

Grading template for laboratory exercise 3

EL2820, Modeling of Dynamical Systems

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	Pass	Fail	
The report is handed in on time?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Number of authors	≤ 2 st <input type="checkbox"/>	> 2 <input type="checkbox"/>	
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The report is well structured? The language is understandable?	yes <input type="checkbox"/>	often <input type="checkbox"/>	sometimes <input type="checkbox"/> no <input type="checkbox"/>
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The preparation task is solved and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The working region is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The sampling time is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A detailed description of the input signal is given and the choice is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The amount of data used for estimation and validation is specified?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Models of three different model structures have been estimated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The model order of each model is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A top 4 ranking of the estimated models have been made?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The best and worst models have been compared using Bode plots, and analyze through pole/zeros and correlation(residuals) analysis using (compare) and correlation analysis(resid)?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
First review	Pass <input type="checkbox"/>	Fail <input type="checkbox"/>	Sign:
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Second review (if failed in the first review)	Pass <input type="checkbox"/>	Fail <input type="checkbox"/>	Sign:

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1 Preparation task

- Derivation of a physical model of the magnetic levitator in state-space form:

From figure in the lab description, we can get:

$$m\ddot{z} = \gamma\dot{z} + E_r - F_{ul} - mg \quad (1)$$

$$m\ddot{y} = \gamma\dot{y} - E_a + F_{lu} - mg \quad (2)$$

Repulsive magnet force is proportional to $m|y - z|^{-4}$, and here, constant T is given to represent the proportion:

$$F_{ul} = F_{lu} = Tm \frac{1}{(y - z)^4} \quad (3)$$

From exercise 2.5, the electromagnetic force can be obtained, where K is a constant and A is the area of the coil:

$$B = \mu_0 \frac{N\gamma^2 I(t)}{2y^3} \quad (4)$$

$$E = \frac{1}{2} A \frac{B^2(t)}{\mu_0} \quad (5)$$

$$E = \frac{AN^2\mu_0\gamma^4}{8} \frac{I^2(t)}{y^6(t)} = Km \frac{I^2(t)}{y^6(t)} \quad (6)$$

$$\text{So: } E_a = Km \frac{I^2(t)}{y^6(t)} \quad E_r = Km \frac{I^2(t)}{z^6(t)} \quad (7)$$

Then we can obtain state space equations:

$$\dot{a} = \frac{\gamma}{m} \dot{z} + K \frac{I^2}{z^6} - T \frac{1}{(y - z)^4} - g \quad (8)$$

$$\dot{b} = \frac{\gamma}{m} \dot{y} + K \frac{I^2}{y^6} - T \frac{1}{(y - z)^4} - g \quad (9)$$

$$\dot{z} = a \quad (10)$$

$$\dot{y} = b \quad (11)$$

- Suggestion for a suitable model order for a linear model:

In stationarity, we have:

$$x = \begin{bmatrix} a \\ b \\ z \\ y \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{z} \\ \dot{y} \end{bmatrix} = 0 = \begin{bmatrix} f_1(a, b, z, y, I_0) \\ \vdots \\ f_4(a, b, z, y, I_0) \end{bmatrix} \quad (12)$$

The linearized system is given by:

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}I(t) \quad (13)$$

$$\mathbf{Y} = \mathbf{C}\mathbf{X} + \mathbf{D} \quad (14)$$

$$\mathbf{A} = \left[\begin{array}{ccc|c} \frac{\partial f_1}{\partial a} & \cdots & \frac{\partial f_1}{\partial y} & \\ \vdots & \ddots & \vdots & \\ \frac{\partial f_4}{\partial a} & \cdots & \frac{\partial f_4}{\partial y} & \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \left[\begin{array}{ccc|c} \frac{\gamma}{m} & 0 & -\frac{4T}{(y^0-z^0)^5} - \frac{6KI^2}{z^{07}} & -\frac{4T}{(y^0-z^0)^5} \\ 0 & \frac{\gamma}{m} & \frac{4T}{(y^0-z^0)^5} & -\frac{4T}{(y^0-z^0)^5} + \frac{6KI^2}{y^{07}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array} \right] \quad (15)$$

$$\mathbf{B} = \left[\begin{array}{c} \frac{\partial f_1}{\partial I} \\ \vdots \\ \frac{\partial f_4}{\partial I} \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \left[\begin{array}{c} \frac{2KI}{z^{06}} \\ -\frac{2KI}{z^{06}} \\ 0 \\ 0 \end{array} \right] \quad (16)$$

$$\mathbf{C} = [0 \ 0 \ 0 \ 1]^T \quad \mathbf{D} = [0 \ 0 \ 0 \ 0]^T \quad (17)$$

