Limits of Computation

7 - A universal program (Self-interpreter)

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So far...

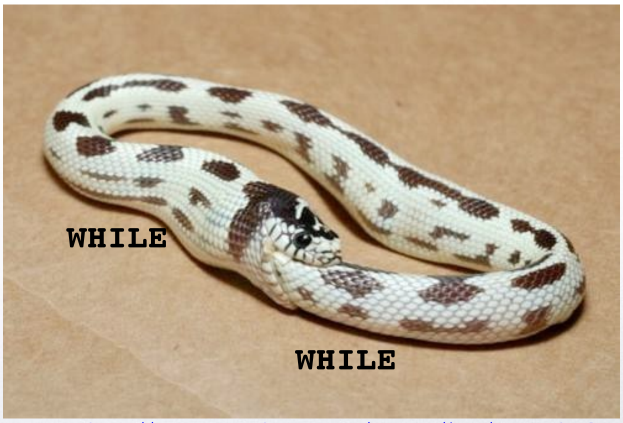
- ... we have learned the `WHILE`-language…
- ...that we have chosen to represent our notion of computation (to write “effective procedures”).
- We learned how to represent programs-as-data...
- ...so now we **can write interpreters**.

Eating your own tail?

- We look at a special form of interpreter:

THIS TIME

- **self-interpreter**
  - WHILE-interpreter in WHILE
  - and first an WH^{1}LE-interpreter in WHILE
- a very important concept for computability theory *(used later)*



http://www.strangedangers.com/content/item/158424.html

Compare to TMs

- Turing defined a "universal Turing machine" *U*
- that can take TM program description *D* and a word *W* as input on its tape
- and simulate the run of TM *D* with given input *W*
- *so U is a TM program which is an interpreter for TM programs*

a self-interpreter in TM

let's use WHILE instead!

Use of self-interpreter?

- in practice:
  "cheap" way to extend your programming language with extra features (interpret them in smaller language)
- in computability theory:
  we will explain this soon. Stay tuned!

First consider `WH`^{1}`LE`

- ...is like `WHILE`...
- ...but programs can only have ***one*** variable.
- simpler "memory management"
- Can we solve fewer problems in language `WH`^{1}`LE` than in `WHILE`?

Interpret WH^{1}LE in WHILE

- Since it is simpler, we first look at an interpreter of WH^{1}LE written in WHILE.
- Then we generalise to arbitrarily many variables and obtain a WHILE-interpreter in WHILE.

Recipe

Tree Traversal of ASTs

(with intermediate results)

NO RECURSION !

initialise tree and value stack to be empty
push tree (to be traversed) on tree stack
while tree stack not empty
  pop a tree t from tree stack
  if t is just an opcode o with arity n // a marker
  then pop n results $r_1,\ldots,r_n$ from value stack
    $r := o(r_1,\ldots,r_n)$ // compute intermediate result
    push r on value stack
  else // t proper tree
  if t's opcode has n arguments
  then push t's opcode on tree stack // (as marker!)
    push n subtrees of t on tree stack
  else // o is leaf
    compute result and push on value stack

order of arguments important

WHILE-interpreter

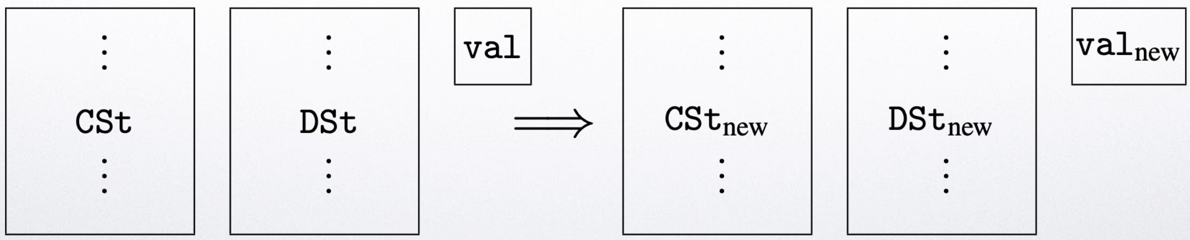
```
read PD  {                        (* input is a list [P,D] *)
  P := hd PD ;                    (* P = [X,B,X] *)
  D := hd tl PD;                  (* D input data *)
  B := hd tl P;                   (* B is program block *)
  CSt := B;                       (* CSt is code stack  *)
                                  (* initially  commands of B *)
  DSt := nil;                     (* DSt is computation stack for *)
                                 (* intermediate results *)
  val := D;                       (* D is initial value of variable *)
  state := [ CSt, DSt, val ];
                                   (* wrap up state for STEP macro *)
  while CSt {                     (* main loop for interpretation *)
    state := <STEP> state;(* loop body macro *)
    CSt := hd state               (* get command stack *)
    }
  val := hd tl tl state           (* get final value of variable *)
}
write val                         (* return value of the one variable *)
```

