

# Grading template for laboratory exercise 3

## EL2820, Modeling of Dynamical Systems

### August 2017

Chang, Hongsheng  951228-2451  Jiang, Sifan  961220-8232
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	Pass	Fail	
The report is handed in on time?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Number of authors	$\leq 2^{\text{st}}$ <input type="checkbox"/>	$> 2$ <input type="checkbox"/>	
Author names and personal identity number filled out?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The report is well structured? The language is understandable?	yes <input type="checkbox"/>	often <input type="checkbox"/>	sometimes <input type="checkbox"/>
The figures are clear? (Captions, high resolution, etc.)	yes <input type="checkbox"/>	often <input type="checkbox"/>	no <input type="checkbox"/>
The preparation task is solved and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The working region is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The sampling time is defined and motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A detailed description of the input signal is given and the choice is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The amount of data used for estimation and validation is specified?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
Models of more than one model structure have been estimated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The model order of each model is motivated?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
A ranking of the estimated models have been made?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
The ranking is well motivated according to the requirements?	yes <input type="checkbox"/>	no <input type="checkbox"/>	
<b>First review</b>	<b>Pass</b> <input type="checkbox"/>	<b>Fail</b> <input type="checkbox"/>	Sign:
<b>Second review</b> (if failed in the first review)	<b>Pass</b> <input type="checkbox"/>	<b>Fail</b> <input type="checkbox"/>	Sign:

**Pass**

Signature:
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# 1 Preparation task

- Derivation of a physical model of the magnetic levitator in state-space form:

From figure in the lab description, we can get:

$$\begin{aligned} m\ddot{z} &= \gamma\dot{z} + E_r - F_{ul} - mg \\ m\ddot{y} &= \gamma\dot{y} - E_a + F_{lu} - mg \end{aligned}$$

Repulsive magnet force is proportional to  $m|y - z|^{-4}$ , and here, constant  $C$  is given to represent the proportion:

$$F_{ul} = F_{lu} = Cm \frac{1}{(y - z)^4}$$

From exercise 2.5, the electromagnetic force can be obtained, where  $K$  is a constant:

$$\begin{aligned} B &= \mu_0 \frac{N\gamma^2 I(t)}{2y^3} \\ E &= \frac{1}{2} A \frac{B^2(t)}{\mu_0} \\ E &= \frac{AN^2\mu_0\gamma^4}{8} \frac{I^2(t)}{y^6(t)} = Km \frac{I^2(t)}{y^6(t)} \end{aligned}$$

So:  $E_a = Km \frac{I^2(t)}{y^6(t)}$        $E_r = Km \frac{I^2(t)}{z^6(t)}$

Then we can obtain state space equations:

$$\begin{aligned} \dot{a} &= \frac{\gamma}{m} \dot{z} + K \frac{I^2}{z^6} - C \frac{1}{(y - z)^4} - g \\ \dot{b} &= \frac{\gamma}{m} \dot{y} + K \frac{I^2}{y^6} - C \frac{1}{(y - z)^4} - g \\ \dot{z} &= a \\ \dot{y} &= b \end{aligned}$$

- Suggestion for a suitable model order for a linear model (based on a linearization of the model):

In stationarity, we have:

$$x = \begin{bmatrix} a \\ b \\ z \\ y \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{a} \\ \dot{b} \\ \dot{z} \\ \dot{y} \end{bmatrix} = 0 = \begin{bmatrix} f_1(a, b, z, y, I_0) \\ \vdots \\ f_4(a, b, z, y, I_0) \end{bmatrix}$$

The linearized system is given by:

$$\dot{X} = AX + BI(t)$$

$$\begin{aligned} A &= \left[ \begin{array}{ccc} \frac{\partial f_1}{\partial a} & \cdots & \frac{\partial f_1}{\partial y} \\ \vdots & \ddots & \vdots \\ \frac{\partial f_4}{\partial a} & \cdots & \frac{\partial f_4}{\partial y} \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \begin{bmatrix} \frac{\gamma}{m} & 0 & -\frac{4C}{(y^0-z^0)^5} - \frac{6KI^2}{z^{07}} & \frac{4C}{(y^0-z^0)^5} \\ 0 & \frac{\gamma}{m} & \frac{4C}{(y^0-z^0)^5} - \frac{6KI^2}{z^{07}} & -\frac{4C}{(y^0-z^0)^5} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \\ B &= \left[ \begin{array}{c} \frac{\partial f_1}{\partial I} \\ \vdots \\ \frac{\partial f_4}{\partial I} \end{array} \right] \bigg|_{a=a^0, b=b^0, z=z^0, y=y^0, I=I_0} = \begin{bmatrix} \frac{2KI}{z^{06}} \\ -\frac{2KI}{z^{06}} \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