Then the transform function is given as, where \mathbf{I} is the identity matrix and N_i^a is a polynomial of i degree with label a on nominator:

$$G(q) = C^T(q\mathbf{I} - A)^{-1}B + D \quad (18)$$

$$= \frac{N_3^a(q)}{D_4^a(q)} + \frac{N_0^b(q)}{D_4^b(q)} \quad (19)$$

Approximating $G(q)$ with $D_4^a(q) = D_4^b(q)$, the transfer function would be $G(q) = \frac{N_3(q)}{D_4(q)}$. Plus, since our sampling time would be $2ms$ and the delay time would be $9ms$ in our experiment, base on the equation $n_k = \lfloor \frac{\tau}{h} \rfloor + 1$ from the lecture slides, we get $n_k = 5$. In all, our suggestion of the model order for ARX model could be ARX(4 3 5). Then the ARMAX and BJ models can be modified accordingly.

- MATLAB codes for the required functions are attached in the section 7.
- Plot for the spectrum of the binary random signal for the required values of α is shown in Fig. 1.

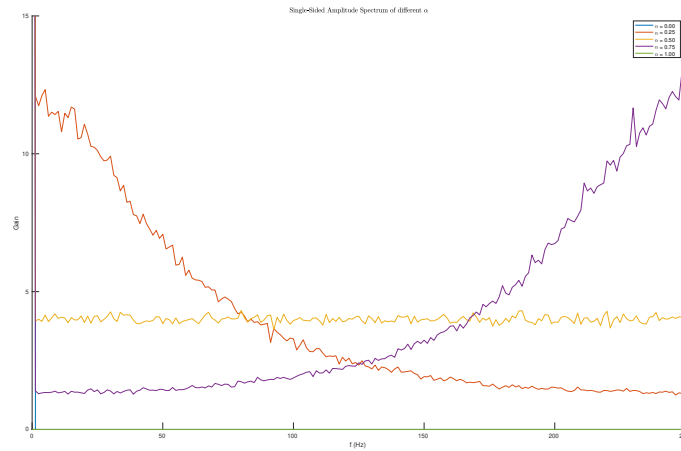


Figure 1: Spectrum of binary random signal.

2 Working region

- After the experiment, we choose working region as $1.5A \sim 3.6A$.
- Motivation for working region:
From Fig. 2, we can see that when $u \in [1.5A, 3.6A]$, the plot have linearity.
- Plot for illustrating working region is shown in Fig. 2.

3 Sampling time

- In the experiment, we chose $T_s = 2ms$ as sampling time.
- Motivation for sampling time:
Fig. 3 is obtained with the sampling time of $1ms$, and we can see the rise time is about $13ms$, which means there are about 13 points in one rise time. According to the requirement of "the sampling time is set to give around four to ten samples per rise time" in the lab description, we choose $2ms$ as sampling time so that there will be 6 to 7 samples per rise time.
- Plot of step responses yielding sampling time is shown in Fig.3.

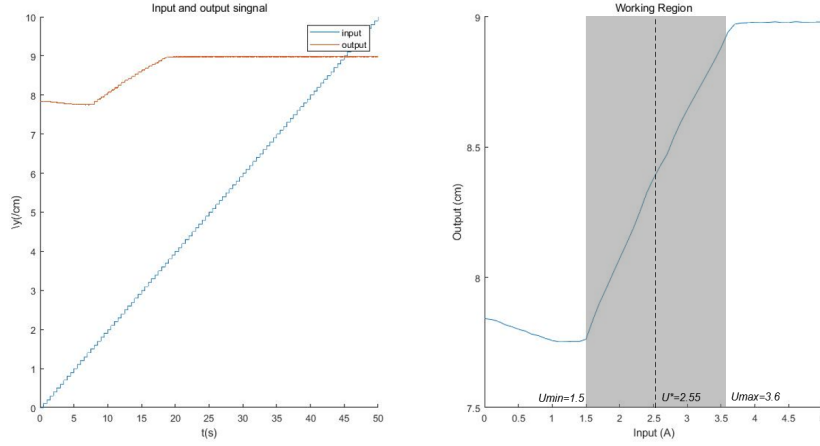


Figure 2: Working region.

