Grading template for laboratory exercise 3
EL2820, Modeling of Dynamical Systems
August 2017

Lihao Guo

961212-T054

Ching-an Wu

9210110632

	Pass		Fail	
The report is handed in on time?		yes	no	
Number of authors		$\leq$ 2st	$>$ 2	
Author names and personal identity number filled out?		yes	no	
The report is well structured? The language is understandable?	yes	often	sometimes	no
The figures are clear? (Captions, high resolution, etc.)	yes	often	sometimes	no
The preparation task is solved and motivated?		yes	no	
The working region is defined and motivated?		yes	no	
The sampling time is defined and motivated?		yes	no	
A detailed description of the input signal is given and the choice is motivated?		yes	no	
The amount of data used for estimation and validation is specified?		yes	no	
Models of more than one model structure have been estimated?		yes	no	
The model order of each model is motivated?		yes	no	
A ranking of the estimated models have been made?		yes	no	
The ranking is well motivated according to the requirements?		yes	no	

	Pass	Fail	
First review			Sign:
Second review (if failed in the first review)			Sign:

Pass

Signature:

Notion: due to mistakes in our experiments, all original data used is borrowed from other group with the permission of TA.

1 Preparation Task

1.1 Physical Model

Given the setting, neglecting induced voltage, the model is of form:

$$\frac{dy}{dt} = \frac{p_y}{m_y} \tag{1}$$
$$\frac{dp_y}{dt} = -g - \gamma_y \frac{p_y}{m_y} - E_a + F_{lu} \tag{2}$$
$$\frac{dz}{dt} = \frac{p_z}{m_z} \tag{3}$$
$$\frac{dp_z}{dt} = -g - \gamma_z \frac{p_z}{m_z} + E_r + F_{ul} \tag{4}$$

As $z >> r_{disc}$ and $y - z >> h_{disc}$, therefore:

$$E_a = \alpha \frac{i^2}{y^6} \tag{5}$$
$$E_r = \alpha \frac{i^2}{z^6} \tag{6}$$
$$F_{ul} = F_{lu} = \beta \frac{1}{(y-z)^4} \tag{7}$$

The linearized model around working state is, expressed with matrice of state-space form:

$$A = \begin{bmatrix} 0 & \frac{1}{m_y} & 0 & 0 \\ 6\alpha \frac{i_0^2}{y_0^7} - 4\beta \frac{1}{(y_0-z_0)^5} & -\frac{\gamma_y}{m_y} & 4\beta \frac{1}{(y_0-z_0)^5} & 0 \\ 0 & 0 & 0 & \frac{1}{m_z} \\ 4\beta \frac{1}{(y_0-z_0)^5} & 0 & -6\alpha \frac{i_0^2}{z_0^7} - 4\beta \frac{1}{(y_0-z_0)^5} & -\frac{\gamma_z}{m_z} \end{bmatrix} B^T = \begin{bmatrix} 0 & -2\alpha \frac{i_0}{y_0^6} & 0 & 2\alpha \frac{i_0}{z_0^6} \end{bmatrix}$$

$$C^T = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \; D^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

Given the matrice, tranform function is solved as(P_k^m is a polynomial of degree k with label m, e.g. $N_2^a(q)$ is a polynomial of degree 2 with label a):

$$G(q) = C^T (qI - A)^{-1} B + D \tag{8}$$
$$= \frac{N_2^a(q)}{D_4^a(q)} + \frac{N_0^b(q)}{D_4^b(q)} \tag{9}$$

Approximating $G(q)$ with $D_4^a(q) = D_4^b(q)$, the transfer function would be $G(q) = \frac{N_2(q)}{D_4(q)}$, which could be modeled with an ARX(4 3 2) model theoretically. ARX could be enhanced with ARMAX then.

1.2 Required Functions

1. Binary Signal

```
function [ u ] = generateBinarySignal( alpha, lims, N )
ws = (rand(N, 1) > alpha) * 2 - 1;% w
s(1) = 1;% s0
for i = 2:N
    s(i) = s(i-1) * ws(i);% s
end
limsList = num2cell(lims);
[lower, upper] = limsList{:};
u = s * (upper - lower)/2 + (upper + lower)/2;% u
end
```

2. Get Average

```
function [ avg ] = getAverage( v, tail )
avg = mean(v(length(v)-tail+1:end));
end
```

3. Get Stationary Averages

```
function [ bar_y ] = getStationaryAverage( y_step, Nwr, tail )
stepWidth = length(y_step) / Nwr;
bar_y(Nwr) = 0;
for i = 1:Nwr
  index = stepWidth*(i-1)+1:stepWidth*i;
  bar_y(i) = getAverage(y_step(index), stepWidth*tail);
end
end
```

