Natural Language Models and Interfaces

BSc Artificial Intelligence

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Institute for Logic, Language, and Computation

2018, week 5, lecture b

Context-Free Grammars

A **CFG** grammar G is denoted by

- a finite set of **nonterminal** symbols $\mathscr{V}$
- a finite set of **terminal** symbols Σ with $\Sigma \cap \mathscr{V} = \varnothing$
- a finite set $\mathscr{R}$ of **rules** of the form $X \rightarrow \beta$ where
  - $X \in \mathscr{V}$ and $\beta \in (\Sigma \cup \mathscr{V})^*$
- $S \in \mathscr{V}$ a distinguished **start** symbol

Let ε denote an **empty** string

Example CFG

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Generative Device

Left-most derivation

- sequence of strings α_1 ... α_n
  - $\alpha_1 = \langle S \rangle$
  - $\alpha_n \in \Sigma^*$
  - $\alpha_{i \geq 2}$ derived from α_{i-1} by picking the left-most nonterminal X
    - and replacing it by some α such that $X \rightarrow \beta \in \mathscr{R}$

Example of Derivation

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String	Substitution

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$\alpha_1 =$	S	S → NP VP

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	String	Substitution
α_1 =	S	S → NP VP
α_2 =	NP VP	NP → DT NN

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	String	Substitution
$\alpha_1 =$	S	S → NP VP
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	String	Substitution
α_1 =	S	S → NP VP
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α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man

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$\alpha_1 =$	S	S → NP VP
$\alpha_2 =$	NP VP	NP → DT NN
$\alpha_3 =$	DT NN VP	DT → the
$\alpha_4 =$	the NN VP	NN → man
$\alpha_5 =$	the man VP	VP → Vi

Example of Derivation

	String	Substitution
α_1 =	S	S → NP VP
α_2 =	NP VP	NP → DT NN
α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man
α_5 =	the man VP	VP → Vi
α_6 =	the man Vi	Vi → sleeps

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$\alpha_6 =$	the man Vi	Vi → sleeps
$\alpha_7 =$	the man sleeps	

Example of Derivation

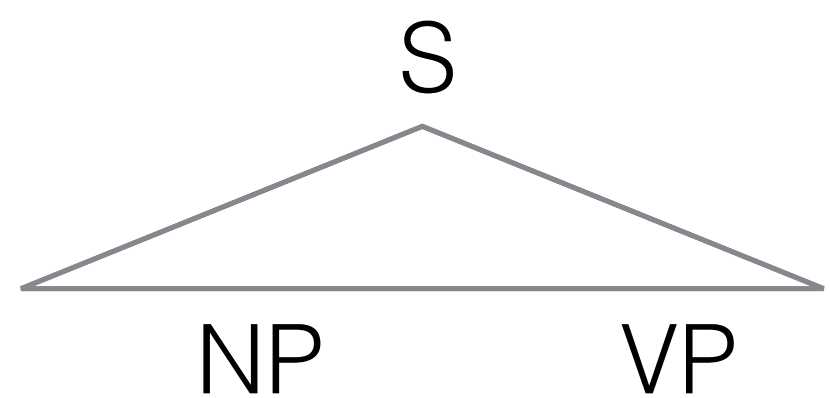
	String	Substitution
α_1 =	S	S → NP VP
α_2 =	NP VP	NP → DT NN
α_3 =	DT NN VP	DT → the
α_4 =	the NN VP	NN → man
α_5 =	the man VP	VP → Vi
α_6 =	the man Vi	Vi → sleeps
α_7 =	the man sleeps	
α_7 =	S $\Rightarrow^*$ the man sleeps	

Example of Generation

Example of Generation

S

Example of Generation



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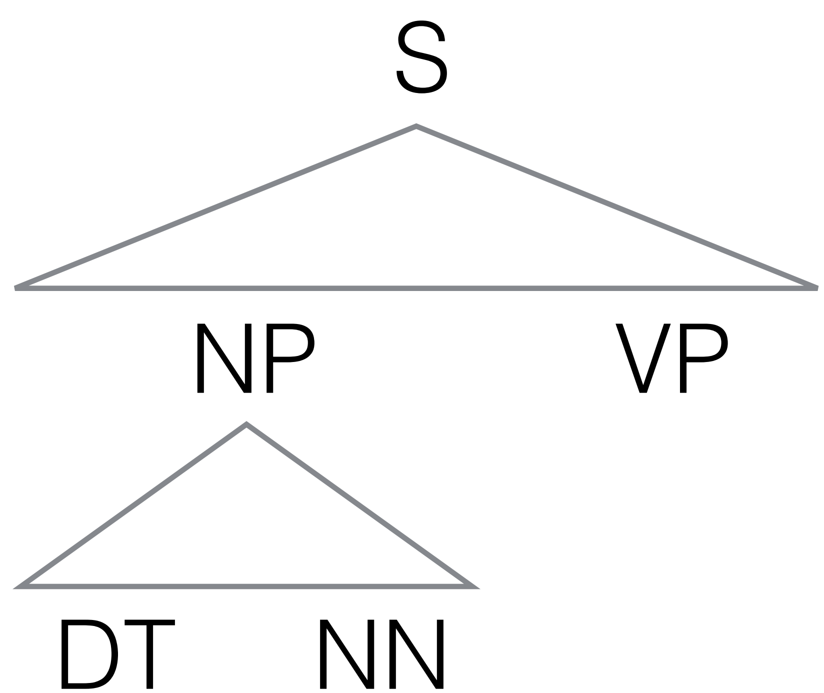

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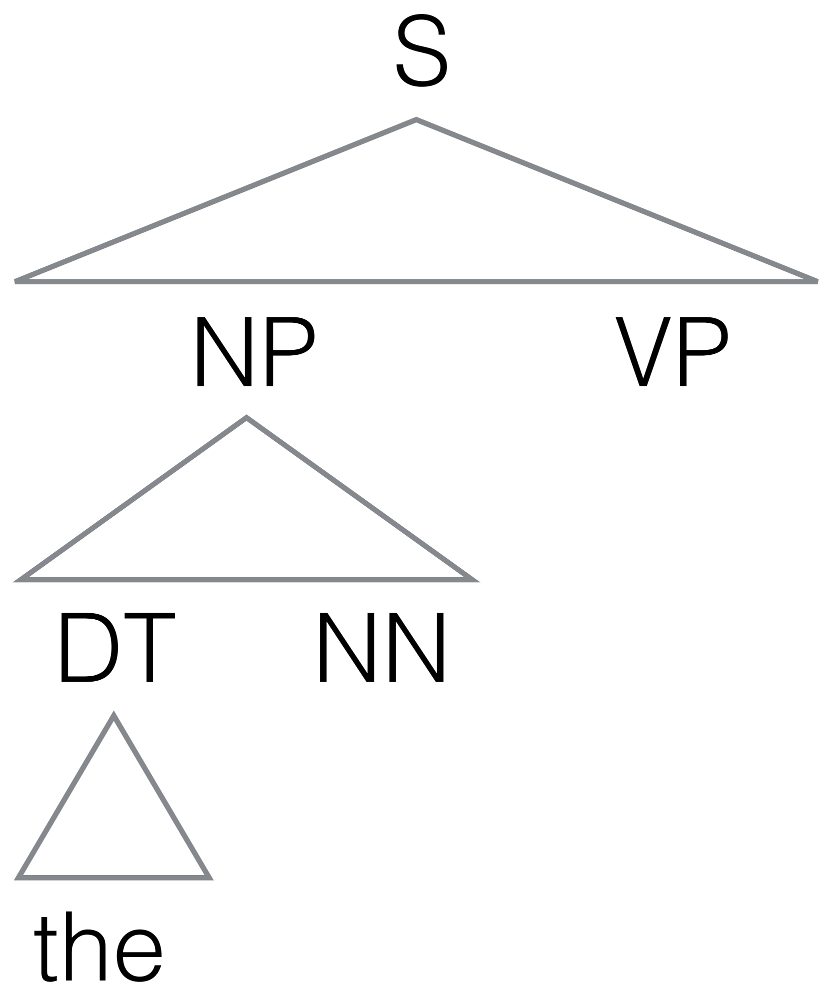

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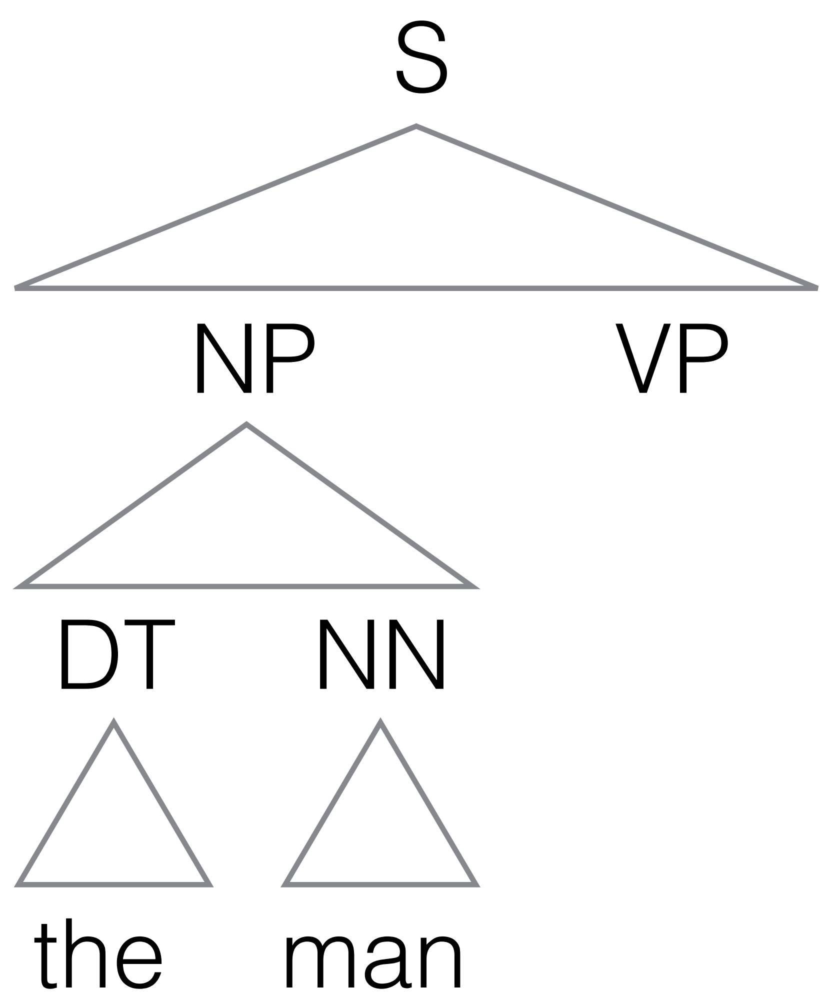

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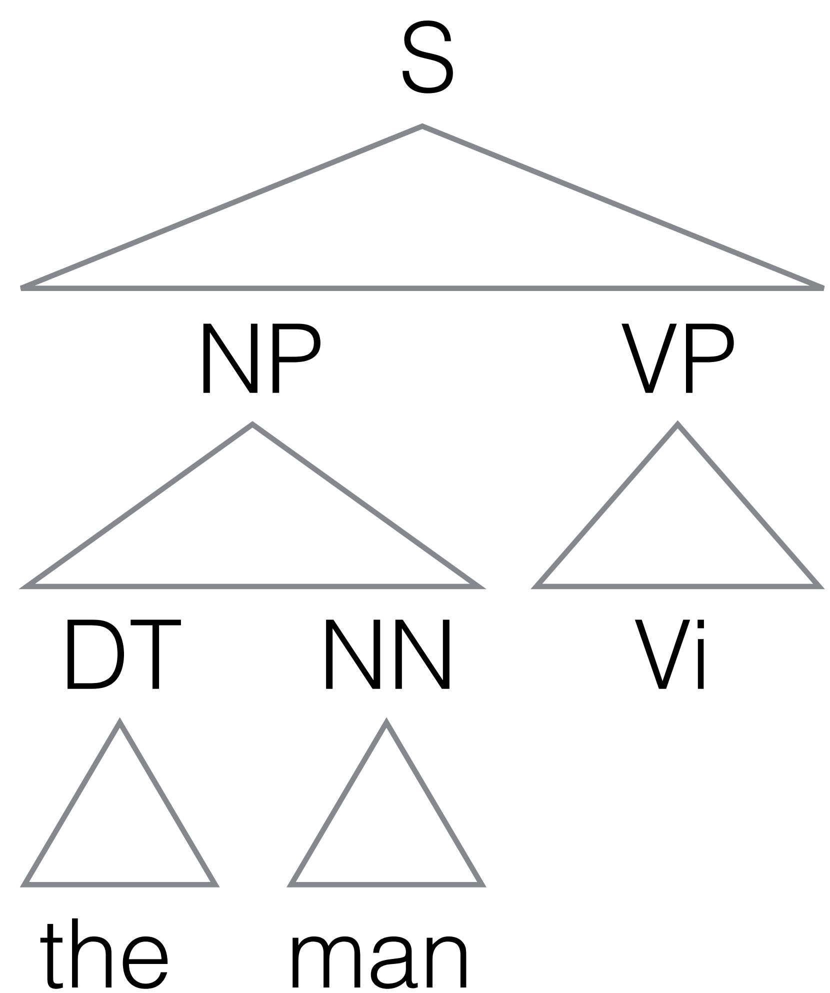

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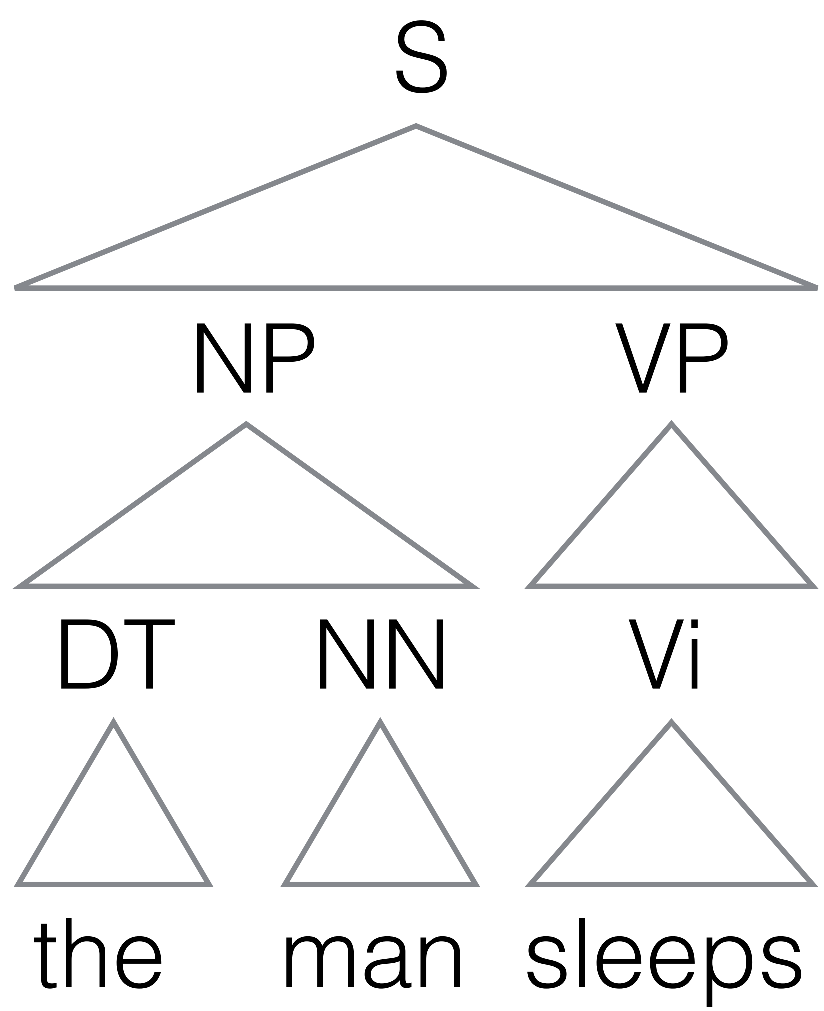
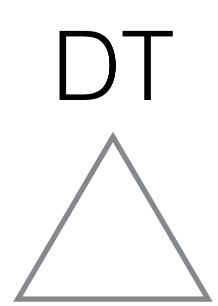


Example of Recognition

Example of Recognition

The man saw the dog

Example of Recognition



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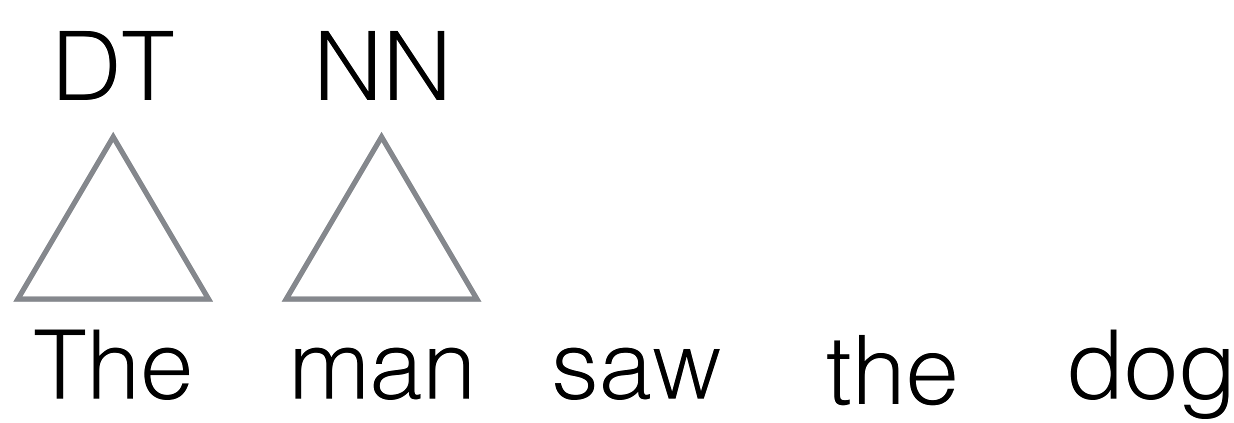

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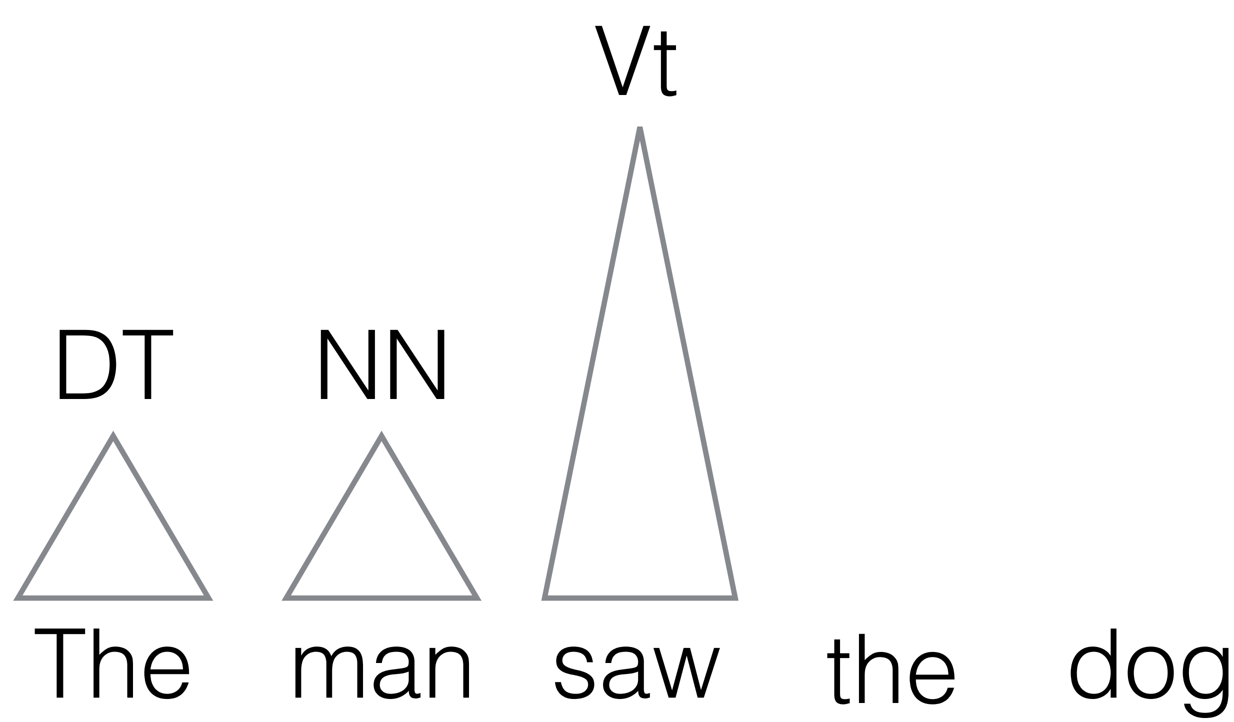

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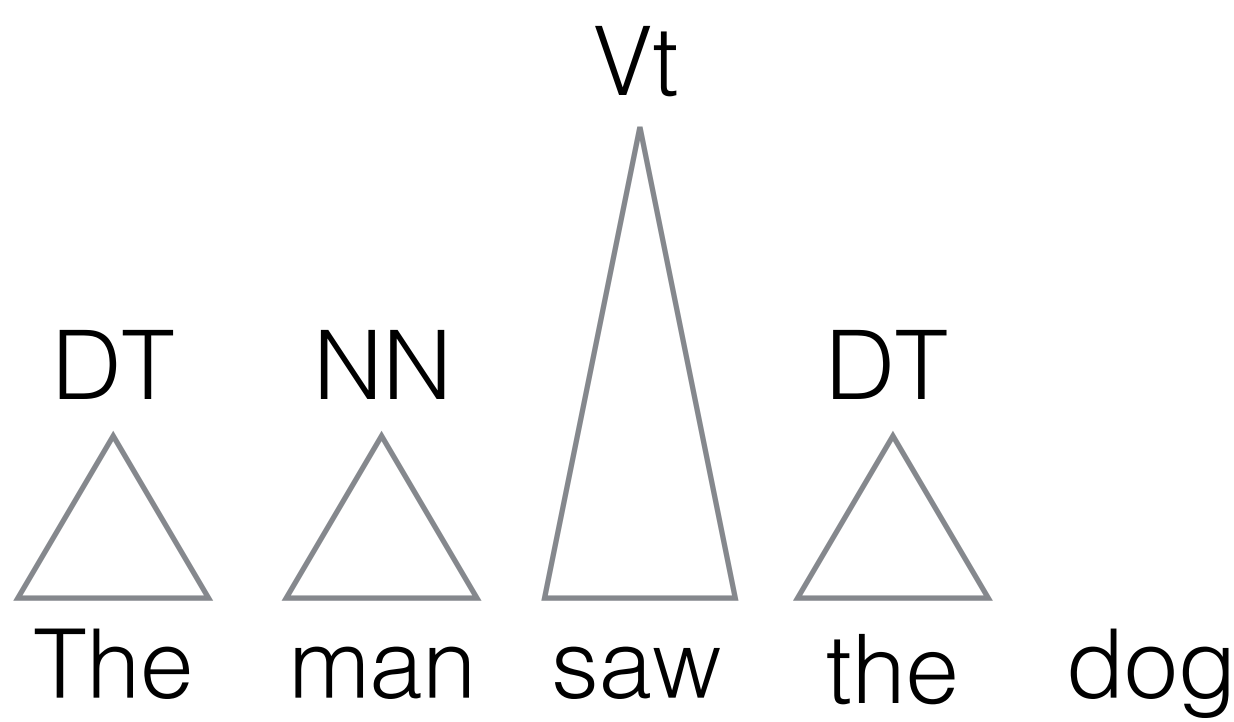

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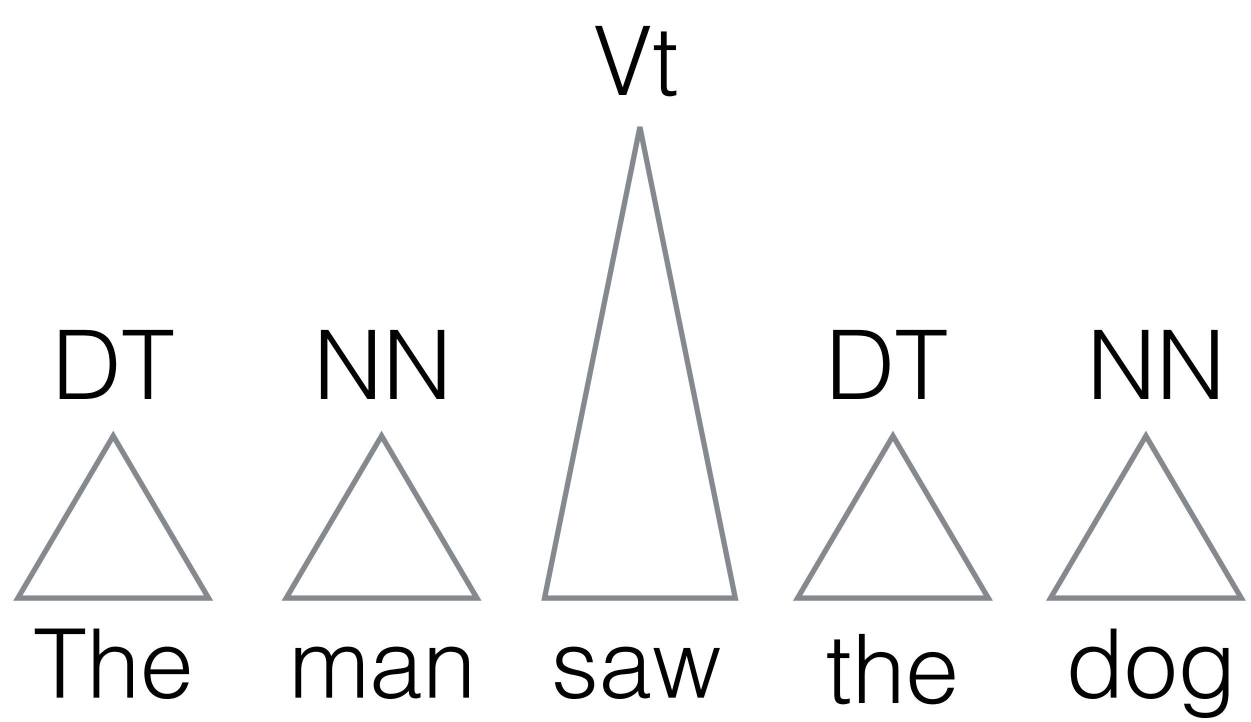