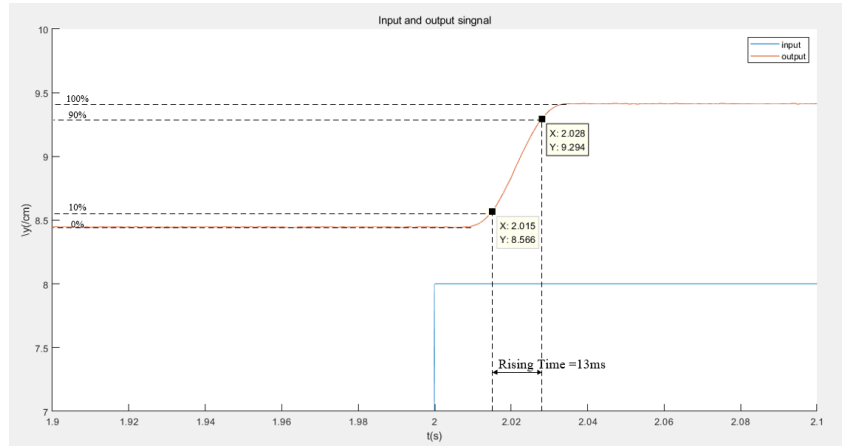


Figure 3: Step Response.

4 Input signals

- The binary random signal we choose have the similar output spectrum as the one of the white noises, which can be seen from Fig. 1 and Fig. 4.
- Motivation for chosen input signals:

Comparing Fig. 1 and Fig. 4, when $\alpha = 0.25$, the spectrum of binary random signal is similar to the spectrum of white noise.

5 Estimation and validation data

Information about the amount of data used for identification and validation:

Table 1: Data used for identification and validation.

Input signal	Value (A)	Amount of data	Estimation (s)	Validation (s)
Uniform white noise	[2, 3]	10000	[0, 10]	[10, 20]
Binary random signal with $\alpha = 0.25$	[2, 3]	10000	[0, 10]	[10, 20]

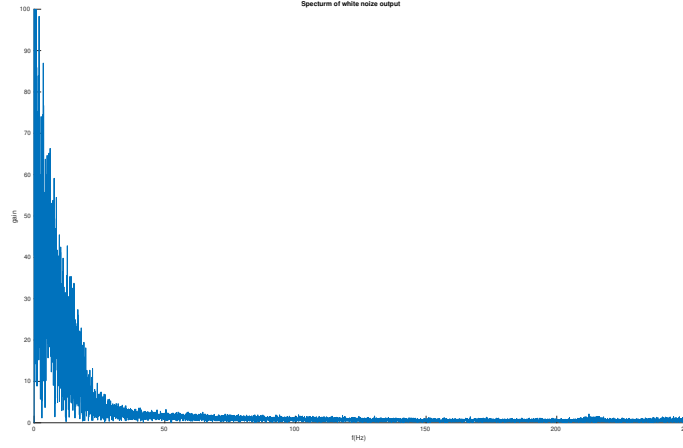


Figure 4: Spectrum of white noise.

6 Models

6.1 Model Description

- ARX model: $A(q)y[t] = B(q)u[k] + e[k]$, performs a filtering on both input and error with $\frac{1}{A}$ and adjust the transform function with B.
- ARMAX model: $A(q)y[t] = B(q)u[k] + C(q)e[k]$, is more generalized than ARX model with error being adjusted with C.
- BJ model: $y[k] = \frac{B(q)}{F(q)}u[k] + \frac{C(q)}{D(q)}e[k]$, includes all special cases.

6.2 Motivation of Model Orders

- ARX model: As mentioned in the “Preparation task” section, ARX(4 3 5) is first chosen for both white noise and binary random signal. In the case of white noise, it performs well enough. In the case of binary random signal, ARX(5 3 5) is chosen for obvious performance improvement.
- ARMAX model: Based on ARX model, we try different values on n_c and finally choose ARMAX(4 3 4 5) since performance drops when $n_c > 4$. For same reason, we choose ARMAX(5 3 2 5) for binary random signal.
- BJ model: Based on ARMAX model, we get BJ(3 4 4 4 5) for white noise and BJ(3 2 5 5 5) for binary random signal since we don’t need to change the order for each polynomial in each transform function.

6.3 White Noise Signal Validation

In this part, we are going to use white noise signal 2 as the validation data since the process of white noise signal 1 is similar and the performance of white noise signal 2 is better.

The best and worst models in validating with binary random signal are compared in Fig. 5, where the blue curves is the best model BJ(3 4 4 4 5) estimated from white noise 1 and the red curves is the worst model ARX(5 3 5) estimated from binary random signal.

