

Natural Language Models and Interfaces

BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2018, week 2, lecture a

NLMI

Probability of a sentence

Language models

Smoothing

Evaluating language models

Chomsky once said

It must be recognised that the notion “probability of a sentence” is an entirely useless one, under any known interpretation of this term — Chomsky, 1969

- ▶ Chomsky is the father of modern linguistics
- ▶ he also made significant contributions to formal language theory

Yet here we are discussing how we will be assigning probabilities to sentences

- ▶ should we listen to authority?

Objective probability

Perhaps Chomsky only acknowledges **objective probability**

- ▶ a notion of *frequency* or *propensity*
- ▶ a tendency of a given situation to yield a certain outcome

Example

The winged Irritator Challengeri pounced upon the hapless Bambiraptor

- ▶ how many times have you heard this sentence in your lifetime?

Under such a view, Chomsky's observation (or claim) is pretty reasonable

Subjective probability

Under the **subjective view**, probability has nothing to do with the frequency or propensity of outcomes

- ▶ it is a notion of **reasonable expectation**
- ▶ represents a **state of knowledge**
or a **quantification of personal belief**

Example

The winged Irritator Challengeri pounced upon the hapless Bambiraptor

- ▶ according to your knowledge of English, is it at all reasonable to think of this as a sentence and expect it may be uttered?

Subjective vs objective probability

It's unlikely we will ever hear either

- ▶ The winged Irritator Challengeri pounced upon the hapless Bambiraptor
- ▶ Upon the pounced winged hapless Bambiraptor the Challengeri Irritator winged

yet it's far more reasonable to **expect** one than the other

- ▶ and that's why we will not listen to authority at least this time ;)

Chomsky's theories and contributions are pretty important though!

Random sentences

We will model probability distributions over **sentences**!

- ▶ let's start with a vocabulary of **words** Σ
- ▶ and let's assume a *language is a subset of strings* in Σ^*

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Let's start with **random words**

- ▶ define an rv X that maps from Σ to \mathbb{R}
- ▶ the mapping is an arbitrary enumeration of Σ
some word is mapped to 1, some other word is mapped to 2, ..., some other word is mapped to $v = |\Sigma|$
- ▶ with $v = |\Sigma|$ we say
let X take on values in an index set $\mathcal{X} = \{1, \dots, v\}$ of Σ

▶ Quiz

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A random sentence S of length n is a **sequence** of random words

$$\langle X_1, \dots, X_n \rangle \quad \text{which we also denote } X_1^n$$

Probability of a sentence

The probability of a sentence $\langle x_1, \dots, x_n \rangle$

$$P_S(\langle x_1, \dots, x_n \rangle) = P_N(n) \underbrace{\prod_{i=1}^n P_{X|H}(x_i | x_{<i})}_{\text{chain rule}}$$

- ▶ x_1^n shorthand for the **sequence** $\langle x_1, \dots, x_n \rangle$
and x_1^n is $\langle x_1 \rangle$ if $n = 1$
- ▶ $x_{<i}$ shorthand for the **prefix sequence** $\langle x_1, \dots, x_{i-1} \rangle$
and $x_{<i}$ is the empty sequence $\langle \rangle$ if $i = 1$
- ▶ we call the random sequence H the **history**
like S , its sample space is Σ^*

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A sentence is a **structured object** — for example it has an *order*

- ▶ above is a **factorisation** of the distribution $P_S(S)$
- ▶ it *chops* the structure **generating** one piece (a word) at a time
- ▶ a factor $P_{X|H}$ is a **conditional probability distribution** (cpd)

Generative story

The stochastic procedure that yields a sentence is

▶ also known as **generative story**

1. Sample a length $N \sim P_N$

2. For $i = 1, \dots, n$

▶ $X_i | x_{<i} \sim P_{X|H}$

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Quiz

Notation mess

You will often find in LM literature

$$\underbrace{P(x_1, \dots, x_n)}_{\text{joint probability}} = \underbrace{P(x_1) \prod_{i=2}^n P(x_i | x_1, \dots, x_{i-1})}_{\text{chain rule}}$$