CSt is code stack (code in list format),
DSt is Stack of intermediate values,
val contains value D of the one variable

Step Macro

performs tree traversal based on `CSt`, `DSt`, and `val`.

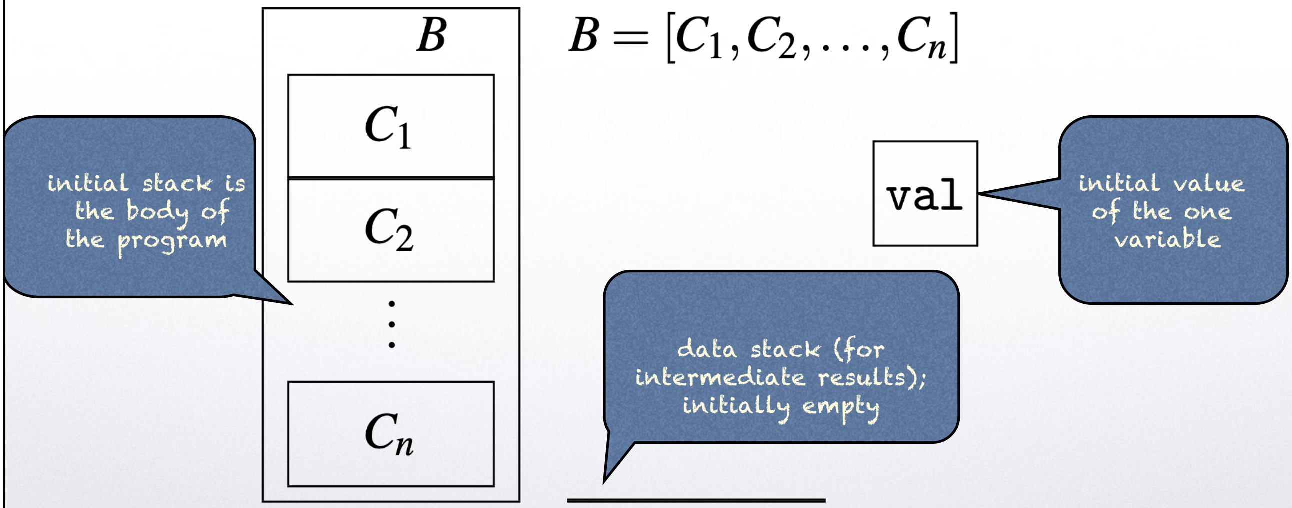
$$[\mathtt{CSt}, \mathtt{DSt}, \mathtt{val}] \Rightarrow [\mathtt{CSt}_{\mathrm{new}}, \mathtt{DSt}_{\mathrm{new}}, \mathtt{val}_{\mathrm{new}}]$$



