Limits of Computation

6 - Programs as Data Objects

Bernhard Reus

So far...

- "effective procedure" = `WHILE`-program
- introduced `WHILE`-language with binary tree data type ...
- ... that can also be viewed as a type of (arbitrary deeply) nested lists
- and extended `WHILE` for convenience

WHILE-programs as lists

THIS TIME

- We show how WHILE-programs can be ***data objects*** usable in another WHILE-program

```
[0,
 [[:=,1,[quote,nil]],
  [while,[var,0],
      [  [:=,1,[cons,[hd,[var,0]],[var,1]]],
         [:=,0,[tl,[var,0]]]
      ]
 ]],
 1]
```

A WHILE-program abstract syntax tree encoded as list

Programs as Input or Output

- Compiler
  program transformer which takes a program and translates it into an *equivalent* program, most likely in another language;
- Interpreter
  takes a program *and* its input data, and returns the result of applying the program to that input.
- Program Specialiser
  takes a *program with two inputs and* one data for one of the inputs and *partially evaluates* the program with the one given data producing a new program with one input only (more on that later).

Programming Languages

our notion, formally

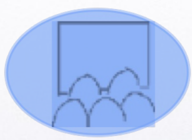
Definition A *programming language* $\mathtt{L}$ consists of

1. two sets: $\mathtt{L}$-programs (the set of $\mathtt{L}$-programs) and $\mathtt{L}$-data (the set of data values described by the datatype used by this language) .
2. A function $[\![\_]\!]^{\mathtt{L}} : \mathtt{L}\text{-programs} \to (\mathtt{L}\text{-data} \to \mathtt{L}\text{-data}_{\bot})$ which maps $\mathtt{L}$-programs into their semantic behaviour, namely a partial function mapping inputs to outputs, which are both in $\mathtt{L}$-data.

PL with Pairing

Definition A programming language $\mathtt{L}$ defined as above *has pairing* if its data type, $\mathtt{L}$-data, permits the encoding of pairs. For a general (unknown) language that has pairing we denote pairs (a,b), i.e. using parenthesis and a comma.

Does `WHILE` have pairing?



Answer: Yes, use [a,b] or <a.b>

PL with Programs As Data

Definition A programming language L defined as above *has programs as data* if its data type, L-data, permits the encoding of L-programs. For a general (unknown) language that has programs as data the encoding of a program p is denoted $\ulcorner p \urcorner$.

The purpose of this session is
to show that WHILE has programs as data.

Programs as Data

- If language L has "***programs as data***" we can write compilers, interpreters, and specialisers in L.
- We want WHILE to have "*programs as data*".
- Thus we need a representation of WHILE programs as binary tree
- It is natural to use ***abstract syntax trees***

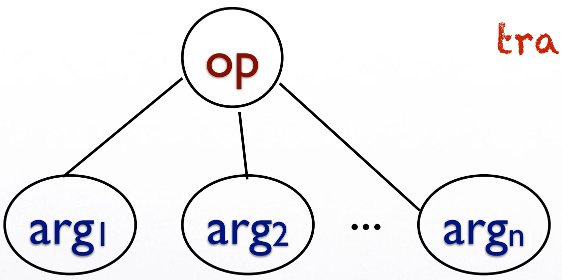
Interpreter

our notion, formally

Definition Assume S has programs as data, S-data $\subseteq$ L-data and L has pairing. An interpreter `int` for a language S written in L must fulfil the following equation for any given S-program p and $d \in$ S-data:

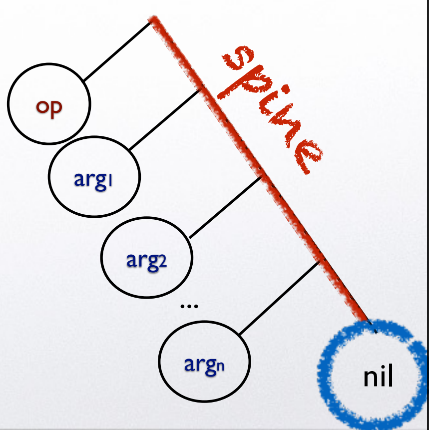
$$\boxed{[\![\texttt{int}]\!]^{\mathtt{L}}(\ulcorner p \urcorner, d) = [\![p]\!]^{\mathtt{S}}(d)}$$