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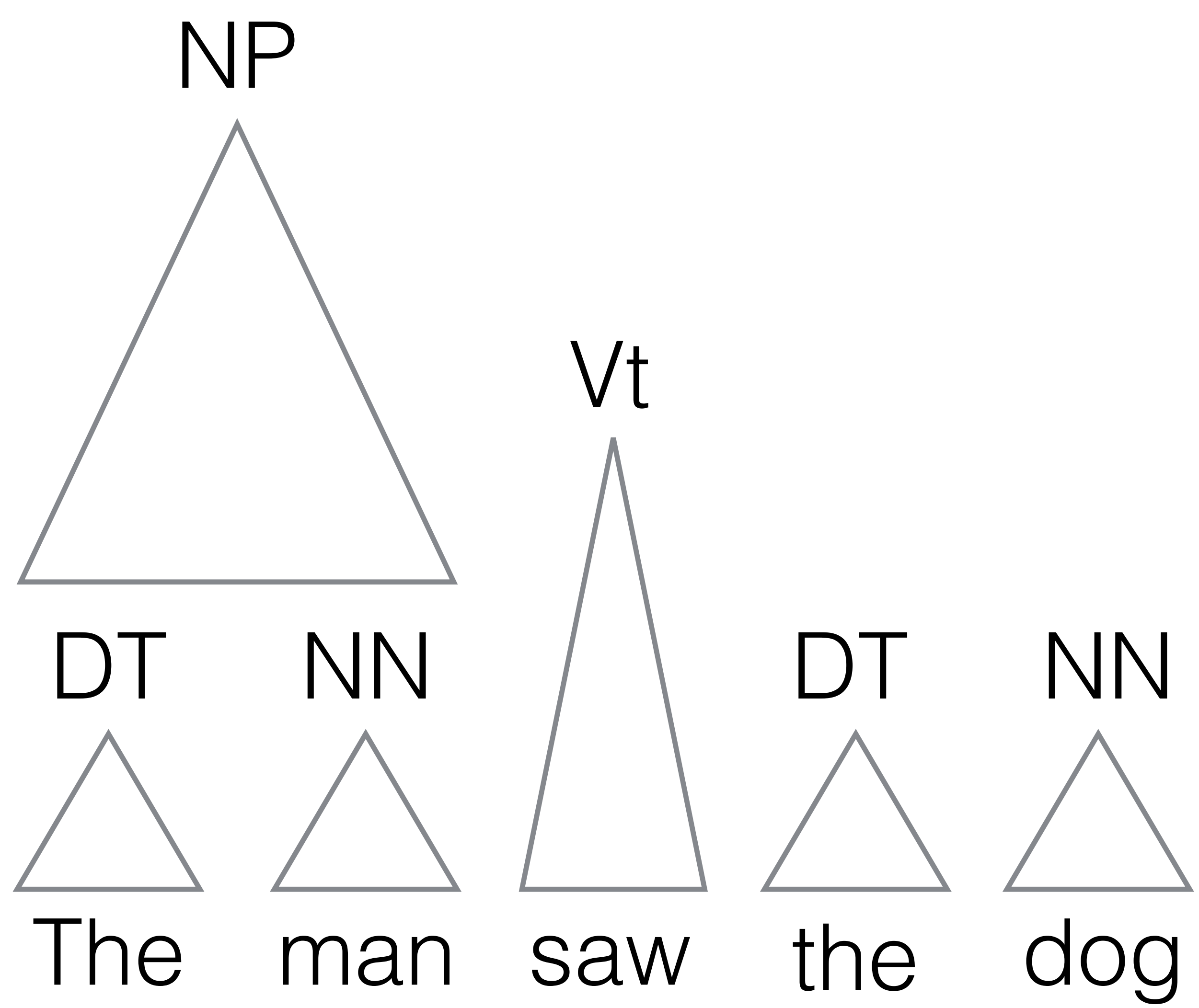

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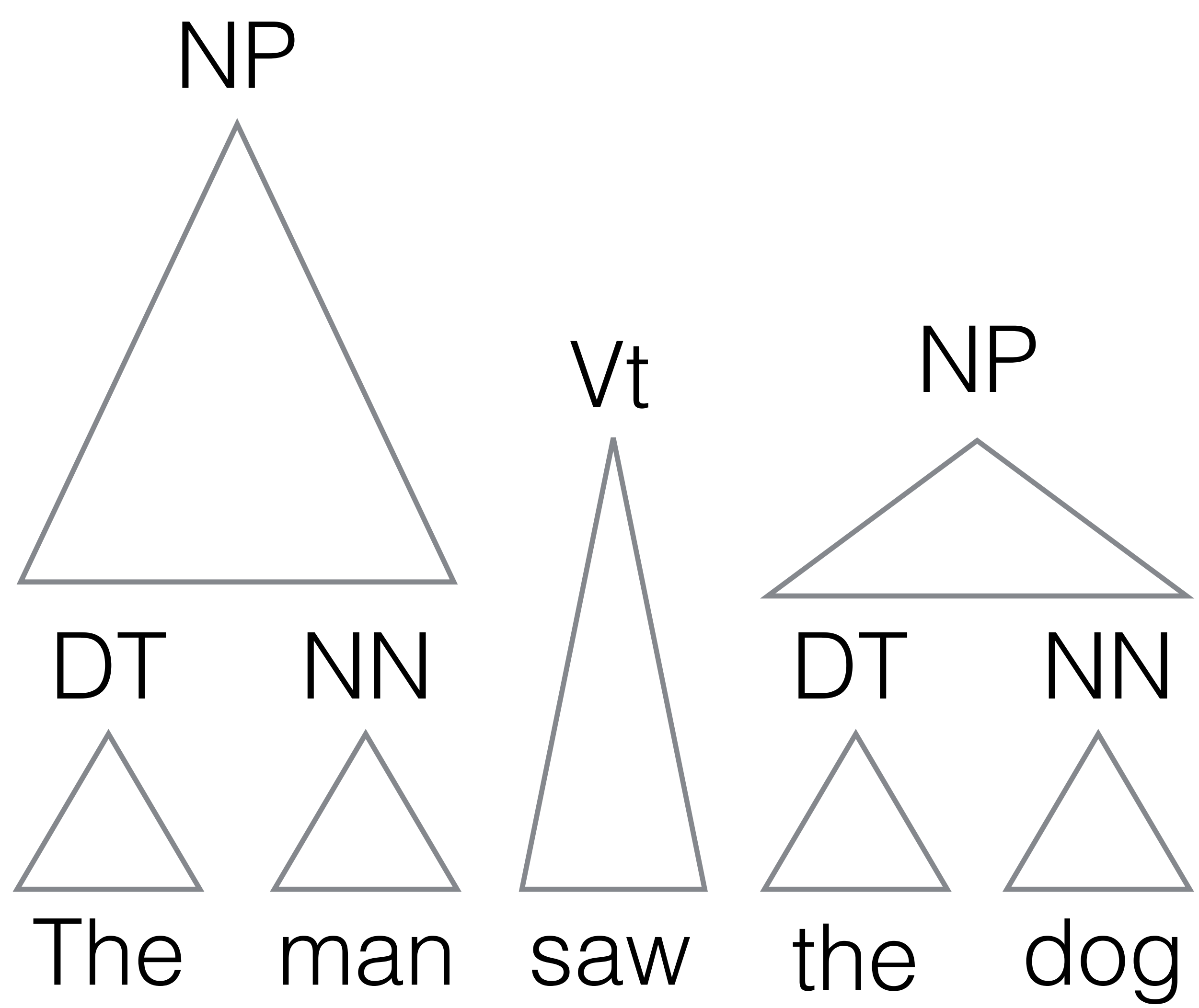

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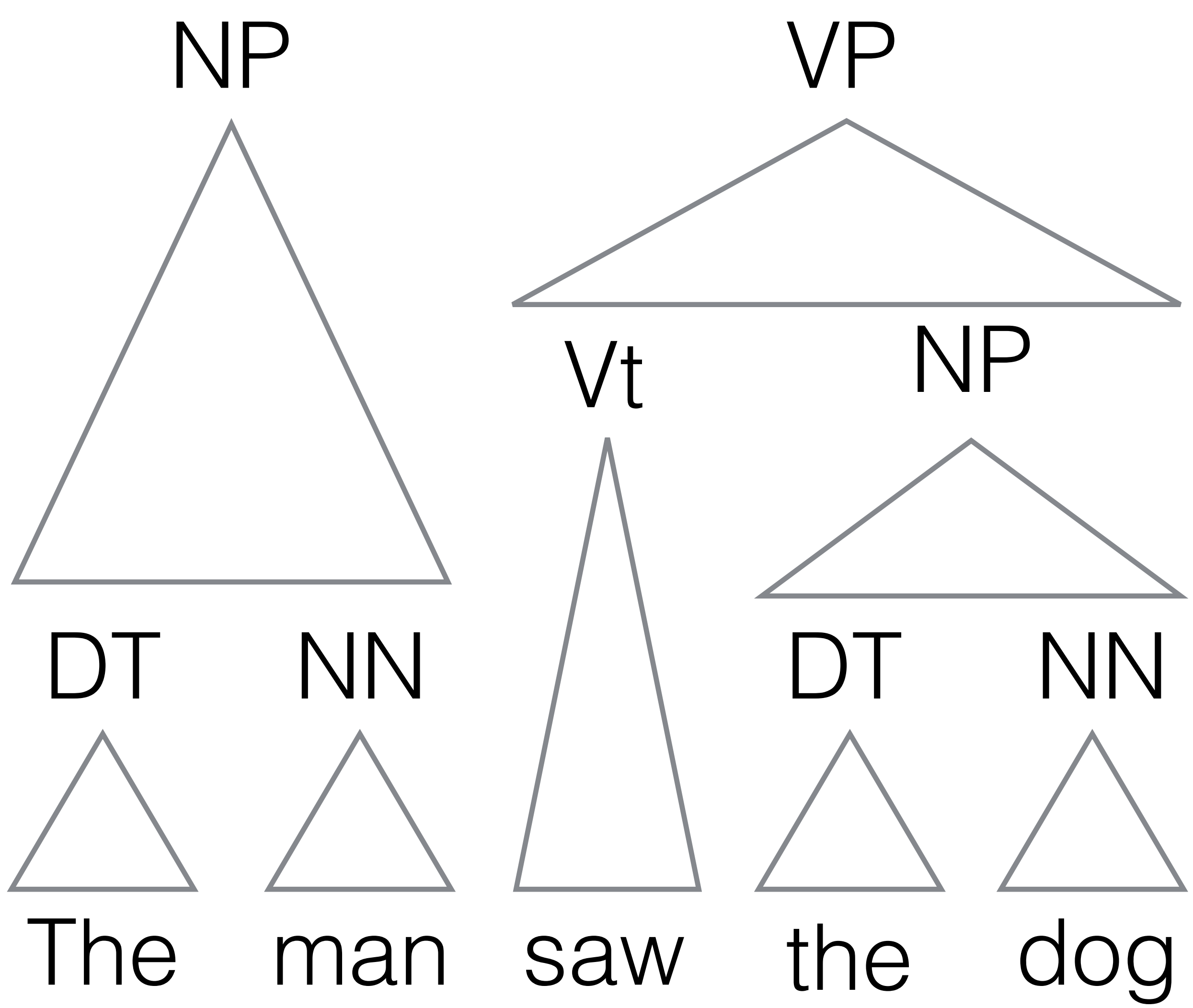

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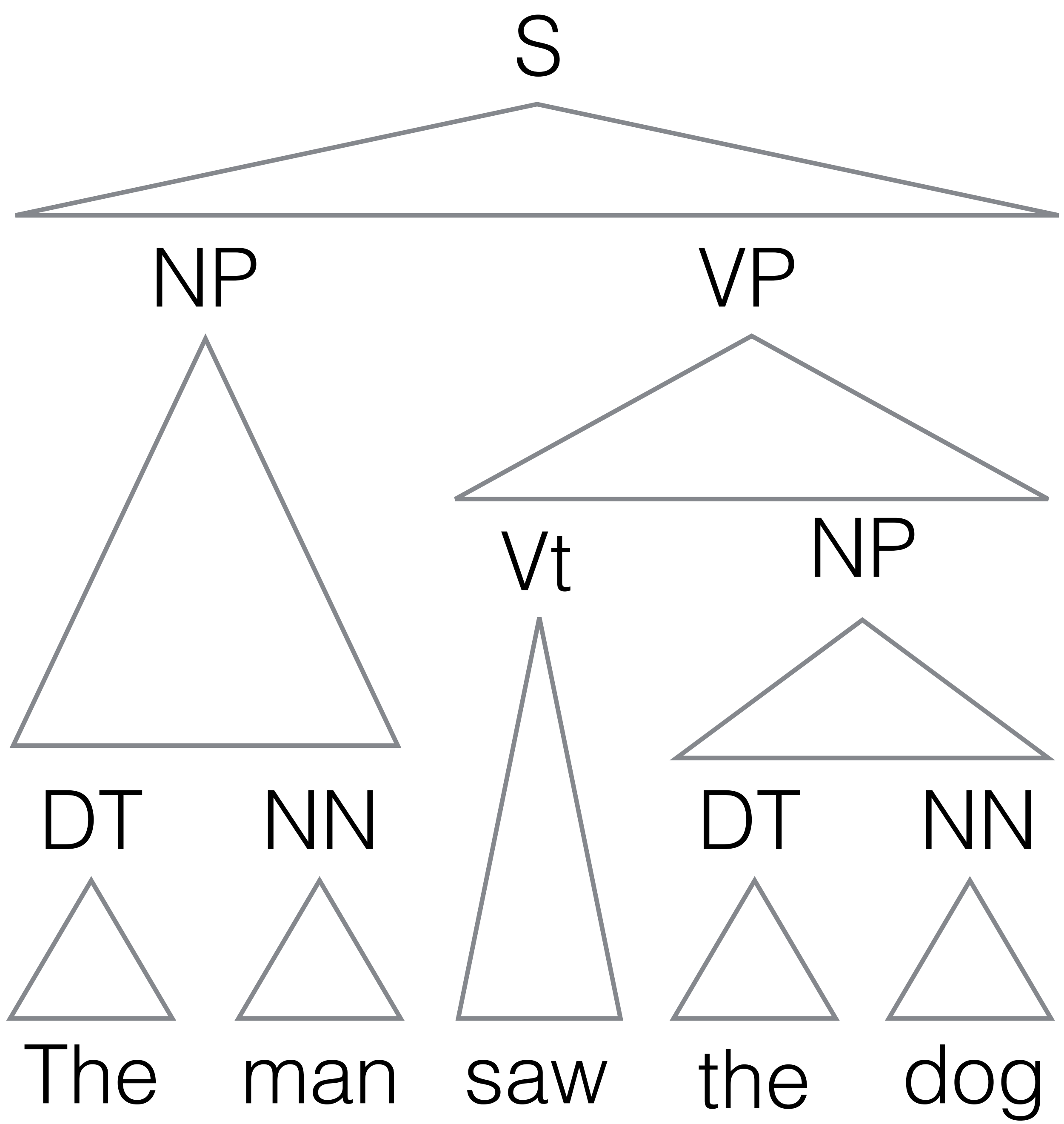

Language

A string $\omega \in \Sigma^*$ is generated/accepted by G if

$$S \Rightarrow^* \omega$$

$\Rightarrow^*$ denotes a sequence of rule applications

Language of G

$$L(G) = \{\omega: S \Rightarrow^* \omega\} \subseteq \Sigma^*$$

Chomsky Normal Form

Every CFG is weakly equivalent to another such that

- $X \rightarrow Y\ Z$ where $X, Y, Z \in \mathscr{V}$
- $X \rightarrow w$ where $w \in \Sigma$
- and possibly $S \rightarrow \varepsilon$

[Hopcroft and Ullman, 1979]

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Deductive process to prove claims about grammaticality
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- dynamic program follows directly

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Deductive process to prove claims about grammaticality

[Shieber et al., 1995]

- focus on strategy rather than implementation
- soundness/completeness easier to prove
- complexity determined by inspection
- dynamic program follows directly
- generality

Deductive systems

Item: a statement / intermediate sound result

- formula or schemata expressed with variables

Inference rule: statement derived from existing items

- $\dfrac{A_1 \ldots A_m}{B}$ (condition) where A_i and B are items
  - Ai are called antecedents
  - B is called consequent

Deductive program

Axioms: trivial items

- do not depend on previous statements

Goal: states that a proof exists

Proof:

- start from axioms
- exhaustively deduce items
  - never twice under the same premises
- accept if goal is proven

Bottom-up: Shift-Reduce

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The man saw the dog

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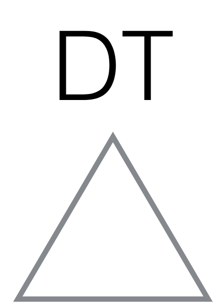

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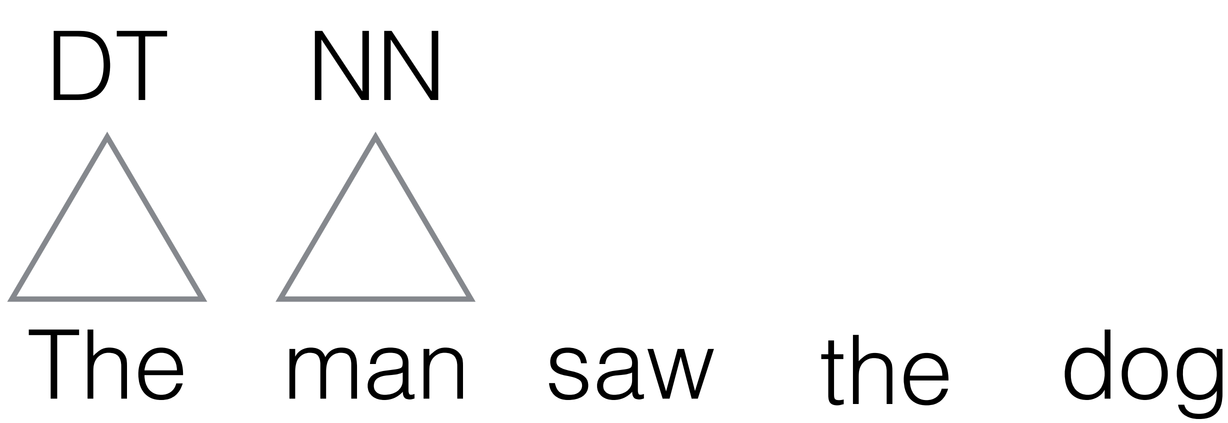

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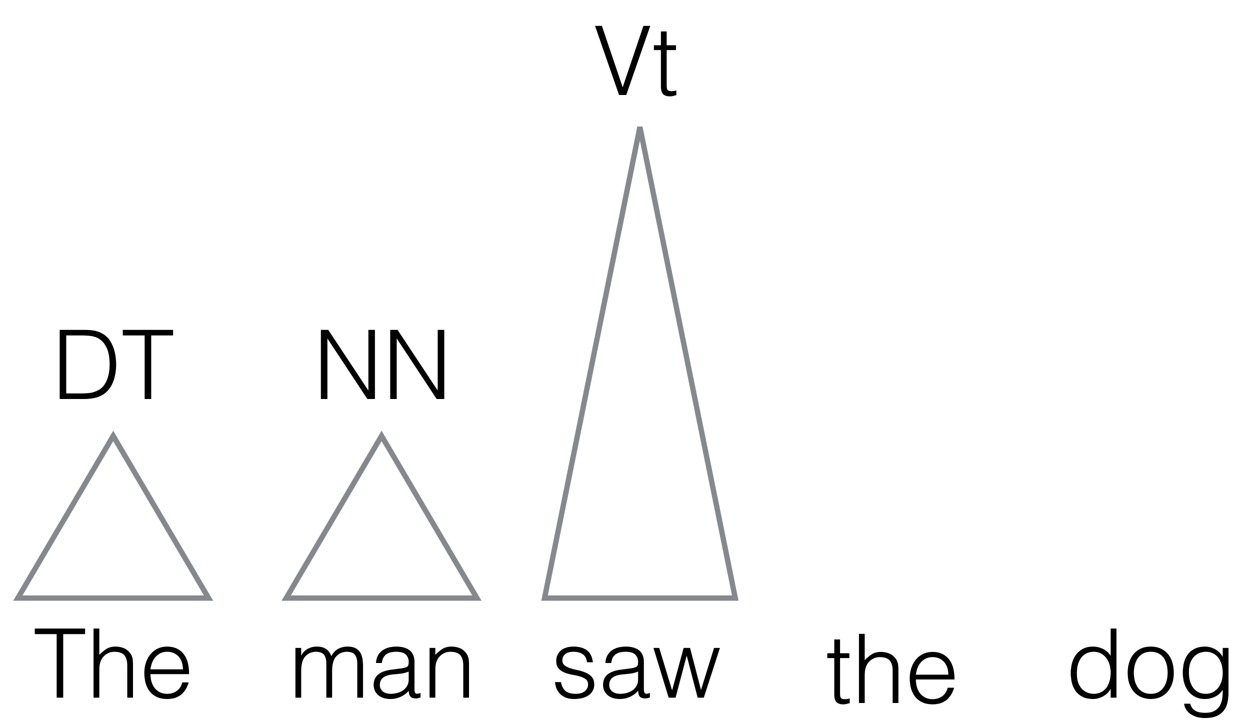

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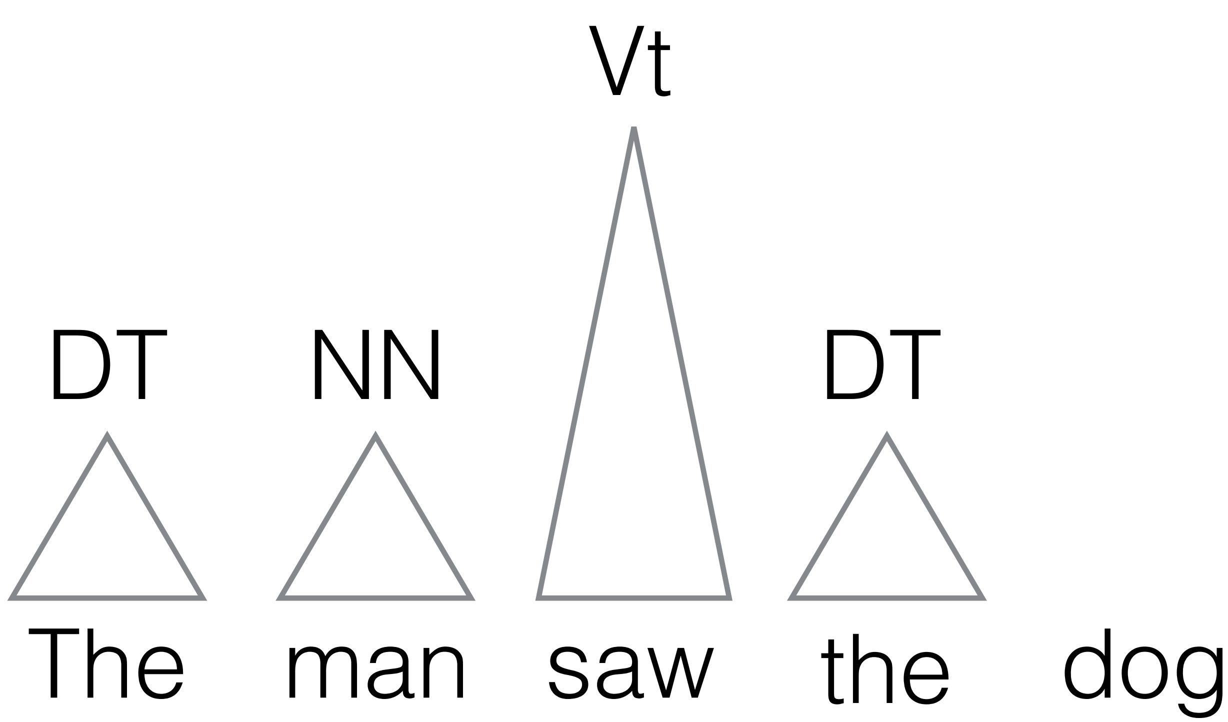

