Natural Language Models and Interfaces

BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2019, week 2

NLMI

Probability of a sentence

Language models

Smoothing

Evaluating language models

Chomsky once said

It must be recognised that the notion "probability of a sentence" is an entirely useless one, under any known interpretation of this term — Chomsky, 1969

- Chomsky is the father of modern linguistics
- he also made significant contributions to formal language theory

Yet here we are discussing how we will be assigning probabilities to sentences

- should we listen to authority?

Objective probability

Perhaps Chomsky only acknowledges objective probability

- a notion of *frequency* or *propensity*
- a tendency of a given situation to yield a certain outcome

Example

The winged Irritator Challengeri pounced upon the hapless Bambiraptor

- how many times have you heard this sentence in your lifetime?

Under such a view, Chomsky's observation (or claim) is pretty reasonable

---

Propensity probability

Subjective probability

Under the subjective view, probability has nothing to do with the frequency or propensity of outcomes

- it is a notion of reasonable expectation
- represents a state of knowledge
  or a quantification of personal belief

Example

The winged Irritator Challengeri pounced upon the hapless Bambiraptor

- according to your knowledge of English, is it at all reasonable to think of this as as sentence and expect it may be uttered?

Bayesian probability

Subjective vs objective probability

It's unlikely we will ever hear either

- The winged Irritator Challengeri pounced upon the hapless Bambiraptor
- Upon the pounced winged hapless Bambiraptor the Challengeri Irritator winged

yet it's far more reasonable to expect one than the other

- and that's why we will not listen to authority
  at least this time ;)

Chomsky's theories and contributions are pretty important though!

Random sentences

We will model probability distributions over sentences!

- let's start with a vocabulary of words Σ
- our *language is a subset of strings* in Σ^*

Kleene star Index set

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- define an rv X that maps from Σ to $\mathbb{R}$
- the mapping is an arbitrary enumeration of Σ
  some word is mapped to 1, some other word is mapped to 2, ..., some other word is mapped to $v = |\Sigma|$
- with $v = |\Sigma|$ we say
  let X *take on values in an index set* $\mathcal{X} = \{1, \ldots, v\}$ *of* Σ

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Quiz

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▸ Quiz

A random sentence S of length n is a **sequence** of random words

$\langle X_1, \ldots, X_n \rangle$ which we also denote X_1^n

Kleene star Index set

Probability of a sentence

The probability of a sentence $\langle x_1, \ldots, x_n \rangle$

$$P_S(\langle x_1, \ldots x_n \rangle) = P_N(n) \underbrace{\prod_{i=1}^{n} P_{X|H}(x_i | x_{<i})}_{\text{chain rule}}$$

- x_1^n shorthand for the **sequence** $\langle x_1, \ldots, x_n \rangle$ and x_1^n is $\langle x_1 \rangle$ if $n = 1$
- $x_{<i}$ shorthand for the **prefix sequence** $\langle x_1, \ldots, x_{i-1} \rangle$ and $x_{<i}$ is the empty sequence $\langle \rangle$ if $i = 1$
- we call the random sequence H the history like S, its sample space is Σ^*

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A sentence is a **structured object** — for example it has an *order*

- above is a *factorisation* of the distribution $P_S(S)$
- it *chops* the structure *generating* one piece (a word) at a time
- a factor $P_{X|H}$ is a *conditional probability distribution* (cpd)

Generative story

The stochastic procedure that yields a sentence is

- also known as **generative story**
  1. Sample a length $N \sim P_N$
  2. For $i = 1, \ldots, n$
     - $X_i | x_{<i} \sim P_{X|H}$

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- $\langle X_1, X_2, X_3, X_4, X_5 \rangle$

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Quiz

Notation mess

You will often find in LM literature

$$\underbrace{P(x_1, \ldots, x_n)}_{\text{joint probability}} = \underbrace{P(x_1) \prod_{i=2}^{n} P(x_i | x_1, \ldots, x_{i-1})}_{\text{chain rule}}$$