Abstract Syntax Trees as lists

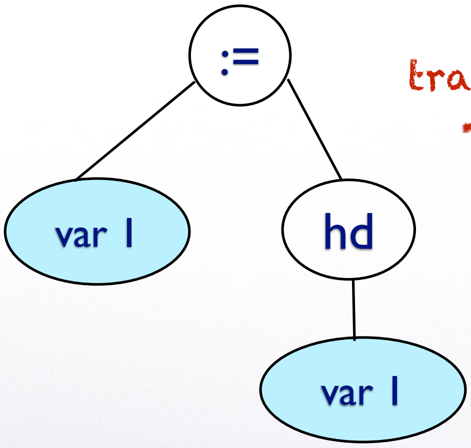


translates to

[op, arg1, arg2,..., argn]

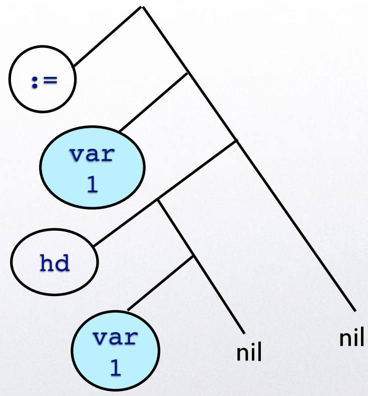


AST as list `Y:= hd Y` (Y is 1st variable)



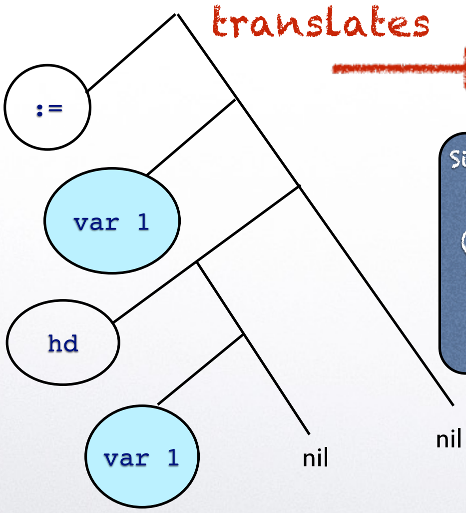
translates to

`[:=,var1,[hd,var1]]`



var 1 needs to be unfolded as list too, see next slide

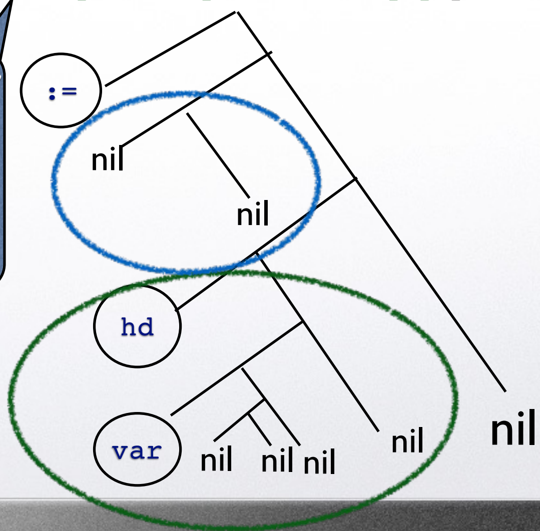
AST as list `Y:= hd Y` (Y is var 1)



translates to

`[:=,1,[hd,[var,1]]]`

Simplification: we do only need to store the variable name (i.e. number), as we can only assign to variables



What to do with var etc?

```
[:=,1,[hd,[var,1]]]
```

These are not yet trees/lists:

:= var hd

Answer: either introduce them as *additional atoms* or *encode them* (uniquely) as numbers.