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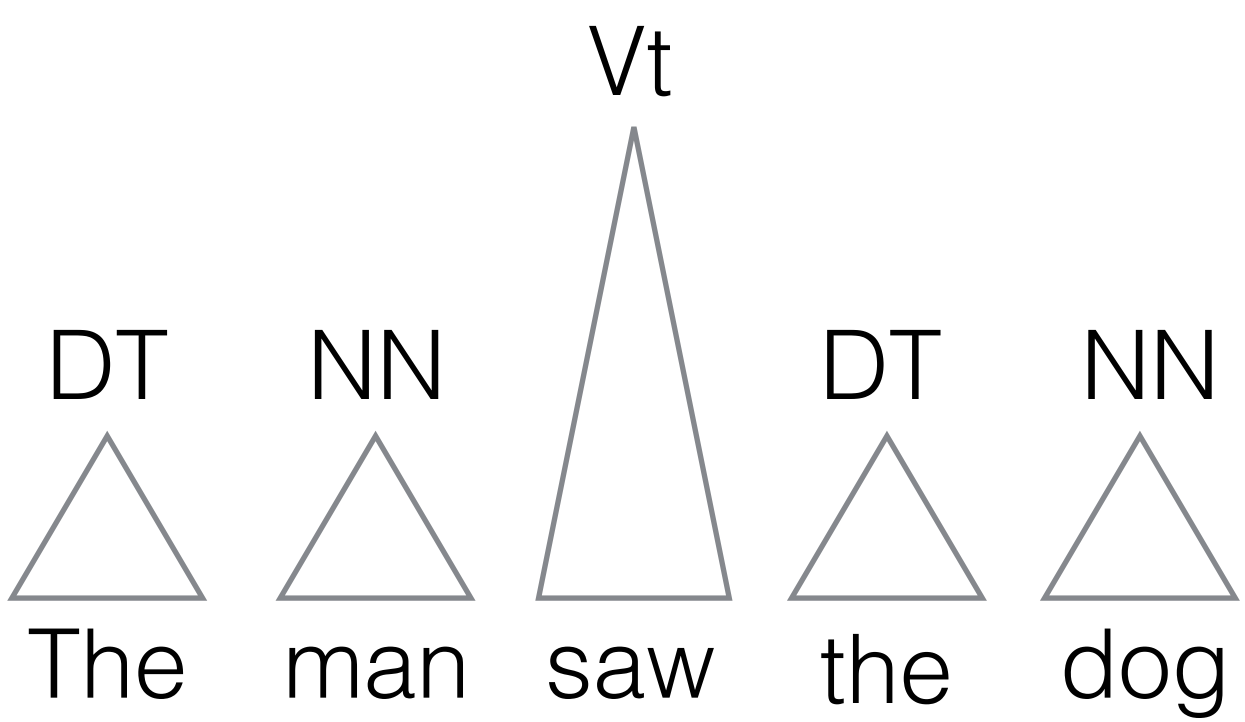

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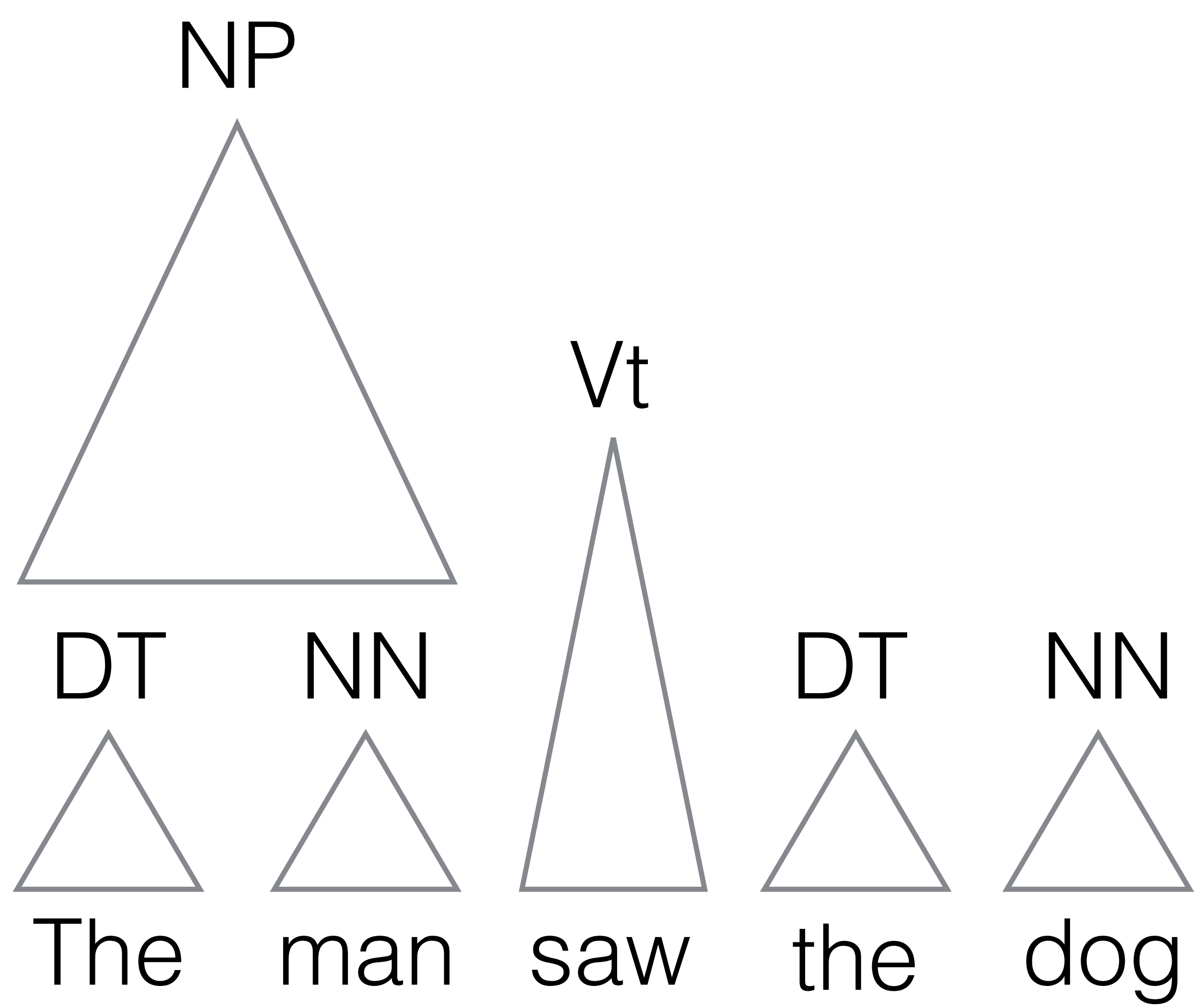

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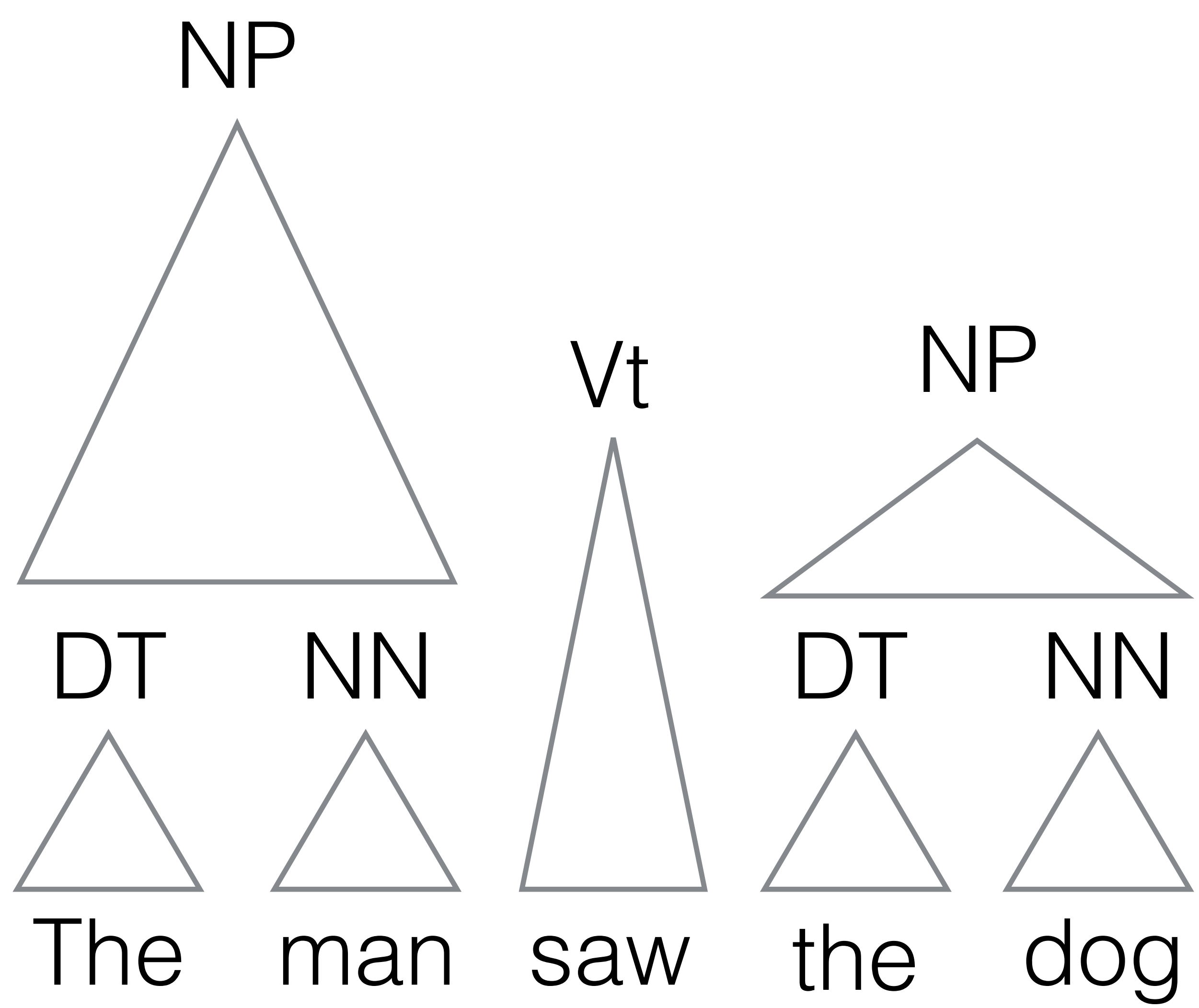

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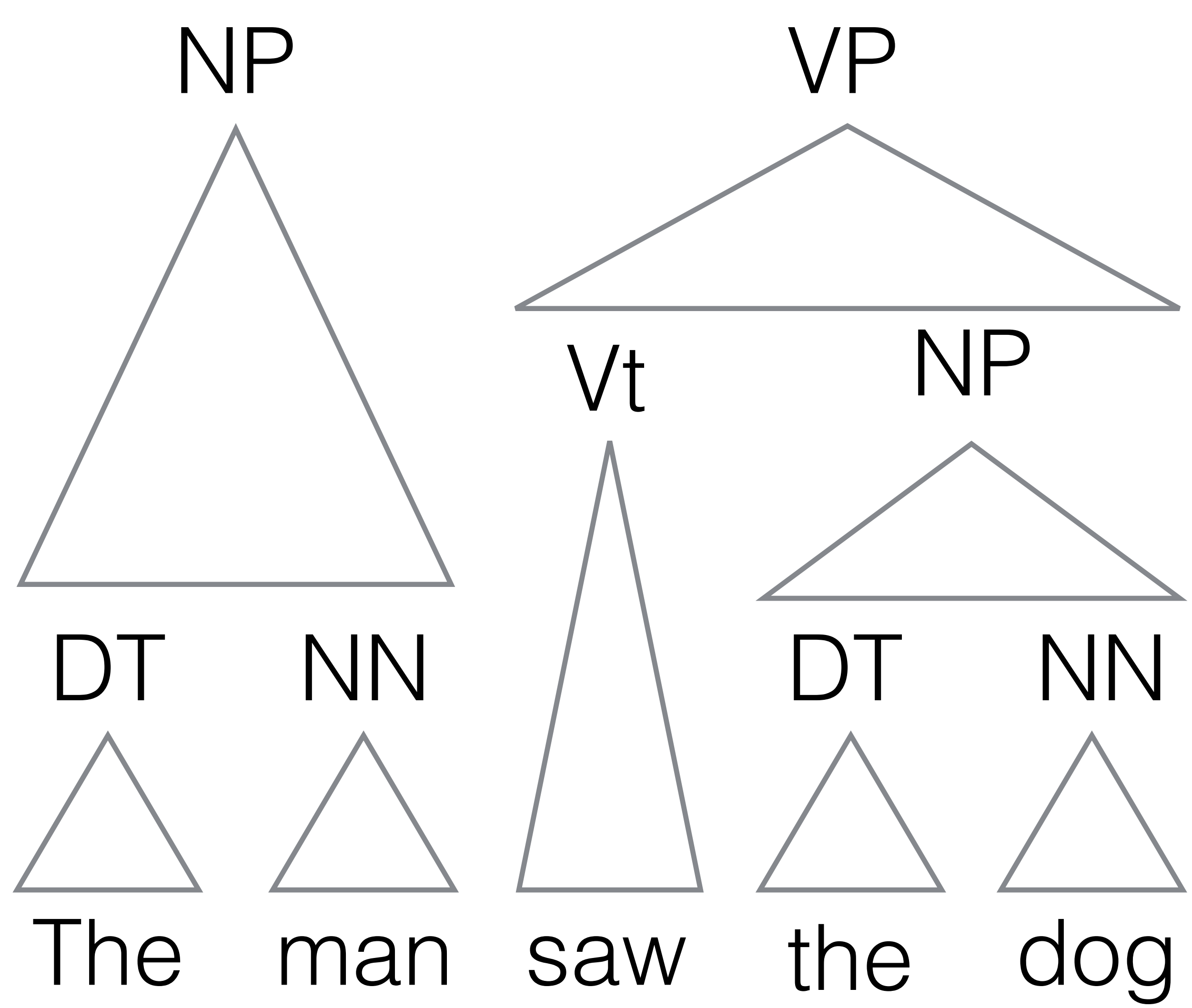

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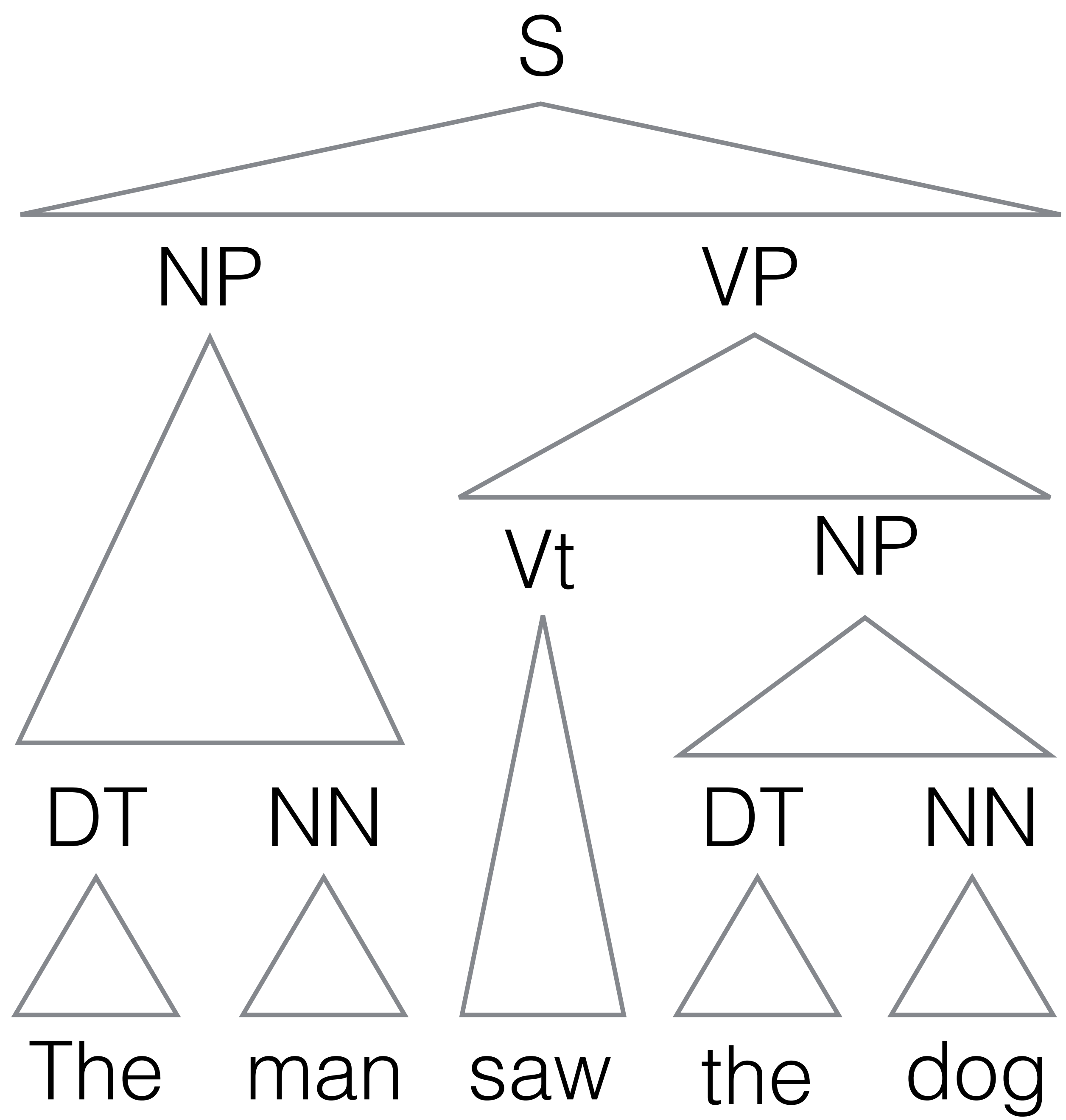

Shift-Reduce Example

Input: *the man sleeps*

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

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Rule	Condition	Statement	Queue
Axiom	1	[•,0]	1
Shift: [1]	2	[the•,1]	2

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Reduce: [4]	NN → man	5	[DT NN •, 2]	5

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Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7

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Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8

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Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9

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Shift: [1]		2	[the•,1]	2
Reduce: [2]	DT → the	3	[DT•,1]	3
Shift: [3]		4	[DT man •, 2]	4
Reduce: [4]	NN → man	5	[DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Shift-Reduce Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[•,0]	1
Shift: [1]		2	[the•,1]	2
Reduce: [2]	DT → the	3	[DT•,1]	3
Shift: [3]		4	[DT man •, 2]	4
Reduce: [4]	NN → man	5	[DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9
Reduce: [9]	S → NP VP	10	[S •, 3]	10

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Shift-Reduce Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[•,0]	1
Shift: [1]		2	[the•,1]	2
Reduce: [2]	DT → the	3	[DT•,1]	3
Shift: [3]		4	[DT man •, 2]	4
Reduce: [4]	NN → man	5	[DT NN •, 2]	5
Reduce: [5]	NP → DT NN	6	[NP •, 2]	6
Shift: [6]		7	[NP sleeps •, 3]	7
Reduce: [7]	Vi → sleeps	8	[NP Vi •, 3]	8
Reduce: [8]	VP → Vi	9	[NP VP •, 3]	9
Reduce: [9]	S → NP VP	10	[S •, 3]	10
GOAL: [10]				∅

S → NP VP
VP → Vi
VP → Vt NP
VP → VP PP
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Shift-Reduce

Input: G and $x_1 \ldots x_n$

Item form: $[\alpha\bullet, j]$
asserts that $\alpha \Rightarrow^* x_1 \ldots x_j$ or
that $\alpha\, x_{j+1} \ldots x_n \Rightarrow^* x_1 \ldots x_n$

Axiom: $[\bullet, 0]$

Goal: $[S\bullet, n]$

$$\textsc{Shift}\ \frac{[\alpha\bullet, j]}{[\alpha\, x_{j+1}, j+1]}$$

Scan (shift)
asserts that $\alpha\, x_{j+1} \Rightarrow^* x_1 \ldots x_j\, x_{j+1}$

$$\textsc{Reduce}\ \frac{[\alpha\,\beta\bullet, j]}{[\alpha\, B, j]}\ B \to \beta \in \mathcal{R}$$

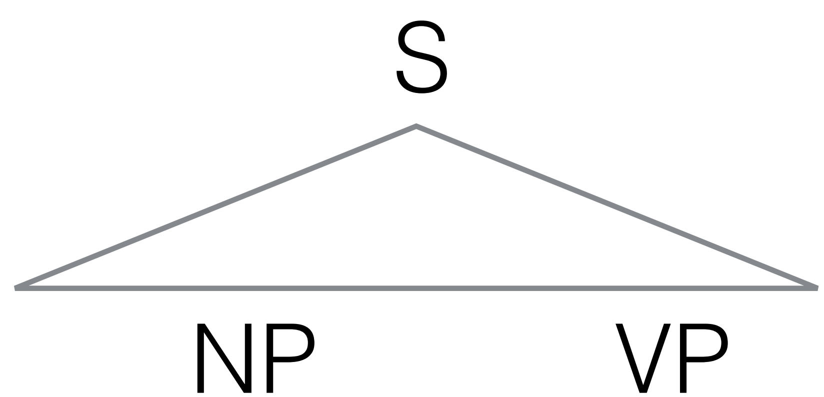
Complete (reduce)
asserts that $\alpha B \Rightarrow^* x_1 \ldots x_j$