Initial set-up

```
state := [ CSt, DSt, val ];
```

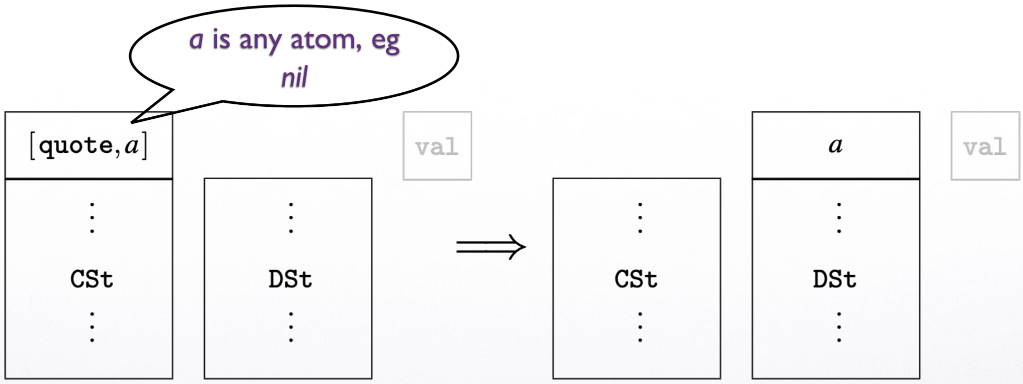
$$B = [C_1, C_2, \ldots, C_n]$$



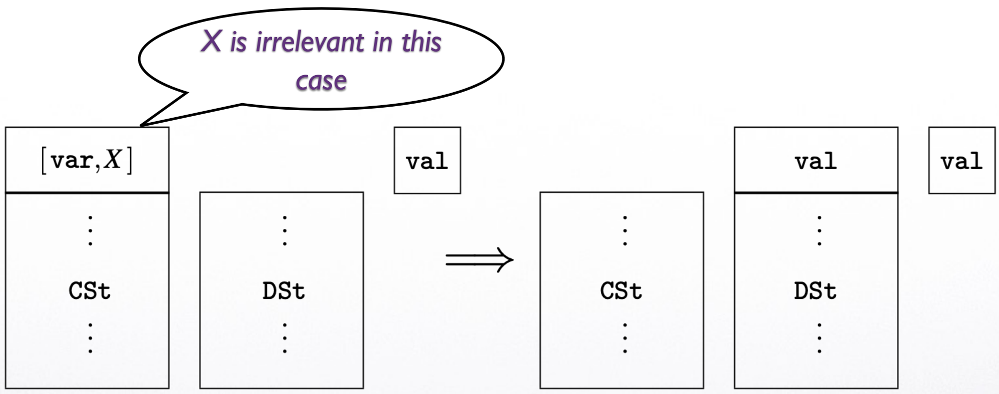
AST Leaves

(expressions without arguments)

Atoms



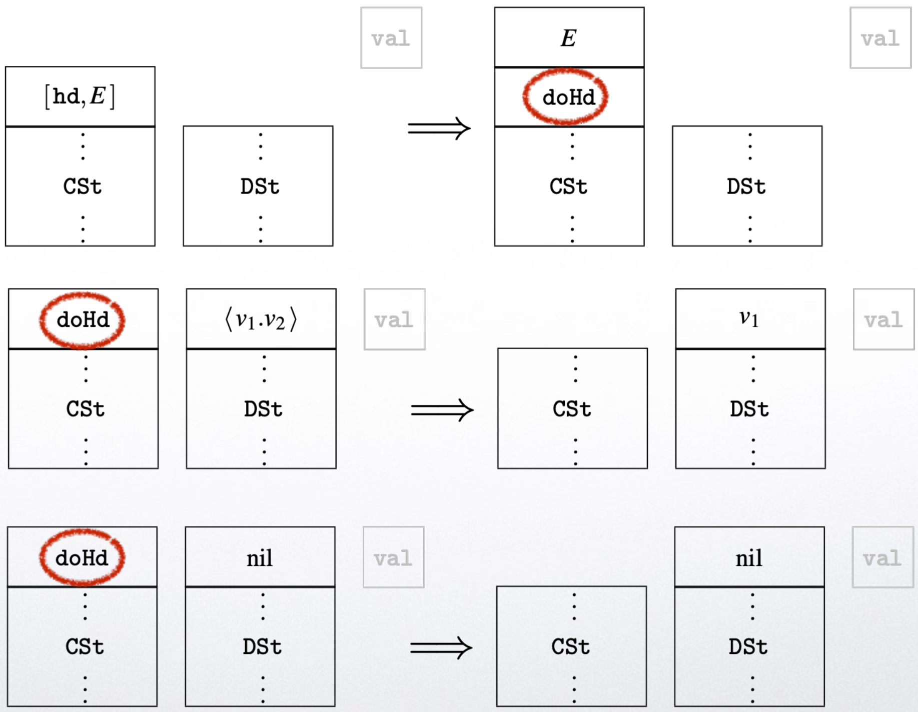
Variable



Compound Expressions

(unary and binary)

hd



(similarly for `tl`)

additional atoms used as mnemonic markers: what needs to be done when this marker is popped off the stack

