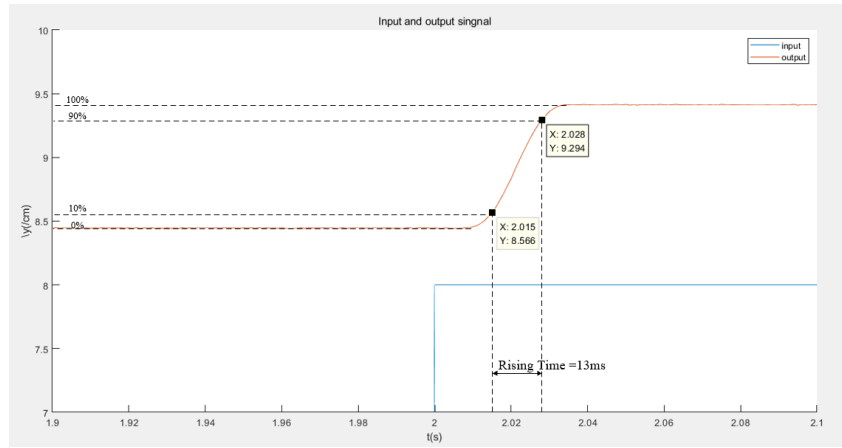


Figure 3: Step Response.

4 Input signals

- The binary random signal we choose have the similar output spectrum as the one of the white noises, which can be seen from Fig. 1 and Fig. 4.

- Motivation for chosen input signals:

Comparing Fig. 1 and Fig. 4, when $\alpha = 0.25$, the spectrum of binary random signal is similar to the spectrum of white noise.

5 Estimation and validation data

Information about the amount of data used for identification and validation:

Table 1: Data used for identification and validation.

Input signal	Value (A)	Amount of data	Estimation (s)	Validation (s)
Uniform white noise	[2, 3]	10000	[0, 10]	[10, 20]
Binary random signal with $\alpha = 0.25$	[2, 3]	10000	[0, 10]	[10, 20]

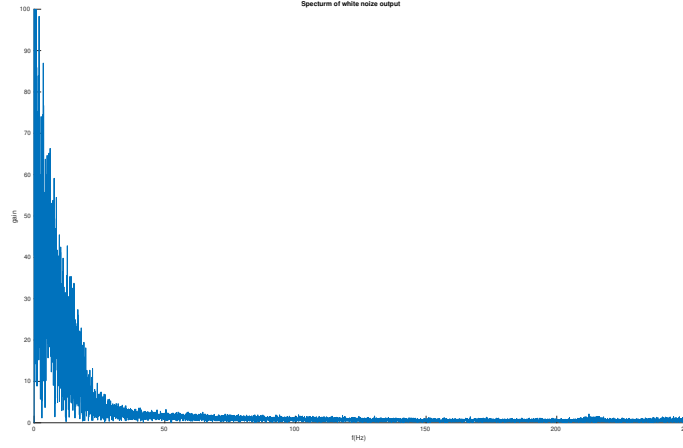


Figure 4: Spectrum of white noise.

6 Models

6.1 Model Description

- ARX model: $A(q)y[t] = B(q)u[k] + e[k]$, performs a filtering on both input and error with $\frac{1}{A}$ and adjust the transform function with B.
- ARMAX model: $A(q)y[t] = B(q)u[k] + C(q)e[k]$, is more generalized than ARX model with error being adjusted with C.
- BJ model: $y[k] = \frac{B(q)}{F(q)}u[k] + \frac{C(q)}{D(q)}e[k]$, includes all special cases.

6.2 Motivation of Model Orders

- ARX model: As mentioned in the “Preparation task” section, ARX(4 3 5) is first chosen for both white noise and binary random signal. In the case of white noise, it performs well enough. In the case of binary random signal, ARX(5 3 5) is chosen for obvious performance improvement.
- ARMAX model: Based on ARX model, we try different values on n_c and finally choose ARMAX(4 3 4 5) since performance drops when $n_c > 4$. For same reason, we choose ARMAX(5 3 2 5) for binary random signal.
- BJ model: Based on ARMAX model, we get BJ(3 4 4 4 5) for white noise and BJ(3 2 5 5 5) for binary random signal since we don’t need to change the order for each polynomial in each transform function.