- joint distribution does not care about the order of its arguments
  - thus this actually hides that $x_1, \ldots, x_n$ is a **sequence**
  - indices are naming the rvs (not an arbitrary enumeration)

$$\begin{aligned} P(X_1 = x_1, X_2 = x_2, X_3 = x_3) &= P(X_3 = x_3, X_1 = x_1, X_2 = x_2) \\ &= P_{X_1}(x_1) P_{X_2|X_1}(x_2|x_1) P_{X_3|X_1 X_2}(x_3|x_1, x_2) \end{aligned}$$

- while a correct application of chain rule, in a modelling context, this hides the fact that the length of the sequence is itself random

Less ambiguous notation

Instead of

$$P(x_1, \ldots, x_n) = P(x_1) \prod_{i=2}^{n} P(x_i | x_1, \ldots, x_{i-1})$$

We prefer

- $\langle x_1, \ldots, x_n \rangle$ or x_1^n for sequences
- $\langle x_1, \ldots, x_{i-1} \rangle$ or $x_{<i}$ for prefix sequences

and the joint probability factorises as

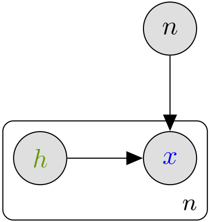
$$P_S(x_1^n) = P_N(n) P_{S|N}(x_1^n | n)$$

- with $P_{S|N}(x_1^n | n) = \prod_{i=i}^{n} P_{X|H}(x_i | x_{<i})$

Language models are directed graphical models

Probabilistic directed graphical model (or Bayesian net)

- directed acyclic graph
- nodes are random variables
- a plate represents a loop
- a directed arrow denotes **conditional dependence**



$$P_S(\langle x_1, \ldots x_n \rangle) = P_N(n) \prod_{i=1}^{n} P_{X|H}(x_i|x_{<i})$$

DAG PGM course PGM book

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Parameterisation

We are very close to defining a complete model

- we need to choose parametric families for P_N and $P_{X|H}$

Length distribution P_N

- for simplicity we pick some uniform distribution
  thus $P_N(n) = c$

Next word distribution $P_{X|H}$

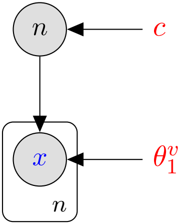
- we start by making a very unrealistic simplifying assumption
  - we assume the next word is independent of the history $X \perp H$
    i.e. $P_{X|H} = P_X$
- and make P_X a categorical distribution $\mathrm{Cat}(\theta_1, \ldots, \theta_v)$

Realistic length distributions can be rather complex (Sichel 1974)

Unigram language model

Unigram factorisation and categorical parameterisation

$$\begin{aligned} P_S(\langle x_1, \ldots x_n \rangle) &= P_N(n) \prod_{i=1}^{n} P_{X|H}(x_i|x_{<i}) \\ &\approx c \prod_{i=1}^{n} P_X(x_i) \\ &\propto \prod_{i=1}^{n} \mathrm{Cat}(X = x_i|\theta_1, \ldots, \theta_v) \end{aligned}$$



Recall the Categorical pmf

- $\mathrm{Cat}(X = a|\theta_1, \ldots, \theta_v) = \prod_{x=1}^{v} \theta_x^{\delta xa}$

Kronecker delta

Unigram LM - illustration

$N \sim P_N$

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Let's see what's wrong with the unigram model

Consider the probability of the sentences

- the winged irritator challengeri pounced upon the hapless bambiraptor
- upon the pounced hapless bambiraptor the challengeri irritator winged

Does the model quantify a *reasonable* expectation?

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Unigram language models see a sentence as a *multiset*

- but before we fix them, let's get to estimation of θ_1^v

Multiset

MLE for unigram LMs

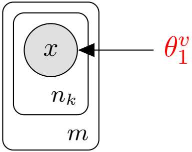
The key cpd in the unigram LM is

$$P_X(X; \theta_1^v) = \mathrm{Cat}(\theta_1, \ldots, \theta_v)$$

which is specified by v parameters (word probabilities)

Say we have a dataset of m observations $\left(\left\langle x_1^{(k)}, \ldots, x_{n_k}^{(k)} \right\rangle\right)_{k=1}^{m}$

- what's the MLE solution for θ_x?



