Natural Language Models and Interfaces

BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2018, week 5, lecture a

NLMI

Trees and grammars

Context-free grammars

Probabilistic context-free grammars

Modelling language so far

Bag-of-word models (or unigram LMs)

- ignore word order entirely

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n-gram models

- capture a shortened fixed-length history

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HMM models

- capture a shortened fixed-length history
- by abstracting away from word form through word classes

Long-distance dependencies

The form of one word often depends on (agrees with) another even when arbitrarily long material intervenes

- Sam sleeps soundly
- Dogs sleep soundly

Adapted from T. Deoskar

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- Sam, who is my cousin, sleeps soundly
- Dogs often play around my house and then sleep soundly

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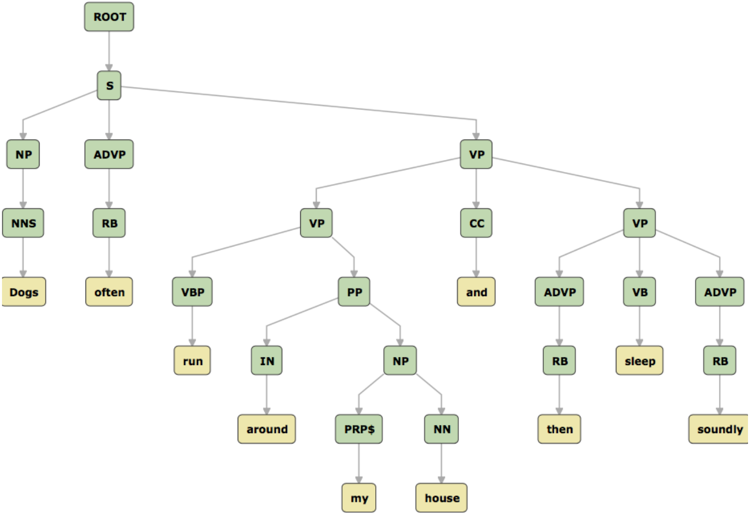
We want models that can capture these dependencies

- and are less sensitive to distance in linear order

Adapted from T. Deoskar

What if we organise words and phrases in a tree?



CoreNLP

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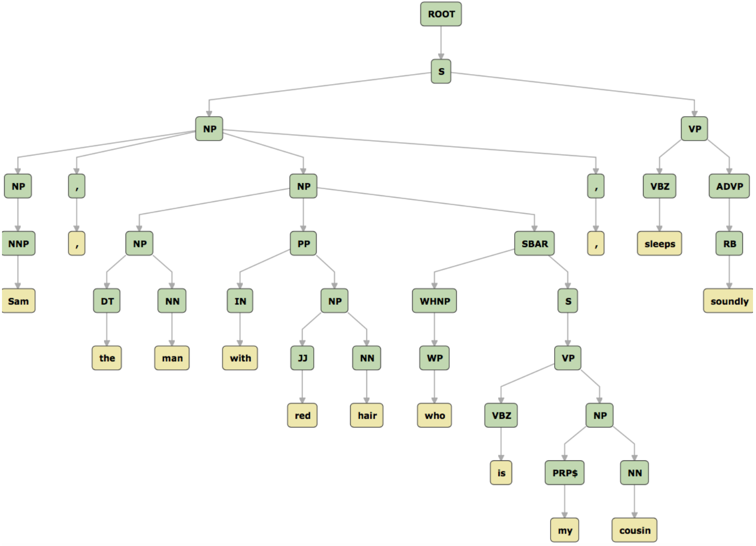

CoreNLP

Phrases

Words are organised into groups (phrases) which function as a unit

- POS categories indicate which words are substitutable.
  e.g., substituting adjectives
  - I saw a red cat
  - I saw a former cat
  - I saw a sleepy cat
- Phrasal categories indicate which phrases are substitutable
  e.g., substituting noun phrase
  - Dogs sleep soundly
  - My next-door neighbours sleep soundly
  - Green ideas sleep soundly

Phrasal categories: noun phrase (NP), verb phrase (VP), prepositional phrase (PP), etc.

Adapted from T. Deoskar

Heads and Phrases

The class that a word belongs to is closely linked to the name of the phrase it customarily appears in.

