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Robotics Topic 5 – Optimisation

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1 Optimisation

Optimisation means achieving the ‘best’ outcome in a given situation, or the ‘best’ solution to a given problem. For example, if you were to mislay your keys, then the best outcome would be to find them again in the minimum time possible. Alternatively, for some students studying an OU module, the best outcome is to pass the exam with the highest mark possible.

So, optimisation requires a goal – e.g. find the car keys – and a means of measuring how well this goal has been achieved – e.g. in the minimum time possible. Of course, not all solutions can be described as ‘best’. They can also be ‘better’ or just plain ‘good’ – usually any solution is good compared with not finding a solution at all. So even if it takes you an hour to find your car keys, this is probably better than not finding them at all. Not always though. If losing your keys means missing an appointment, then it might be better to abandon that goal and to call for a taxi. In this case the goal of finding your keys has lower priority than the goal of getting to the appointment on time.

The same assumptions apply for optimisation in robotics as they do in real life. In artificial intelligence systems and in robotics, one of the major objectives is to get intelligent systems to make relatively good decisions given the information they have available to them. For example, an autonomous car plotting a route between multiple locations needs to consider factors it knows about – such as the distance to each location, the speed limit on that route and the current traffic situation – to try to pick the optimal route. Many situations are so complex that there is rarely a ‘best decision’, but rather a choice between alternative ‘good decisions’. The technique of optimisation makes it possible to quantify different alternatives, and provides a basis on which to make operational decisions.

Sometimes the best course of action may depend on what decisions have already been taken by others. In these situations an approach known as ‘game theory’ may be used to help in the decision-making process. For example, consider two drones on a collision course, flying at the same altitude. The best course of action for both drones is to avoid the other drone by either flying higher, lower or staying at the same height – but the correct decision is also based on what the other drone decides to do.

Another approach makes use of the idea of a ‘potential field’. This can be used to describe an imaginary force or set of forces that act on a robot to modify its behaviour. Potential field approaches allow robots to navigate through an environment by imagining various force fields acting on them. The ‘potential’ of the physical points around the robot relate to the ease or difficulty with which the robot can reach those points. More traditional approaches require the robot to plan its course, often using optimisation techniques to choose between different possible plans that lead to the same outcome, but at different costs (such as time or energy used).

1.1 Learning outcomes

By the end of Topic 5 you should be able to:

- explain what is meant by optimisation
- explain the meaning of local and global optima
- explain what is meant by the terms landscape, maximum and minimum
- appreciate how game theory can help in the decision-making process
- give the example of Braitenberg's vehicles responding to potentials
- explain the interaction of Asimov's laws in terms of their potential.

2 Search

Many of the problems faced by people – and robots – in a complex environment do not have obvious solutions, so we have to search for them. For example, what should a robot football player do at any given moment? What should a robot astronaut do if it damages itself in a fall? What should a robot nanny do if its child has a temper tantrum? How can a robot get its work done safely in the minimum time? How can a robot use its power supply most efficiently?

Solution space and search space

It has proven very productive to think of such questions in terms of searching for solutions. In a general way, one can hypothesise the set of all possible solutions to a problem. Consider the problem of finding a pair of dominoes with spots adding up to 21. The four solutions to this problem are shown in Figure 2.1. Rather abstractly, this set of four solutions is called a 'solution space': it contains everything that could be considered to be a solution to this problem.

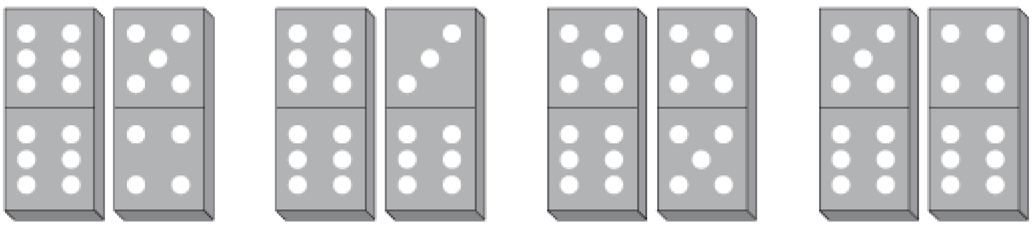

Figure 2.1 The solution space for the 21-spot domino problem

Show description

The issue of finding a solution to a problem can then be rephrased as searching for a member of the solution space. Where to search? In Figure 2.2, are there any candidate solutions for the 21-spot domino problem?

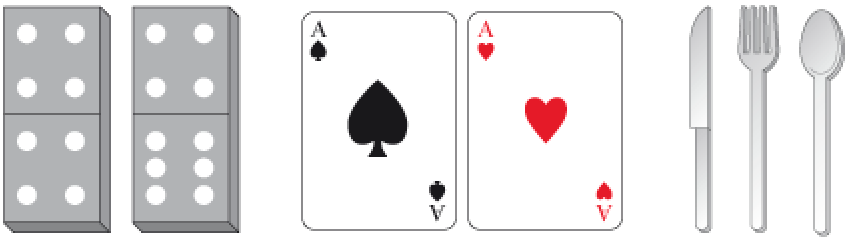

Figure 2.2 Which of these is a candidate solution for the 21-spot domino problem?