1.2.1 Spectrum of Binary Random Signal

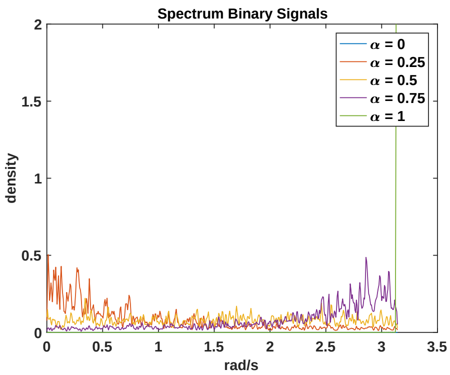


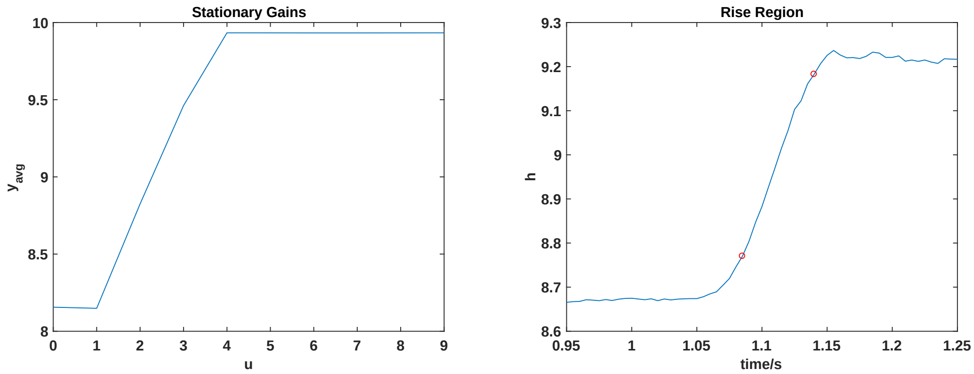
Figure 1: Spectrum of Binary Signal

2 Working Region

10 uniformly distributed points, each lasting 0.25s, in region $[0, 10)$ are experimented. As Fig.2a shows, the system's working region can be chosen as $[2, 3]$.

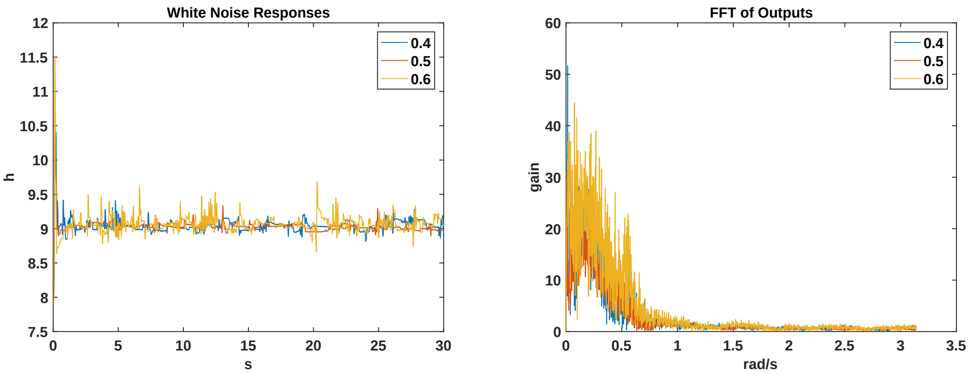
3 Sampling Time

Initialized sampling time is 5ms, experiment of the step response in working region($u = 2, t < 1s$ and $u = 3, t > 1s$) is enlarged in Fig.2b. To sample enough points(4 to 10) within rise time(55ms), the sampling time can be chosen as 10ms.



(a) Working Region Data (b) Enlarged Rise Region

Figure 2: Working Region and Sampling Time



(a) White Noise Responses (b) FFT of Output

Figure 3: Input Signal Selection

4 Input Signals

Initialized test signal is white noise signals around working point 2.5 with three different amplitudes [0.4, 0.5, 0.6], experiment result is shown in Fig.3a. To have the largest data-noise ratio while having a relatively smooth fft, the amplitude should be chosen as 0.5. Comparing Fig.3b with Fig.1, α could be chosen as 0.25 to put most energy in the system's frequency band.

5 Estimation and Validation Data

Collected data and usage is shown in Tab.1.