- In a X-phrase (e.g. NP), the key occurrence of X (e.g. N) is called the head.

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English NPs are commonly of the form

- (Det) Adj* **Noun** (PP — RelClause)*
  NP: the angry duck that tried to bite me ; **head: duck**

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VPs are commonly of the form

- (Aux) Adv* **Verb** Arg* Adjunct*
  VP: usually eats pasta for dinner ; **head : eat**

Adapted from T. Deoskar

Heads and Phrases

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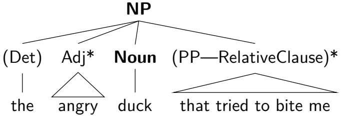

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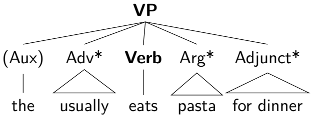

Theories of Syntax

A theory of syntax should explain which sentences are well-formed (grammatical) and which are not

- *well-formed* is distinct from *meaningful*.
- Example from Chomsky
  Colorless green ideas sleep furiously

Adapted from T. Deoskar

Theories of Syntax

A theory of syntax should explain which sentences are well-formed (grammatical) and which are not

- *well-formed* is distinct from *meaningful*.
- Example from Chomsky
  Colorless green ideas sleep furiously
- However, the reason we care about syntax is mainly for interpreting meaning

Adapted from T. Deoskar

Desirable properties of a grammar

Chomsky specified two properties that make a grammar "interesting and satisfying"

- It should be a finite specification of the strings of the language, rather than a list of its sentences.
- It should be revealing, in allowing strings to be associated with meaning (semantics) in a systematic way.

Adapted from T. Deoskar

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- It should be revealing, in allowing strings to be associated with meaning (semantics) in a systematic way.

We can add another desirable property

- It should capture structural and distributional properties of the language
  e.g. where heads of phrases are located
  e.g. how a sentence transforms into a question
  e.g. which phrases can move around the sentence

Adapted from T. Deoskar

Desirable properties of a grammar

- Context-free grammars (CFGs) provide a pretty good approximation
- Some features of NLs are more easily captured using *mildly context-sensitive* grammars
  - Combinatory Categorial Grammar (CCG)
  - Lexicalised Tree Adjoining Grammar (LTAG)

Adapted from T. Deoskar

A small fragment of English

Let's say we want to capture in a grammar the structural and distributional properties that give rise to sentences like this:

A duck walked in the park.	NP,V,PP
The man walked with a duck.	NP,V,PP
You made a duck.	Pro,V,NP
You made her duck.	? Pro,V,NP
A man with a telescope saw you.	NP,PP,V,Pro
A man saw you with a telescope.	NP,V,Pro,PP
You saw a man with a telescope.	Pro,V,NP,PP

- write lexical rules that generate the words appearing in them
- write grammatical rules that generate these phrase structures

Adapted from T. Deoskar

Grammar for the small fragment of English

Grammar G1 generates the sentences on the previous slide:

Grammatical rules	**Lexical rules**
S $\rightarrow$ NP VP	Det $\rightarrow$ a $\vert$ the $\vert$ her (determiners)
NP $\rightarrow$ Det N	N $\rightarrow$ man $\vert$ park $\vert$ duck $\vert$ telescope (nouns)
NP $\rightarrow$ Det N PP	Pro $\rightarrow$ you (pronoun)
NP $\rightarrow$ Pro	V $\rightarrow$ saw $\vert$ walked $\vert$ made (verbs)
VP $\rightarrow$ V NP PP	Prep $\rightarrow$ in $\vert$ with $\vert$ for (prepositions)
VP $\rightarrow$ V NP	
VP $\rightarrow$ V	
PP $\rightarrow$ Prep NP	

Does G1 produce a finite or an infinite number of sentences?