- Motivation for suggested model order:  
?????
- MATLAB codes for the required functions:  
MATLAB codes are attached in the end of the report.
- One plot for the spectrum of the binary random signal for the required values of  $\alpha$ :

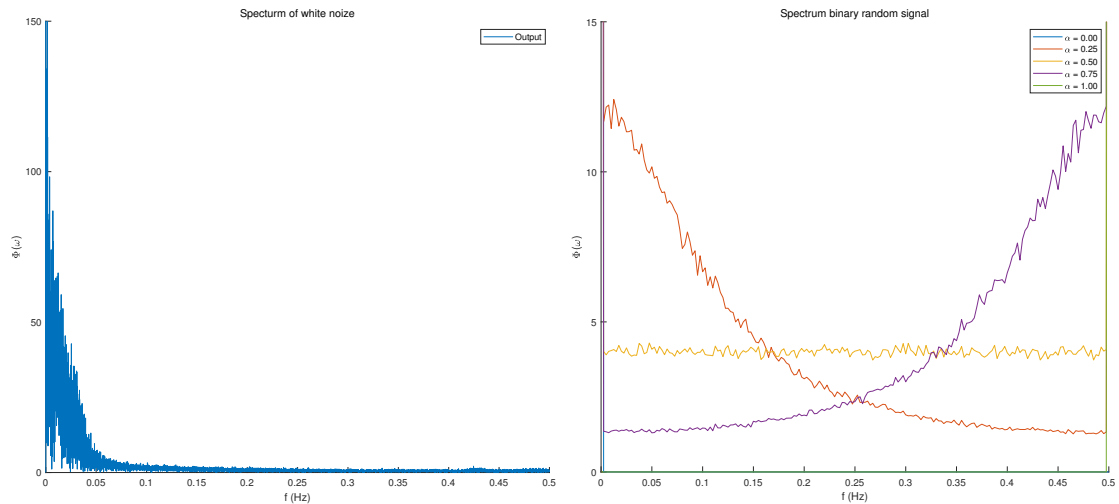


Figure 1: Spectrum of white noise signal and binary random signal.

From figure, we can see that when  $\alpha = 0.25$ , the spectrum of binary random signal is most similar to the one of white noise.

## 2 Working region

This section should include:

- chosen working region.
- motivation for working region.
- plot illustrating working region, for example as shown in Fig. 4. (Always reference all figures in the text and use captions.)

## 3 Sampling time

This section should include:

- sampling time.
- motivation for sampling time.
- plot of step responses yielding sampling time.

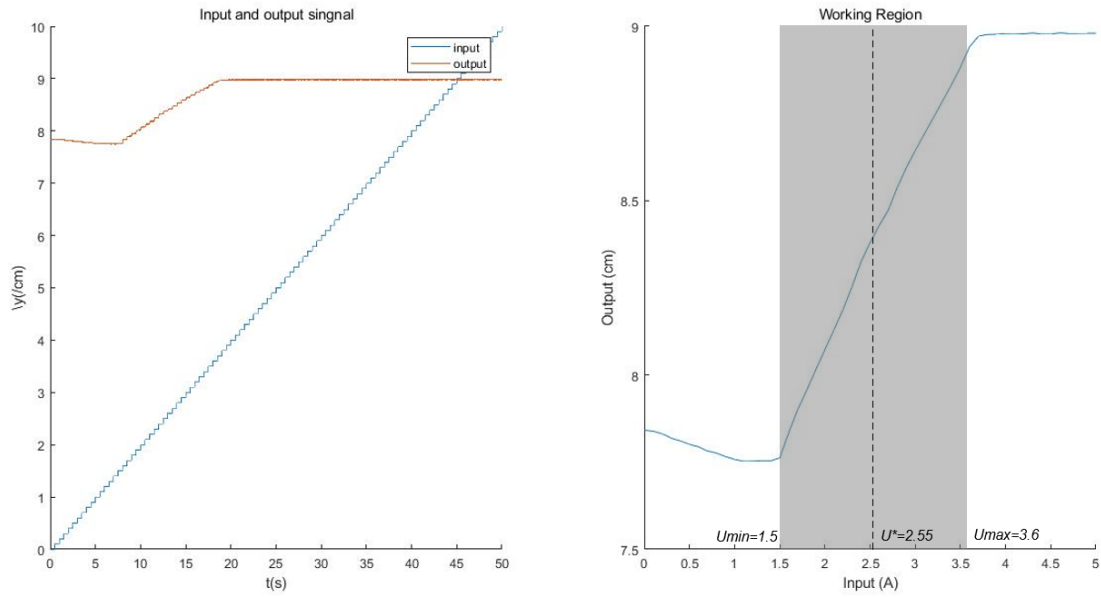


Figure 2: Working region.