- ▶ joint distribution does not care about the order of its arguments
 - ▶ thus this actually hides that x_1, \dots, x_n is a **sequence**
 - ▶ indices are naming the rvs (not an arbitrary enumeration)

$$\begin{aligned} P(X_1 = x_1, X_2 = x_2, X_3 = x_3) &= P(X_3 = x_3, X_1 = x_1, X_2 = x_2) \\ &= P_{X_1}(x_1) P_{X_2|X_1}(x_2|x_1) P_{X_3|X_1 X_2}(x_3|x_1, x_2) \end{aligned}$$

- ▶ while a correct application of chain rule, in a modelling context, this hides the fact that the **length** of the sequence is itself random

Less ambiguous notation

Instead of

$$P(x_1, \dots, x_n) = P(x_1) \prod_{i=2}^n P(x_i | x_1, \dots, x_{i-1})$$

We prefer

- ▶ $\langle x_1, \dots, x_n \rangle$ or x_1^n for sequences
- ▶ $\langle x_1, \dots, x_{i-1} \rangle$ or $x_{<i}$ for prefix sequences

and the joint probability factorises as

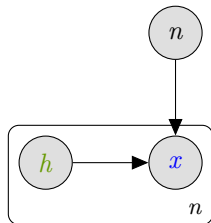
$$P_S(x_1^n) = P_N(n) P_{S|N}(x_1^n | n)$$

- ▶ with $P_{S|N}(x_1^n | n) = \prod_{i=1}^n P_{X|H}(x_i | x_{<i})$

Language models are directed graphical models

Probabilistic directed graphical model (or Bayesian net)

- ▶ directed acyclic graph
- ▶ nodes are **random variables**
- ▶ a plate represents a loop
- ▶ a directed arrow denotes **conditional dependence**



$$P_S(\langle x_1, \dots, x_n \rangle) = P_N(n) \prod_{i=1}^n P_{X|H}(x_i | x_{<i})$$

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Parameterisation

We are very close to defining a complete model

- ▶ we need to choose parametric families for P_N and $P_{X|H}$

Length distribution P_N

- ▶ for simplicity we pick some uniform distribution
thus $P_N(n) = c$

Next word distribution $P_{X|H}$

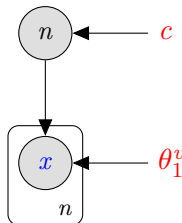
- ▶ we start by making a **very unrealistic** simplifying assumption
 - ▶ we assume the next word is independent of the history $X \perp H$
i.e. $P_{X|H} = P_X$
- ▶ and make P_X a categorical distribution $\text{Cat}(\theta_1, \dots, \theta_v)$

Realistic length distributions can be rather complex (Sichel 1974)

Unigram language model

Unigram factorisation and categorical parameterisation

$$\begin{aligned}P_S(\langle x_1, \dots, x_n \rangle) &= P_N(n) \prod_{i=1}^n P_{X|H}(x_i | x_{<i}) \\&\approx c \prod_{i=1}^n P_X(x_i) \\&\propto \prod_{i=1}^n \text{Cat}(X = x_i | \theta_1, \dots, \theta_v)\end{aligned}$$



Recall the Categorical pmf

$$\blacktriangleright \text{Cat}(X = a | \theta_1, \dots, \theta_v) = \prod_{x=1}^v \theta_x^{\delta_{xa}}$$

Unigram LM - illustration

$$N \sim P_N$$

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Let's see what's wrong with the unigram model

Consider the probability of the sentences

- ▶ the winged irritator challenger_i pounced upon the hapless bambiraptor
- ▶ upon the pounced winged hapless bambiraptor the challenger_i irritator winged

Does the model quantify a *reasonable* expectation?

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Unigram language models see a sentence as a *multiset*

- ▶ but before we fix them, let's get to estimation of θ_1^v

MLE for unigram LMs

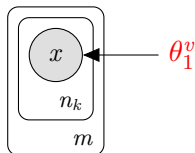
The key cpd in the unigram LM is

$$P_X(X; \theta_1^v) = \text{Cat}(\theta_1, \dots, \theta_v)$$

which is specified by v **parameters** (**word probabilities**)

Say we have a dataset of m observations $\left(\langle x_1^{(k)}, \dots, x_{n_k}^{(k)} \rangle \right)_{k=1}^m$

► what's the MLE solution for θ_x ?



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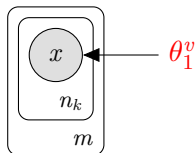
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$$\begin{aligned}\theta_x &= \frac{\sum_{k=1}^m \sum_{i=1}^{n_k} [x = x_i^{(k)}]}{\sum_{k=1}^m n_k} \\ &= \frac{\text{count}(x)}{\text{number of tokens}}\end{aligned}$$

How do we fix a unigram LM?