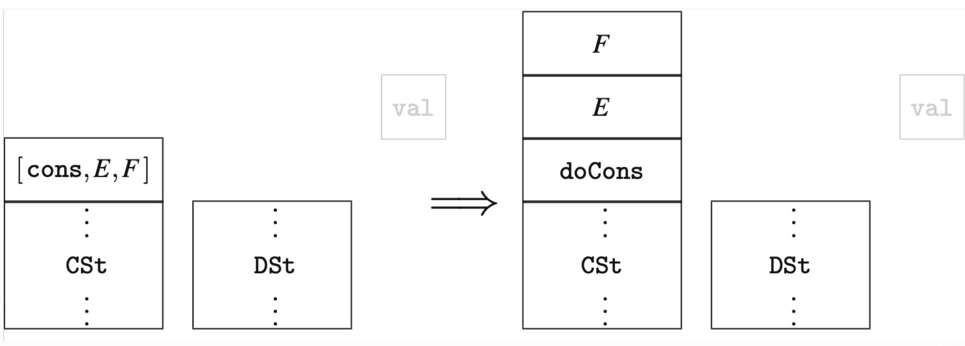
Auxiliary atoms

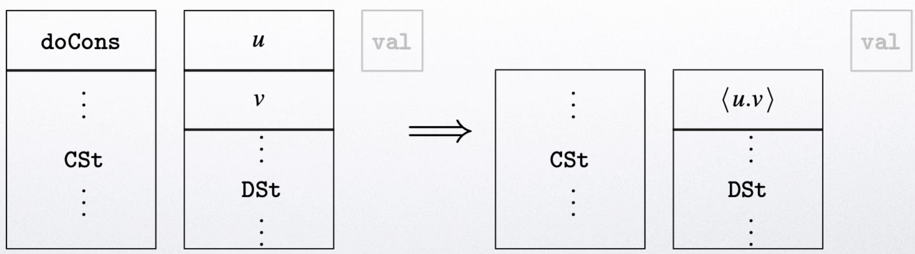
We now add new (encoded) atoms to $\mathbb{D}$