Top-down: Predict-Scan

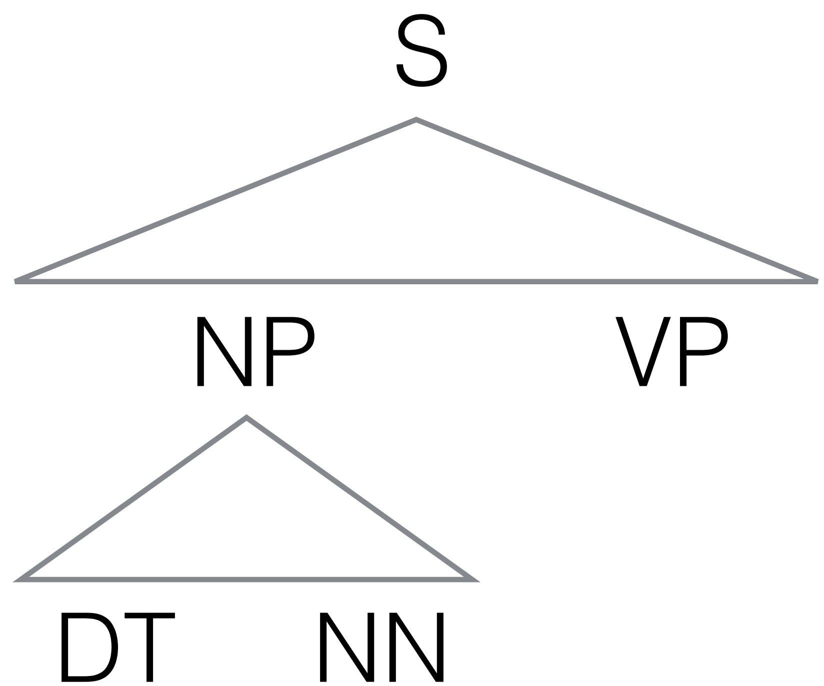
Top-down: Predict-Scan

S

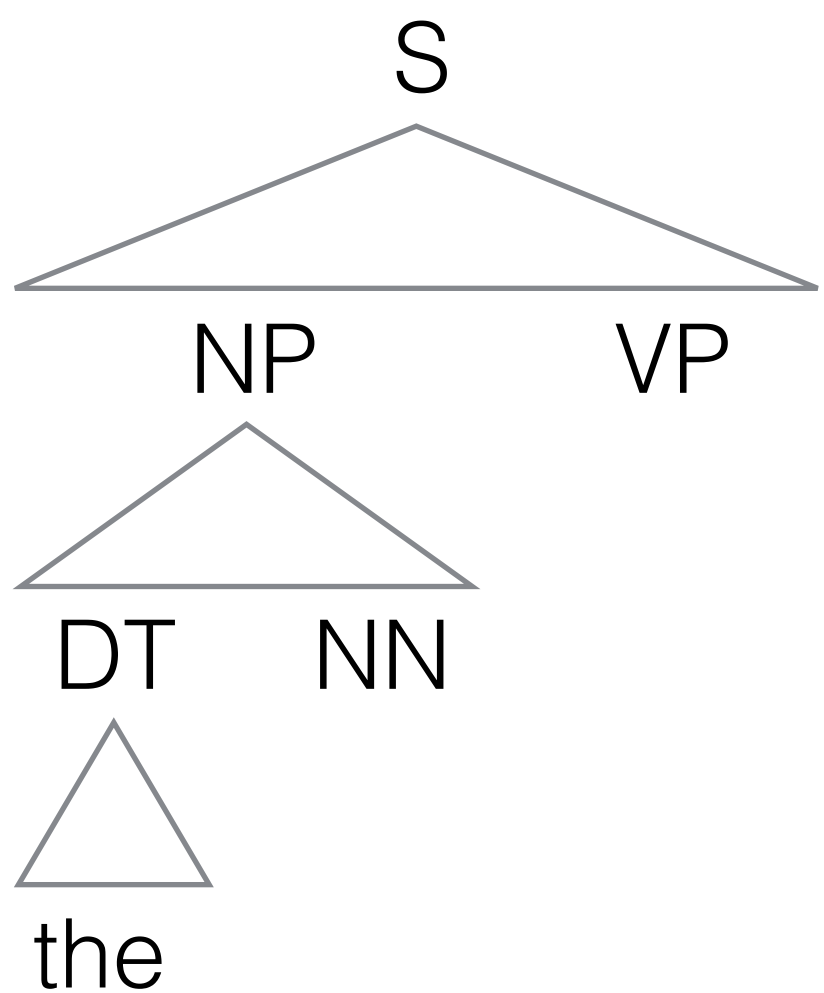
Top-down: Predict-Scan



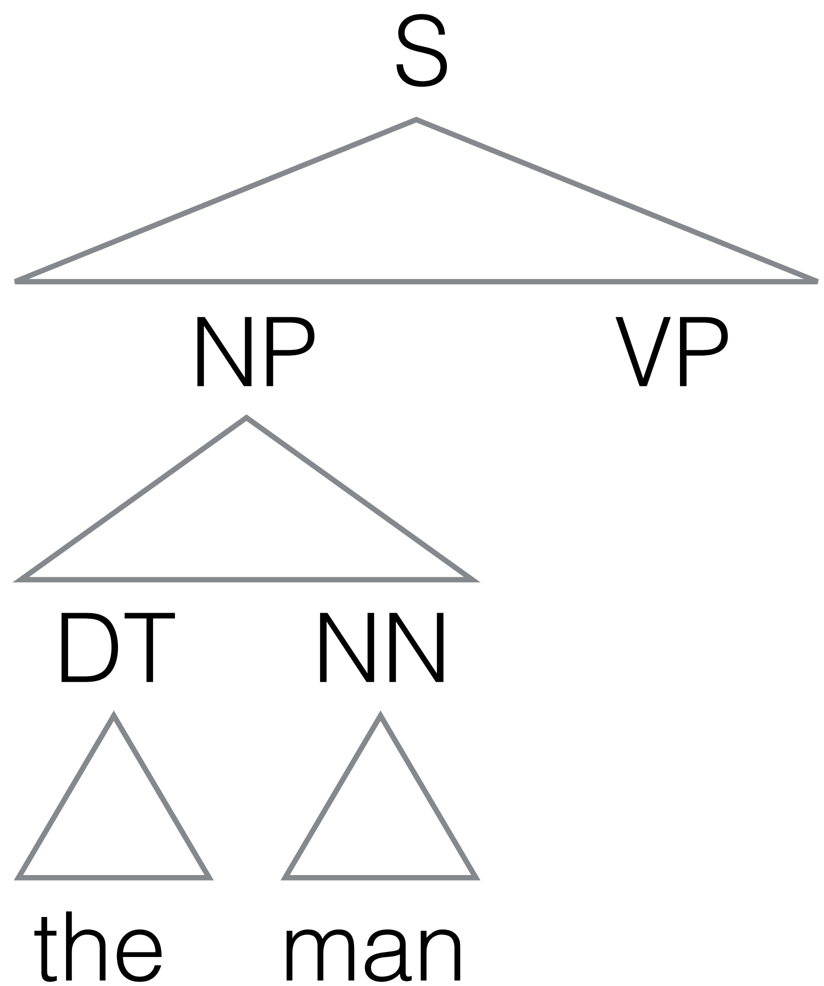
Top-down: Predict-Scan



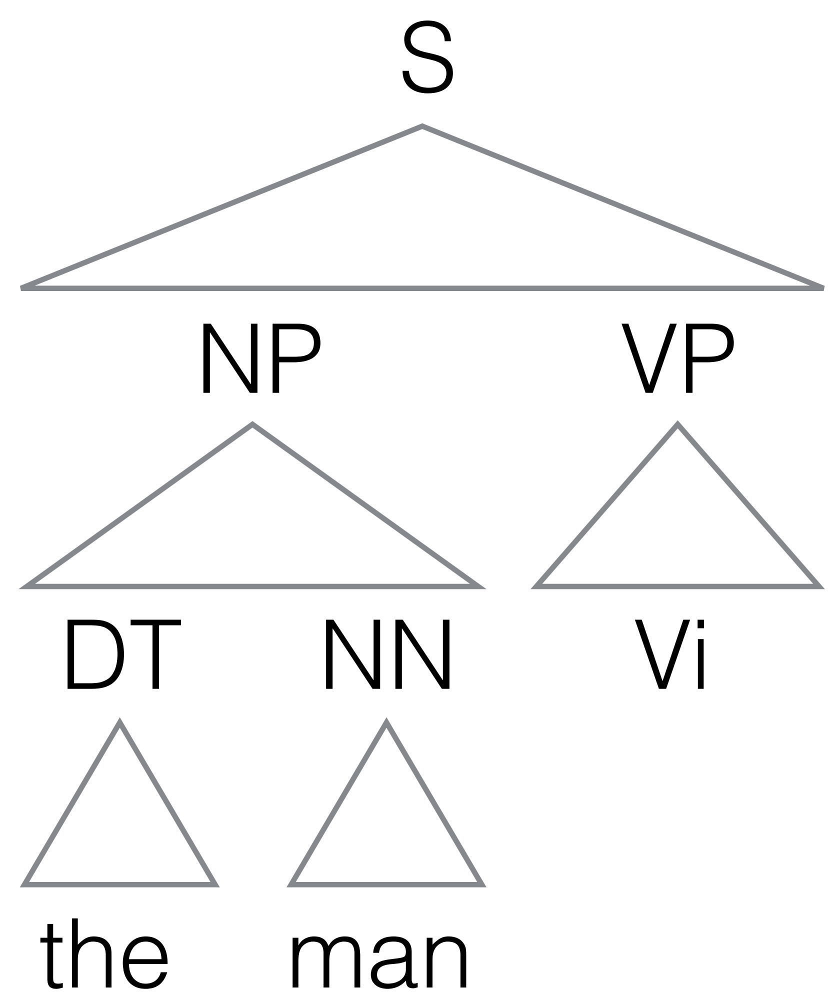
Top-down: Predict-Scan



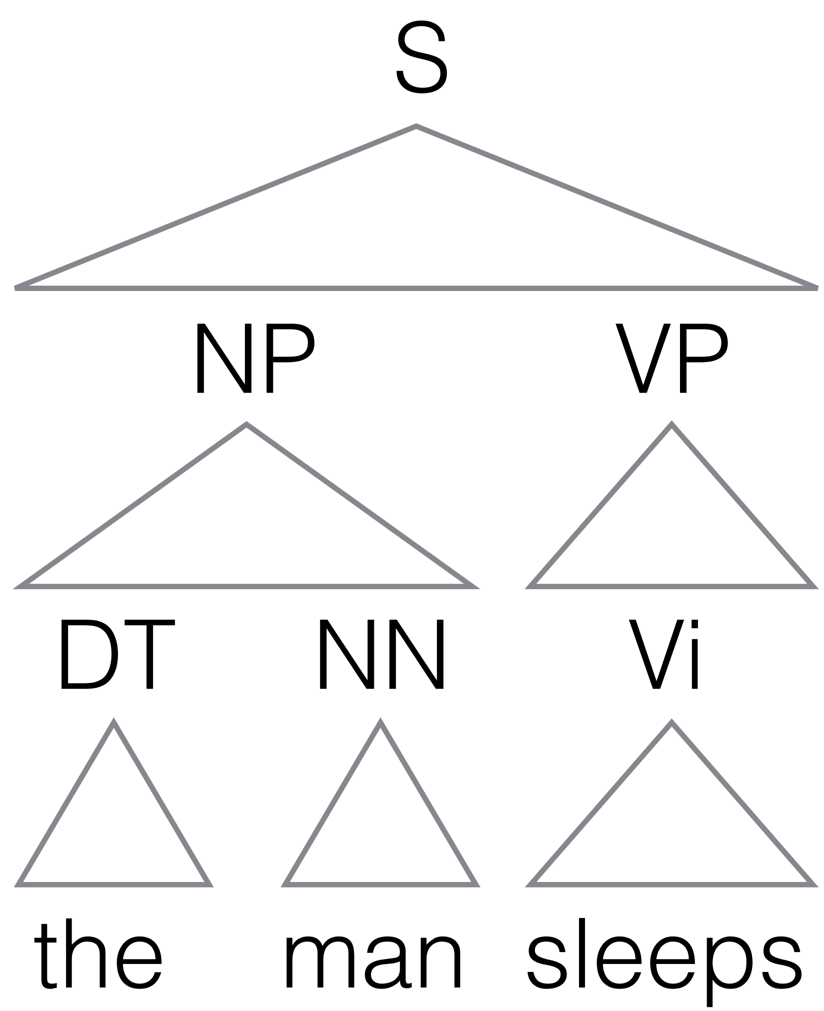
Top-down: Predict-Scan



Top-down: Predict-Scan



Top-down: Predict-Scan



Top-Down Example

Input: *the man sleeps*

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom	1	[• S, 0]	1

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition	Statement	Queue
Axiom	1	[• S, 0]	1

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10
Scan: [10]		11	[•, 3]	11

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down Example

Input: *the man sleeps*

Rule	Condition		Statement	Queue
Axiom		1	[• S, 0]	1
Predict: [1]	S → NP VP	2	[• NP VP, 0]	2
Predict: [2]	NP → DT NN	3	[• DT NN VP, 0]	3
Predict: [3]	DT → the	4	[• the NN VP,0]	4
Scan: [4]		5	[• NN VP,1]	5
Predict: [5]	NN → man	6	[• man VP, 1]	6
Scan: [6]		7	[• VP, 2]	7
Predict: [7]	VP → Vi	8	[• Vi, 2]	8, 9
	VP → Vt NP	9	[• Vt NP, 2]	
Predict: [8]	Vi → sleeps	10	[• sleeps, 2]	9, 10
[9]				10
Scan: [10]		11	[•, 3]	11
GOAL: [11]				∅

S → NP VP
VP → Vi
VP → Vt NP
~~VP → VP PP~~
NP → DT NN
NP → NP PP
PP → IN NP
Vi → sleeps
Vt → saw
NN → man
NN → dog
NN → telescope
DT → the
IN → with

Top-Down recognition

Input: G and x_1 ... x_n

Item form: [•β, j]
asserts that $S \Rightarrow^* x_1 \ldots x_j\ \beta$

Axiom: [•S,0]

Goal: [•,n]

$$\textsc{Scan}\ \frac{[\bullet x_{j+1}\,\beta, j]}{[\bullet\beta, j+1]}$$

Scan
asserts that $S \Rightarrow^* x_1 \ldots x_j\, x_{j+1}\ \beta$

$$\textsc{Predict}\ \frac{[\bullet A\,\beta, j]}{[\bullet\alpha\,\beta, j]}\ A \to \alpha \in \mathcal{R}$$

Predict
asserts that $S \Rightarrow^* x_1 \ldots x_j\, B\ \beta$

Bottom-Up for CNF: CKY

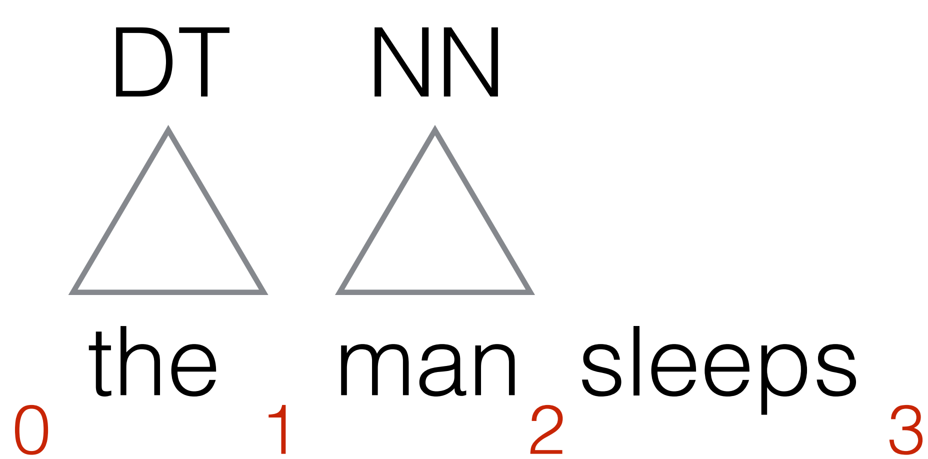
0 1 2 3

Bottom-Up for CNF: CKY

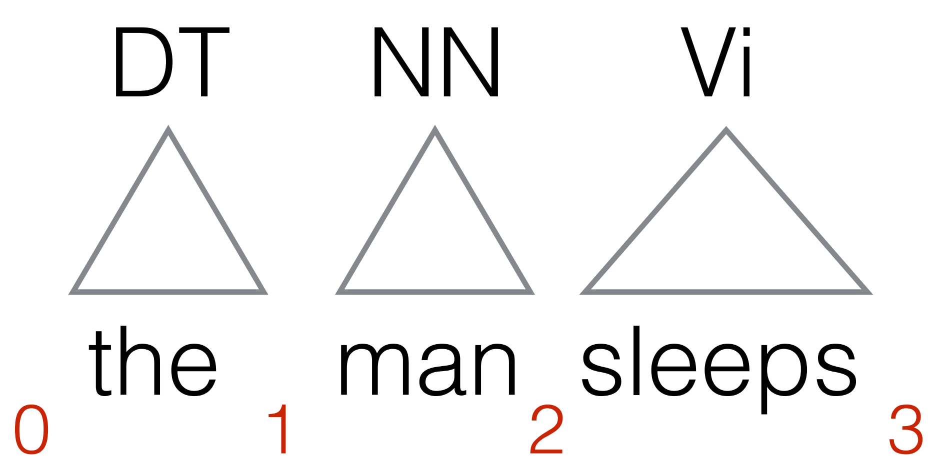
$_0$ the $_1$ man $_2$ sleeps $_3$

Bottom-Up for CNF: CKY

Bottom-Up for CNF: CKY



Bottom-Up for CNF: CKY



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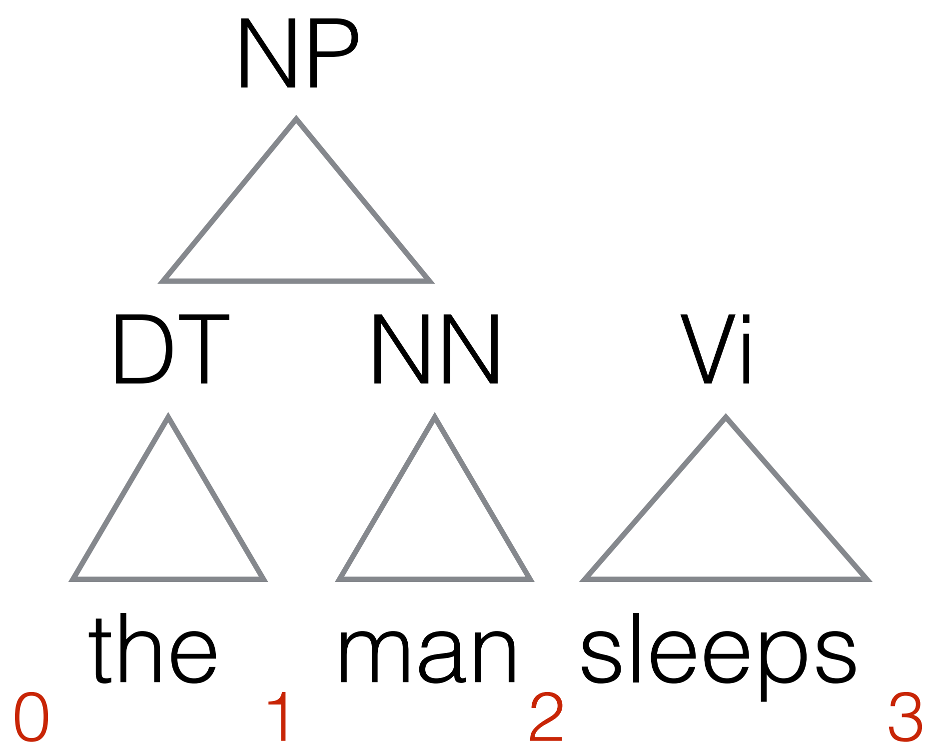

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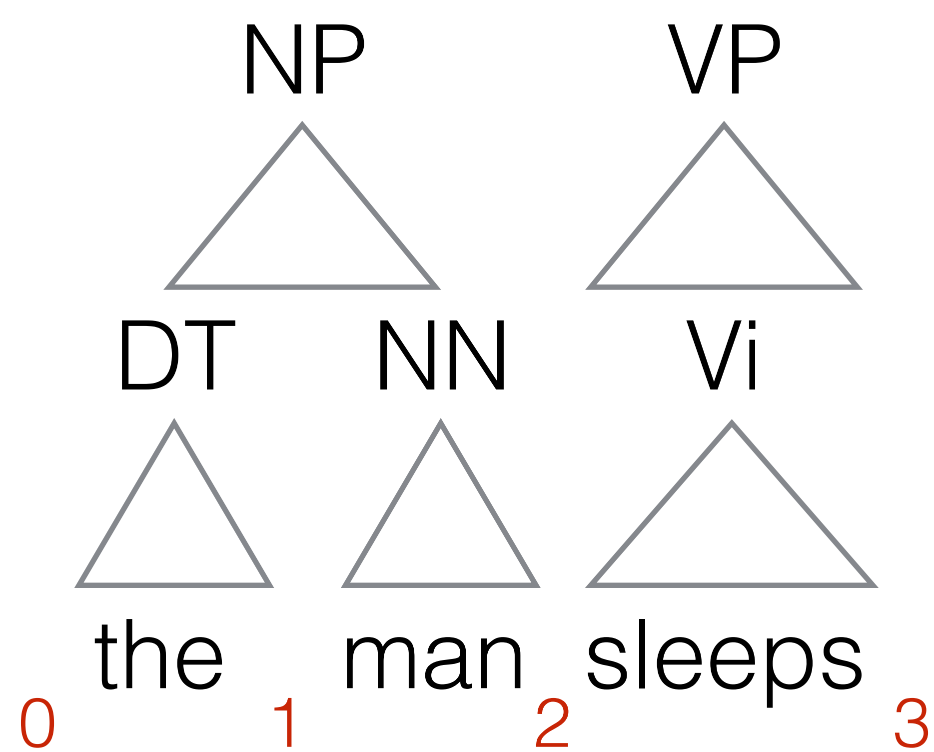
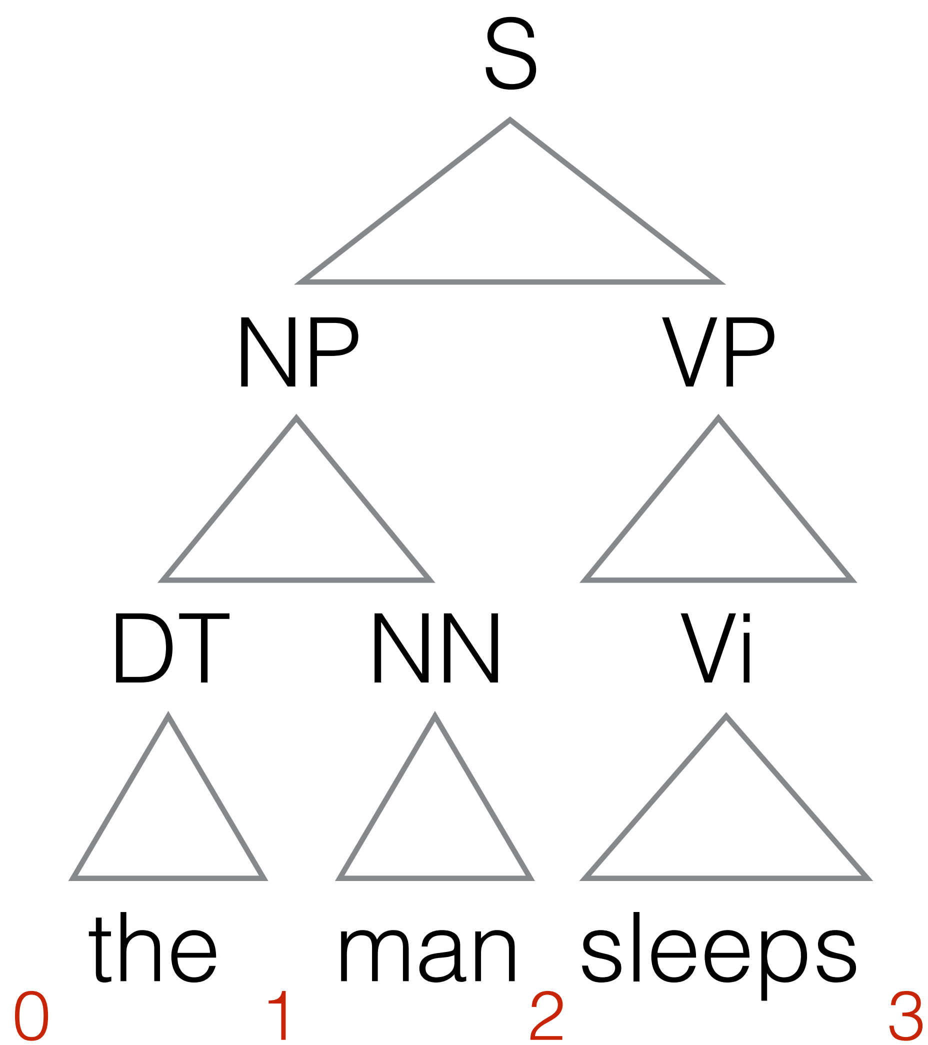


Bottom-Up for CNF: CKY



CKY - CNF only

Input: G and $s = x_1 \dots x_n$

Item form: [i, X, j]
asserts that $X \Rightarrow^* x_{i+1} \dots x_j$

Axioms: [i, X, i+1] $X \rightarrow x_i \in \mathscr{R}$

Goal: [0, S, n]

Merge:
asserts that
$x_{i+1} \dots x_k \, x_{k+1} \dots x_j \Rightarrow^* x_{i+1} \dots x_j$

$$\frac{[i, A, k][k, B, j]}{[i, C, j]} \; C \rightarrow A \, B \in \mathcal{R}$$

CKY Example

Input: *the man saw the dog*

S → NP VP	Vi → sleeps
~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

CKY Example

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Rule	Condition	Statement	Queue	Passive

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Rule	Condition	Statement	Queue	Passive
Axiom	DT → the	1 [0, DT, 1]	1	

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	

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Rule	Condition		Statement	Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	

CKY Example

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VP → Vt NP	NN → man
VP → VP PP	NN → dog
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	

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	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1

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Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2

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Rule	Condition		Statement	Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3

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Axiom	DT → the	1	[0, DT, 1]	1	
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	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4

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Axiom	DT → the	1	[0, DT, 1]	1	
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	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
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	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7

CKY Example

Input: *the man saw the dog*

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~~VP → Vi~~	Vt → saw
VP → Vt NP	NN → man
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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7
Merge: [6] [8]	S → NP VP	9	[0, S, 5]	9	8

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Rule	Condition	Statement		Queue	Passive
Axiom	DT → the	1	[0, DT, 1]	1	
	NN → man	2	[1, NN, 2]	1, 2	
	Vt → saw	3	[2, Vt, 3]	1, 2, 3	
	DT → the	4	[3, DT, 4]	1, 2, 3, 4	
	NN → dog	5	[4, NN, 5]	1, 2, 3, 4, 5	
				2, 3, 4, 5	1
Merge: [1][2]	NP → DT NN	6	[0, NP, 2]	3, 4, 5, 6	2
				4, 5, 6	3
				5, 6	4
Merge: [4][5]	NP → DT NN	7	[3, NP, 5]	6, 7	5
				7	6
Merge: [3] [7]	VP → Vt NP	8	[2, VP, 5]	8	7
Merge: [6] [8]	S → NP VP	9	[0, S, 5]	9	8
GOAL: [9]				∅	9

Rule Segmentation: "Split Points"

$_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

$_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps

0 1 2 3

Rule Segmentation: "Split Points"

$_0$ the $_1$ man $_2$ sleeps $_3$

$_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

$_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps

Rule Segmentation: "Split Points"

${}_0DT_1$

${}_0$ the ${}_1$ man ${}_2$ sleeps ${}_3$

$${}_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$$

$${}_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$$

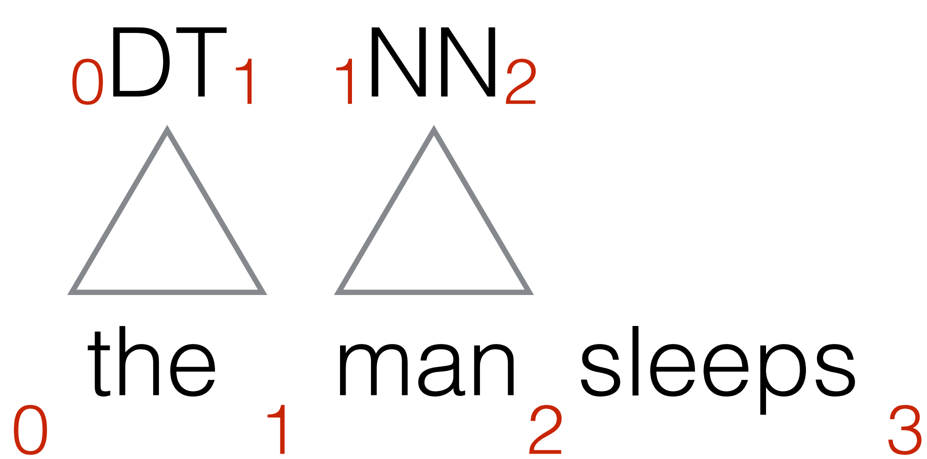
$${}_2VP_3 \rightarrow {}_2Vi_3$$

$${}_0DT_1 \rightarrow \text{the}$$

$${}_1NN_2 \rightarrow \text{man}$$

$${}_2Vi_3 \rightarrow \text{sleeps}$$

Rule Segmentation: "Split Points"



$_0S_3 \rightarrow {_0NP_2}\ {_2VP_3}$

$_0NP_2 \rightarrow {_0DT_1}\ {_1NN_2}$

$_2VP_3 \rightarrow {_2Vi_3}$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps

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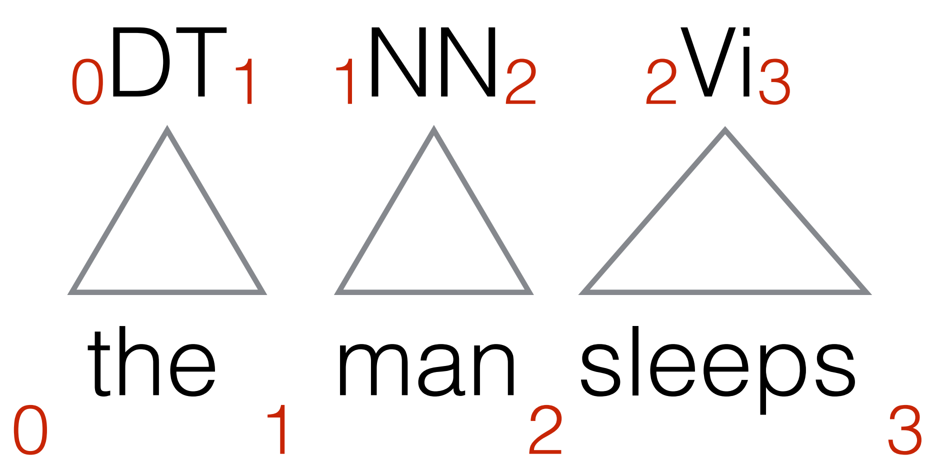

