


Limits of Computation

I - Intro & Motivation

Bernhard Reus

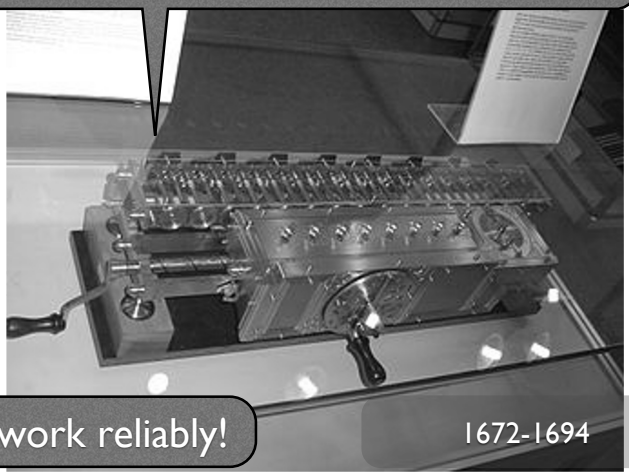

Let's step back in time

This man had a dream

1646-1716

maybe I'm the first computer scientist!

The "stepped reckoner" (Staffelwalze): the first digital (mechanical) calculator performing arithmetic



Alas, it did not work reliably!

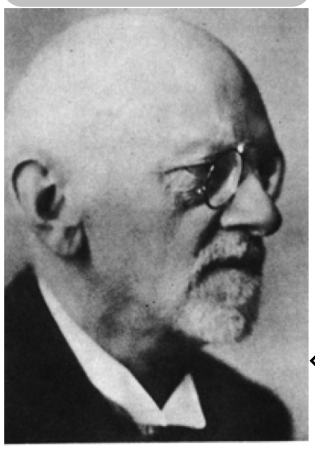
1672-1694

Gottfried Wilhelm Leibniz

invented differential & integral calculus (independently of Newton), modern formal logic, and more

This man had a dream too

1928



"The *Entscheidungsproblem (German for "decision problem")* asks for a procedure which allows one to decide, using a finite number of operations, on the validity, respectively the satisfiability, of a given logical expression (in number theory)."

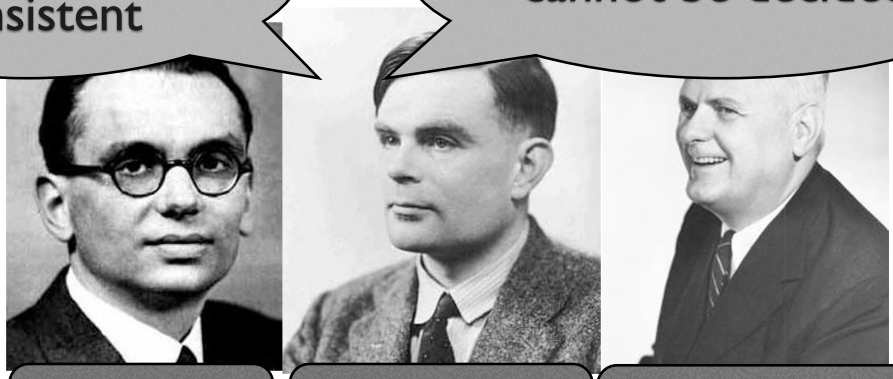
David Hilbert believed strongly that there *exists a solution* to the "*Entscheidungs*problem"

1932/1936

These guys shattered it

(formalisation of) arithmetic cannot be complete **and** consistent

true sentences of arithmetic cannot be decided



Kurt Gödel | Alan M Turing | Alonzo Church

"*Entscheidungs*problem"

There is no program/machine/procedure that can decide whether any given formula in arithmetic is actually true. Sorry David Hilbert!

Alan M. Turing: *'On computable numbers, with an application to the Entscheidungsproblem' from Proceedings of the London Mathematical Society, (Ser. 2, Vol. 42, 1937);*


Major Impact on Science

- not every problem (that can be clearly formalised and understood) can be solved (in this case the answer is yes/no only) by a program.
- So, there are obviously limits to what programs can do!!


1. Sometimes we cannot compute it!

e.g. Hilbert's *Decision Problem* from earlier


Computability

- Are there problems that cannot be solved by a program on a computer?
- Does it matter which computer/language we use?
- Can one identify classes of problems that can be solved by programs or that can't be solved by programs?
- How can one see or find out that a problem can't be solved by a program?