Adapted from T. Deoskar

Recursion

Recursion in a grammar makes it possible to generate an infinite number of sentences

- Direct recursion: a non-terminal on the LHS of a rule also appears on its RHS
  VP $\rightarrow$ VP Conj VP
  Conj $\rightarrow$ and — or
- Indirect recursion: some non-terminal can be expanded (in several steps) to a sequence of symbols containing that non-terminal
  NP $\rightarrow$ Det N PP
  PP $\rightarrow$ Prep NP

NLMI

Trees and grammars

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Context-Free Grammar

A rewriting system with two types of symbols

- Terminals (or *constants*): words
- Nonterminals (or *variables*): phrasal categories
  e.g. S, NP, VP
  with S being the start symbol

Rules of the form X $\rightarrow \beta$
where β is any string of nonterminals and terminals
indicate that X can be replaced by β anywhere where X occurs

CFG example

S → NP VP	**(Sentences)**
NP → D N | Pro | PropN	**(Noun phrases)**
D → PosPro | Art | NP 's	**(Determiners)**
VP → Vi | Vt NP | Vp NP VP	**(Verb phrases)**
Pro → i | we | you | he | she | him | her	**(Pronouns)**
PosPro → my | our | your | his | her	**(Possessive pronouns)**
PropN → Robin | Jo	**(Proper nouns)**
Art → a | an | the	**(Articles)**
N → man | duck | saw | park | telescope	**(Nouns)**
Vi → sleep | run | duck	**(Intransitive verbs)**
Vt → eat | break | see | saw	**(Transitive verbs)**
Vp → see | saw | heard	**(Verbs with NP VP args)**

CFGs formally

Let Σ be a finite set of terminal symbols (aka words)
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Let $\mathcal{R} \subseteq \mathcal{V} \times (\Sigma \cup \mathcal{V})^*$ be a finite set of rules of the form
$\mathsf{X} \rightarrow \beta$ where $\mathsf{X} \in \mathcal{V}$ and $\beta \in (\Sigma \cup \mathcal{V})^*$

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A CFG is the tuple $\mathfrak{G} = \langle \Sigma, \mathcal{V}, \mathsf{S}, \mathcal{R} \rangle$

CFG terminology

The number of symbols on the RHS is the arity of the rule

- unary: $\mathsf{X} \rightarrow \mathsf{Y}$
- binary: $\mathsf{X} \rightarrow \mathsf{Y}\,\mathsf{Z}$
- n-ary: $\mathsf{X} \rightarrow \mathsf{X}_1 \cdots \mathsf{X}_n$
- if the longest rule has arity a we say the grammar has arity a

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- unary rules such as $\mathsf{X} \rightarrow \mathsf{x}$
  where $\mathsf{x} \in \Sigma$ is a terminal

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- phrase rules?

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How many?

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- phrase rules? $O(|\mathcal{V}| \times |\Sigma \cup \mathcal{V}|^a)$

Derivation

We can use CFGs to derive strings

A derivation is a sequence of strings

- we start from the string $\langle \mathsf{S} \rangle$
- and at each step we rewrite the leftmost nonterminal X by application of a rule $\mathsf{X} \to \beta$
- until only terminals remain
  which we denote $\mathsf{S} \overset{*}{\Rightarrow} \mathsf{x}_1 \cdots \mathsf{x}_n$

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Example

1. $\langle \mathsf{S} \rangle$

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1. $\langle \mathsf{S} \rangle$
2. $\langle \mathsf{NP}\ \mathsf{VP} \rangle$

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Example

1. $\langle \mathsf{S} \rangle$
2. $\langle \mathsf{NP\ VP} \rangle$
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Example

1. $\langle \mathrm{S} \rangle$
2. $\langle \mathrm{NP\ VP} \rangle$
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Example

1. ⟨S⟩
2. ⟨NP VP⟩
3. ⟨D N VP⟩
4. ⟨the N VP⟩
5. ⟨the dog VP⟩

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Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$

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Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog barks$\rangle$

Derivation - examples

Example

1. $\langle S \rangle$

Example

Derivation - examples

Example

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$

Example

Derivation - examples

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1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{D N VP} \rangle$

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Derivation - examples

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Example

Derivation - examples

Example

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Example

Derivation - examples

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Derivation - examples

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2. $\langle \text{NP VP} \rangle$
3. $\langle \text{D N VP} \rangle$
4. $\langle \text{the N VP} \rangle$
5. $\langle \text{the dog VP} \rangle$
6. $\langle \text{the dog V} \rangle$
7. $\langle \text{the dog runs} \rangle$