6.3 White Noise Signal Identification

6.3.1 Plots and Analysis

6.3.2 Validation and Correlation Analysis

6.4 Binary Random Signal Identification

6.4.1 Plots and Analysis

6.4.2 Validation and Correlation Analysis

6.5 Ranking and Comparison

- validation performed with simulation (compare with $M=\inf$) and correlation analysis (resid).
- analysis/interpretations of validation and correlation analysis.
- a comparison between the accuracy of models obtained with different input signals (binary random signals v/s uniformly distributed white noise)
- A ranking of estimated models with respect to how well they describe the process, along with a rigorous motivation of chosen ranking. (Compare the analysis of plots, validation and correlation analysis for the different models.)

Table 2: Ranking of the Estimated Models with 1-step Prediction

Rank	Validation Data		
	White Noise 1	White Noise 2	Binary Random Signal
1	ARMAX(4 3 4 5) WH2: 89.33%	BJ(3 4 4 4 5) WH1: 92.15%	BJ(3 2 5 5 5) BR: 98.54%
2	BJ(3 4 4 4 5) WH2: 89.11%	ARMAX(4 3 4 5) WH1: 91.90%	ARMAX(5 3 2 5) BR: 98.53%
3	ARX(4 3 5) WH2: 88.91%	BJ(3 4 4 4 5) WH2: 91.73%	ARX(5 3 5) BR: 98.36%
4	BJ(3 4 4 4 5) WH1: 88.80%	ARMAX WH2: 91.71%	ARMAX(4 3 4 5) WH1: 97.93%

Attachments

generateBinarySignal.m

```
1 function u = generateBinarySignal(alpha, lims, N)
2     %% Initialization
3     w = 2 * (rand(N,1) > alpha) - 1;
4     u = zeros(N,1);
5
6     %% Initial value of u
7     if w(1) == -1
8         u(1) = lims(2);
9     else
10        u(1) = lims(1);
11    end
12
13    %% Rest values of u
14    for i = 2 : N
15        if w(i) == -1
16            if u(i-1) == lims(1)
17                u(i) = lims(2);
18            else
19                u(i) = lims(1);
20            end
21        else
22            u(i) = u(i-1);
23        end
24    end
25 end
```

getAverage.m

```
1 function bar_v = getAverage(v, tail)
2     bar_v = mean(v(length(v)*tail/100 + 1 : end));
3 end
```

getStationaryAverages.m

```
1 function bar_y = getStationaryAverages(y_step, Nwr, tail)
2     wrLen = length(y_step)/Nwr;
3     for i=1:Nwr
4         bar_y(i) = getAverage(y_step((i-1)*wrLen+1:(i)*wrLen),tail);
5     end
6 end
```

findWorkingRegion.m

```
1 close all; clear all; clc;
2 %% Initialization
3 Ts = 0.001;
4 t = 0;
5 i = 1;
6 maxCurrent = 10;
7 stairHeight = 0.1;
8 stairDuration = 0.2;
9
10 %% Create Stair Signal
11 stairSignal(i) = 0;
12 while true
```

```

13     t = t + Ts;
14     i = i + 1;
15     if t < stairDuration
16         stairSignal(i) = stairSignal(i-1);
17     else
18         if stairSignal(i-1) + stairHeight > maxCurrent
19             break
20         end
21         stairSignal(i) = stairSignal(i-1) + stairHeight;
22         t = 0;
23     end
24 end
25 t = 0 : Ts : (length(stairSignal)-1) * Ts;
26 t = t.';
27 stairSignal = stairSignal.';
28
29 %% Plot Stair Signal and Time
30 figure
31 plot(t, stairSignal)
32 xlabel('Time (s)')
33 ylabel('Stair Input (A)')
34
35 %% Input Signal Amplitudes
36 averagedU = getStationaryAverages(stairSignal, (maxCurrent/stairHeight+1),
    0.6);
37
38 %% Conducting Experiment
39 y = getData(stairSignal, Ts);
40
41 %% Averaged Output Signal
42 averagedY = getStationaryAverages(y, (10/stairHeight+1), 0.6);
43
44 %% Plot to Find Working Region
45 figure
46 plot(averagedU, averagedY);
47 xlabel('Input (A)')
48 ylabel('Output (cm)')

```