


Limits of Computation

14 - Robustness of **P**

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The story so far

- We have discussed "computability",
- Church-Turing Thesis: it does not matter which notion of computation we use.
- in Complexity: does this matter if we restrict to class **P**?

THIS TIME

Robustness of **P**

- we discuss *theses* similar to Church-Turing Thesis but now with *added time complexity*.
- "*robust*" means *resilient*, *hard-wearing*, so
- for a complexity class this means *resilient under compilation into other languages*.
- we focus on class **P**, as it turns out it is "*robust*" compared to other classes.

robust

not as robust

Cook's (Invariance)Thesis

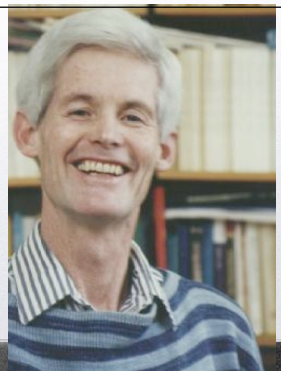
"Extended CTT"

Definition (*Invariance Thesis*) All "reasonable" *sequential* notions of computation can simulate each other up to a polynomial factor.

in other words

> $\mathbf{P}^{\mathrm{L}}$ is the same class of problems for all reasonable sequential (that is, nonparallel) computational models L.

- Like CTT, this is a (widely believed if one excludes Quantum Computing) *thesis ("reasonable computational models" is not a formal notion)*.
- We will give some evidence now.



Stephen Arthur Cook
Turing Award Winner 1982

Evidence for Cook's thesis

Lemma

$$TM \preceq^{lintime-pg-ind} GOTO \preceq^{lintime-pg-ind} SRAM \preceq^{ptime} TM$$

proof by careful analysis of compilation results

Lemma

$$TM \preceq^{lintime} CA \preceq^{ptime} TM$$

proof by careful analysis of compilation results

Now use lifting lemma(s) from Lecture 13, slide 14

Theorem *It holds that*

$$P^{CA} = P^{TM} = P^{GOTO} = P^{SRAM}$$

LIN is not so robust

Lemma

$$GOTO \preceq^{lintime-pg-ind} WHILE \preceq^{lintime} WH^1LE$$
$$WH^1LE \preceq^{lintime-pg-ind} WHILE \preceq^{lintime} GOTO$$

Theorem

$$LIN^{GOTO} = LIN^{WHILE} = LIN^{WH^1LE}$$

- Linear time only robust for "similar" languages
- Too restrictive for all notions of computation.

Is P the bee's knees then?

Can we even go as far a this:

Definition *(Cobham-Edmonds Thesis)* The tractable (feasible) problems are exactly those in **P**.

also often called Cook-Karp thesis

- only a thesis (what is a "tractable/feasible problem")?
- not widely believed. Why not? (next slide)

Why we do not believe Cobham-Edmonds/Cook-Karp

- Is every polynomial time bound really a good time bound indicating feasibility?
- Is every time bound beyond polynomial really a bad time bound indicating intractability?


END

Next time:
Hierarchy Theorems