Grading template for laboratory exercise 3
EL2820, Modeling of Dynamical Systems
August 2018

Chang, Hongsheng

951228-2451

Jiang, Sifan

961220-8232

	Pass		Fail	
The report is handed in on time?		yes ☐	no ☐	
Number of authors		$\leq$ 2st ☐	$>$ 2 ☐	
Author names and personal identity number filled out?		yes ☐	no ☐	
The report is well structured? The language is understandable?	yes ☐	often ☐	sometimes ☐	no ☐
The figures are clear? (Captions, high resolution, etc.)	yes ☐	often ☐	sometimes ☐	no ☐
The preparation task is solved and motivated?		yes ☐	no ☐	
The working region is defined and motivated?		yes ☐	no ☐	
The sampling time is defined and motivated?		yes ☐	no ☐	
A detailed description of the input signal is given and the choice is motivated?		yes ☐	no ☐	
The amount of data used for estimation and validation is specified?		yes ☐	no ☐	
Models of more than one model structure have been estimated?		yes ☐	no ☐	
The model order of each model is motivated?		yes ☐	no ☐	
A ranking of the estimated models have been made?		yes ☐	no ☐	
The ranking is well motivated according to the requirements?		yes ☐	no ☐	

	Pass	Fail	
First review	☐	☐	Sign:
....		....	
Second review (if failed in the first review)	☐	☐	Sign:

Pass

Signature:

1 Preparation task

- Derivation of a physical model of the magnetic levitator in state-space form:
  From figure in the lab description, we can get:

$$m\ddot{z} = \gamma\dot{z} + E_r - F_{ul} - mg$$
$$m\ddot{y} = \gamma\dot{y} - E_a + F_{lu} - mg$$

Repulsive magnet force is proportional to $m|y - z|^{-4}$, and here, constant C is given to represent the proportion:

$$F_{ul} = F_{lu} = Cm\frac{1}{(y-z)^4}$$

From exercise 2.5, the electromagnetic force can be obtained, where K is a constant:

$$B = \mu_0 \frac{N\gamma^2 I(t)}{2y^3}$$
$$E = \frac{1}{2}A\frac{B^2(t)}{\mu_0}$$
$$E = \frac{AN^2\mu_0\gamma^4}{8}\frac{I^2(t)}{y^6(t)} = Km\frac{I^2(t)}{y^6(t)}$$
$$\text{So:} \quad E_a = Km\frac{I^2(t)}{y^6(t)} \qquad E_r = Km\frac{I^2(t)}{z^6(t)}$$

Then we can obtain state space equations:

$$\dot{a} = \frac{\gamma}{m}\dot{z} + K\frac{I^2}{z^6} - C\frac{1}{(y-z)^4} - g$$
$$\dot{b} = \frac{\gamma}{m}\dot{y} + K\frac{I^2}{y^6} - C\frac{1}{(y-z)^4} - g$$
$$\dot{z} = a$$
$$\dot{y} = b$$

- Suggestion for a suitable model order for a linear model:
  In stationarity, we have:

$$x = \begin{bmatrix} a \\ b \\ z \\ y \end{bmatrix} \qquad \dot{x} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{z} \\ \dot{y} \end{bmatrix} = 0 = \begin{bmatrix} f_1(a, b, z, y, I_0) \\ \vdots \\ f_4(a, b, z, y, I_0) \end{bmatrix}$$

The linearized system is given by:

$$\dot{X} = AX + BI(t)$$

$$A = \left.\begin{bmatrix} \frac{\partial f_1}{\partial a} & \cdots & \frac{\partial f_1}{\partial y} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_4}{\partial a} & \cdots & \frac{\partial f_4}{\partial y} \end{bmatrix}\right|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \begin{bmatrix} \frac{\gamma}{m} & 0 & -\frac{4C}{(y^0-z^0)^5} - \frac{6KI^2}{z^{07}} & \frac{4C}{(y^0-z^0)^5} \\ 0 & \frac{\gamma}{m} & \frac{4C}{(y^0-z^0)^5} & -\frac{4C}{(y^0-z^0)^5} + \frac{6KI^2}{y^{07}} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \left.\begin{bmatrix} \frac{\partial f_1}{\partial I} \\ \vdots \\ \frac{\partial f_4}{\partial I} \end{bmatrix}\right|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \begin{bmatrix} \frac{2KI}{z^{06}} \\ -\frac{2KI}{z^{06}} \\ 0 \\ 0 \end{bmatrix}$$

- Motivation for suggested model order:

  ?????

- MATLAB codes for the required functions are attached in the page 5.

- Plot for the spectrum of the binary random signal for the required values of α is shown in Fig. 1.