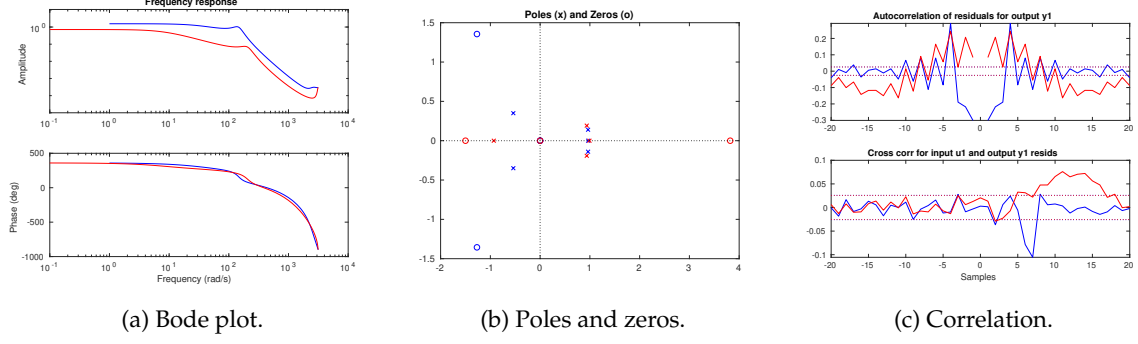


Figure 5: Comparison between the best and worst models using white noise as validation data.

According to Bode plot in Fig. 5(a), there is a resonance at around frequency 100 rad/s. By comparison, BJ(3 4 4 4 5) has higher gains around resonance frequency than ARX(5 3 5). Also, according to the poles and zeros in Fig. 5(b), poles are in the unit circle and both models have effective pairs of conjugate poles that contribute to the peak in Bode plot.

Table 2: 1-step Prediction of Validation

Model	Binary Signal Fit	White Noise 2 Fit
BJ(3 4 4 4 5) WN1	97.25%	92.15%
ARX(5 3 5) BR	98.36%	88.83%

Base on the 1-step prediction, both signal performs perfect on random binary validation data. But the ARX(5 3 5) model preforms not so good with white noise 2 data which can also be seen from Fig.5(c).

6.4 Binary Random Signal Validation

The best and worst models in validating with binary random signal are compared in Fig. 6, where the red curves is the best model BJ(3 2 5 5 5) estimated from binary random signal and the blue curves is the worst model BJ(3 4 4 4 5) estimated from WN2.

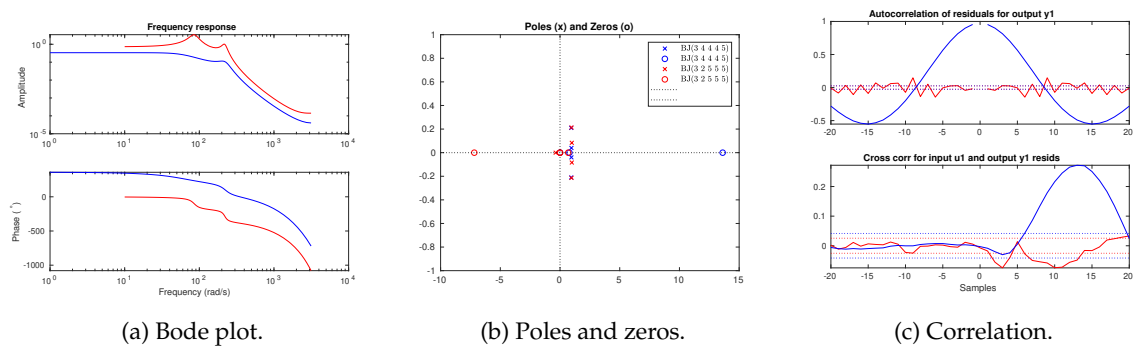


Figure 6: Comparison between the best and worst models using binary random signal as validation data.

According to Bode plot in Fig. 6(a), there is a resonance at around frequency 100 rad/s. As the input binary random signal focuses on low frequencies, the variance is relatively small around the peak frequency and relatively big. By comparison, BJ(3 2 5 5 5) estimated from binary random signal has higher gains around resonance frequency than BJ(3 4 4 4 5) estimated from WN2.

According to poles and zeros in Fig. 6(b), the poles of both models are both in the unit circle. Plus, both

models have effective pairs of conjugate poles that contribute to the peak in Bode plot.

Table 3: 1-step Prediction of Validation

Model	Binary Signal Fit	White Noise 2 Fit
BJ(3 2 5 5 5) BR	98.54%	90.38%
BJ(3 4 4 4 5) WN2	95.11%	91.73%

Base on the 1-step prediction, both signal performs perfect on random binary validation data and also good with on white noise validation data. However, it is clear according to correlation plots in Fig. 6(c), most correlation of BJ(3 2 5 5 5) is inside the confidence region while there is much outlier in the case of BJ(3 4 4 4 5). Therefore, BJ(3 4 4 4 5) estimated from WN2 is over fitted to binary random data signal.