$_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

$_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps

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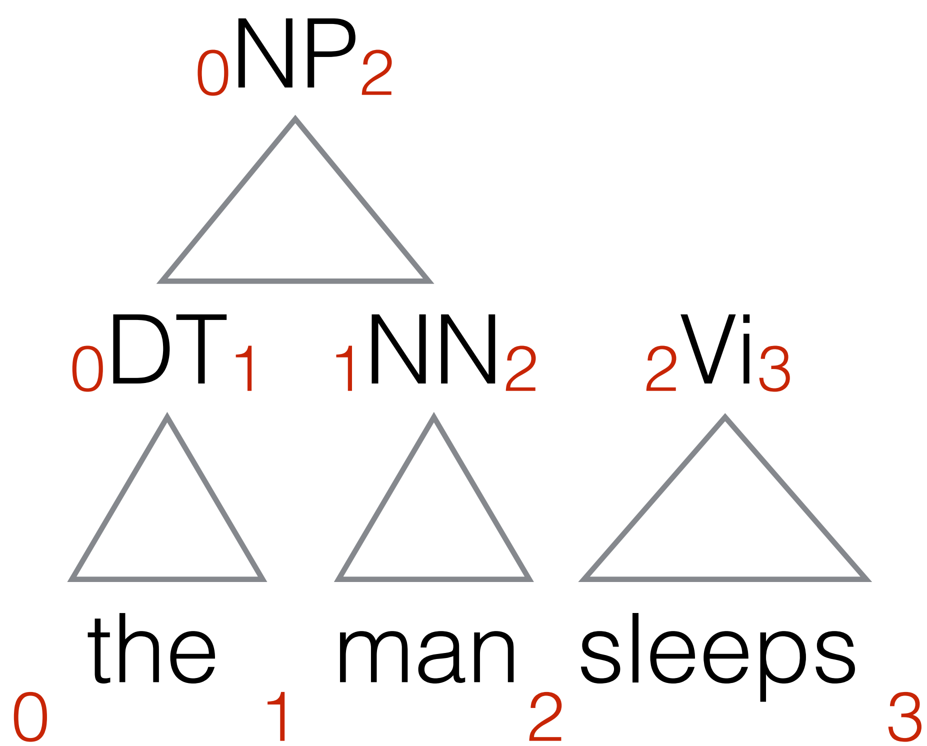

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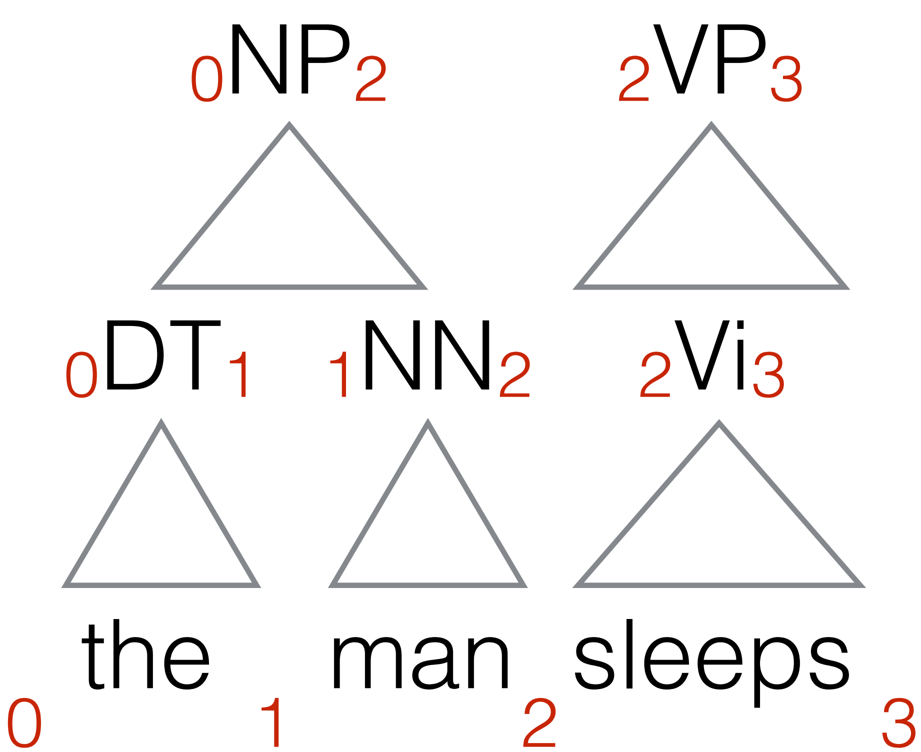


$_0\mathrm{S}_3 \rightarrow {_0\mathrm{NP}_2}\ {_2\mathrm{VP}_3}$

$_0\mathrm{NP}_2 \rightarrow {_0\mathrm{DT}_1}\ {_1\mathrm{NN}_2}$

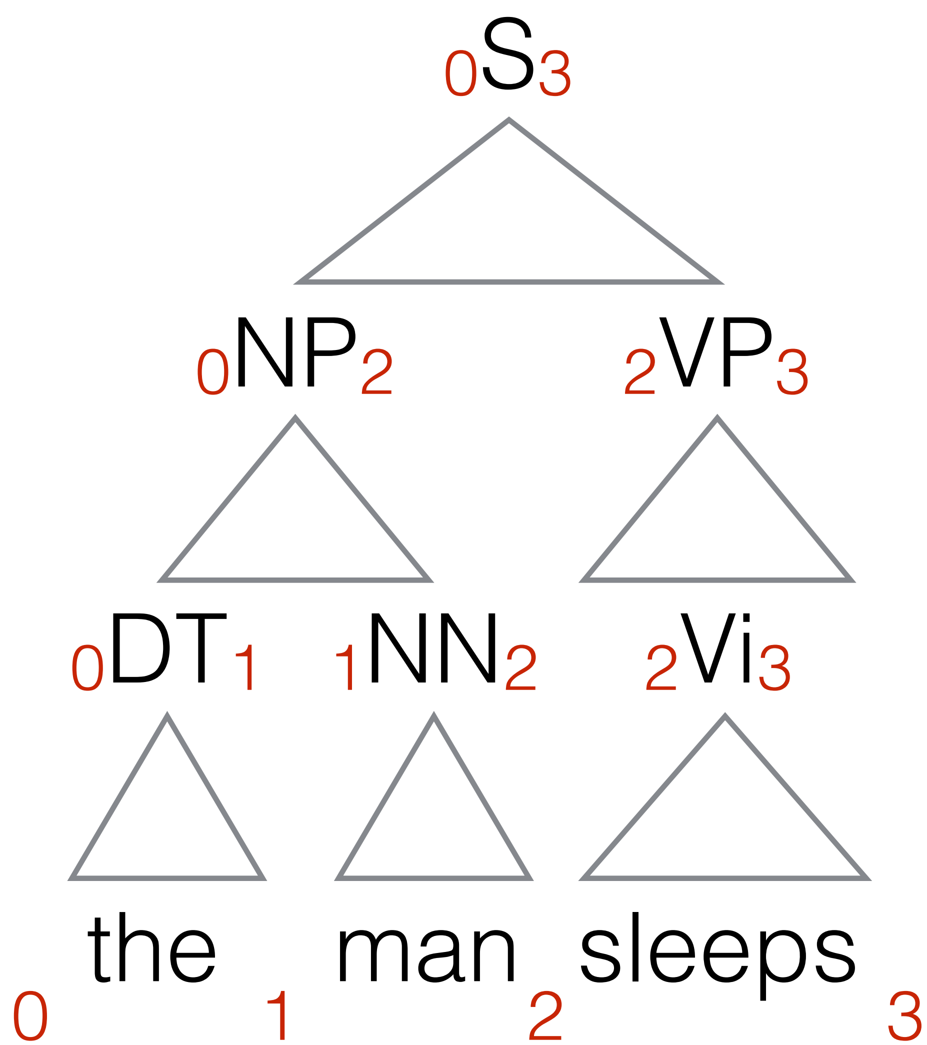
$_2\mathrm{VP}_3 \rightarrow {_2\mathrm{Vi}_3}$

$_0\mathrm{DT}_1 \rightarrow$ the

$_1\mathrm{NN}_2 \rightarrow$ man

$_2\mathrm{Vi}_3 \rightarrow$ sleeps

Rule Segmentation: "Split Points"



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$${}_2\mathrm{VP}_3 \rightarrow {}_2\mathrm{Vi}_3$$

$${}_0\mathrm{DT}_1 \rightarrow \text{the}$$

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"Dotted items"

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Parsing a CNF grammar is easy because we know the shape of rules

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- In general, we segment rules with respect to the input $x_1 \ldots x_n$
- The dot represents progress through the rule's right-hand side (RHS)
- The prefix α has already been parsed and we are waiting for β
- The filled box represents a segmentation of $[0 .. j]$ into $|\alpha|$ adjacent parts
- The empty box has no actual role, it's just a reminder that the segmentation beyond j is unknown

CKY+

Input: G and $s = x_1 \ldots x_n$

Item form: $[i, X \to \alpha_\blacksquare \bullet \beta_\square, j]$
asserts that $X \Rightarrow^* x_{i+1} \ldots x_j \beta$

Axioms: $[i, X \to x_i \bullet \alpha_\square, i+1]$ $\quad X \to x_i\, \alpha \in \mathscr{R}$
$[i, X \to \varepsilon \bullet, i]$ $\quad X \to \varepsilon \in \mathscr{R}$

Goal: $[0, S \to \alpha_\blacksquare \bullet, n]$

Scan

$$\frac{[i, X \to \alpha_\blacksquare \bullet x_{j+1}\, \beta_\square, j]}{[i, X \to \alpha_\blacksquare\, x_{j+1} \bullet \beta_\square, j+1]}$$

Prefix

$$\frac{[i, Y \to \alpha_\blacksquare \bullet, j]}{[i, X \to Y_{i,j} \bullet \beta_\square, j]} X \to Y\, \beta \in \mathcal{R}$$

Complete

$$\frac{[i, X \to \alpha_\blacksquare \bullet Y\, \beta_\square, k][k, Y \to \gamma_\blacksquare \bullet, j]}{[i, X \to \alpha_\blacksquare\, Y_{k,j} \bullet \beta_\square, j]}$$

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
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Rule	Condition	Item	Active	Passive

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Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	

CKY+ Example

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Input: *the man sleeps*

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	

CKY+ Example

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Input: *the man sleeps*

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1

CKY+ Example

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Input: *the man sleeps*

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $Vi_{2,3}$ •, 3]	4, 5	3

CKY+ Example

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Input: *the man sleeps*

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $\text{DT}_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $\text{Vi}_{2,3}$ •, 3]	4, 5	3
Complete: [4] [2]		6	[0, NP → $\text{DT}_{0,1}$ $\text{NN}_{1,2}$ •, 2]	5, 6	4

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
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Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $Vi_{2,3}$ •, 3]	4, 5	3
Complete: [4] [2]		6	[0, NP → $DT_{0,1}$ $NN_{1,2}$ •, 2]	5, 6	4
				6	5

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
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Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $Vi_{2,3}$ •, 3]	4, 5	3
Complete: [4] [2]		6	[0, NP → $DT_{0,1}$ $NN_{1,2}$ •, 2]	5, 6	4
				6	5
Prefix: [6]	S → NP VP	7	[0, S → $NP_{0,2}$ • VP, 2]	7	6

CKY+ Example

Input: *the man sleeps*

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $Vi_{2,3}$ •, 3]	4, 5	3
Complete: [4] [2]		6	[0, NP → $DT_{0,1}$ $NN_{1,2}$ •, 2]	5, 6	4
				6	5
Prefix: [6]	S → NP VP	7	[0, S → $NP_{0,2}$ • VP, 2]	7	6
Complete: [7] [5]		8	[0, S → $NP_{0,2}$ $VP_{2,3}$ •, 3]	8	7

CKY+ Example

S → NP VP	Vi → sleeps
VP → Vi	Vt → saw
VP → Vt NP	NN → man
VP → VP PP	NN → dog
NP → DT NN	NN → telescope
NP → NP PP	DT → the
PP → IN NP	IN → with

Input: *the man sleeps*

Rule	Condition		Item	Active	Passive
Axiom	DT → the	1	[0, DT → the •,1]	1	
	NN → man	2	[1, NN → man •, 2]	1, 2	
	Vi → sleeps	3	[2, Vi → sleeps •, 3]	1, 2, 3	
Prefix: [1]	NP → DT NN	4	[0, NP → $DT_{0,1}$ • NN, 2]	2, 3, 4	1
				3, 4	2
Prefix: [3]	VP → Vi	5	[2, VP → $Vi_{2,3}$ •, 3]	4, 5	3
Complete: [4] [2]		6	[0, NP → $DT_{0,1}$ $NN_{1,2}$ •, 2]	5, 6	4
				6	5
Prefix: [6]	S → NP VP	7	[0, S → $NP_{0,2}$ • VP, 2]	7	6
Complete: [7] [5]		8	[0, S → $NP_{0,2}$ $VP_{2,3}$ •, 3]	8	7
GOAL: [8]				∅	

Correctness of Parsing Strategy

Soundness: if a goal item is proven for α

- then $\omega \in L(G)$

Completeness: if $\alpha \in L(G)$

- then a goal item can be proven for α

Parse Forest

Efficient representation of the whole space $T_G(\omega)$

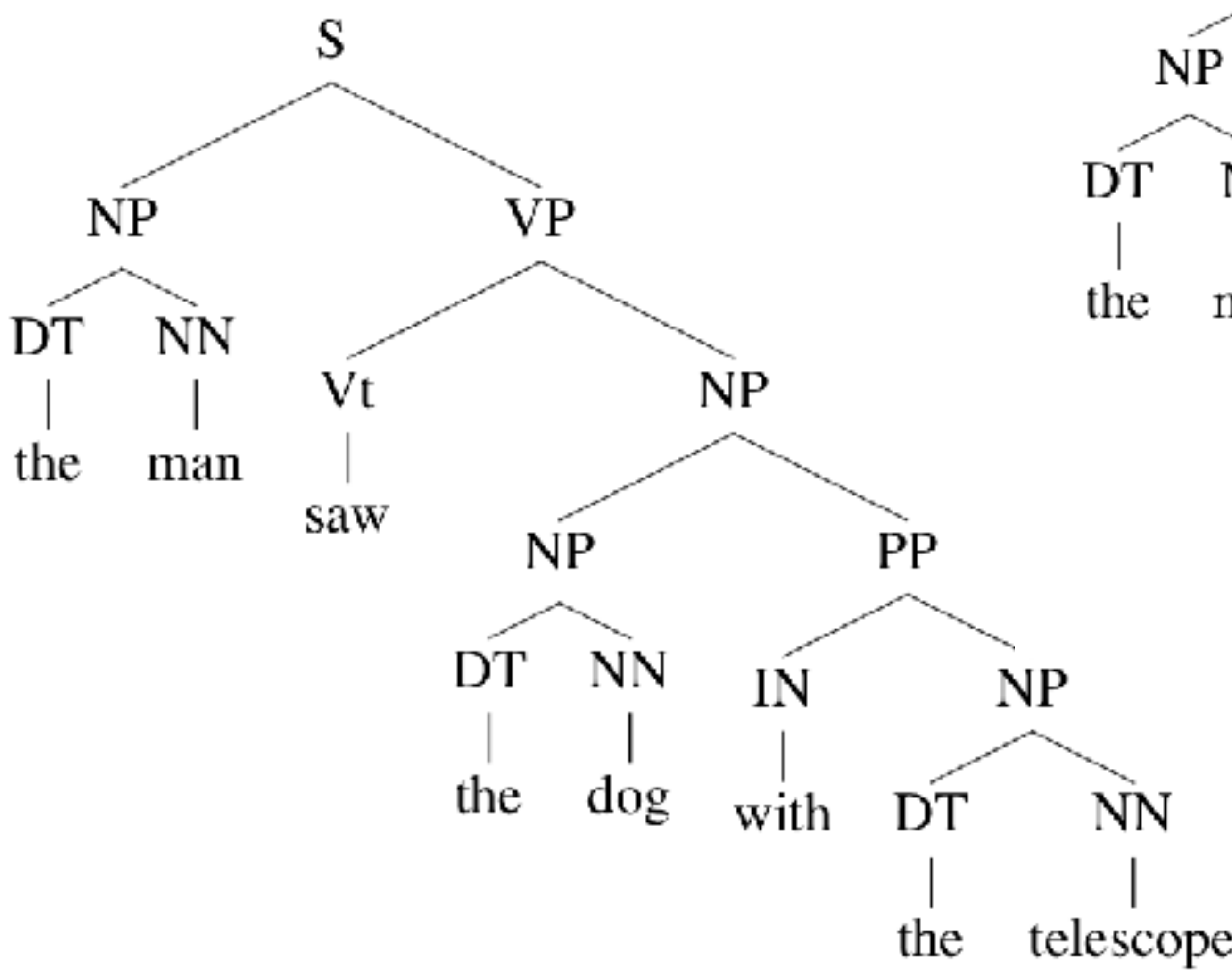
- each and every possible tree yielding ω

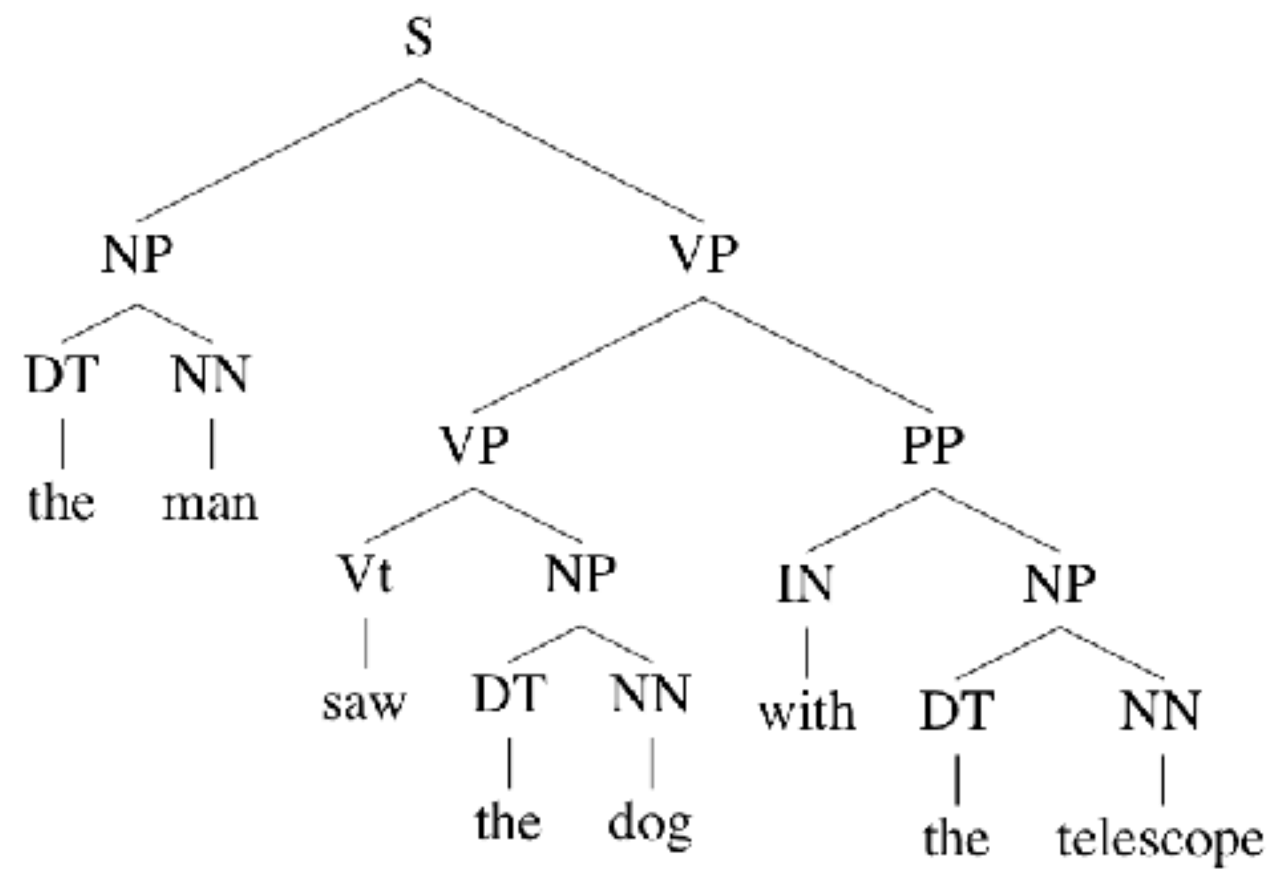
We must be able to represent partial derivations

- including alternative ones

Ambiguity

Some strings may have more than one derivation in G





Dealing with Ambiguity

Statistical model: weight steps in a derivation

- induces a partial ordering over derivations
- can be used to make a decision
  - e.g. best tree under the model

Probabilistic CFG

CFG extended with parameters $0 \leq \theta_r \leq 1$

- where $r \in \mathscr{R}$ and

$$\sum_{\alpha : X \to \alpha \in R} \theta_{X \to \alpha} = 1$$

Probabilistic CFG

Distribution over trees and their yields

$$P_{DS|NM}(R_1^m = r_1^m, X_1^n = \text{yield}(r_1^m)|n, m)$$
$$= \prod_{i=1}^{m} \theta_{r_i} = \prod_{i=1}^{m} \theta_{v_i \to \beta_i}$$
$$\text{where } r_i \text{ corresponds to } v_i \to \beta_i$$

Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ the

${}_1NN_2 \rightarrow$ man

${}_2Vi_3 \rightarrow$ sleeps

Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

${}_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow$ the

${}_1NN_2 \rightarrow$ man

${}_2Vi_3 \rightarrow$ sleeps

the man sleeps

Joint Distribution

$_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

$_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps

Joint Distribution

${}_0\mathrm{S}_3 \rightarrow {}_0\mathrm{NP}_2\ {}_2\mathrm{VP}_3$

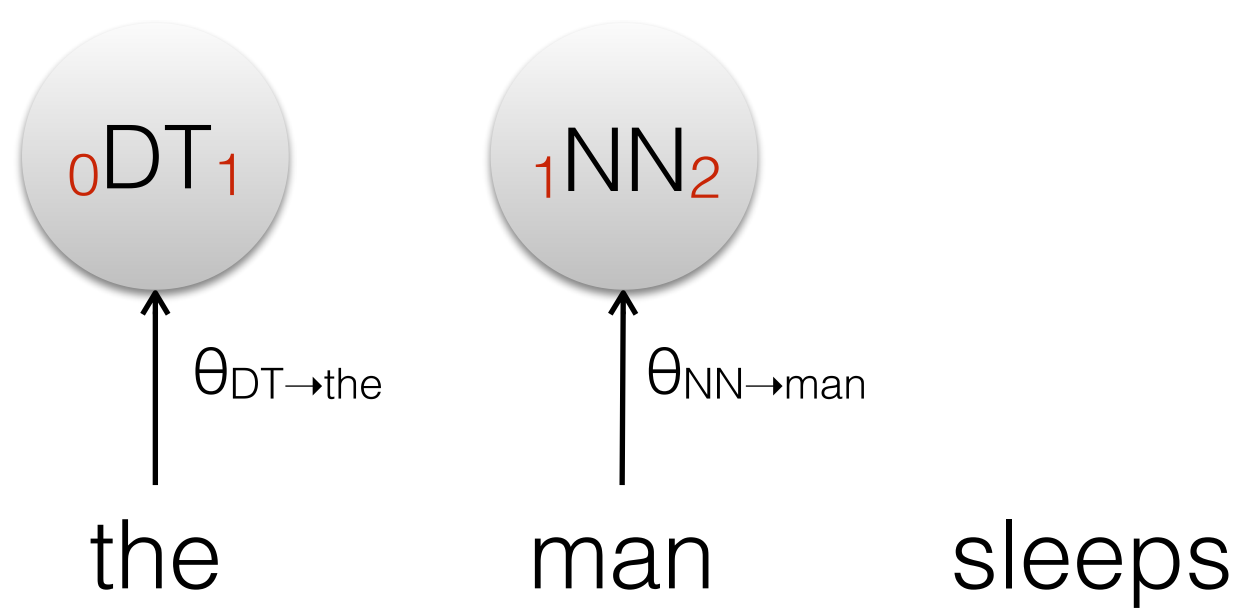
${}_0\mathrm{NP}_2 \rightarrow {}_0\mathrm{DT}_1\ {}_1\mathrm{NN}_2$

${}_2\mathrm{VP}_3 \rightarrow {}_2\mathrm{Vi}_3$

${}_0\mathrm{DT}_1 \rightarrow \mathrm{the}$

${}_1\mathrm{NN}_2 \rightarrow \mathrm{man}$

${}_2\mathrm{Vi}_3 \rightarrow \mathrm{sleeps}$



Joint Distribution

${}_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

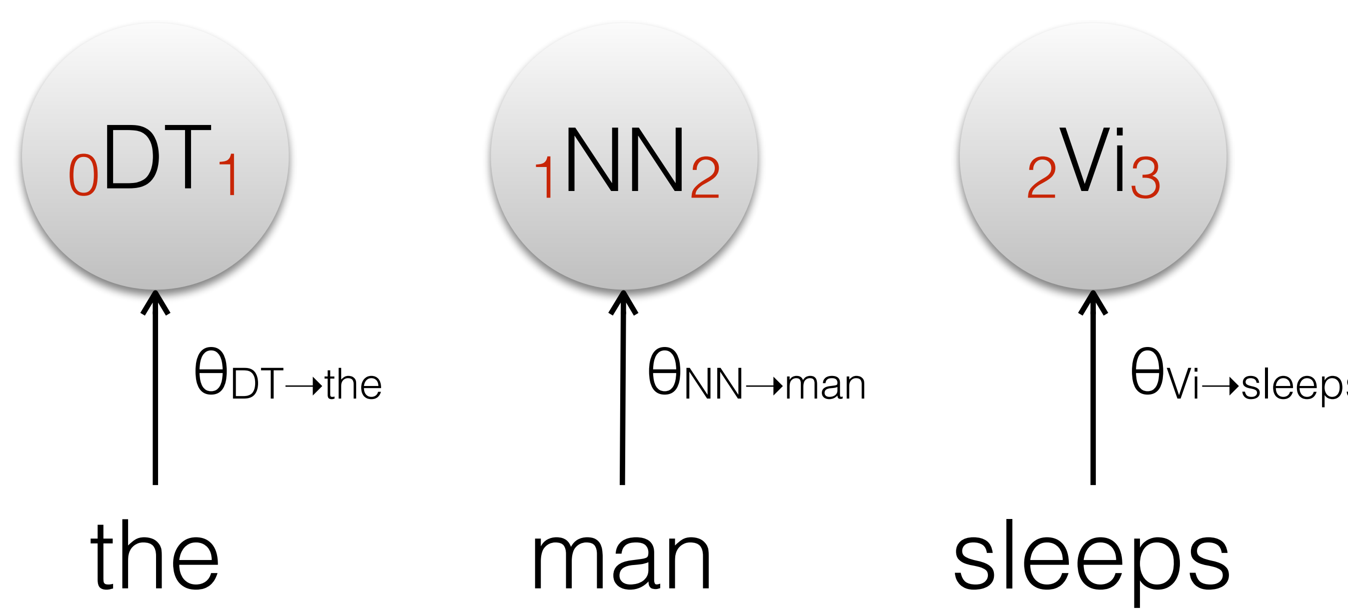
${}_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