What is a *program* anyway?

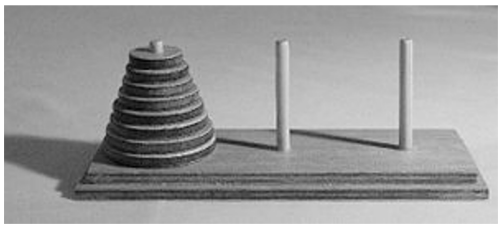

2. Sometimes we cannot afford it!


Annoyingly...

- ... even decidable (computable) problems sometimes are problematic.
- They may take a long time to compute.
  TOO LONG ACTUALLY!


Examples of *computable* problems that are *intractable*


Towers of Hanoi




- move all disks from 1st to 3rd rod
- only take/put disks from/on top of rods
- never put disk on a smaller disk.


Towers of Hanoi (cont'd)

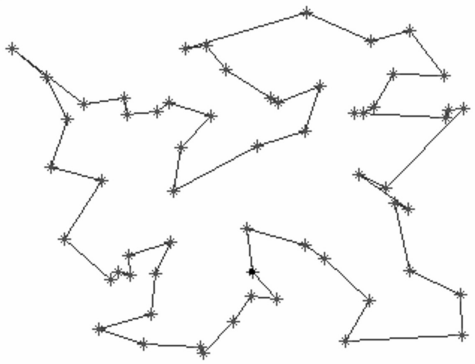
- for *N* disks it takes 2^N-1 moves to complete the task
- if you could move one million disks per second (!), for *N*=64 you'd still need about **585,000 years** to finish the job!

disks	moves
3	7
4	15
8	255
16	64535
32	4,294 bn
64	18,44 bn bn

Even for meditating monks this is too long!


TravellingSalesmanProblem

- Given a number of cities and the costs of travelling from any city to any other city, what is the least-cost round-trip route that visits each city exactly once and then returns to the start?




TravellingSalesmanProblem

- the number of tours to search through grows exponentially with the number of cities *N, namely (n-1)! / 2*
- Any brute force technique must fail. But are there clever ways?

cities	tours
3	1
4	3
8	2520
16	653.8 bn
32	4.11×10^{33}
64	0.99×10^{87}

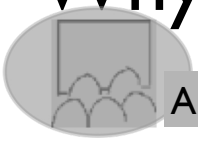
There are only about 10^{80} atoms in the known universe.


Computational Complexity

Questions

- Which problems can be solved within certain limits of time (and space)?
- Are there resource limits within which certain combinatorial problems cannot be solved?
- Are intractable problems good for anything?
- Does adding resources allow one to solve more problems?
- How does one deal with intractable problems in practice?
- Can new emerging computing paradigms like *Quantum computing* and *Molecular computing* help?


Why are these important questions?

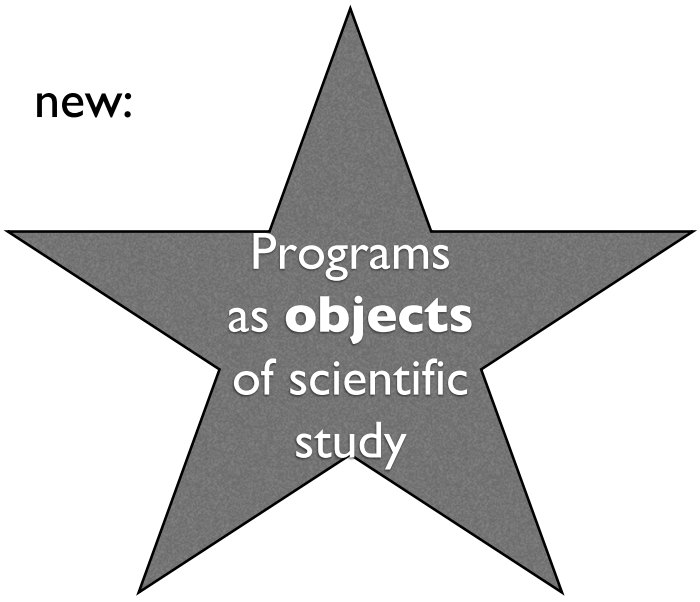


Audience Participation


So what is this module for?