Programs as data in `WHILE`

- We are now in a position to define more exactly how the list encoding of abstract syntax trees work.
- Lists are themselves encoded as binary trees.
- Let's go:

WHILE programs in $\mathbb{D}$

$$\ulcorner \texttt{progname read X \{S\} write Y} \urcorner = [\mathit{varnum}_{\texttt{X}}, \ulcorner \texttt{S} \urcorner, \mathit{varnum}_{\texttt{Y}}]$$

commands

$$\ulcorner \texttt{while E B} \urcorner = [\texttt{while}, \ulcorner \texttt{E} \urcorner, \ulcorner \texttt{B} \urcorner]$$
$$\ulcorner \texttt{X := E} \urcorner = [:=, \mathit{varnum}_{\texttt{X}}, \ulcorner \texttt{E} \urcorner]$$
$$\ulcorner \texttt{if E } \texttt{B}_{\texttt{T}} \texttt{ else } \texttt{B}_{\texttt{E}} \urcorner = [\texttt{if}, \ulcorner \texttt{E} \urcorner, \ulcorner \texttt{B}_{\texttt{T}} \urcorner, \ulcorner \texttt{B}_{\texttt{E}} \urcorner]$$
$$\ulcorner \texttt{if E B} \urcorner = [\texttt{if}, \ulcorner \texttt{E} \urcorner, \ulcorner \texttt{B} \urcorner, [\,]]$$

$$\ulcorner \{ \texttt{C}_1; \texttt{C}_2; \ldots; \texttt{C}_n \} \urcorner = [\ulcorner \texttt{C}_1 \urcorner, \ulcorner \texttt{C}_2 \urcorner, \ldots, \ulcorner \texttt{C}_n \urcorner]$$

expressions

$$\ulcorner \texttt{nil} \urcorner = [\texttt{quote}, \texttt{nil}]$$
$$\ulcorner \texttt{X} \urcorner = [\texttt{var}, \mathit{varnum}_{\texttt{X}}]$$
$$\ulcorner \texttt{cons E F} \urcorner = [\texttt{cons}, \ulcorner \texttt{E} \urcorner, \ulcorner \texttt{F} \urcorner]$$
$$\ulcorner \texttt{hd E} \urcorner = [\texttt{hd}, \ulcorner \texttt{E} \urcorner]$$
$$\ulcorner \texttt{tl E} \urcorner = [\texttt{tl}, \ulcorner \texttt{E} \urcorner]$$

Example

```
reverse read X {
 Y:= nil;
 while X {
   Y:= cons hd X Y;
   X:= tl X
   }
}
write Y
```

```
X is var 0
Y is var 1
```

translate program into data

```
[0,
 [[:=,1,[quote,nil]],
  [while,[var,0],
      [ [:=,1,[cons,[hd,[var,0]],[var,1]]],
        [:=,0,[tl,[var,0]]]
      ]
 ]],
 1]
```

Programs-as-data in *hwhile*

- We can now write compilers, interpreters, specializers in `WHILE` using abstract syntax trees in list notation ("programs-as-data") instead of string representation.
- Thus we do not have to care about parsing programs.
- In *hwhile* (see Canvas) we can use the -u flag to produce this list representation:


```
hwhile -u reverse.while
```

```
[ 0
,
    [ [@:=, 1, [@quote, nil]]
    ,
        [ @while, [@var, 0]
        ,
            [ [@:=, 1, [@cons, [@hd, [@var, 0]], [@var, 1]]]
            , [@:=, 0, [@tl, [@var, 0]]]
            ]
        ]
    ]
, 1
]
```

hWhile uses @ to indicate special atoms

A note on `hwhile` output

- `hwhile` output by default is given as binary tree:

```
./hwhile add [3,4]
<nil.<nil.<nil.<nil.<nil.<nil.<nil.nil>>>>>>>
```

- *use flags to determine the "type" in which it is presented*

```
./hwhile -i add [3,4]
7
```

integer

```
./hwhile -l add [3,4]
[nil,nil,nil,nil,nil,nil,nil]
```

list of trees

```
./hwhile -li add [3,4]
[0, 0, 0, 0, 0, 0, 0]
```

list of integers

A note on `hwhile` output

- *There are more output formats, to see them all run:*

```
./hwhile -h
```

- *Look at this one, can you explain it?*

```
/hwhile -La  add [3,4]
@doWhile
```

`-La` ?

`hwhile` Demo

(if there is time)

END

Next time:
A special interpreter