${}_2VP_3 \rightarrow {}_2Vi_3$

${}_0DT_1 \rightarrow \text{the}$

${}_1NN_2 \rightarrow \text{man}$

${}_2Vi_3 \rightarrow \text{sleeps}$



Joint Distribution

$_0S_3 \rightarrow {}_0NP_2 \; {}_2VP_3$

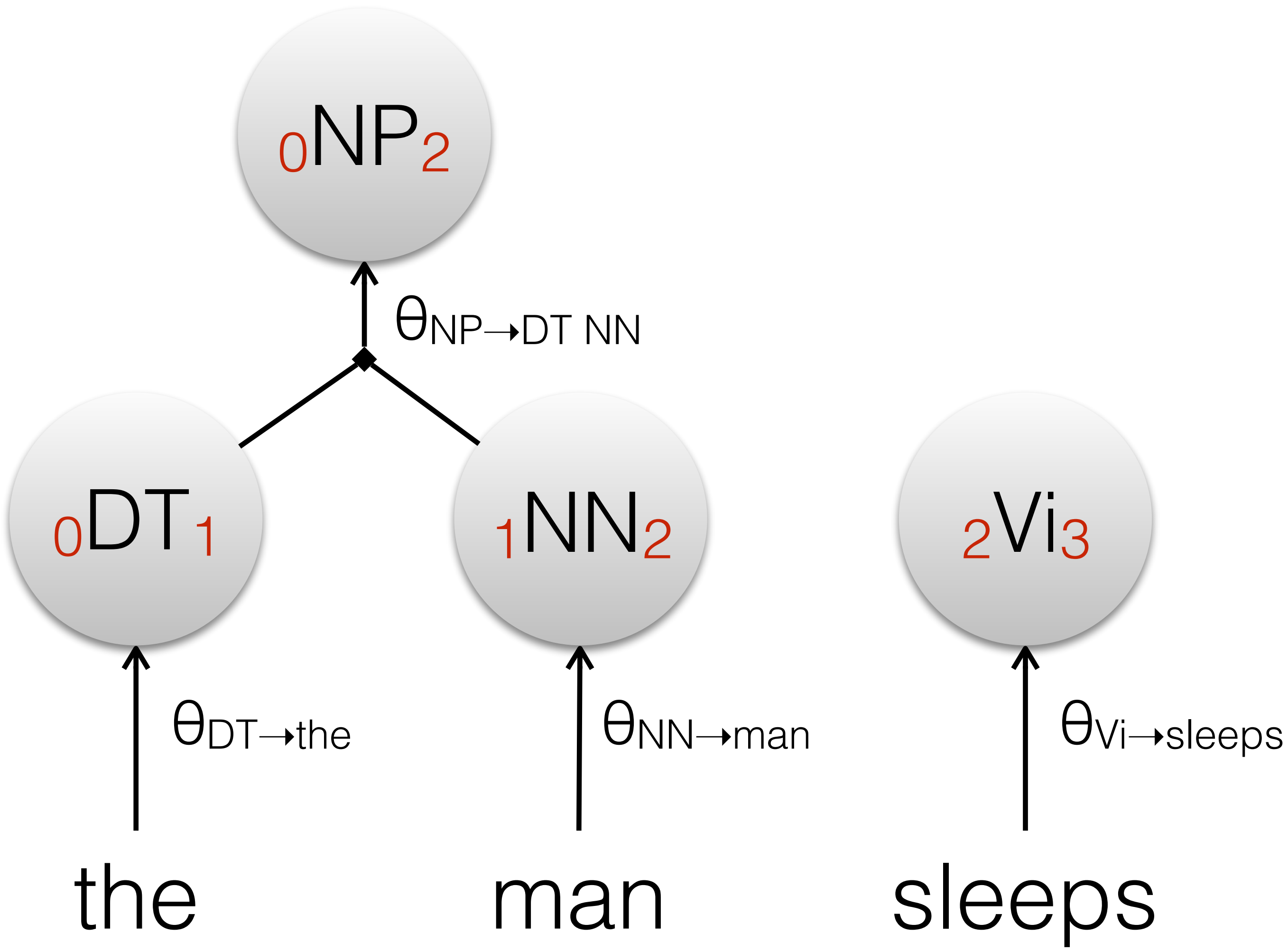
$_0NP_2 \rightarrow {}_0DT_1 \; {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps



Joint Distribution

$_0S_3 \rightarrow {}_0NP_2\ {}_2VP_3$

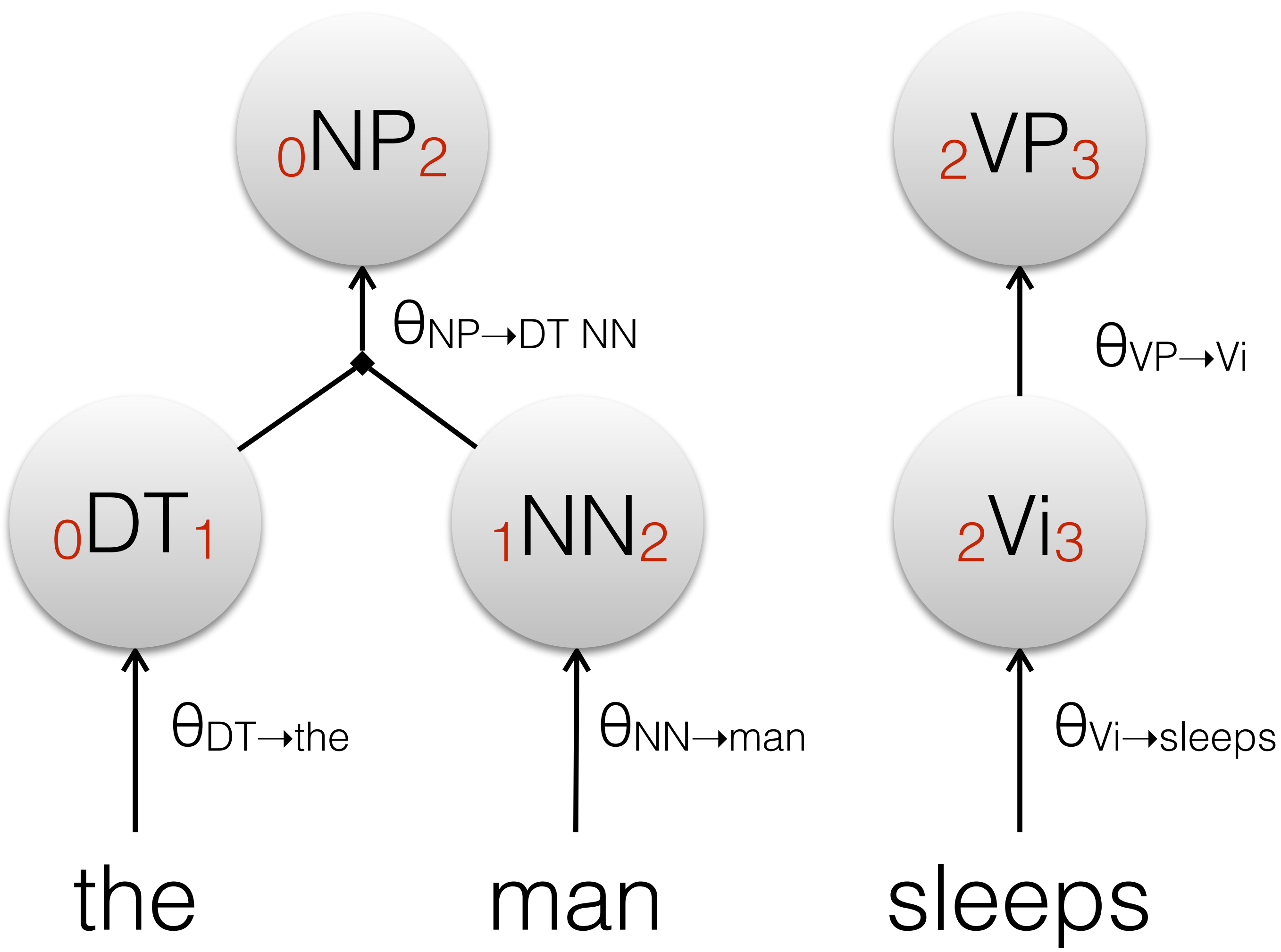
$_0NP_2 \rightarrow {}_0DT_1\ {}_1NN_2$

$_2VP_3 \rightarrow {}_2Vi_3$

$_0DT_1 \rightarrow$ the

$_1NN_2 \rightarrow$ man

$_2Vi_3 \rightarrow$ sleeps



Joint Distribution

$_0S_3 \rightarrow {_0NP_2}\ {_2VP_3}$

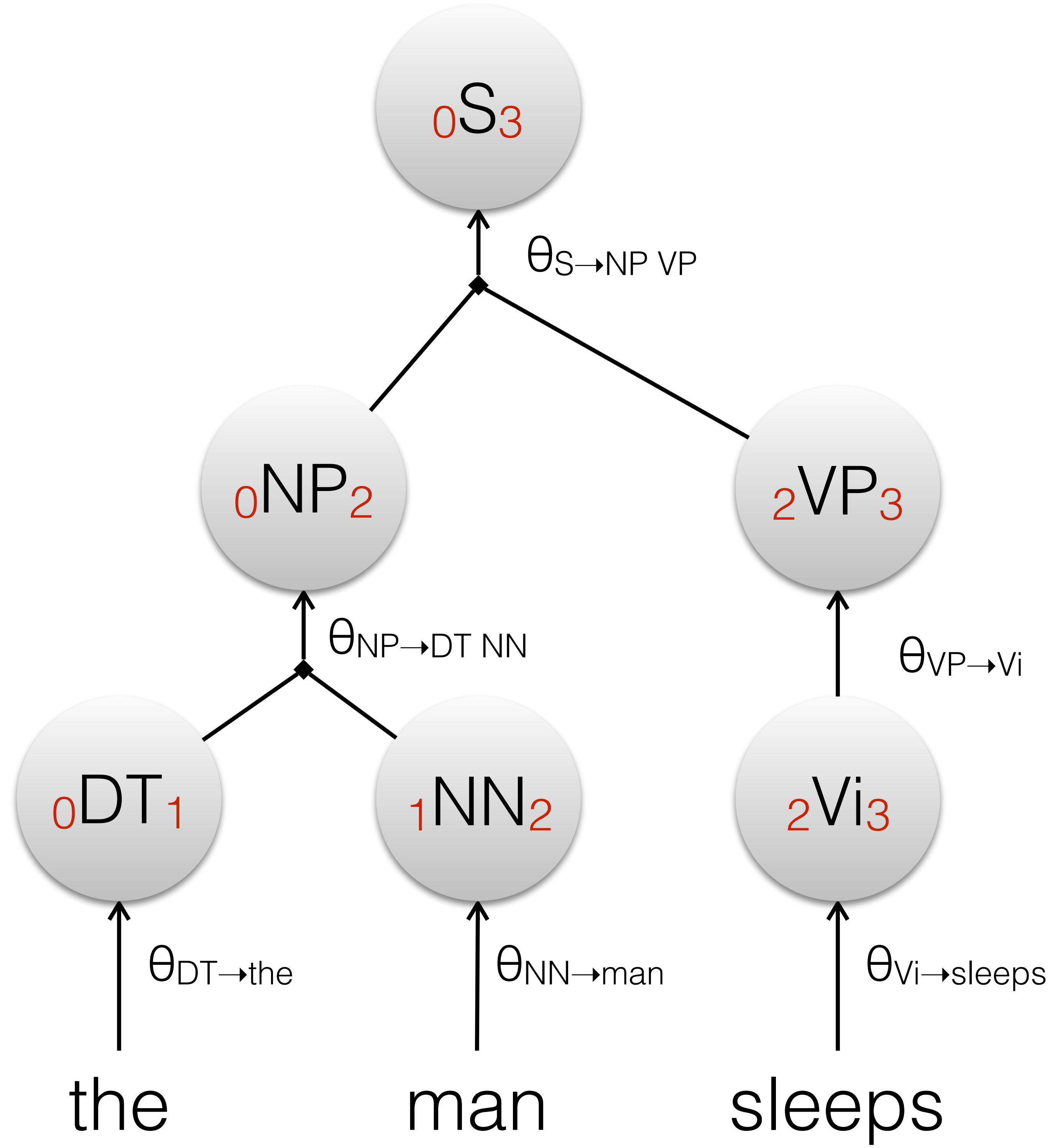
$_0NP_2 \rightarrow {_0DT_1}\ {_1NN_2}$

$_2VP_3 \rightarrow {_2Vi_3}$

$_0DT_1 \rightarrow \text{the}$

$_1NN_2 \rightarrow \text{man}$

$_2Vi_3 \rightarrow \text{sleeps}$



Ambiguity

Ambiguity

the man

saw

the dog

with the telescope

Ambiguity

${}_{0}\mathrm{DT}_{1}$

$\theta_{\mathrm{DT}\rightarrow\mathrm{the}}$

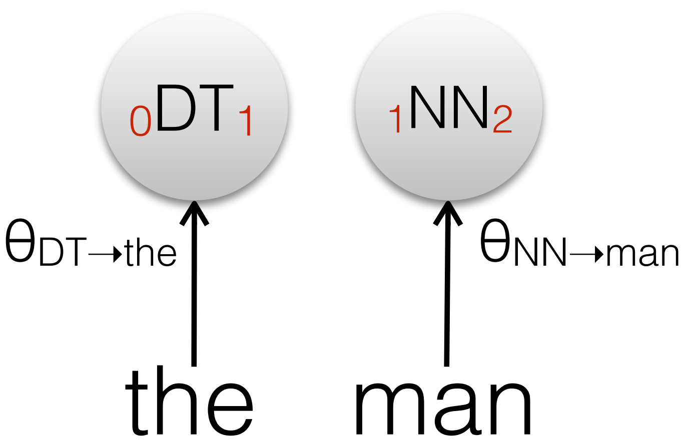
the man

saw

the dog

with the telescope

Ambiguity

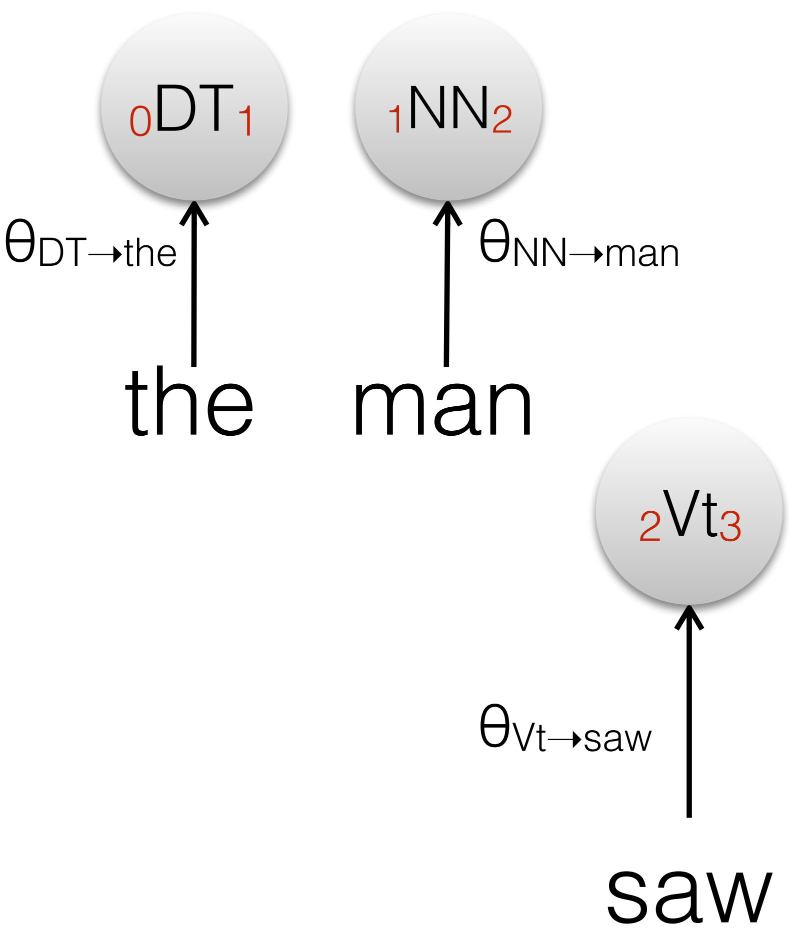


saw

the dog

with the telescope

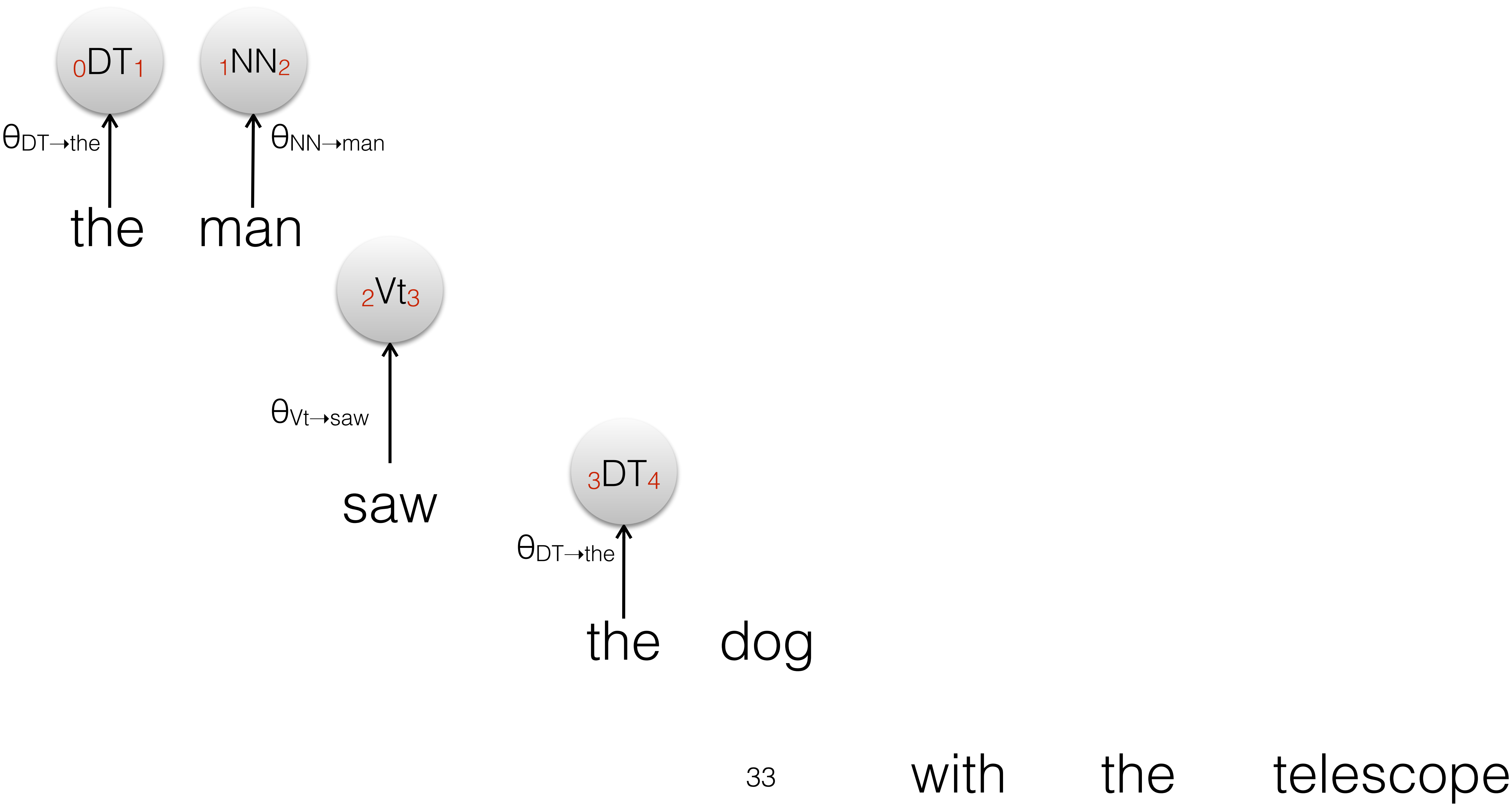
Ambiguity



the dog

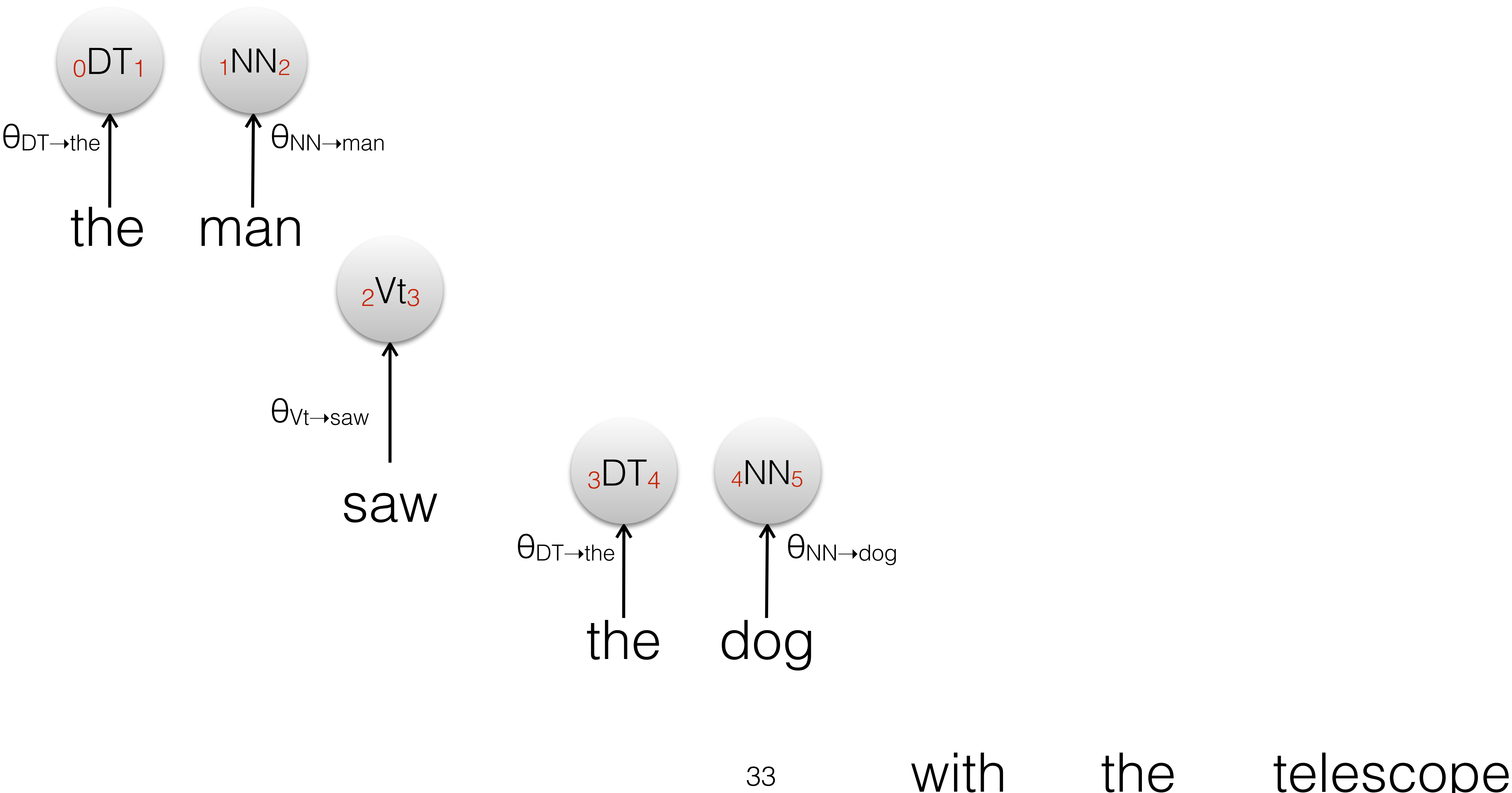
with the telescope

Ambiguity

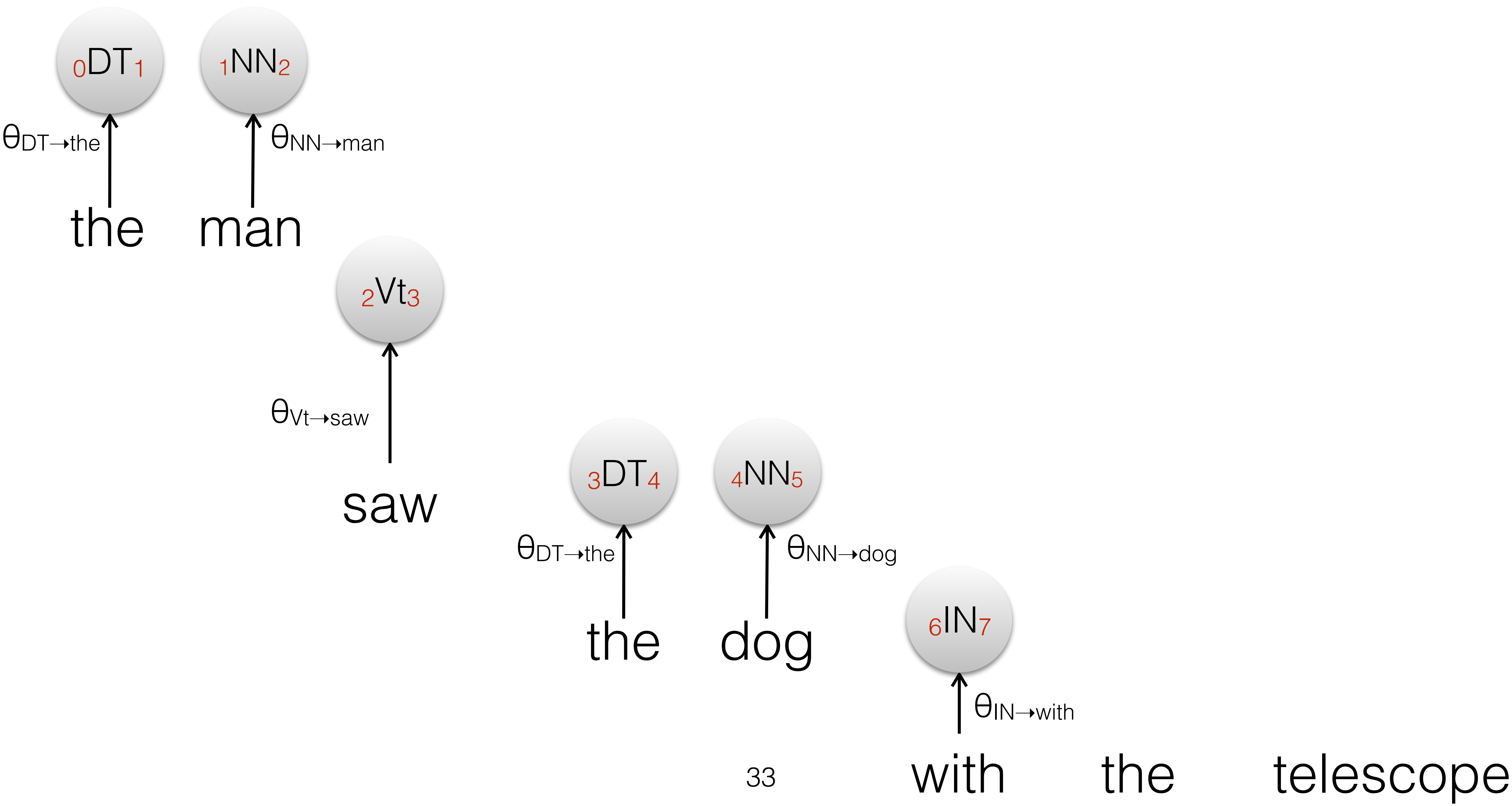


$_0DT_1$
$_1NN_2$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_2Vt_3$
$\theta_{Vt \to saw}$
saw
$_3DT_4$
$\theta_{DT \to the}$
the
dog
with
the
telescope