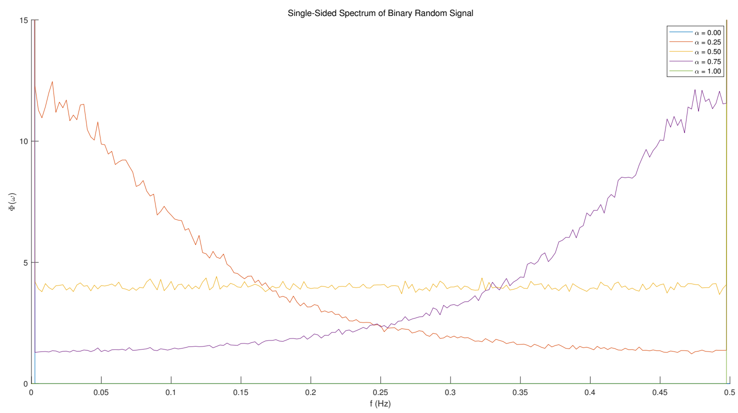

Figure 1: Spectrum of binary random signal.

2 Working region

- After the experiment, we choose working region as 1.5A $\sim$ 3.6A.

- Motivation for working region:

  From Fig. 2, we can see that when $u \in [1.5A, 3.6A]$, the plot have linearity.

- Plot for illustrating working region is shown in Fig. 2.

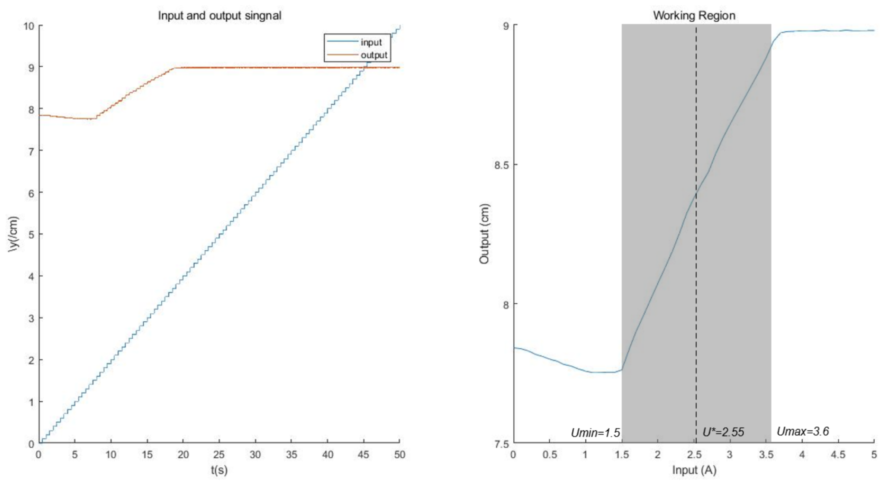

Figure 2: Working region.

3 Sampling time

- In the experiment, we chose $Ts = 0.002s$ as sampling time.
- Motivation for sampling time:

  Fig. 3 is obtained with the sampling time of $0.001s$, and we can see the rise time is about $13ms$, which means there are about 13 points in one rise time. According to the requirement of "the sampling time is set to give around four to ten samples per rise time" in the lab description, we choose $2ms$ as sampling time so that there will be 6 to 7 samples per rise time.
- Plot of step responses yielding sampling time:

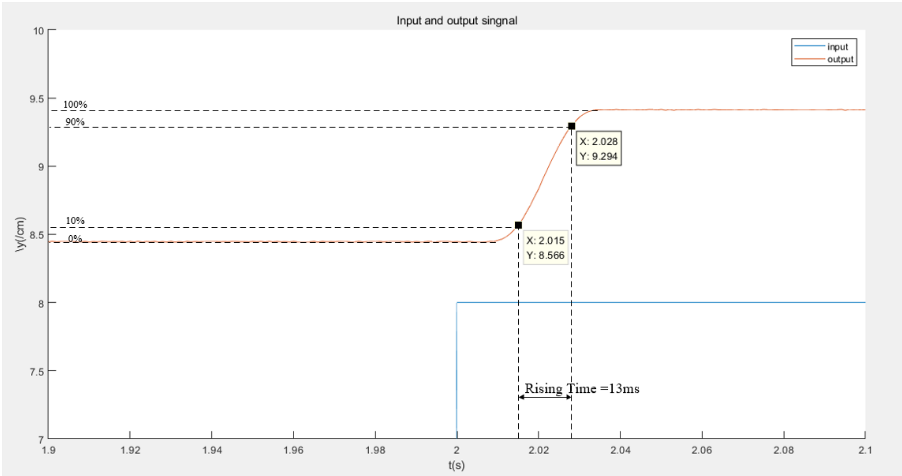

Figure 3: Working region.