6.5 Ranking and Comparison

According to validation result shown in Tab.2 and Tab.3, models estimated from binary random signal has relatively higher accuracy than with white noise.

Table 4: Ranking of the Estimated Models with 1-step Prediction

Rank	Validation Data		
	White Noise 1	White Noise 2	Binary Random Signal
1	ARMAX(4 3 4 5) WN2: 89.33%	BJ(3 4 4 4 5) WN1: 92.15%	BJ(3 2 5 5 5) BR: 98.54%
2	BJ(3 4 4 4 5) WN2: 89.11%	ARMAX(4 3 4 5) WN1: 91.90%	ARMAX(5 3 2 5) BR: 98.53%
3	ARX(4 3 5) WN2: 88.91%	BJ(3 4 4 4 5) WN2: 91.73%	ARX(5 3 5) BR: 98.36%
4	BJ(3 4 4 4 5) WN1: 88.80%	ARMAX WN2: 91.71%	ARMAX(4 3 4 5) WN1: 97.93%

Table 5: Accuracy comparison of models with different input signals (binary random signal v.s. uniformly distributed white noise).

Identification data	Input Validation Signal		
	Model	White Noise 2 (%)	Binary Random Signal (%)
White Noise 1	ARX(4 3 5)	88.23	97.31
	ARMAX(4 3 4 5)	88.26	97.93
	BJ(3 4 4 4 5)	88.8	97.25
White Noise 2	ARX(4 3 5)	88.91	96.38
	ARMAX(4 3 4 5)	89.33	96.85
	BJ(3 4 4 4 5)	89.11	95.11
Binary Random Signal	ARX(5 3 5)	84.85	98.36
	ARMAX(5 3 2 5)	86.48	98.53
	BJ(3 2 5 5 5)	86.6	98.54

7 Attachment

generateBinarySignal.m

```
1 function u = generateBinarySignal(alpha, lims, N)
2     %% Initialization
3     w = 2 * (rand(N,1) > alpha) - 1;
4     u = zeros(N,1);
5
6     %% Initial value of u
7     if w(1) == -1
8         u(1) = lims(2);
9     else
10        u(1) = lims(1);
11    end
12
13    %% Rest values of u
14    for i = 2 : N
15        if w(i) == -1
16            if u(i-1) == lims(1)
17                u(i) = lims(2);
18            else
19                u(i) = lims(1);
20            end
21        else
22            u(i) = u(i-1);
23        end
24    end
25 end
```

getAverage.m

```
1 function bar_v = getAverage(v, tail)
2     bar_v = mean(v(length(v)*tail/100 + 1 : end));
3 end
```

getStationaryAverages.m

```
1 function bar_y = getStationaryAverages(y_step, Nwr, tail)
2     wrLen = length(y_step)/Nwr;
3     for i=1:Nwr
4         bar_y(i) = getAverage(y_step((i-1)*wrLen+1:(i)*wrLen),tail);
5     end
6 end
```

findWorkingRegion.m

```
1 close all; clear all; clc;
2 %% Initialization
3 Ts = 0.001;
4 t = 0;
5 i = 1;
6 maxCurrent = 10;
7 stairHeight = 0.1;
8 stairDuration = 0.2;
9
10 %% Create Stair Signal
11 stairSignal(i) = 0;
12 while true
```



```

13     t = t + Ts;
14     i = i + 1;
15     if t < stairDuration
16         stairSignal(i) = stairSignal(i-1);
17     else
18         if stairSignal(i-1) + stairHeight > maxCurrent
19             break
20         end
21         stairSignal(i) = stairSignal(i-1) + stairHeight;
22         t = 0;
23     end
24 end
25 t = 0 : Ts : (length(stairSignal)-1) * Ts;
26 t = t.';
27 stairSignal = stairSignal.';
28
29 %% Plot Stair Signal and Time
30 figure
31 plot(t, stairSignal)
32 xlabel('Time (s)')
33 ylabel('Stair Input (A)')
34
35 %% Input Signal Amplitudes
36 averagedU = getStationaryAverages(stairSignal, (maxCurrent/stairHeight+1),
    0.6);
37
38 %% Conducting Experiment
39 y = getData(stairSignal, Ts);
40
41 %% Averaged Output Signal
42 averagedY = getStationaryAverages(y, (10/stairHeight+1), 0.6);
43
44 %% Plot to Find Working Region
45 figure
46 plot(averagedU, averagedY);
47 xlabel('Input (A)')
48 ylabel('Output (cm)')

```