Ambiguity

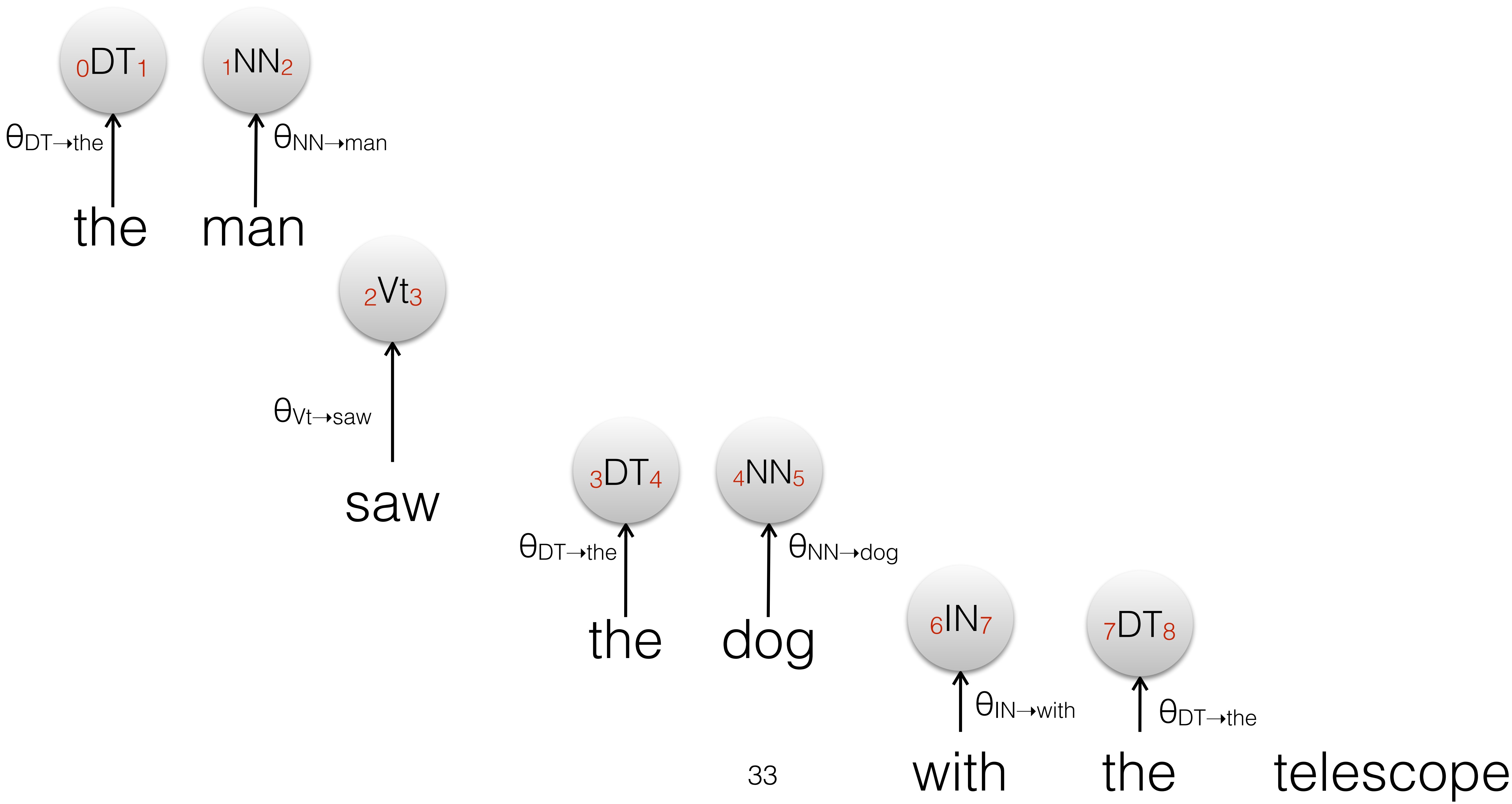

$_{0}DT_{1}$
$_{1}NN_{2}$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_{2}Vt_{3}$
$\theta_{Vt \to saw}$
saw
$_{3}DT_{4}$
$_{4}NN_{5}$
$\theta_{DT \to the}$
$\theta_{NN \to dog}$
the
dog
with
the
telescope

Ambiguity



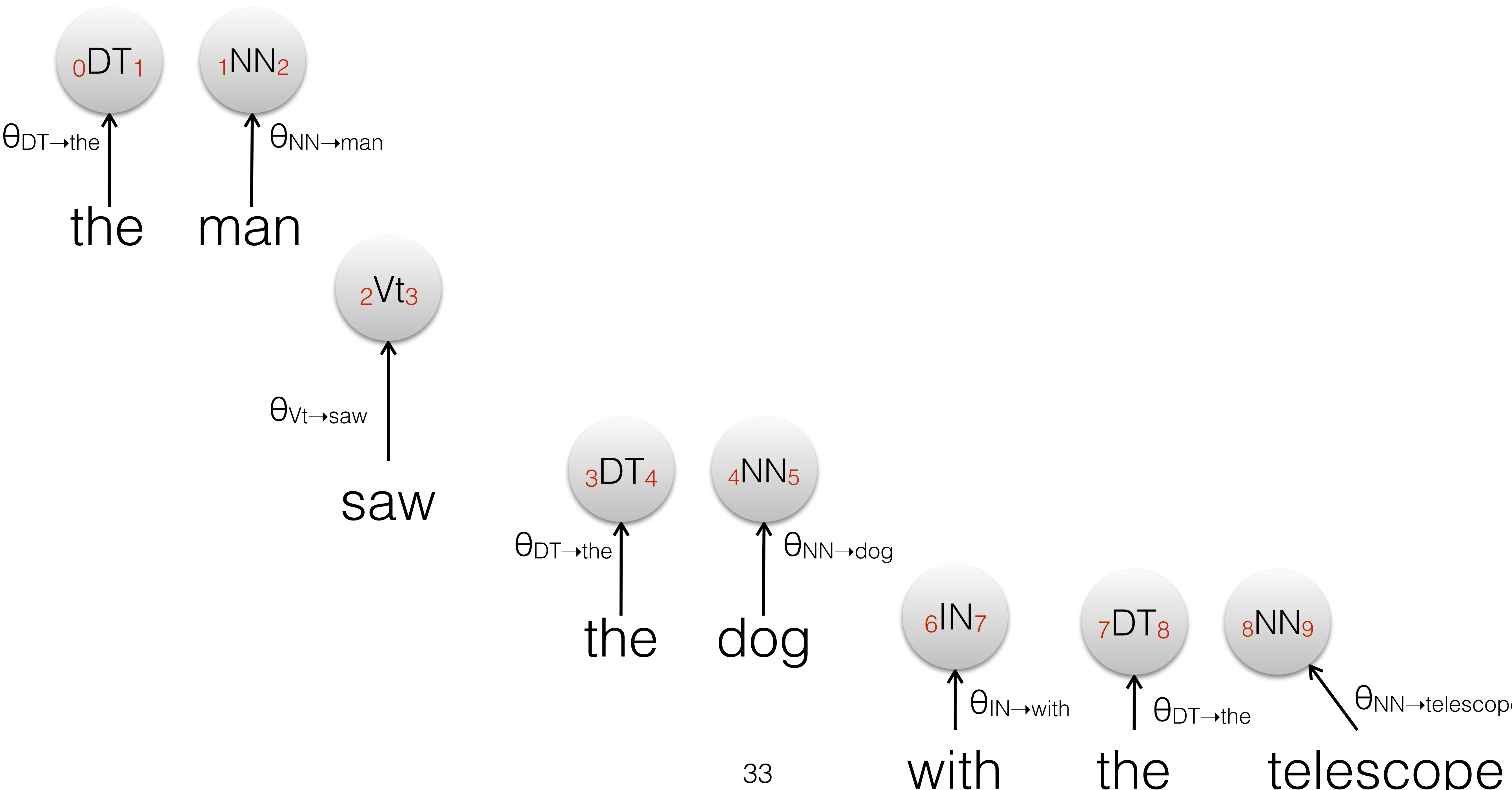
0DT1
1NN2
θDT→the
θNN→man
the
man
2Vt3
θVt→saw
saw
3DT4
4NN5
θDT→the
θNN→dog
the
dog
6IN7
θIN→with
with
the
telescope

Ambiguity



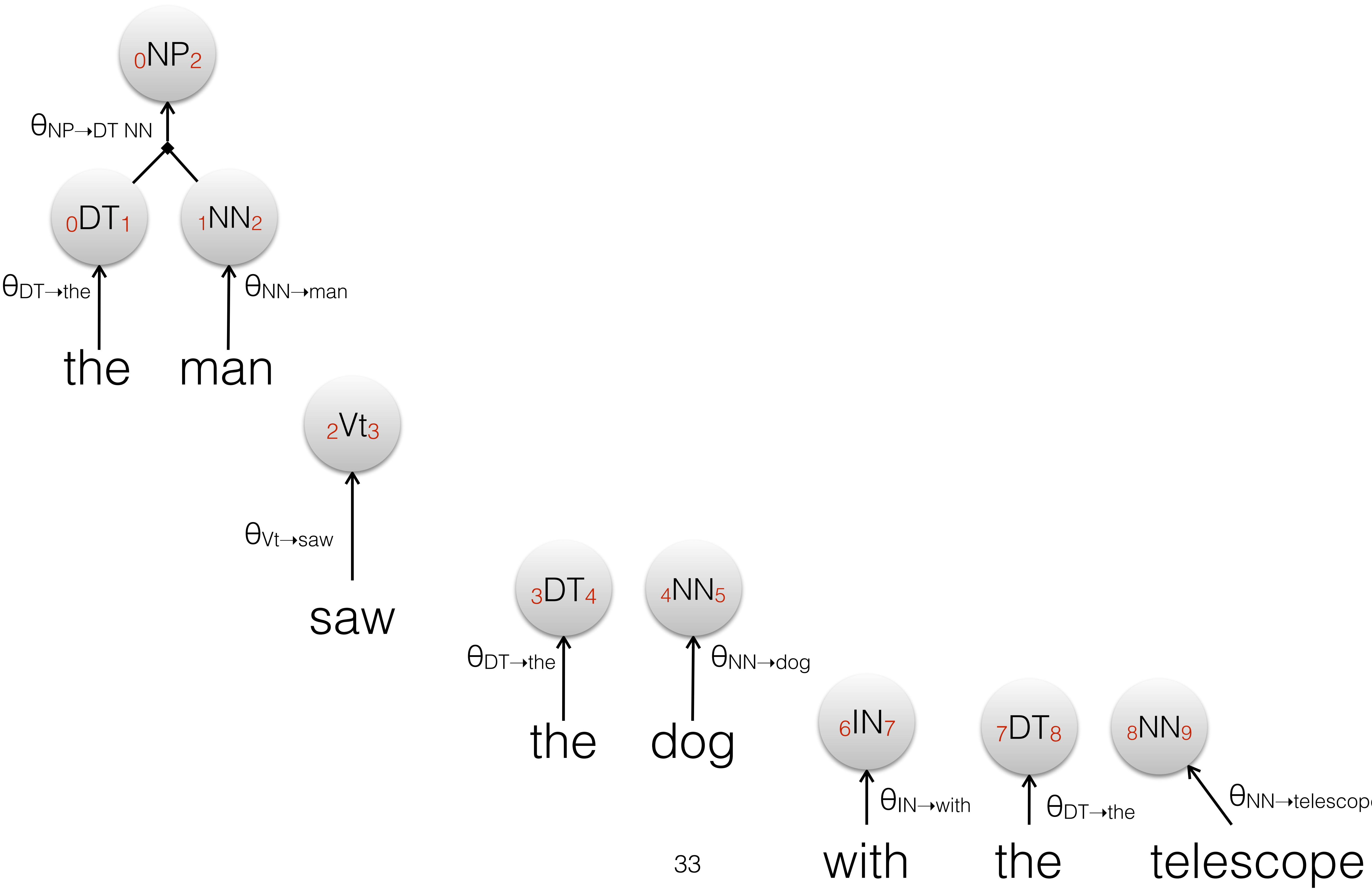
$_0DT_1$
$_1NN_2$
$\theta_{DT\to the}$
$\theta_{NN\to man}$
the
man
$_2Vt_3$
$\theta_{Vt\to saw}$
saw
$_3DT_4$
$_4NN_5$
$\theta_{DT\to the}$
$\theta_{NN\to dog}$
the
dog
$_6IN_7$
$_7DT_8$
$\theta_{IN\to with}$
$\theta_{DT\to the}$
with
the
telescope

Ambiguity



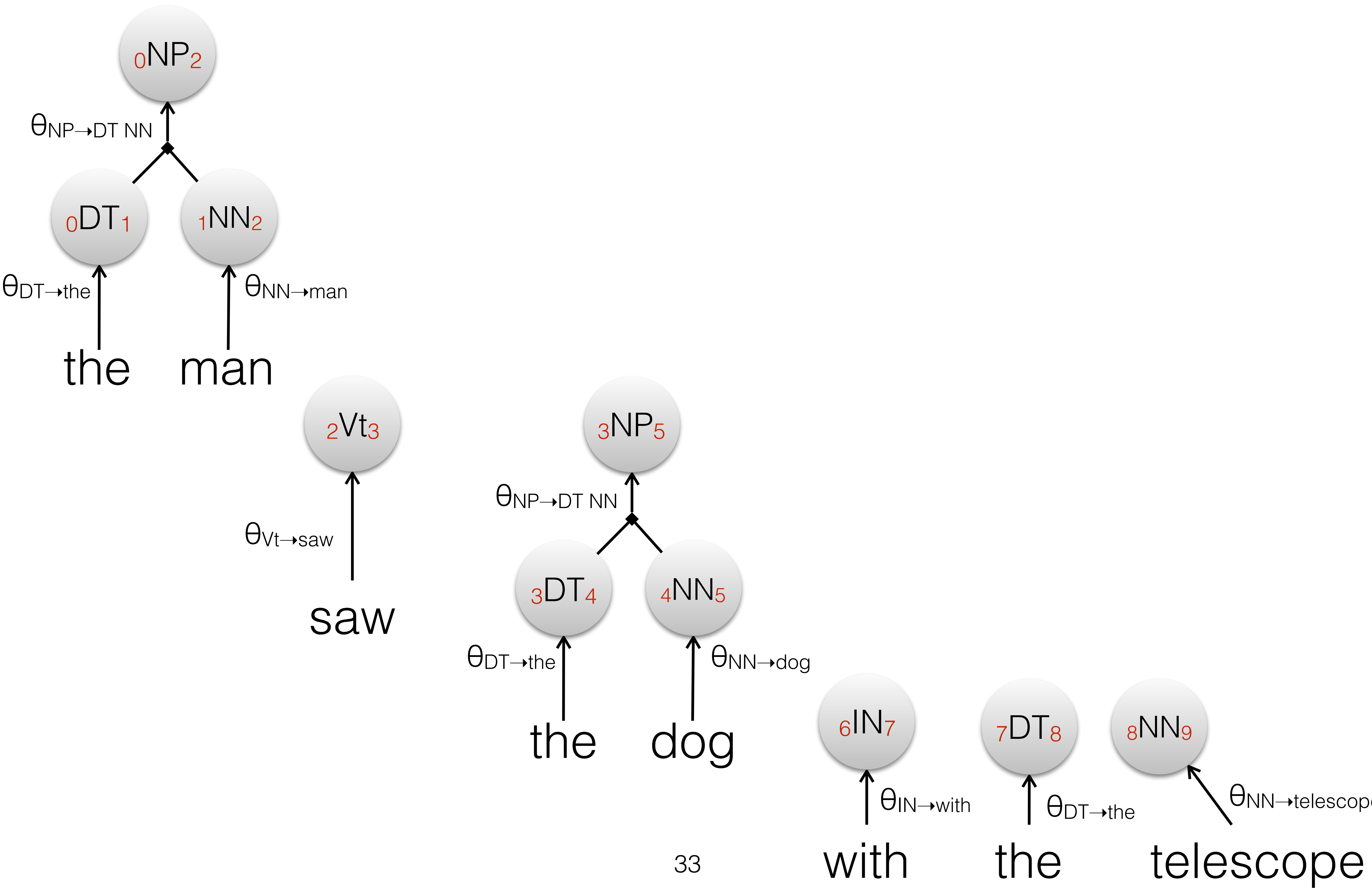
$_0DT_1$
$_1NN_2$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_2Vt_3$
$\theta_{Vt \to saw}$
saw
$_3DT_4$
$_4NN_5$
$\theta_{DT \to the}$
$\theta_{NN \to dog}$
the
dog
$_6IN_7$
$_7DT_8$
$_8NN_9$
$\theta_{IN \to with}$
$\theta_{DT \to the}$
$\theta_{NN \to telescope}$
with
the
telescope

Ambiguity



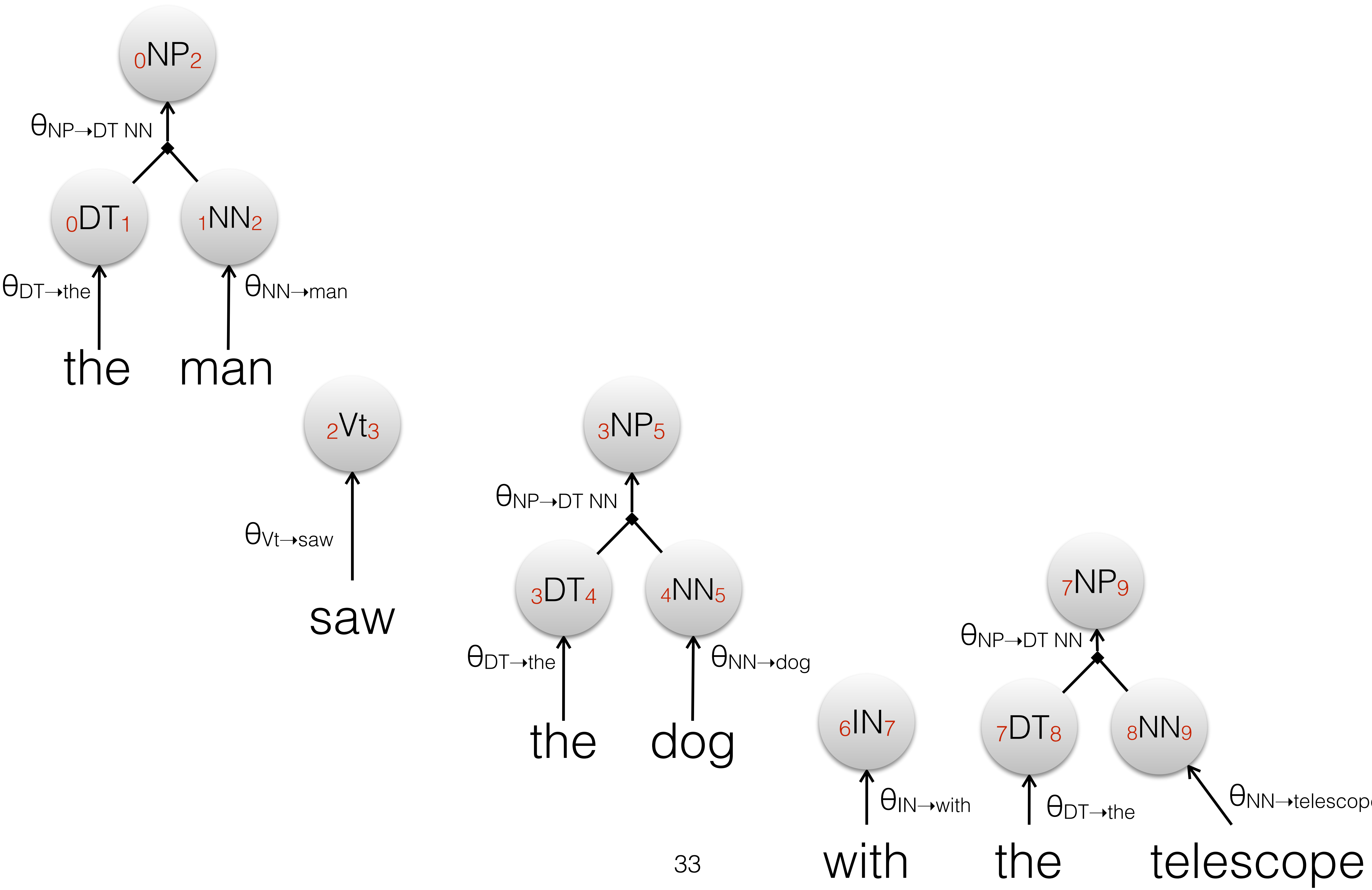
$_0NP_2$
$\theta_{NP \to DT\ NN}$
$_0DT_1$
$_1NN_2$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_2Vt_3$
$\theta_{Vt \to saw}$
saw
$_3DT_4$
$_4NN_5$
$\theta_{DT \to the}$
$\theta_{NN \to dog}$
the
dog
$_6IN_7$
$_7DT_8$
$_8NN_9$
$\theta_{IN \to with}$
$\theta_{DT \to the}$
$\theta_{NN \to telescope}$
with
the
telescope