Table 1: Collected Data and Usage

Type	Value	Estimation	Validation
Uniform White Noise	[2 3]	0 ~15s	15 ~30s
Binary Random($\alpha = 0.25$)	{2 3}	0 ~15s	15 ~30s

6 Models

6.1 Model Descriptions

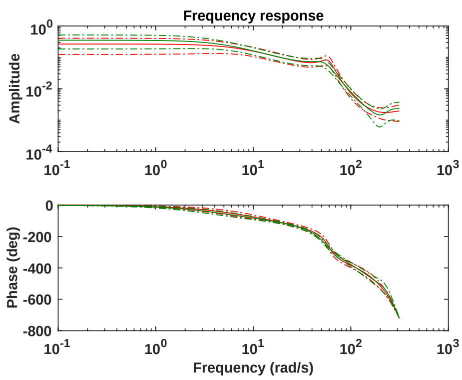
- ARX model $Ay = Bu + e$ performs a filtering on both input and error with $\frac{1}{A}$ and adjust the the transform function with B.
- ARMAX model $y = \frac{B}{A}u + \frac{C}{A}e$ is more generalized as error can be adjusted with C as well.

6.2 White Noise Signal Identification

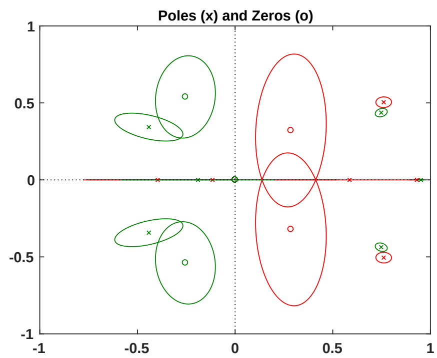
6.2.1 Degree Selection

Starting with ARX model, the initialized model order is ARX(4 3 2) as described in 1.1. By increasing order, ARX(6 3 4) is a proper choice as performance improves little with higher order. An enhanced ARMAX(6 3 2 4) is chosen, similarly, as the performance drops when $n_c > 2$.

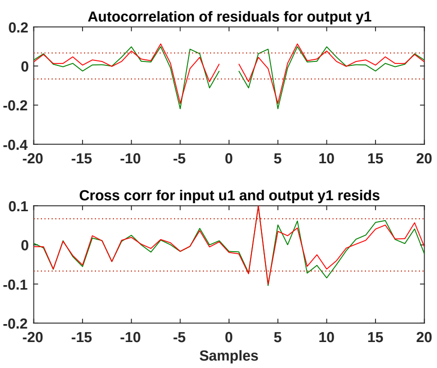
6.2.2 Analysis of Plots



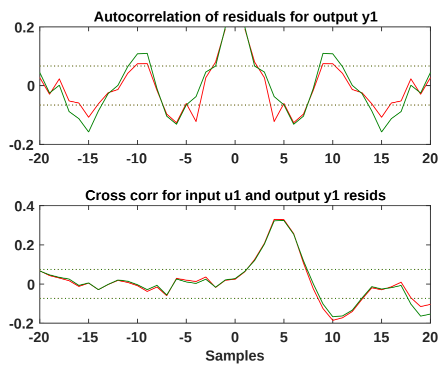
(a) Bode Plot



(b) Zero-Pole Plot



(c) Correlation on White Noise Validation



(d) Correlation on Random Binary Validation

Figure 4: Uniform White Noise Signal Identification

In Fig.4, green represents ARX(6 3 4) and red represents ARMAX(6 3 2 4).

According to Bode plot in Fig.4a, there is a resonance at around frequency 60 rad/s. As the input is uniform white noise signal, the variance doesn't change much on all frequencies. By comparison, ARMAX(6 3 2 4) has higher gains around resonance frequency than ARX(6 3 4).

According to Zero-Pole plot in Fig.4b, there are pairs of conjugate poles which contribute to the peak in Bode plot. Both models have large uncertainty on conjugate zeros. Cancellation of poles with zeros is more significant in ARX[6 3 4] which explains the lower peak.