Example

Derivation - examples

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

1. $\langle$S$\rangle$

Derivation - examples

Example

1. $\langle S \rangle$
2. $\langle NP\ VP \rangle$
3. $\langle D\ N\ VP \rangle$
4. $\langle \text{the } N\ VP \rangle$
5. $\langle \text{the dog } VP \rangle$
6. $\langle \text{the dog } V \rangle$
7. $\langle \text{the dog runs} \rangle$

Example

1. $\langle S \rangle$
2. $\langle NP\ VP \rangle$

Derivation - examples

Example

1. $\langle \mathrm{S} \rangle$
2. $\langle \mathrm{NP\ VP} \rangle$
3. $\langle \mathrm{D\ N\ VP} \rangle$
4. $\langle \text{the N VP} \rangle$
5. $\langle \text{the dog VP} \rangle$
6. $\langle \text{the dog V} \rangle$
7. $\langle \text{the dog runs} \rangle$

Example

1. $\langle \mathrm{S} \rangle$
2. $\langle \mathrm{NP\ VP} \rangle$
3. $\langle \mathrm{N\ VP} \rangle$

Derivation - examples

Example

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{D N VP} \rangle$
4. $\langle \text{the N VP} \rangle$
5. $\langle \text{the dog VP} \rangle$
6. $\langle \text{the dog V} \rangle$
7. $\langle \text{the dog runs} \rangle$

Example

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{N VP} \rangle$
4. $\langle \text{cats VP} \rangle$

Derivation - examples

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$D N VP$\rangle$
4. $\langle$the N VP$\rangle$
5. $\langle$the dog VP$\rangle$
6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$N VP$\rangle$
4. $\langle$cats VP$\rangle$
5. $\langle$cats V$\rangle$

Derivation - examples

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4. $\langle$the N VP$\rangle$
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6. $\langle$the dog V$\rangle$
7. $\langle$the dog runs$\rangle$

Example

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$N VP$\rangle$
4. $\langle$cats VP$\rangle$
5. $\langle$cats V$\rangle$
6. $\langle$cats run$\rangle$

Derivation - more examples

1. $\langle \mathsf{S} \rangle$

Derivation - more examples

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$

Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$

Derivation - more examples

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{NP CC NP VP} \rangle$
4. $\langle \text{N CC NP VP} \rangle$

Derivation - more examples

1. $\langle \text{S} \rangle$
2. $\langle \text{NP VP} \rangle$
3. $\langle \text{NP CC NP VP} \rangle$
4. $\langle \text{N CC NP VP} \rangle$
5. $\langle \text{cats CC NP VP} \rangle$

Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$
4. $\langle$N CC NP VP$\rangle$
5. $\langle$cats CC NP VP$\rangle$
6. $\langle$cats and NP VP$\rangle$

Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$
4. $\langle$N CC NP VP$\rangle$
5. $\langle$cats CC NP VP$\rangle$
6. $\langle$cats and NP VP$\rangle$
7. $\langle$cats and N VP$\rangle$

Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$
4. $\langle$N CC NP VP$\rangle$
5. $\langle$cats CC NP VP$\rangle$
6. $\langle$cats and NP VP$\rangle$
7. $\langle$cats and N VP$\rangle$
8. $\langle$cats and dogs VP$\rangle$

Derivation - more examples

1. $\langle$S$\rangle$
2. $\langle$NP VP$\rangle$
3. $\langle$NP CC NP VP$\rangle$
4. $\langle$N CC NP VP$\rangle$
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7. $\langle$cats and N VP$\rangle$
8. $\langle$cats and dogs VP$\rangle$
9. $\langle$cats and dogs V$\rangle$

Derivation - more examples

1. $\langle$S$\rangle$
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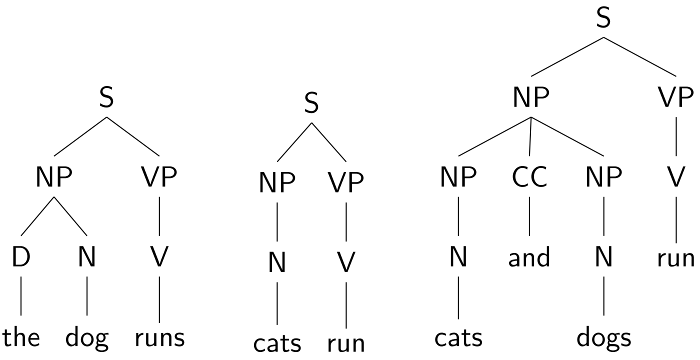
8. $\langle$cats and dogs VP$\rangle$
9. $\langle$cats and dogs V$\rangle$
10. $\langle$cats and dogs run$\rangle$