We need to relax our **way-too-strong** independence assumption

- ▶ that is, we need to condition on history

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We need to relax our **way-too-strong** independence assumption

- ▶ that is, we need to condition on history

but there's a problem with conditioning on the complete history

- ▶ and to understand it we need to look into **tabular cpds**

Tabular cpds

A tabular cpd is a set of probability distributions,
for each **conditioning context** we get a new distribution

Recall the example involving grades and recommendation letters

$P_{G L}$				$P_{G L}$			
Letter (L)	Grade (G)			Letter (L)	Grade (G)		
	[0, 6)	[6, 8)	[8, 10]		[0, 6)	[6, 8)	[8, 10]
	1	2	3		1	2	3
0	0.27	0.71	0.02	0	$\theta_{0,1}$	$\theta_{0,2}$	$\theta_{0,3}$
1	0.10	0.68	0.22	1	$\theta_{1,1}$	$\theta_{1,2}$	$\theta_{1,3}$

Table: Conditional distribution: $G|L = l \sim \text{Cat}(\theta_{l,1}, \dots, \theta_{l,3})$

Data sparsity

If we have 1 cpd per assignment of the conditioning variable
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The more parameters we need to estimate,
the more data we need

- ▶ many valid histories will never be seen

The winged Irritator Challengeri pounced →?

Conditional independence

o th order Markov assumption

- ▶ we forget some—but not all—history
- ▶ make **next word** independent of **all but** o preceding words
- ▶ we call this class of models n -gram language models
we use $o = n - 1$ to avoid confusion with sentence length

Next word distribution $X_i | x_{<i} \approx x_{i-o}^{i-1} \sim \text{Cat}(\theta_1^{(h_i)}, \dots, \theta_v^{(h_i)})$

- ▶ with x_{i-o}^{i-1} a shorthand for $\langle x_{i-o}, \dots, x_{i-1} \rangle$
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Conditional independence

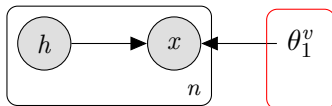
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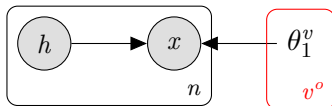
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How many cpds can we have?



Bigram LM (1-order assumption) - illustration

$$N \sim P_N$$

More systematic with BoS and EoS padding: o leading BoS and 1 trailing EoS

Bigram LM (1-order assumption) - illustration

$$N \sim P_N$$

$$\blacktriangleright \langle \text{BoS}, X_1, X_2, X_3, X_4, X_5, X_6 = \text{BoS} \rangle$$

More systematic with BoS and EoS padding: o leading BoS and 1 trailing EoS

Bigram LM (1-order assumption) - illustration

$$N \sim P_N$$

$$\blacktriangleright \langle \text{BoS}, X_1, X_2, X_3, X_4, X_5, X_6 = \text{BoS} \rangle$$

$$X_i | x_{<i} \approx \langle x_{i-1} \rangle \sim P_{X|H} \text{ for } i = 1, \dots, 6$$

More systematic with BoS and EoS padding: o leading BoS and 1 trailing EoS

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Factorisation of n -gram LMs

Unigram LM – 0-order Markov model (MM)

the winged irritator challenger_i pounced upon the hapless bambiraptor

Bigram LM – 1st order MM

1. the | BoS
2. winged | the
3. irritator | winged
4. challenger_i | irritator
5. pounced | challenger_i
6. upon | pounced
7. the | upon
8. hapless | the
9. bambiraptor | hapless
10. EoS | bambiraptor

Trigram LM – 2nd order MM

1. the | BoS BoS
2. winged | BoS the
3. irritator | the winged
4. challenger_i | winged irritator
5. pounced | irritator challenger_i
6. upon | challenger_i pounced
7. the | pounced upon
8. hapless | upon the
9. bambiraptor | the hapless
10. EoS | hapless bambiraptor

MLE for n -gram language models

0th order factorisation and categorical parameterisation

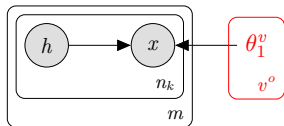
$$\begin{aligned}P_S(\langle x_1, \dots, x_n \rangle) &= P_N(n) \prod_{i=1}^n P_{X|H}(x_i | x_{<i}) \\&\approx c \prod_{i=1}^n P_{X|H}(x_i | x_{i-o}^{i-1}) \\&\propto \prod_{i=1}^n \text{Cat}(X = x_i | \theta_1^{(h_i)}, \dots, \theta_v^{(h_i)})\end{aligned}$$

MLE for n -gram language models

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What's the MLE solution for $\theta_x^{(h)}$?

