Homework 1
VE475 Introduction to Cryptography

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Ex. 1 - Simple questions

1. Since Alice uses Caesar cipher and we already known the ciphertext, we can apply CCA to decrypt the ciphertext, "EVIRE". So, the 25 possible names of the place are: *duhqd*, *ctgpc*, *bsfob*, *arena*, *zqdmz*, *ypcly*, *xobkx*, *wnajw*, *vmziv*, *ulyhu*, *tkxgt*, *sjwfs*, *river*, *qhudq*, *pgtcp*, *ofsbo*, *neran*, *mdqzm*, *lcpyl*, *kboxk*, *janwj*, *izmvi*, *hyluh*, *gxktg*, or *fwjsf*. However, only "*arena*" and "*river*" are meaningful, which could be the meeting place.

2. Since the length of plaintext *dont* is 4, reasonable size of the key should be 2×2. Label letters as integers from 0 to 25, the plaintext then is $\begin{pmatrix}3 & 14 & 13 & 19\end{pmatrix}$ and the ciphertext is $\begin{pmatrix}4 & 11 & 13 & 8\end{pmatrix}$.

$$\underbrace{\begin{pmatrix}3 & 14\\ 13 & 19\end{pmatrix}}_{A} \cdot \begin{pmatrix}a & b\\ c & d\end{pmatrix} \equiv \begin{pmatrix}4 & 11\\ 13 & 8\end{pmatrix} \mod 26$$

   Since $\det(A) = -125$ and $\gcd(-125, 26) = 1$, A is invertible modulo 26. Also, we can obtain that -5 is the multiplicative inverse of -125 modulo 26 by applying extended Euclidean algorithm. We can then calculate

$$\begin{aligned}
K = \begin{pmatrix}a & b\\ c & d\end{pmatrix} &\equiv \begin{pmatrix}3 & 14\\ 13 & 19\end{pmatrix}^{-1} \cdot \begin{pmatrix}4 & 11\\ 13 & 8\end{pmatrix} \mod 26\\
&\equiv \frac{1}{-125} \cdot \begin{pmatrix}19 & -14\\ -13 & 3\end{pmatrix} \cdot \begin{pmatrix}4 & 11\\ 13 & 8\end{pmatrix} \mod 26\\
&\equiv \frac{1}{-125} \cdot \begin{pmatrix}-106 & 97\\ -13 & -119\end{pmatrix} \mod 26\\
&\equiv \begin{pmatrix}530 & -485\\ 65 & 595\end{pmatrix} \mod 26\\
&\equiv \begin{pmatrix}10 & 9\\ 13 & 23\end{pmatrix} \mod 26
\end{aligned}$$

   So, the encryption matrix is $K = \begin{pmatrix}10 & 9\\ 13 & 23\end{pmatrix}$.

3.

Ex. 2 -