Parse trees

Parse trees compactly represent derivations

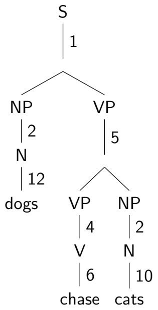


Derivation: a sequence of rule applications

A derivation can be seen as a sequence of rule applications $\langle r_1, \ldots, r_m \rangle$

- starts from S
- and after m steps yields a string $\mathrm{yield}(r_1^m) = x_1^n$
- the sequence can be read off of a tree by a depth-first traversal

Derivations as trees



1. S → NP VP
2. NP → N
3. NP → D N
4. VP → V
5. VP → VP NP
6. V → chase
7. V → chases
8. D → the
9. N → cat
10. N → cats
11. N → dog
12. N → dogs

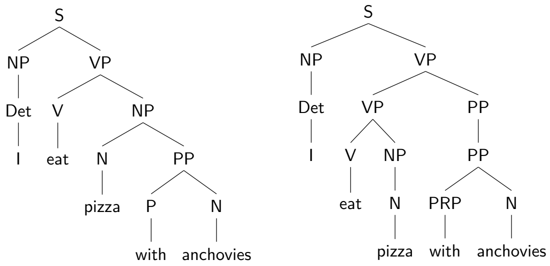
Sequence of rule applications (depth-first traversal)

$\langle r_1 = 1, r_2 = 2, r_3 = 12, r_4 = 5, r_5 = 4, r_6 = 6, r_7 = 2, r_8 = 10 \rangle$

The sentence is the yield of the derivation

Structural ambiguity

Different structure leads to different interpretation



Structural ambiguity

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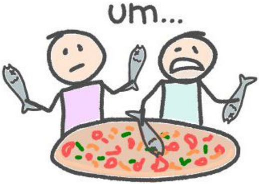

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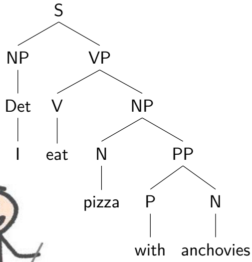

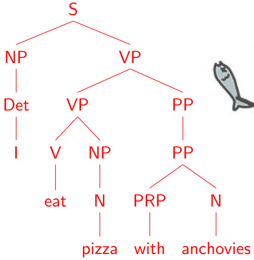

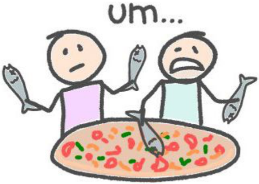

How should we deal with this?

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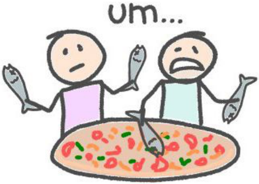

How should we deal with this? Probabilities!

NLMI

Trees and grammars

Context-free grammars

Probabilistic context-free grammars

Probability distributions over derivations

We define a random derivation D

- a sequence $\langle R_1, \ldots, R_m \rangle$ of random rule applications
- where R is a random variable indexing rules of the grammar

The probability over a sequence of m rules can be written

$$P_{D|M}(r_1^m|m) = \underbrace{\prod_{i=1}^{m} P_{R_i|R_{<i}}(r_i|r_{<i})}_{\text{chain rule}}$$

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$$\approx \underbrace{\prod_{i=1}^{m} P_R(r_i)}_{\text{independence assumption}}$$

Conditional independence

A rule rewrites a LHS nonterminal symbol into a RHS string

- A rule $R : v \to \beta$ corresponds to a random pair $(\mathrm{LHS}, \mathsf{RHS})$
- LHS corresponds to a random nonterminal symbol $v \in \mathcal{V}$
- RHS corresponds to a random sequence of terminals and nonterminals $\beta \in (\Sigma \cup \mathcal{V})^a$

Then we re-write the probability of a derivation as

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&= \prod_{i=1}^{m} P_{\text{RHS}|\text{LHS}}(\beta_i | v_i)
\end{aligned}
$$