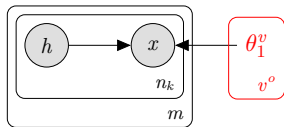


MLE for n -gram language models

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 \end{aligned}$$

What's the MLE solution for $\theta_x^{(h)}$?



$$\begin{aligned}
 \theta_x^{(h)} &= \frac{\sum_{k=1}^m \sum_{i=1}^{n_k} [h = h_i^{(k)} \wedge x = x_i^{(k)}]}{\sum_{k=1}^m \sum_{i=1}^{n_k} [h = h_i^{(k)}]} \\
 &= \frac{\text{count}(\langle x_{i-o}, \dots, x_i \rangle)}{\text{count}(\langle x_{i-o}, \dots, x_{i-1} \rangle)}
 \end{aligned}$$

NLMI

Probability of a sentence

Language models

Smoothing

Evaluating language models

Have we really beaten data sparsity?

Dinosaurs have been long extinct

the winged irritator challengeri pounced upon the hapless bambiraptor

- ▶ How many of these words do you expect to find in newswire corpora?
- ▶ What about higher-order n -grams?
- ▶ The probability of the sentence is likely 0
 - ▶ it takes one **unseen** n -gram
e.g. *challengeri* or *bambiraptor*

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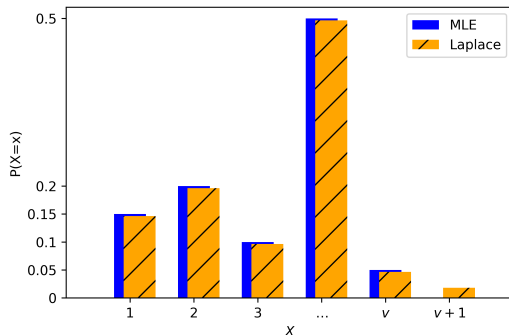
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MLE assigns probability to **observed** n -grams only

Smoothing — Rationale

We can take probability mass away from **seen** n -grams and reserve such mass to **unseen** n -grams

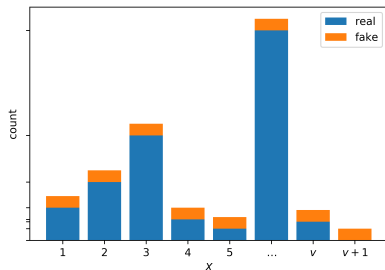
Example unigram distribution



Laplace smoothing: “add 1 smoothing”

1. Sample space: redefine to include an UNK token
 $\Sigma := \Sigma \cup \{\text{UNK}\}$ and $v := v + 1$
2. RV: redefine X to map unseen symbols to UNK's index
3. MLE: augment counts by 1
 $\text{count}(\text{UNK}) = 0 \rightarrow 1$

Example unigram counts



Laplace smoothing: “add α smoothing”

Unsmoothed MLE

$$\theta_x^{(h)} = \frac{\text{count}(\textcolor{teal}{h}, \textcolor{blue}{x})}{\text{count}(\textcolor{blue}{h})}$$

Laplace-smoothed: $\alpha > 0$

$$\theta_x^{(h)} = \frac{\text{count}(\textcolor{teal}{h}, \textcolor{blue}{x}) + \alpha}{\sum_{x \in \mathcal{X}} \text{count}(\textcolor{blue}{h}, x) + \alpha} = \frac{\text{count}(\textcolor{teal}{h}, \textcolor{blue}{x}) + \alpha}{\text{count}(\textcolor{blue}{h}) + v\alpha}$$

In the lab you will work with another discounting technique:

Good-Turing

Interpolation

Back-off to further shortened histories

- ▶ if we haven't seen *challengeri pounced upon*
we might have seen *pounced upon*