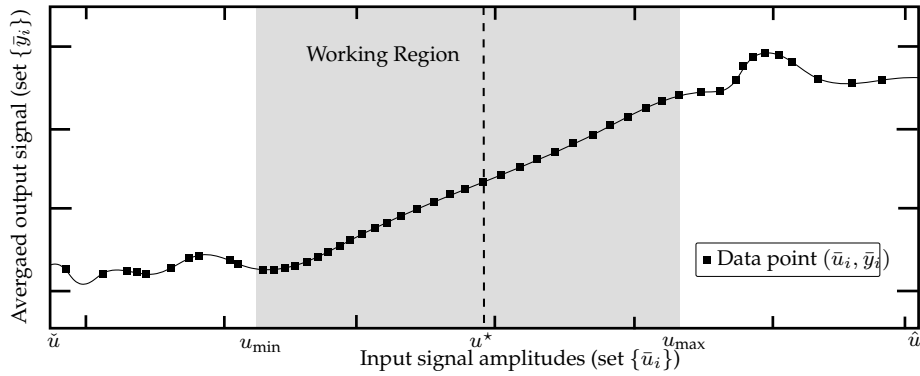


Figure 3: Example of how to choose the working region.

## 4 Input signals

This section should include:

- characteristics of chosen input signals.
- motivation for chosen input signals.

## 5 Estimation and validation data

This section should include:

- information about the amount of data used for identification and validation.

## 6 Models

This section should include:

- descriptions of model structures used. (You should use more than one structure.)
- motivation for choice of model order for each model structure.
- plots of Bode diagrams, poles and zeros.
- analysis/interpretations of plots.
- validation performed with simulation (`compare with M=inf`) and correlation analysis (`resid`).
- analysis/interpretations of validation and correlation analysis.
- a comparison between the accuracy of models obtained with different input signals (binary random signals v/s uniformly distributed white noise)
- a ranking of estimated models with respect to how well they describe the process, along with a rigorous motivation of chosen ranking. (Compare the analysis of plots, validation and correlation analysis for the different models.)

## Attachments

### generateBinarySignal.m

```
1 function u = generateBinarySignal(alpha, lims, N)
2     %% Initialization
3     w = 2 * (rand(N,1) > alpha) - 1;
4     u = zeros(N,1);
5
6     %% Initial value of u
7     if w(1) == -1
8         u(1) = lims(2);
9     else
10        u(1) = lims(1);
11    end
12
13    %% Rest values of u
14    for i = 2 : N
15        if w(i) == -1
16            if u(i-1) == lims(1)
17                u(i) = lims(2);
18            else
19                u(i) = lims(1);
20            end
21        else
22            u(i) = u(i-1);
23        end
24    end
25 end
```

### getAverage.m

```
1 function bar_v = getAverage(v, tail)
2     bar_v = mean(v(length(v)*tail/100 + 1 : end));
3 end
```

### getStationaryAverages.m

```
1 function bar_y = getStationaryAverages(y_step, Nwr, tail)
2     wrLen = length(y_step)/Nwr;
3     for i=1:Nwr
4         bar_y(i) = getAverage(y_step((i-1)*wrLen+1:(i)*wrLen),tail);
5     end
6 end
```

### findWorkingRegion.m

```
1 close all; clear all; clc;
2 %% Initialization
3 Ts = 0.001;
4 t = 0;
5 i = 1;
6 maxCurrent = 10;
7 stairHeight = 0.1;
8 stairDuration = 0.2;
9
10 %% Create Stair Signal
```

```

11 stairSignal(i) = 0;
12 while true
13     t = t + Ts;
14     i = i + 1;
15     if t < stairDuration
16         stairSignal(i) = stairSignal(i-1);
17     else
18         if stairSignal(i-1) + stairHeight > maxCurrent
19             break
20         end
21         stairSignal(i) = stairSignal(i-1) + stairHeight;
22         t = 0;
23     end
24 end
25 t = 0 : Ts : (length(stairSignal)-1) * Ts;
26 t = t.';
27 stairSignal = stairSignal.';
28
29 %% Plot Stair Signal and Time
30 figure
31 plot(t,stairSignal)
32 xlabel('Time (s)')
33 ylabel('Stair Input (A)')
34
35 %% Input Signal Amplitudes
36 averagedU = getStationaryAverages(stairSignal, (maxCurrent/stairHeight
    +1), 0.6);
37
38 %% Conducting Experiment
39 y = getData(stairSignal, Ts);
40
41 %% Averaged Output Signal
42 averagedY = getStationaryAverages(y, (10/stairHeight+1), 0.6);
43
44 %% Plot to Find Working Region
45 figure
46 plot(averagedU,averagedY);
47 xlabel('Input (A)')
48 ylabel('Output (cm)')

```

Note that the report is not in the format of a conference or journal paper. However, the language should be correct, concise and understandable. The figures should be clear, have captions, labels and be referenced in the text. All equations should be punctuated appropriately. (Equations are considered as part of sentences and should be treated accordingly.) All introduced symbols must be defined.