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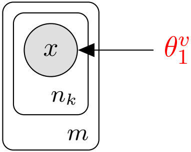

$$\theta_x = \frac{\sum_{k=1}^{m} \sum_{i=1}^{n_k} [x = x_i^{(k)}]}{\sum_{k=1}^{m} n_k} = \frac{\mathrm{count}(x)}{\text{number of tokens}}$$

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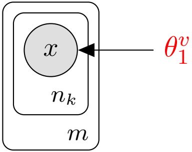

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Quiz

Iverson bracket

How dow we fix a unigram LM?

We need to relax our way-too-strong independence assumption

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but there's a problem with conditioning on the complete history

- and to understand it we need to look into tabular cpds

Tabular cpds

A tabular cpd is a set of probability distributions,
for each **conditioning context** we get a new distribution

Recall the example involving grades and recommendation letters

$P_{G|L}$

Letter (L)	Grade (G) $[0,6)$ 1	$[6,8)$ 2	$[8,10]$ 3
0	0.27	0.71	0.02
1	0.10	0.68	0.22

$P_{G|L}$

Letter (L)	Grade (G) $[0,6)$ 1	$[6,8)$ 2	$[8,10]$ 3
0	$\theta_1^{(0)}$	$\theta_2^{(0)}$	$\theta_3^{(0)}$
1	$\theta_1^{(1)}$	$\theta_2^{(1)}$	$\theta_3^{(1)}$

Table: Conditional distribution: $G|L = l \sim \mathrm{Cat}(\theta_1^{(l)}, \ldots, \theta_3^{(l)})$

Data sparsity

If we have 1 cpd per assignment of the conditioning variable
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The more parameters we need to estimate,
the more data we need

- many valid histories will never be seen
  The winged Irritator Challengeri pounced $\rightarrow ?$

Conditional independence

oth order Markov assumption

- we forget some —but not all—history
- make next word independent of
  all but o preceding words
- we call this class of models n-gram language models
  we use $o = n - 1$ to avoid confusion with sentence length

Next word distribution $X_i | x_{<i} \approx x_{i-o}^{i-1} \sim \mathrm{Cat}\left(\theta_1^{(h_i)}, \ldots, \theta_v^{(h_i)}\right)$

- with x_{i-o}^{i-1} a shorthand for $\langle x_{i-o}, \ldots, x_{i-1} \rangle$
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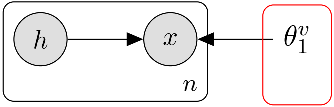
How many cpds can we have?



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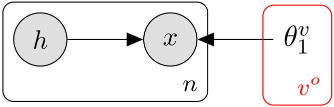

Conditional independence

oth order Markov assumption

- we forget some —but not all—history
- make next word independent of **all but** o preceding words
- we call this class of models n-gram language models

  we use $o = n - 1$ to avoid confusion with sentence length

Next word distribution $\quad X_i|x_{<i} \approx x_{i-o}^{i-1} \sim \mathrm{Cat}\left(\theta_1^{(h_i)}, \ldots, \theta_v^{(h_i)}\right)$

- with x_{i-o}^{i-1} a shorthand for $\langle x_{i-o}, \ldots, x_{i-1}\rangle$

  $\langle\rangle$ if $o = 0$ and $\langle x_{i-1}\rangle$ if $o = 1$
- h_i uniquely identifies the ith shortened random history

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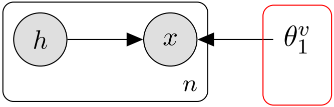

Quiz

Bigram LM (1-order assumption) - illustration

$N \sim P_N$

More systematic with BoS and EoS padding: o leading BoS and 1 trailing EoS

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Factorisation of n-gram LMs

Unigram LM – 0-order Markov model (MM)

the winged irritator challengeri pounced upon the hapless bambiraptor

Bigram LM – 1st order MM

1. the | BoS
2. winged | the
3. irritator | winged
4. challengeri | irritator
5. pounced | challengeri
6. upon | pounced
7. the | upon
8. hapless | the
9. bambiraptor | hapless
10. EoS | bambiraptor