`doHd, doTl, doCons, doAsgn, doIf, doWhile`

Use: push on stack to indicate operation still to be do-ne

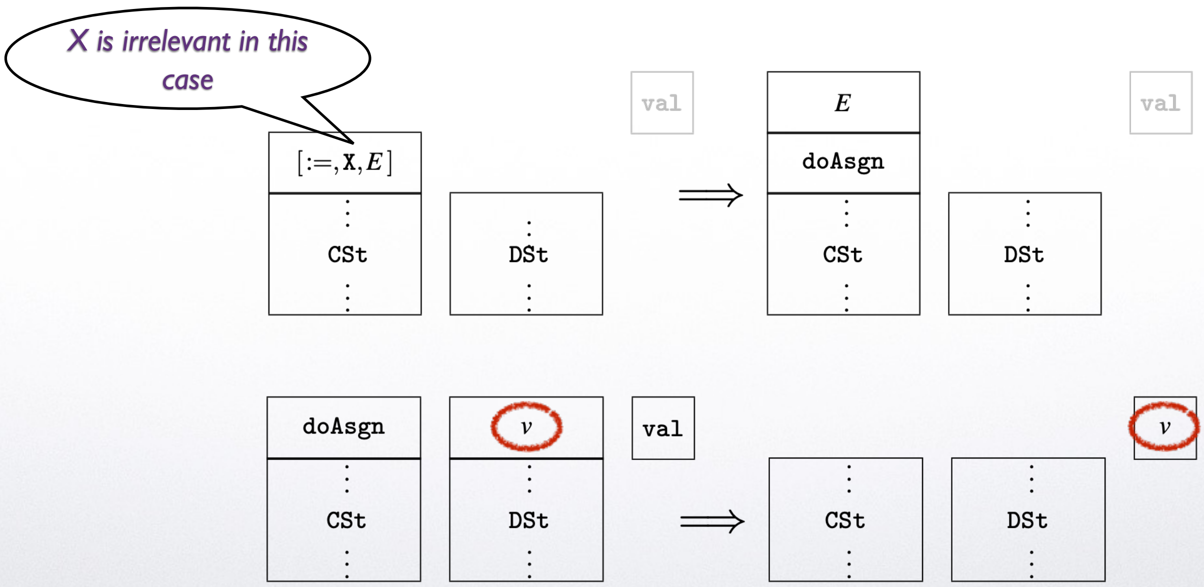
cons



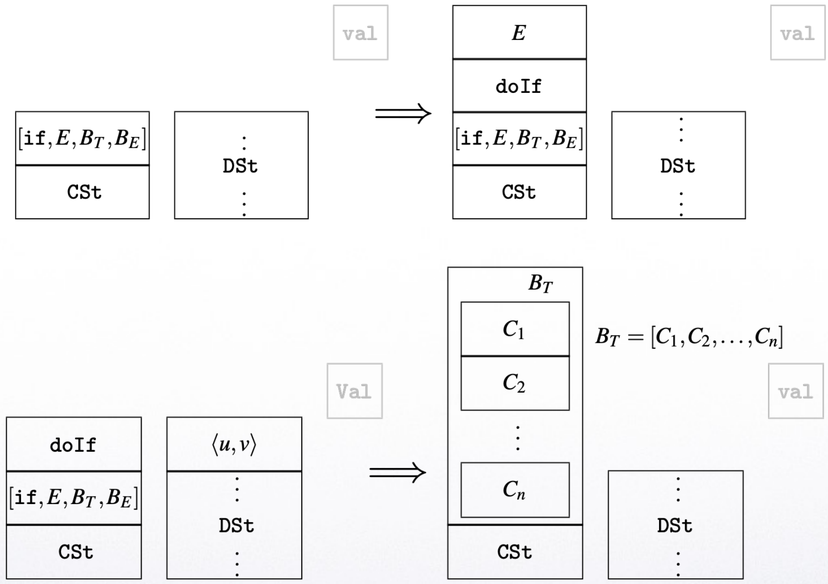


Commands

Assignment

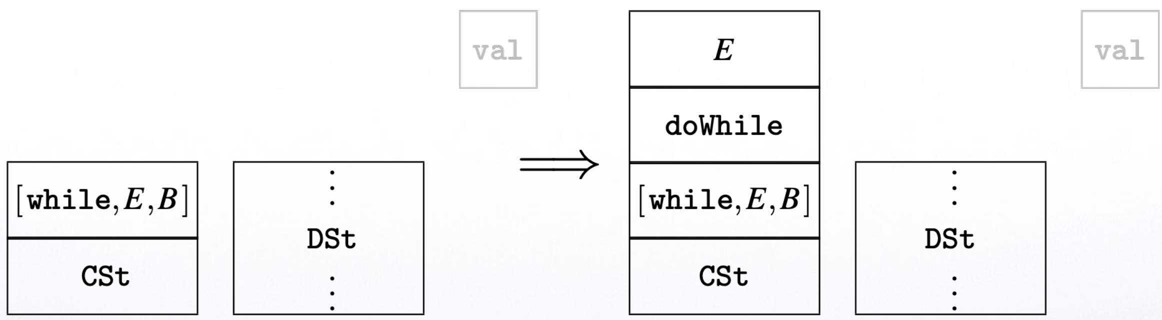


if

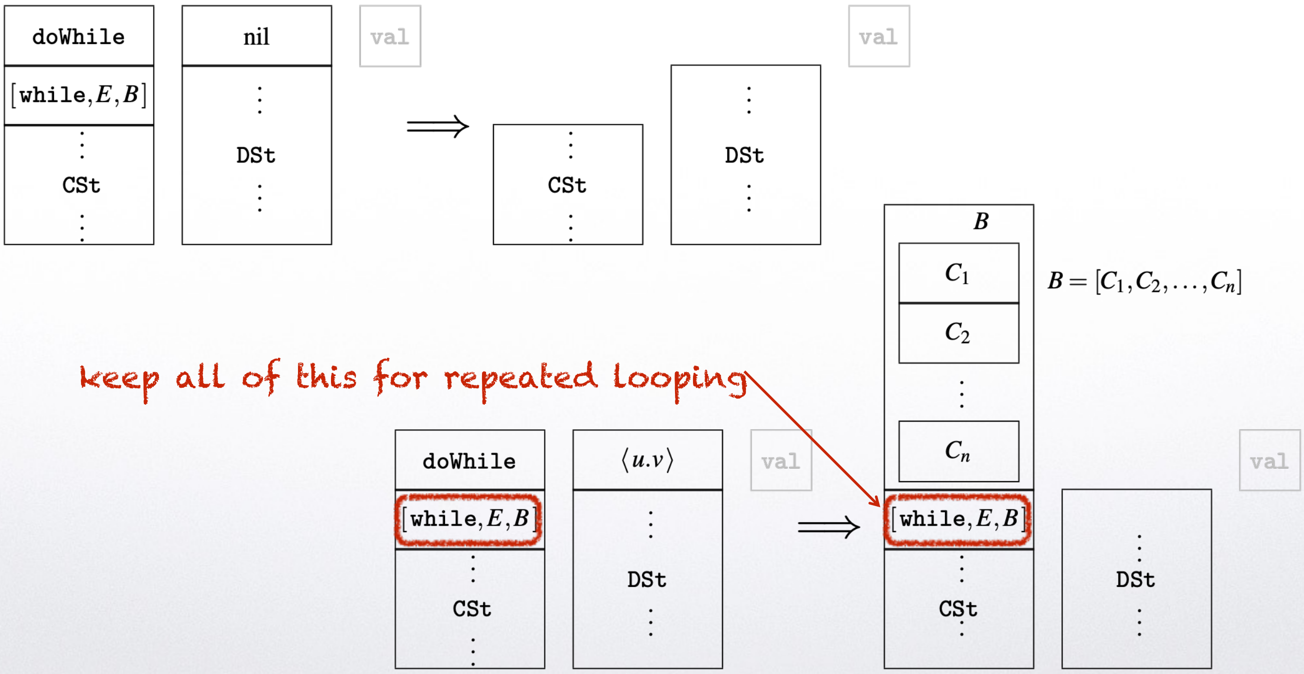


Analogously, if top element of `DSt` is *nil*, B_E is pushed on `CSt`

while

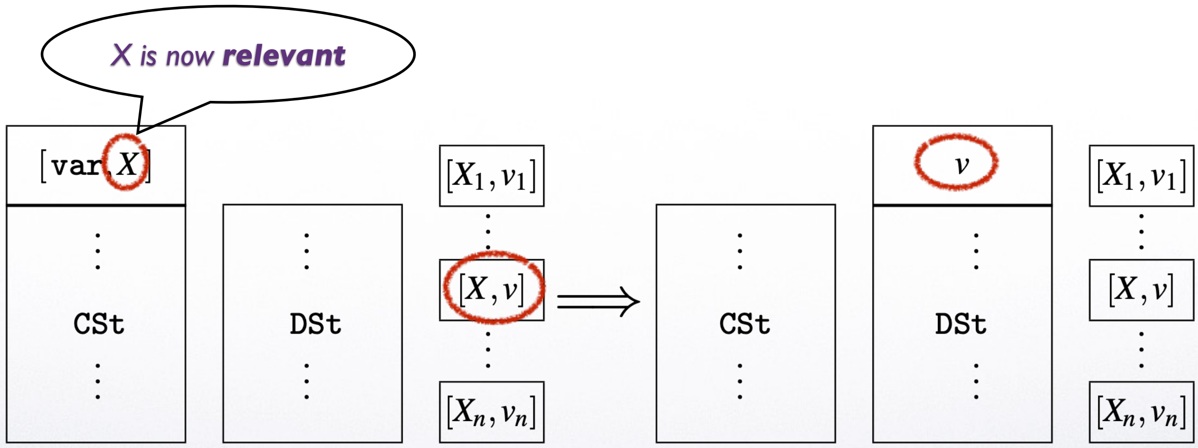


doWhile

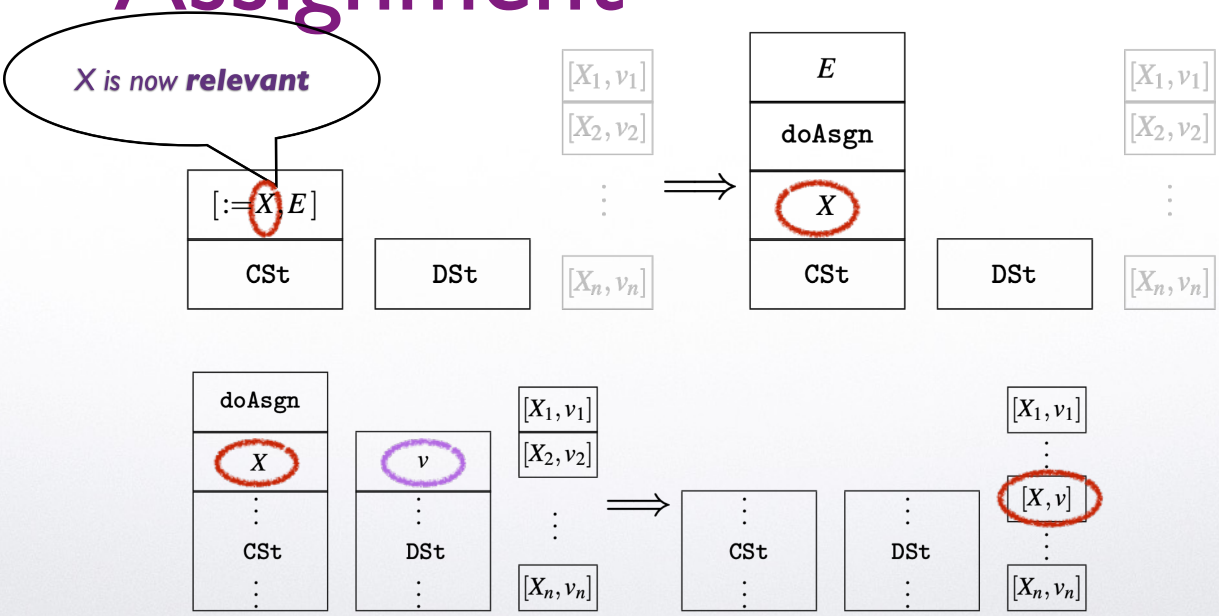


Changes to interpret WHILE

Variable lookup



Assignment



WHILE-interpreter

```
read PD {                       (* input i
  P := hd PD ;                  (*
  D := hd tl PD;
  X := hd P;                                  name *)
  Y := hd t                                 put var name *)
  B                                         is program code block *)
  C                             (* CSt is code stack  *)
                                (* initially contains only B *)
  DS                            (* DSt is data stack for *)
                                (* intermediate results *)
  bind := [ X, D ];
  St  := [ bind ];              (* initialise store *)
  state := [ CSt, DSt, St ];    (* wrap state for STEP macro *)
  while CSt {                   (* main loop for interpretation *)
    state := <STEPn> state;     (* loop body macro  *)
    CSt := hd state             (* get command stack *)
    };
  St  := hd tl tl state;        (* get final store *)
  arg := [ Y, St ];             (* wrap argument for lookup *)
  Out := <lookup> arg           (* lookup output variable value *)
 }
write Out                       (* return value of result variable *)
```

this part is as before but we now keep X and Y

CSt is code stack (code in list format),
DSt is Stack of intermediate values.
St is the the list of variable bindings

Coding the macros

- The update and lookup macro are available from Canvas, as is the main interpreter loop and the STEPn macro.
- The STEP macro for WH^{1}LE we will complete in the exercises.

END

Next time: Our first non-computable problem