4 Input signals

- The binary random signal we choose have the similar output spectrum as the one of the white noises, which can be seen from Fig. 1 and Fig. 4

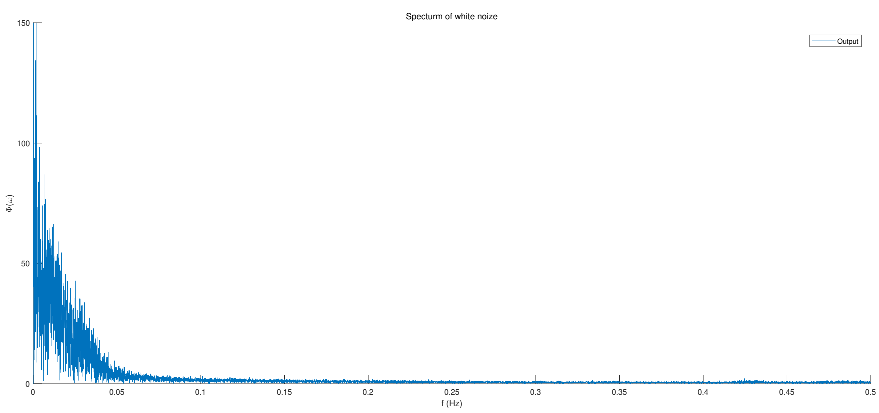

Figure 4: Working region.

- motivation for chosen input signals.

5 Estimation and validation data

- Information about the amount of data used for identification and validation:

Table 1: Data used for identification and validation.

Input signal	Value (A)	Estimation (s)	Validation (s)
Uniform white noise	[1.5, 3.6]	[0, 10]	[10, 20]
Binary random signal with $\alpha = 0.25$	[1.5, 3.6]	[0, 10]	[10, 20]

6 Models

- descriptions of model structures used. (You should use more than one structure.)
- motivation for choice of model order for each model structure.
- plots of Bode diagrams, poles and zeros.
- analysis/interpretations of plots.
- validation performed with simulation (`compare` with `M=inf`) and correlation analysis (`resid`).
- analysis/interpretations of validation and correlation analysis.
- a comparison between the accuracy of models obtained with different input signals (binary random signals v/s uniformly distributed white noise)
- a ranking of estimated models with respect to how well they describe the process, along with a rigorous motivation of chosen ranking. (Compare the analysis of plots, validation and correlation analysis for the different models.)

Note that the report is not in the format of a conference or journal paper. However, the language should be correct, concise and understandable. The figures should be clear, have captions, labels and be referenced in the text. All equations should be punctuated appropriately. (Equations are considered as part of sentences and should be treated accordingly.) All introduced symbols must be defined.

Attachments

generateBinarySignal.m

```
function u = generateBinarySignal(alpha, lims, N)
    %% Initialization
    w = 2 * (rand(N,1) > alpha) - 1;
    u = zeros(N,1);

    %% Initial value of u
    if w(1) == -1
        u(1) = lims(2);
    else
        u(1) = lims(1);
    end

    %% Rest values of u
    for i = 2 : N
        if w(i) == -1
            if u(i-1) == lims(1)
                u(i) = lims(2);
            else
                u(i) = lims(1);
            end
        else
            u(i) = u(i-1);
        end
    end
end
```

getAverage.m

```
function bar_v = getAverage(v, tail)
    bar_v = mean(v(length(v)*tail/100 + 1 : end));
end
```

getStationaryAverages.m

```
function bar_y = getStationaryAverages(y_step, Nwr, tail)
    wrLen = length(y_step)/Nwr;
    for i=1:Nwr
        bar_y(i) = getAverage(y_step((i-1)*wrLen+1:(i)*wrLen),tail);
    end
end
```

findWorkingRegion.m

```
close all; clear all; clc;
%% Initialization
```

```matlab
Ts = 0.001;
t = 0;
i = 1;
maxCurrent = 10;
stairHeight = 0.1;
stairDuration = 0.2;

%% Create Stair Signal
stairSignal(i) = 0;
while true
    t = t + Ts;
    i = i + 1;
    if t < stairDuration
        stairSignal(i) = stairSignal(i-1);
    else
        if stairSignal(i-1) + stairHeight > maxCurrent
            break
        end
        stairSignal(i) = stairSignal(i-1) + stairHeight;
        t = 0;
    end
end
t = 0 : Ts : (length(stairSignal)-1) * Ts;
t = t.';
stairSignal = stairSignal.';

%% Plot Stair Signal and Time
figure
plot(t,stairSignal)
xlabel('Time (s)')
ylabel('Stair Input (A)')

%% Input Signal Amplitudes
averagedU = getStationaryAverages(stairSignal, (maxCurrent/stairHeight
    +1), 0.6);

%% Conducting Experiment
y = getData(stairSignal, Ts);

%% Averaged Output Signal
averagedY = getStationaryAverages(y, (10/stairHeight+1), 0.6);

%% Plot to Find Working Region
figure
plot(averagedU,averagedY);
xlabel('Input (A)')
ylabel('Output (cm)')
```