- you know the principles of programming
  - ➳ Programming Concepts
  - ➳ Data Structures & Algorithms
  - ➳ Program Analysis
- you know how to write programs
  - ➳ Intro to Programming
  - ➳ Further Programming

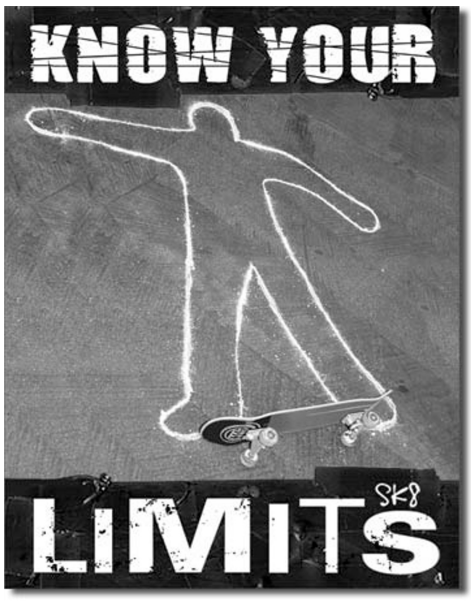
new:



Organisation & Overview

Scientific Questions About Programs

- What are programs? What are their limitations?
- **What** are their limits in terms of what we can achieve with programs?
- How can we spot the limits?
- How can we circumnavigate them?
- Can we exploit those limits for a benefit?



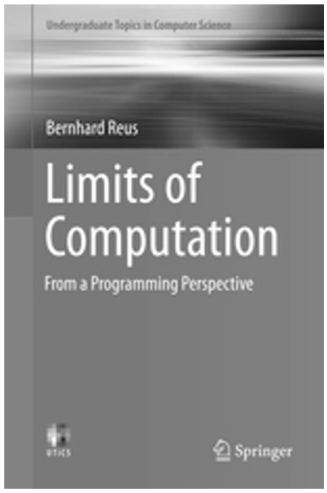
Organisation 2024

- **Lectures**
  Mo 5pm A1 (Arts A), Tue 2pm A1 (Arts A)
- **Classes**
  Three classes, as assigned to you on Sussex Direct, start in week 2
  Sussex Direct tells you which group you're in, please stay there throughout the term
- **Coursework**
  Problem Set (60% week 6, March 7th)
  + Test (40% week 11, tba)
- **Assessment** 50% CWK + 50% CEX (online)
- **Web** Canvas Site (with all info)

DISCLAIMER: dates may change, always consult Sussex Direct!

Course Text

There is a Reading List for this module on Canvas.



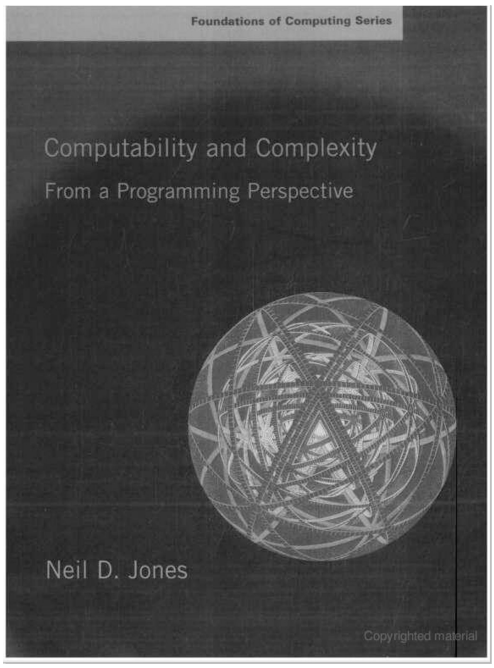
eBook **£26.99**
price for United Kingdom (gross)
Buy eBook
ISBN 978-3-319-27889-6
Digitally watermarked, DRM-free
Included format: EPUB, PDF
ebooks can be used on all reading devices
Immediate eBook download after purchase
Softcover **£33.99**

shameless plug

a few copies in the library

condensed notes will be available for free on Canvas

https://link.springer.com/book/10.1007/978-3-319-27889-6



Other literature

- Neil D Jones: *Computability and Complexity From a Programming Perspective*, MIT Press, 1997. **Online copy** on Canvas Links page.
- Also useful maybe: Hopcroft, Motwani, Ullmann: *Introduction to Automata Theory, Languages, and Computation,* Addison Wesley

END

Next time:
We define "Algorithms and Algorithmic Problems"