Trigram example

$$\begin{aligned}P_{X|H}(x_i|\langle x_{i-2}, x_{i-1} \rangle) &= \lambda_1 P_{X|H}(x_i|\langle x_{i-2}, x_{i-1} \rangle) && \text{trigram} \\&+ \lambda_2 P_{X|H}(x_i|\langle x_{i-1} \rangle) && \text{bigram} \\&+ \lambda_3 P_{X|H}(x_i) && \text{unigram} \\&\text{with } \lambda_1 + \lambda_2 + \lambda_3 = 1\end{aligned}$$

Weights may be a function of history

e.g. $\lambda_1 := \lambda_1(\langle x_{i-2}, x_{i-1} \rangle)$

See for example [Katz's back-off model](#)

Data pre-processing

Map infrequent types to UNK

- ▶ for example, all types that occur once

This also has the effect of reducing the number of parameters

- ▶ you can always use this to reduce memory requirements
including in lab exercises ;)
- ▶ but use sensible thresholds given the size of the data
e.g. 1, 2, or 3
- ▶ and always explain your choices

Bag of tricks

Smoothing techniques

e.g. discounting, interpolation, data pre-processing

- ▶ are tricks to make MLE more useful
- ▶ some are justified by frequentist statistics
- ▶ some are simply necessary hacks

Manning and Schütze (1999) as well as Jurafsky and Martin (2000) discuss more techniques and in greater detail

NLMI

Probability of a sentence

Language models

Smoothing

Evaluating language models

How do we compare language models?

Model \mathcal{M}

- ▶ a set of conditional dependence statements
graphical structure
- ▶ a parameterisation and a set of parameters θ

Intrinsic evaluation setup

- ▶ training data: sentences used to estimate parameters
- ▶ test data: a disjoint set of sentences used to assess models

We would like to compare models by comparing

- ▶ the probability they assign to held-out (iid) data \mathcal{D}

$$\prod_{x_1^n \in \mathcal{D}} P_S(x_1^n | \mathcal{M}_1) \stackrel{?}{>} \prod_{x_1^n \in \mathcal{D}} P_S(x_1^n | \mathcal{M}_2)$$

- ▶ but P_S depends on choice of factorisation

Perplexity

For dataset \mathcal{D} and model \mathcal{M}

$$\text{PP}(\mathcal{D}; \mathcal{M}) = \left(\prod_{x_1^n \in \mathcal{D}} P_S(x_1^n; \mathcal{M}) \right)^{-1/t}$$

where t is the number of tokens in \mathcal{D}

Or in log-domain: $\log \text{PP}(\mathcal{D}; \mathcal{M}) =$

$$\begin{aligned} &= -\frac{1}{t} \left[\sum_{k=1}^m \log P_N(n_k) + \log P_{S|N} \left(\langle x_1^{(k)}, \dots, x_{n_k}^{(k)} \rangle | n_k; \mathcal{M} \right) \right] \\ &= -\frac{1}{t} \left[\log P_{S|N} \left(\langle x_1^{(k)}, \dots, x_{n_k}^{(k)} \rangle | n_k; \mathcal{M} \right) \right] + C \\ &\propto -\frac{1}{t} \left[\sum_{k=1}^m \sum_{i=1}^{n_k} P_{X|H} \left(x_i^{(k)} | x_{<i}^{(k)}; \mathcal{M} \right) \right] \end{aligned}$$

Assuming the length component is the same for every model in the comparison

Perplexity: interpretation

Perplexity can be seen as

- ▶ *average branching factor* of the language according to the estimated model
- ▶ branching factor: number of words that may follow any word

Comparing models using perplexity require

- ▶ their support must overlap
i.e. there is a common set of sentences to which both models assign non-zero probability
- ▶ test sentences must be in that common set
for n -gram models this typically requires
 - ▶ smoothing
 - ▶ shared vocabulary

References I

Daniel Jurafsky and James H. Martin. *Speech and Language Processing: An Introduction to Natural Language Processing, Computational Linguistics and Speech Recognition (Prentice Hall Series in Artificial Intelligence)*. Prentice Hall, 1 edition, 2000.

Christopher D. Manning and Hinrich Schütze. *Foundations of statistical natural language processing*. MIT Press, Cambridge, MA, USA, 1999.

H. S. Sichel. On a distribution representing sentence-length in written prose. *Journal of the Royal Statistical Society. Series A (General)*, 137(1):25–34, 1974. ISSN 00359238. URL <http://www.jstor.org/stable/2345142>.