Trigram LM – 2nd order MM

1. the | BoS BoS
2. winged | BoS the
3. irritator | the winged
4. challengeri | winged irritator
5. pounced | irritator challengeri
6. upon | challengeri pounced
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9. bambiraptor | the hapless
10. EoS | hapless bambiraptor

MLE for n-gram language models

oth order factorisation and categorical parameterisation

$$\begin{aligned} P_S(\langle x_1, \ldots x_n \rangle) &= P_N(n) \prod_{i=1}^{n} P_{X|H}(x_i | x_{<i}) \\ &\approx c \prod_{i=1}^{n} P_{X|H}(x_i | x_{i-o}^{i-1}) \\ &\propto \prod_{i=1}^{n} \mathrm{Cat}(X = x_i | \theta_1^{(h_i)}, \ldots, \theta_v^{(h_i)}) \end{aligned}$$

Iverson bracket

— We use $a \circ b$ to denote sequence concatenation

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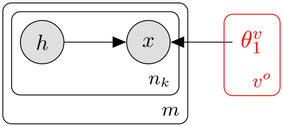

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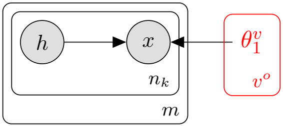

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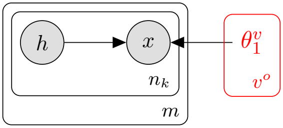

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Iverson bracket Quiz — We use $a \circ b$ to denote sequence concatenation

NLMI

Probability of a sentence

Language models

Smoothing

Evaluating language models

Have we really beaten data sparsity?

Dinosaurs have been long extinct

the winged irritator challengeri pounced upon the hapless bambiraptor

- How many of these words do you expect to find in newswire corpora?
- What about higher-order n-grams?
- The probability of the sentence is likely 0
  - it takes one unseen n-gram
    e.g. *challengeri* or *bambiraptor*

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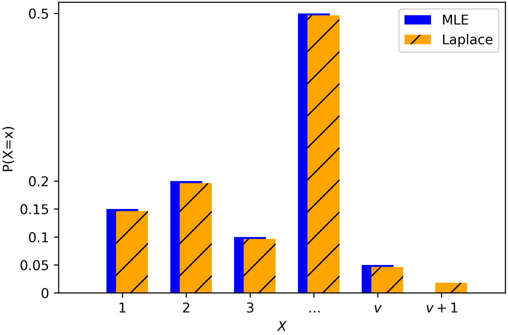
MLE assigns probability to **observed** n-grams only

Smoothing — Rationale

We can take probability mass away from **seen** n-grams
and reserve such mass to **unseen** n-grams

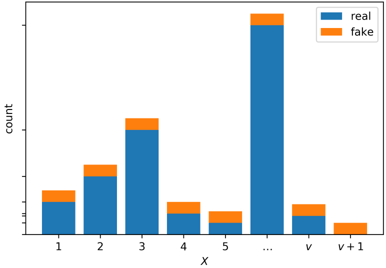
Example unigram distribution



Laplace smoothing: "add 1 smoothing"

1. Sample space: redefine to include an $\textsc{Unk}$ token
   $\Sigma := \Sigma \cup \{\textsc{Unk}\}$ and $v := v + 1$
2. RV: redefine X to map unseen symbols to $\textsc{Unk}$'s index
3. MLE: augment counts by 1
   $\text{count}(\textsc{UnK}) = 0 \to 1$

Example unigram counts



Laplace smoothing: "add α smoothing"

Unsmoothed MLE

$$\theta_x^{(h)} = \frac{\text{count}(h, x)}{\text{count}(h)}$$

Laplace-smoothed: $\alpha > 0$

$$\theta_x^{(h)} = \frac{\text{count}(h, x) + \alpha}{\sum_{x \in \mathcal{X}} \text{count}(h, x) + \alpha} = \frac{\text{count}(h, x) + \alpha}{\text{count}(h) + v\alpha}$$

