VE414 Appendix 1

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Basic properties of probability

The set of all possible outcomes Ω is known as a sample space, and a subset $\mathcal{A}$ of Ω is known as an event. The following properties holds for probability:

1. The numeric bound

$$0 \leq \Pr\left(\mathcal{A}\right) \leq 1$$

2. Th probability of the empty set

$$\Pr\left(\varnothing\right) = 0$$

3. Monotonicity

$$\Pr\left(\mathcal{A}\right) \leq \Pr\left(\mathcal{B}\right) \qquad \text{where} \qquad \mathcal{A} \subset \mathcal{B}$$

4. Additive Law

$$\Pr\left(\mathcal{A} \cup \mathcal{B}\right) = \Pr\left(\mathcal{A}\right) + \Pr\left(\mathcal{B}\right) - \Pr\left(\mathcal{A} \cap \mathcal{B}\right)$$

Conditional Probability

Definition

Let $\mathcal{A}$ and $\mathcal{B}$ be two events with

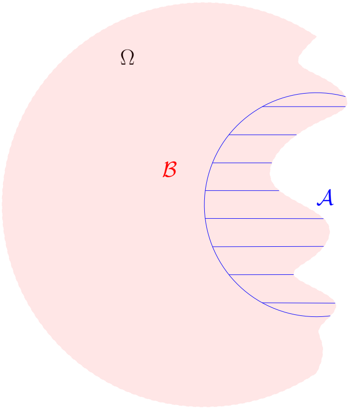
$$\Pr(\mathcal{B}) \neq 0$$

The conditional probability of $\mathcal{A}$ given $\mathcal{B}$ is defined to be

$$\Pr(\mathcal{A} \mid \mathcal{B}) = \frac{\Pr(\mathcal{A} \cap \mathcal{B})}{\Pr(\mathcal{B})}$$

- Note this is a definition, not a property of probability. The idea behind this definition is that if we are given that event $\mathcal{B}$ occurred, the relevant sample space becomes $\mathcal{B}$ rather than Ω, and conditional probability is a probability measure on the "new sample space" $\mathcal{B}$.

Conditional Probability and Multiplication Law



$$\Pr(\mathcal{A} \mid \mathcal{B}) = \frac{\Pr(\mathcal{A} \cap \mathcal{B})}{\Pr(\mathcal{B})}$$

$$\begin{aligned}\Pr(\mathcal{A} \mid \mathcal{B}) \Pr(\mathcal{B}) &= \Pr(\mathcal{A} \cap \mathcal{B}) \\ &= \Pr(\mathcal{B} \mid \mathcal{A}) \Pr(\mathcal{A})\end{aligned}$$

Law of Total Probability

Let $\mathcal{B}_1$, $\mathcal{B}_2$, ..., $\mathcal{B}_n$ be such that $\bigcup_{i=1}^{n} \mathcal{B}_i = \Omega$ and $\mathcal{B}_i \cap \mathcal{B}_j = \varnothing$ for $i \neq j$, with

$$\Pr(\mathcal{B}_i) \neq 0 \qquad \text{for all } i$$

Then, for any even $\mathcal{A}$,

$$\Pr(\mathcal{A}) = \sum_{i=1}^{n} \Pr(\mathcal{A} \mid \mathcal{B}_i) \Pr(\mathcal{B}_i)$$

Proof

$$\Pr(\mathcal{A}) = \Pr(\mathcal{A} \cap \Omega) = \Pr\left(\mathcal{A} \cap \left(\bigcup_{i=1}^{n} \mathcal{B}_i\right)\right) = \Pr\left(\bigcup_{i=1}^{n} (\mathcal{A} \cap \mathcal{B}_i)\right)$$

$$\implies \Pr(\mathcal{A}) = \sum_{i=1}^{n} \Pr(\mathcal{A} \cap \mathcal{B}_i) = \sum_{i=1}^{n} \Pr(\mathcal{A} \mid \mathcal{B}_i) \Pr(\mathcal{B}_i)$$

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