Factorisation

We make each factor $P_{\mathrm{RHS}|\mathrm{LHS}=v}$ a distribution $\mathrm{Cat}(\boldsymbol{\theta}^{(v)})$

- $0 < \theta_{v \to \beta} < 1$
- $\sum_{\beta} \theta_{v \to \beta} = 1$

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- $P_R(v \to \beta) = P_{\mathrm{RHS}|\mathrm{LHS}}(\beta|v) = \theta_{v\to\beta}$ or $P_R(r) = \theta_r$
  e.g. $P_R(\mathsf{S} \to \mathsf{NP\ VP}) = P_{\mathrm{RHS}|\mathsf{LHS}}(\mathsf{NP\ VP}|\mathsf{S}) = \theta_{\mathsf{S}\to\mathsf{NP\ VP}}$

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How many parameters to represent P_R?

- One cpd per LHS, thus $O(|\mathcal{V}| \times |\Sigma \cup \mathcal{V}|^a)$

Probability of a derivation

A simple product over m terms

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Probabilistic CFG Language Model

Generative story

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1. Generate a sentence length $N \sim P_N$

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$$P_{SD}(x_1^n, r_1^m) = P_N(n)P_{M|N}(m|n) \underbrace{P_{D|M}(r_1^m)P_{S|DNM}(x_1^n|r_1^m)}_{P_{SD|NM}(x_1^n, r_1^m|n, m)}$$

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But note that (4) is deterministic

$$P_{S|DNM}(x_1^n|r_1^m) = \begin{cases} 1 & \text{if } \mathrm{yield}(r_1^m) = x_1^n \\ 0 & \text{otherwise} \end{cases}$$

Probability of a sentence

Joint distribution

$$P_{SD}(x_1^n, r_1^m) = P_N(n)P_{M|N}(m|n)P_{D|M}(r_1^m)P_{S|DNM}(x_1^n|r_1^m)$$

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where $\mathfrak{G}(x_1^n)$ is the set of derivations whose yield is x_1^n

Probability of a sentence

Typically P_N and $P_{M|N}$ are ignored (assumed uniform), then

$$P_S(x_1^n) \propto \sum_{r_1^m \in \mathfrak{G}(x_1^n)} P_{D|M}(r_1^m)$$

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Challenge: to express $\mathfrak{G}(x_1^n)$ a task called parsing

Maximum likelihood estimation

We have a treebank, that is, a corpus where

- a sentence x_1^n is annotated with its CFG tree r_1^m

Our distributions $P_{\mathrm{RHS}|\mathrm{LHS}}$ are categorical

- $\mathrm{RHS} \mid \mathrm{LHS} = v \sim \mathrm{Cat}(\boldsymbol{\theta}^{(v)})$

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MLE solution?

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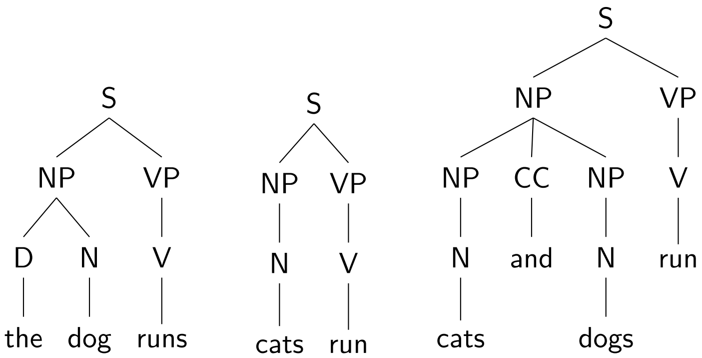
MLE solution?

$$\theta_{v \to \beta} = \frac{\mathrm{count}_R(v \to \beta)}{\sum_{\beta'} \mathrm{count}_R(v \to \beta')} = \frac{\mathrm{count}_R(v \to \beta)}{\mathrm{count}_{\mathrm{LHS}}(v)}$$