Iterpolation

Back-off to further shortened histories

- if we haven't seen *challengeri pounced upon*
  we might have seen pounced upon

Trigram example

$$
\begin{aligned}
P_{X|H}(x_i|\langle x_{i-2}, x_{i-1}\rangle) &= \lambda_1 P_{X|H}(x_i|\langle x_{i-2}, x_{i-1}\rangle) && \text{trigram} \\
&+ \lambda_2 P_{X|H}(x_i|\langle x_{i-1}\rangle) && \text{bigram} \\
&+ \lambda_3 P_X(x_i) && \text{unigram} \\
&\text{with } \lambda_1 + \lambda_2 + \lambda_3 = 1
\end{aligned}
$$

Weights may be a function of history
e.g. $\lambda_1 := \lambda_1(\langle x_{i-2}, x_{i-1}\rangle)$

See for example Katz's back-off model

Data pre-processing

Map infrequent types to UNK

- for example, all types that occur once

This also has the effect of reducing the number of parameters

- you can always use this to reduce memory requirements
  including in lab exercises ;)
- but use sensible thresholds given the size of the data
  e.g. 1, 2, or 3
- and always explain your choices

Bag of tricks

Smoothing techniques

e.g. discounting, interpolation, data pre-processing

- are tricks to make MLE more useful
- some are justified by frequentist statistics
- some are simply necessary hacks

Manning and Schütze (1999) as well as Jurafsky and Martin (2000) discuss more techniques and in greater detail

NLMI

Probability of a sentence

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Smoothing

Evaluating language models

How do we compare language models?

Model $\mathcal{M}$

- a set of conditional dependence statements
  graphical structure
- a parameterisation and a set of parameters $\boldsymbol{\theta}$

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- training data: sentences used to estimate parameters
- test data: a disjoint set of sentences used to assess models

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We would like to compare models by comparing

- the probability they assign to held-out (iid) data $\mathcal{D}$

$$\prod_{x_1^n \in \mathcal{D}} P_S(x_1^n|\mathcal{M}_1) \overset{?}{>} \prod_{x_1^n \in \mathcal{D}} P_S(x_1^n|\mathcal{M}_2)$$

- but P_S depends on choice of factorisation

Perplexity

For dataset $\mathcal{D}$ and model $\mathcal{M}$

$$\mathrm{PP}(\mathcal{D}; \mathcal{M}) = \left(\prod_{x_1^n \in \mathcal{D}} P_S(x_1^n; \mathcal{M}) \right)^{-1/t}$$

where t is the number of tokens in $\mathcal{D}$

Assuming the length component is the same for every model in the comparison

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Or in log-domain: $\log \mathrm{PP}(\mathcal{D};\mathcal{M}) =$

$$= -\frac{1}{t}\left[\sum_{k=1}^{m} \log P_N(n_k) + \log P_{S|N}\left(\langle x_1^{(k)},\ldots,x_{n_k}^{(k)}\rangle | n_k;\mathcal{M}\right)\right]$$

$$= -\frac{1}{t}\left[\log P_{S|N}\left(\langle x_1^{(k)},\ldots,x_{n_k}^{(k)}\rangle | n_k;\mathcal{M}\right)\right] + C$$

$$\propto -\frac{1}{t}\left[\sum_{k=1}^{m}\sum_{i=1}^{n_k} P_{X|H}\left(x_i^{(k)} | x_{<i}^{(k)};\mathcal{M}\right)\right]$$

Assuming the length component is the same for every model in the comparison

Perplexity: interpretation

Perplexity can be seen as

- *average branching factor* of the language according to the estimated model
- branching factor: number of words that may follow any word

Comparing models using perplexity require

- their support must overlap
  i.e. there is a common set of sentences to which both models assign non-zero probability
- test sentences must be in that common set
  for n-gram models this typically requires
  - smoothing
  - shared vocabulary

References I

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