
Question1 (6 points)

Julia should be the only computer language you used in this question,

$$f_{Y|X} \propto \exp\left(-\frac{(x-y)^2}{2}\right) \times \frac{1}{1+y^2}$$

- (a) (2 points) Choose a proposal distribution and implement a rejection sampling scheme for the above posterior, where $x = 0.5$.
- (b) (1 point) Complete the following table using your sampling scheme in part (a).

Sample size n	50	250	750	1500	3000
$\mathbb{E}[Y X = 0.5]$					

- (c) (2 points) Choose a proposal distribution and implement a importance sampling scheme for the above posterior, where $x = 0.5$.
- (d) (1 point) Complete the following table using your sampling scheme in part (c).

Sample size n	50	250	750	1500	3000
$\mathbb{E}[Y X = 0.5]$					

Question2 (2 points)

Consider the following bivariate normal distribution.

$$\mathbf{Y} \sim \text{Normal}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

where

$$\boldsymbol{\mu} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \boldsymbol{\Sigma} = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix}$$

Suppose you can only generate uniform pseudo random numbers, and the inverse of CDF of a normal random variable involves “dark magic” that you are not allowed to touch. Describe in detail how to obtain a sample of \mathbf{Y} .

Question3 (2 points)

Discuss in what sense is Gibbs sampling a special case of Metropolis-Hastings algorithm.