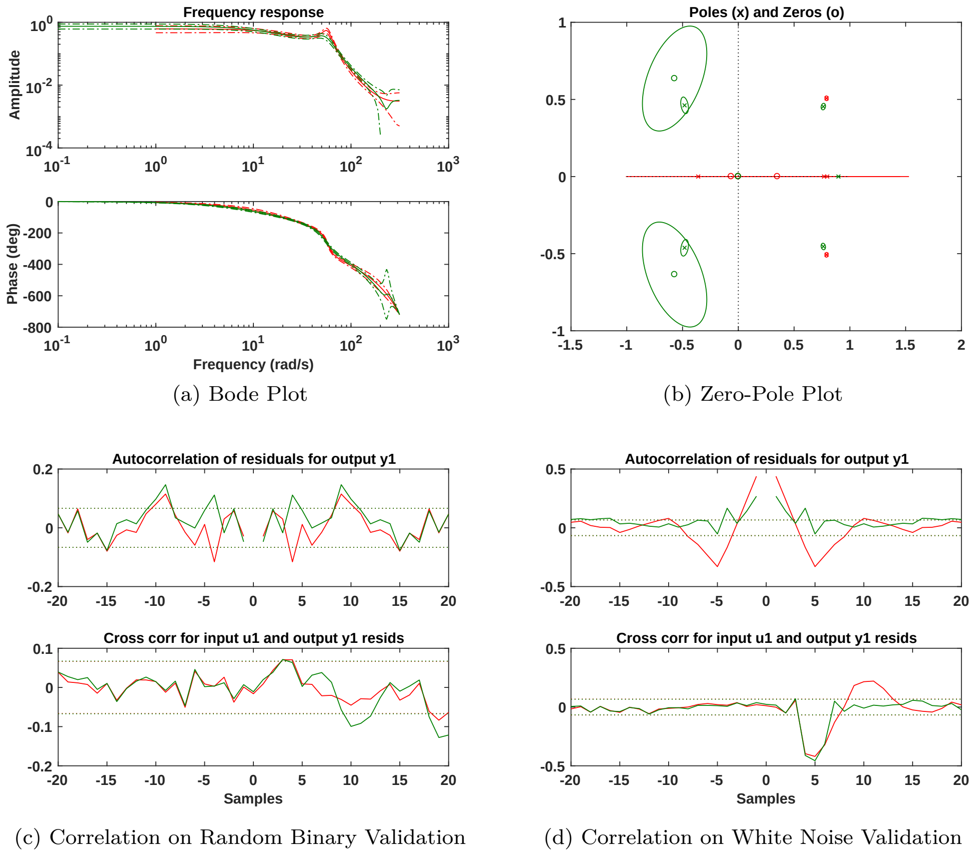
6.2.3 Validation and Correlation

Table 2: FIT of 1-step Prediction

Model	FIT RB	FIT UWN
ARX(4 3 2)	82.24	81.5
ARX(6 3 4)	83.33	82.86
ARMAX(6 3 2 4)	83.53	83.08

Based on FIT of 1-step prediction, both ARX(6 3 4) and ARMAX(6 3 2 4) performs well on white noise validation data and generalizes well on random binary validation data. However, as shown in Fig.4d, quite large outliers indicates that some dynamics are clearly not modeled. It implies that there are intrinstic nonlinearities in the system which couldn't be identified with linear models.

6.3 Random Binary Signal Identification



(a) Bode Plot

(b) Zero-Pole Plot

(c) Correlation on Random Binary Validation

(d) Correlation on White Noise Validation

Figure 5: Random Binary ($\alpha = 0.25$) Signal Identification

6.3.1 Degree Selection

Similar to white noise signal identification, an ARX(5 3 4) and an enhanced ARXMAX(5 3 3 4) are chosen.

6.3.2 Analysis of Plots

In Fig.5, green represents ARX(5 3 4) and red represents ARMAX(5 3 3 4).

According to Bode plot in Fig.5a, there is a resonance at around frequency 60 rad/s. As the input random binary signal focuses on low frequencies, the variance is relatively small around the peak frequency and relatively big otherwise. By comparison, ARXMAX(5 3 3 4) has higher gains around resonance frequency than ARX(5 3 4).

According to Zero-Pole plot in Fig.5b, both models have effective pairs of conjucate poles that contribute to the peak in Bode plot.

6.3.3 Validation and Correlation

Table 3: FIT of 1-step Prediction

Model	FIT RB	FIT UWN
ARX(4 3 2)	83.52	75.62
ARX(5 3 4)	84.93	75.48
ARMAX(5 3 3 4)	85.32	73.37

Based on FIT of 1-step prediction, both ARX(5 3 4) and ARMAX(5 3 4 4) performs well on random binary validation data but generalizes poorly on white noise validation data. It is clearer according to correlation plots in Fig.5c and Fig.5d, in former correlation plot, most correlation falls in confidence region while are large outliers in the latter. Therefore, ARX(5 3 4) and ARMAX(5 3 4 4) are overfitted to random binary data.

6.4 Comparison and Ranking

According to FIT in Tab.2 and Tab.3, models estimated with proper random binary signal has higher accuracy than with uniform white nosie signal but generalize worse. This is due to the concentration of energy around the most excited frequency of the system with random binary signals which loses information on other frequency.

Performance of models depends on both credibilty and working situation.

According to Bode plots in Fig.4a and Fig.5a. For frequencies around resonance, models obtained with random binary signals are clearly more credible. Additionally, as shown in Fig.4b and Fig.5b, models obtained with random binary signals have more certain zeros and poles. However, when considering generalization, models obtained with random binary signals works worse due to overfitting. Therefore, the performance depends on the working frequency range.

In conclusion, the ranking is stated as below:

Table 4: Ranking of Models

Model	Credibility	Generalization
ARX(6 3 4)	4	2
ARMAX(6 3 3 4)	3	1
ARX(5 3 4)	2	3
ARMAX(5 3 3 4)	1	4