Ambiguity

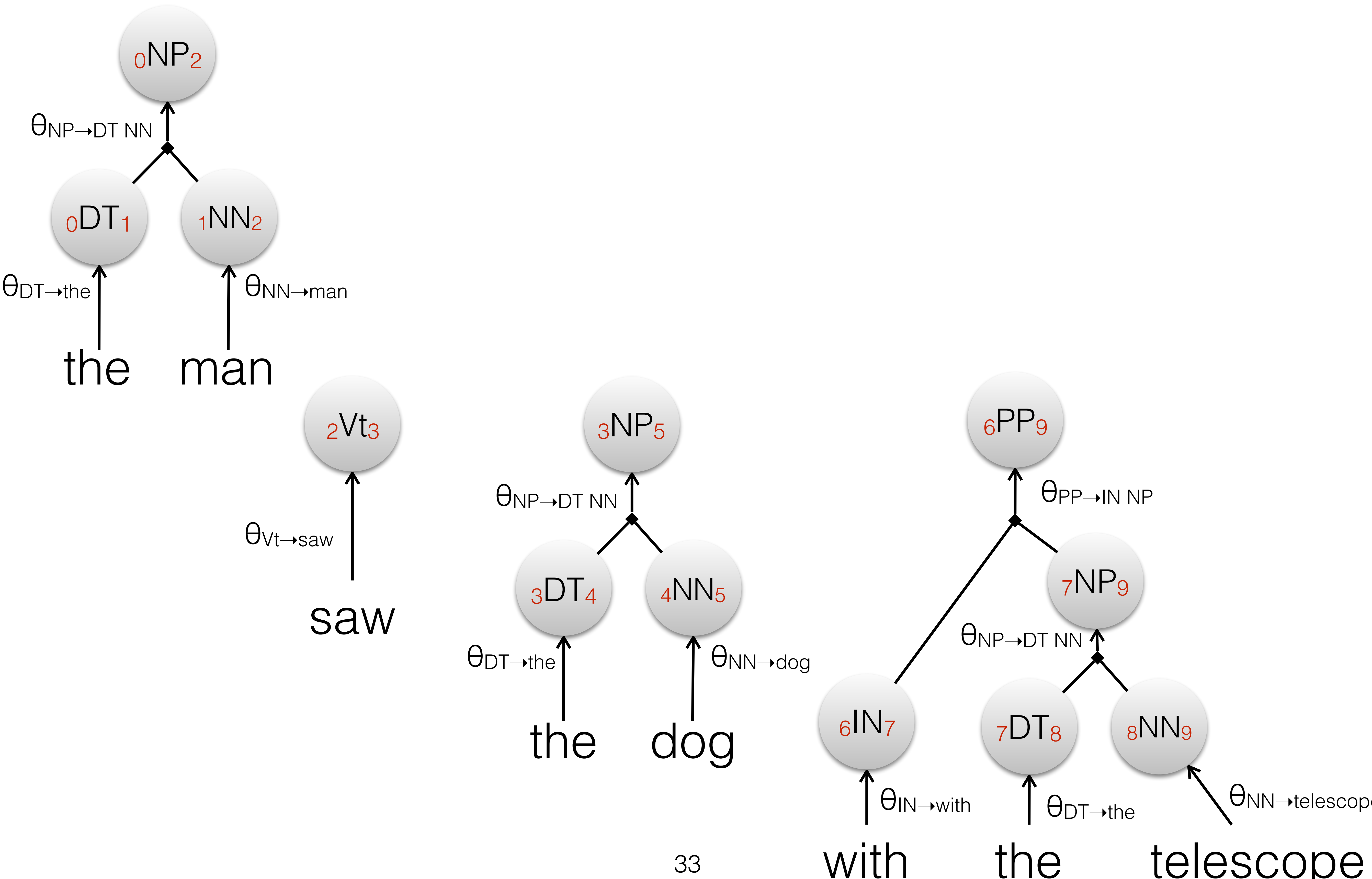


0NP2
θNP→DT NN
0DT1
1NN2
θDT→the
θNN→man
the
man
2Vt3
θVt→saw
saw
3NP5
θNP→DT NN
3DT4
4NN5
θDT→the
θNN→dog
the
dog
6IN7
7DT8
8NN9
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Ambiguity

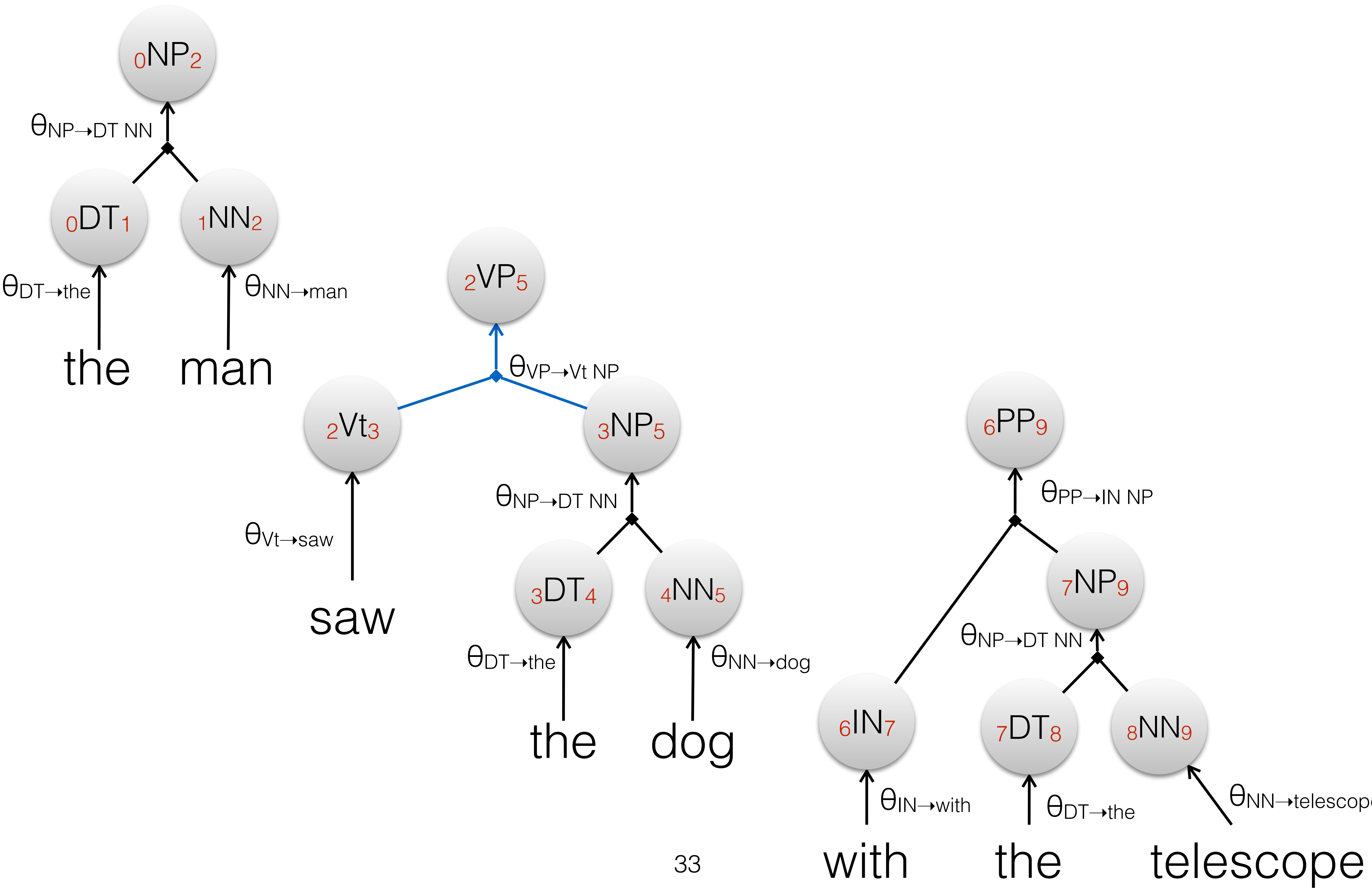

$_0NP_2$
$\theta_{NP \to DT\ NN}$
$_0DT_1$
$_1NN_2$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_2Vt_3$
$\theta_{Vt \to saw}$
saw
$_3NP_5$
$\theta_{NP \to DT\ NN}$
$_3DT_4$
$_4NN_5$
$\theta_{DT \to the}$
$\theta_{NN \to dog}$
the
dog
$_6IN_7$
$\theta_{IN \to with}$
with
$_7NP_9$
$\theta_{NP \to DT\ NN}$
$_7DT_8$
$_8NN_9$
$\theta_{DT \to the}$
$\theta_{NN \to telescope}$
the
telescope

Ambiguity



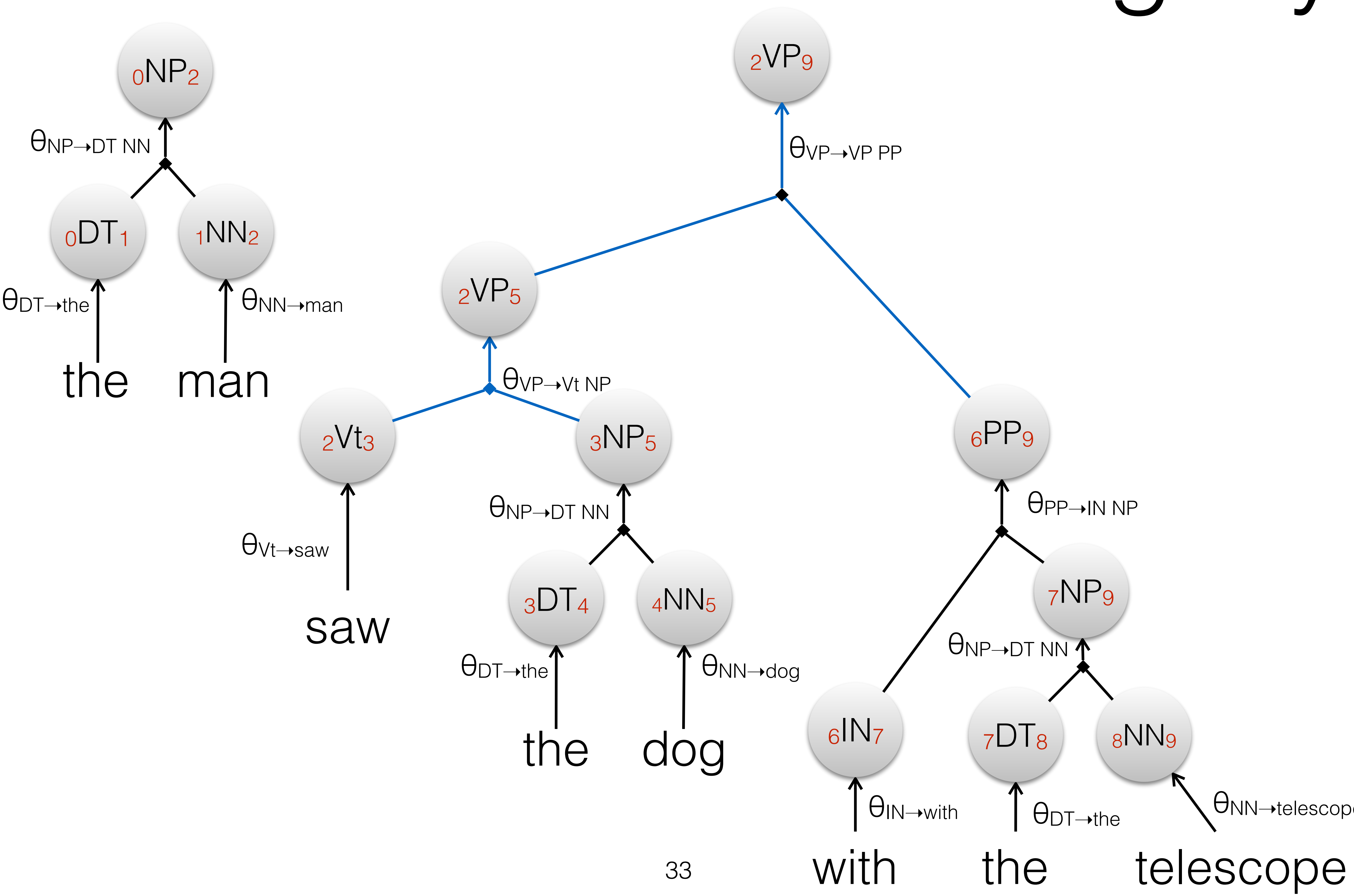
0NP2
θNP→DT NN
0DT1
1NN2
θDT→the
θNN→man
the
man
2Vt3
θVt→saw
saw
3NP5
θNP→DT NN
3DT4
4NN5
θDT→the
θNN→dog
the
dog
6PP9
θPP→IN NP
7NP9
θNP→DT NN
6IN7
7DT8
8NN9
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Ambiguity



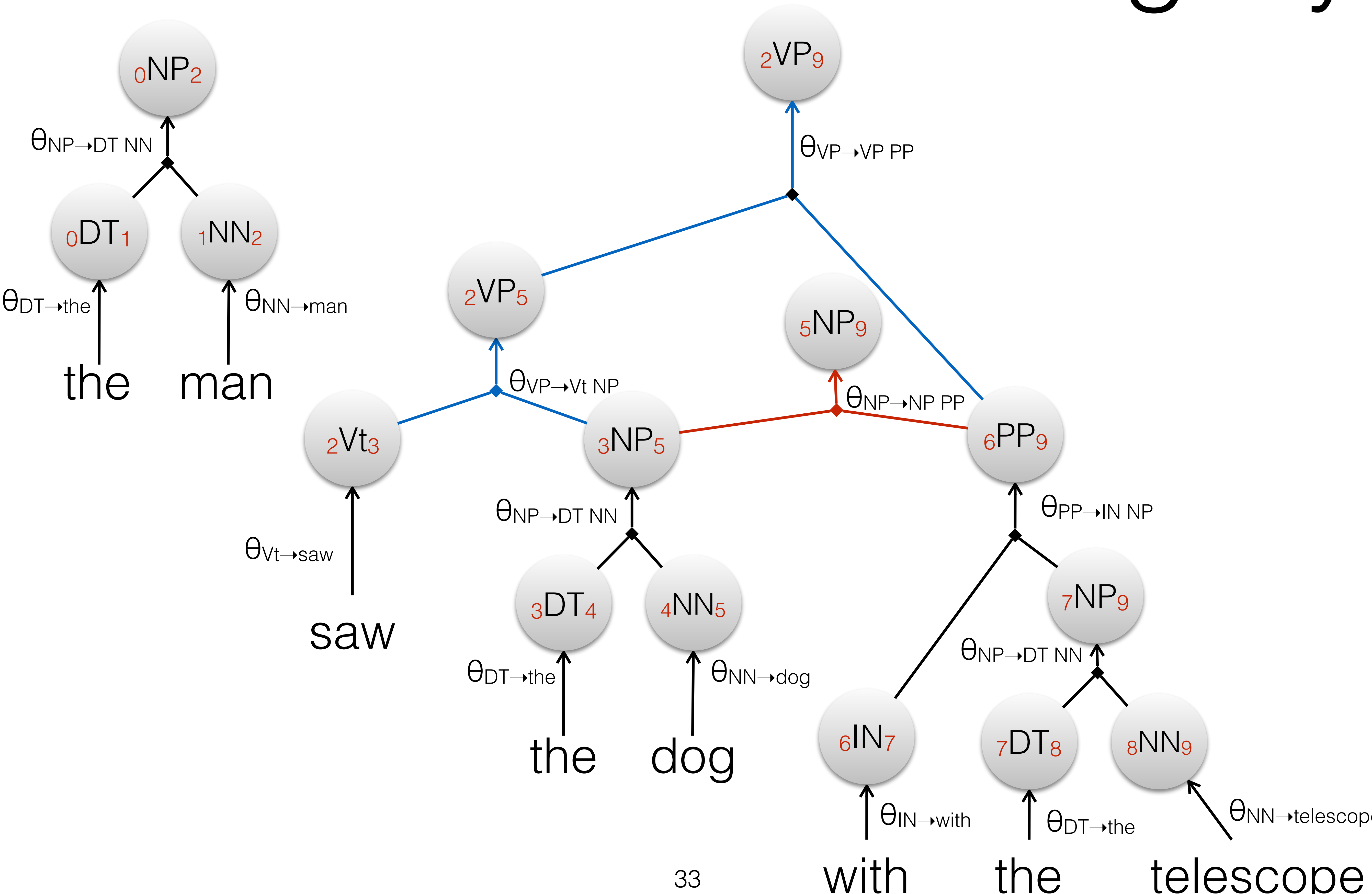
$_0$NP$_2$
θNP→DT NN
$_0$DT$_1$
$_1$NN$_2$
θDT→the
θNN→man
the
man
$_2$VP$_5$
θVP→Vt NP
$_2$Vt$_3$
$_3$NP$_5$
θNP→DT NN
θVt→saw
saw
$_3$DT$_4$
$_4$NN$_5$
θDT→the
θNN→dog
the
dog
$_6$PP$_9$
θPP→IN NP
$_7$NP$_9$
θNP→DT NN
$_6$IN$_7$
$_7$DT$_8$
$_8$NN$_9$
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Ambiguity



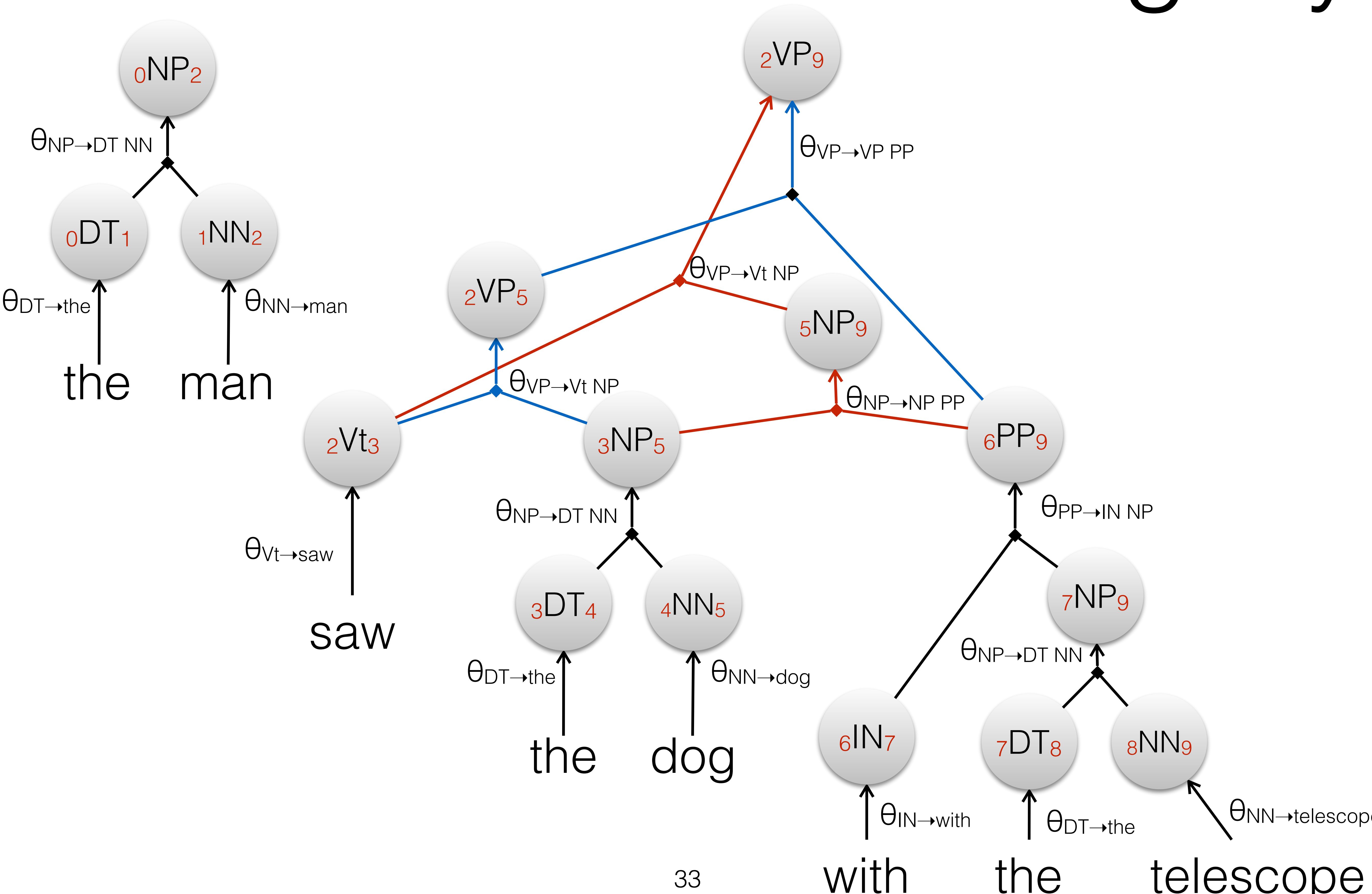
0NP2
θNP→DT NN
0DT1
1NN2
θDT→the
θNN→man
the
man
2VP9
θVP→VP PP
2VP5
θVP→Vt NP
2Vt3
3NP5
6PP9
θVt→saw
saw
θNP→DT NN
3DT4
4NN5
θDT→the
θNN→dog
the
dog
θPP→IN NP
7NP9
θNP→DT NN
6IN7
7DT8
8NN9
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Ambiguity



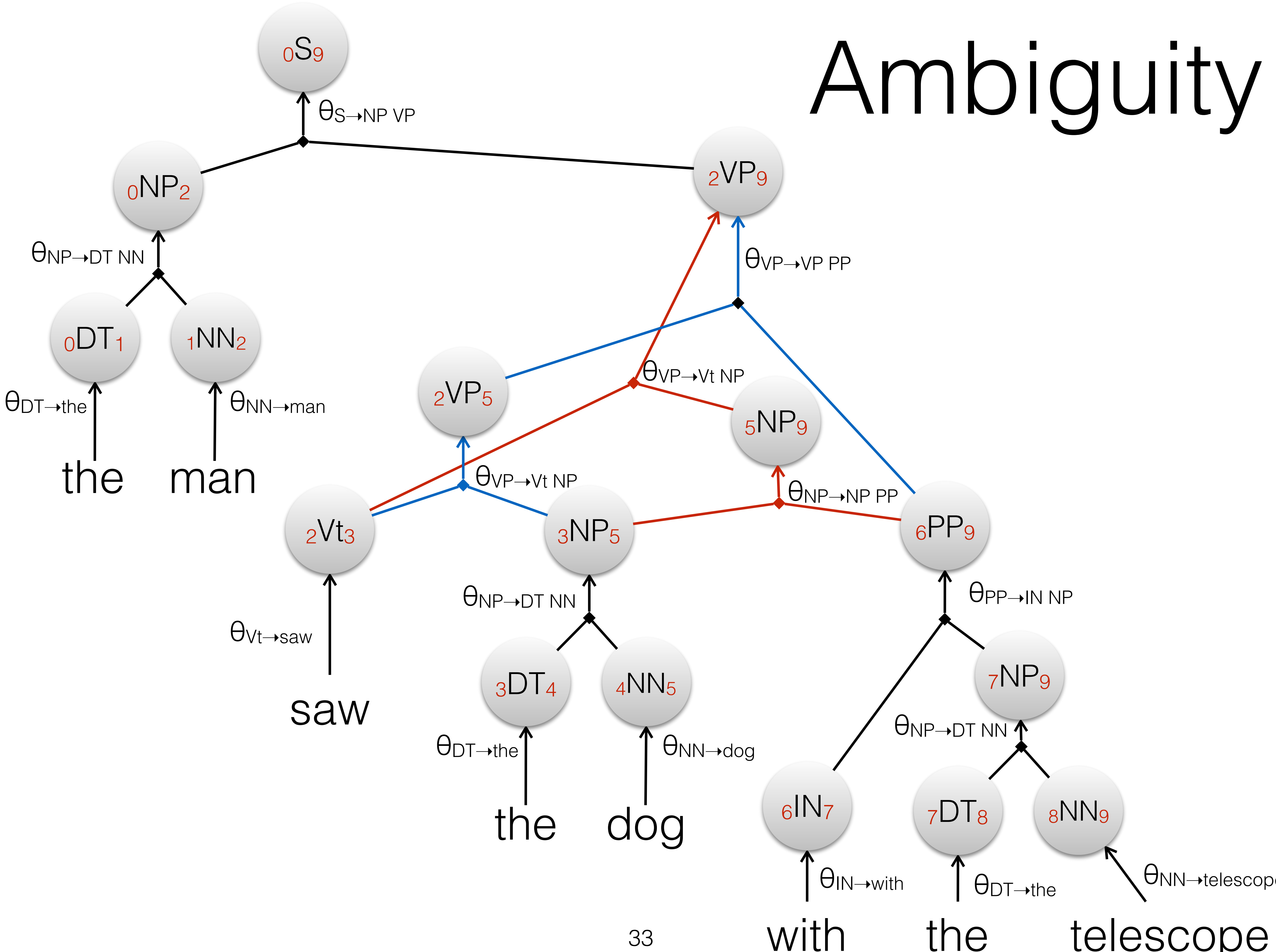
$_0NP_2$
$\theta_{NP \to DT\ NN}$
$_0DT_1$
$_1NN_2$
$\theta_{DT \to the}$
$\theta_{NN \to man}$
the
man
$_2VP_9$
$\theta_{VP \to VP\ PP}$
$_2VP_5$
$_5NP_9$
$\theta_{VP \to Vt\ NP}$
$\theta_{NP \to NP\ PP}$
$_2Vt_3$
$_3NP_5$
$_6PP_9$
$\theta_{NP \to DT\ NN}$
$\theta_{PP \to IN\ NP}$
$\theta_{Vt \to saw}$
saw
$_3DT_4$
$_4NN_5$
$_7NP_9$
$\theta_{DT \to the}$
$\theta_{NN \to dog}$
$\theta_{NP \to DT\ NN}$
the
dog
$_6IN_7$
$_7DT_8$
$_8NN_9$
$\theta_{IN \to with}$
$\theta_{DT \to the}$
$\theta_{NN \to telescope}$
with
the
telescope

Ambiguity



0NP2
θNP→DT NN
0DT1
1NN2
θDT→the
θNN→man
the
man
2VP9
θVP→VP PP
2VP5
θVP→Vt NP
5NP9
θVP→Vt NP
2Vt3
3NP5
θNP→NP PP
6PP9
θNP→DT NN
θPP→IN NP
θVt→saw
saw
3DT4
4NN5
7NP9
θDT→the
θNN→dog
θNP→DT NN
the
dog
6IN7
7DT8
8NN9
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Ambiguity



0S9
θS→NP VP
0NP2
2VP9
θNP→DT NN
θVP→VP PP
0DT1
1NN2
θDT→the
θNN→man
the
man
2VP5
θVP→Vt NP
5NP9
θVP→Vt NP
θNP→NP PP
2Vt3
3NP5
6PP9
θNP→DT NN
θPP→IN NP
θVt→saw
saw
3DT4
4NN5
7NP9
θNP→DT NN
θDT→the
θNN→dog
the
dog
6IN7
7DT8
8NN9
θIN→with
θDT→the
θNN→telescope
with
the
telescope

Complexity

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- $O(n^3)$ annotated rules

Bibliography

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