MLE - Example

Consider the treebank



- $\theta_{\text{NP}\to\text{N}}$
- $\theta_{\text{N}\to\text{dog}}$

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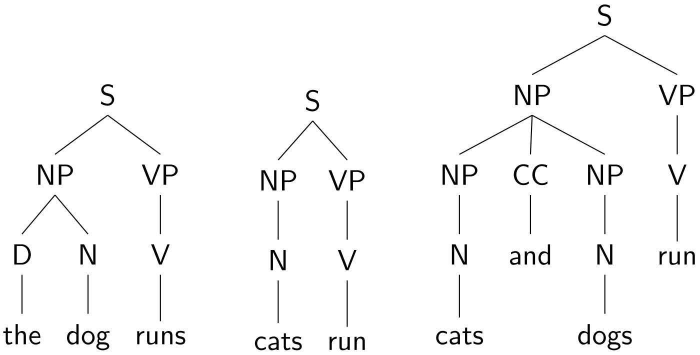

- $\theta_{\text{NP}\to\text{N}} = \frac{\text{count}(\text{NP}\to\text{N})}{\text{count}(\text{NP}\to *)}$
- $\theta_{\text{N}\to\text{dog}}$

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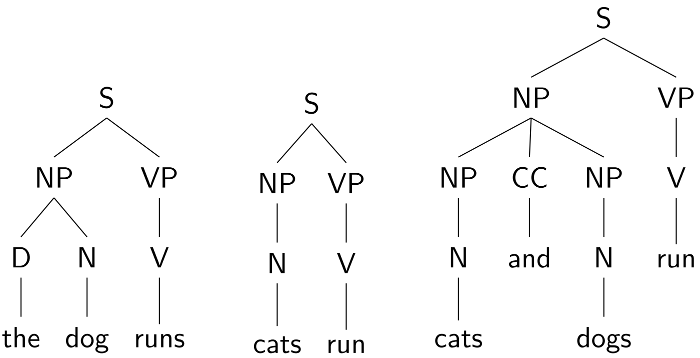

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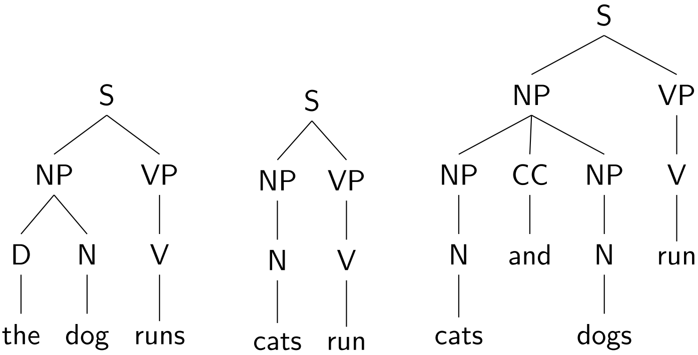

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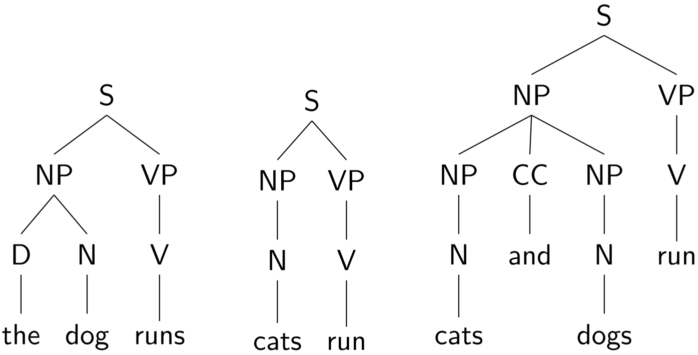
- $\theta_{\mathsf{N}\to\mathsf{dog}} = \frac{\text{count}(\mathsf{N}\to\mathsf{dog})}{\text{count}(\mathsf{N}\to *)}$

MLE - Example

Consider the treebank



- $\theta_{\mathsf{NP}\rightarrow\mathsf{N}} = \frac{\text{count}(\mathsf{NP}\rightarrow\mathsf{N})}{\text{count}(\mathsf{NP}\rightarrow *)} = \frac{3}{5}$
- $\theta_{\mathsf{N}\rightarrow\mathsf{dog}} = \frac{\text{count}(\mathsf{N}\rightarrow\mathsf{dog})}{\text{count}(\mathsf{N}\rightarrow *)} = \frac{1}{4}$

References I