Show description

The pair of dominoes is a candidate: it *could* be a solution, but in this case it is not. The pair of aces is not a candidate: playing cards are not dominoes, as specified. The knife, fork and spoon combination has the wrong number of the wrong things, so is not a candidate.

The set of all candidate solutions to a problem is referred to as the ‘search space’. The search space for the 21-spot domino problem is the set of all pairs of dominoes. The ‘solution space’ is that part of the search space that contains actual solutions to the problem.

SAQ 2.1

a. What is the search space for the problem of finding a spoon in the kitchen? What is the solution space?
b. What is the search space for the problem of finding a holiday? What is the solution space?
c. What is the search space for the problem of passing an OU module? What is the solution space?

Reveal answer

2.1 Brute-force search

The notions of search space and solution space help in problem solving because they help us to focus on where to search. Once we have decided what the problem is and what characterises a possible solution, we also need to know how to search that space to find a solution.

One approach is an ‘exhaustive’ or ‘brute-force’ search. This means looking at every member of the search space until a solution is found. When search spaces are relatively small this can be a good option; when they are large this is a poor option.

To get a feel for how big some search spaces are, consider the proverbial monkey sitting at a computer keyboard. It is said that if you have enough monkeys randomly hitting the keys, then eventually they will type out the complete works of William Shakespeare. This is sometimes referred to as the Infinite Monkey Theorem.

Figure 2.3 A monkey typing at a computer

Show description

Suppose a monkey types five characters. What are the chances that those five characters will be 'Romeo'? My keyboard has 47 character keys and the spacebar. So there are $48 \times 48 \times 48 \times 48 \times 48 = 48^5$ permutations of five characters. That's over 250 million! If the monkey were to hit five keys at random, then we would expect 'Romeo' to occur roughly once in every 250 million attempts. Now think about the chance of 'Romeo and Juliet' appearing in 16 characters. There are $48^{16} = 7.94 \times 10^{26}$ permutations of 16 characters, so the chance is now roughly once in 7.94×10^{26} attempts.

Notice what happens when we consider a larger problem. We first considered five characters and then 16 characters, a little more than three times as many. But the number of permutations is not three or four times as many: instead, it is larger by an enormous factor (to be exact, it is larger by the factor $48^{16} \div 48^5 = 48^{11} = 3.1 \times 10^{18}$).

Let's consider another problem, similar to the 21-spot dominoes problem but this time based on a deck of 52 playing cards rather than the set of 28 dominoes. We'll look for combinations of cards that have numbers adding up to 40, assuming Jacks, Queens, Kings and Aces all count as 10. The search space for this problem is the set of all combinations of any number (1, 2, 3, …, 52) of cards. This number can be calculated as 2^{52} = 4,503,599,627,370,496 possible combinations of cards. If it takes a robot one millionth of a second to examine each combination of playing cards to see if the numbers add up to 40, how long would it take in total? There are over 30 million seconds in a year, so it could examine 30,000,000,000,000 a year. Not bad, but not good enough: it would take around 150 years to examine them all!

Study note – Mathematical aside

I don't expect you to know where I got the figure 2^{52} from, but the following argument might help. Imagine laying all the cards face down in order 2, 3, 4, …, Queen, King, Ace, one suit after another. There would be a line of 52 cards. Each possibility that we must look at consists of turning some of the cards face up and leaving the others face down. We can represent any such arrangement by writing ‘1’ under any card that is face up and ‘0’ under any that is face down. For one combination we might get 10010110…; that is, a string of 52 ones and zeros. We can think of this as a binary number 52 digits long. The number of possible values that a binary number with n digits can store is 2^n, so here the number of possible values is 2^{52} = 4,503,599,627,370,496.

Problems like these involve combinations or permutations of alternatives. We’ve seen that, as the number of things being combined increases, the associated search space becomes very large very quickly. When searching for the optimum solution, you cannot know if you’ve found it until every point in the search space has been tested (or excluded). Problems with very large search spaces are, appropriately, known as ‘hard’ problems!

However, in some problems it is possible to ‘prune’ the search space if we can exclude parts of it from consideration. For example, in the problem of finding playing cards that add up to 40 we can discount some combinations of cards. Since the maximum number per card is 10 and the minimum is 2, a solution must have at least 4 cards and no more than 20 cards. So we could prune all combinations with fewer than 4 or more than 20 cards, reducing the search space considerably. However, even though this pruning reduces the search space by over 90%, the remaining number is still very large (284,508,321,483,260) and the time taken for our robot would still be several years. (You might be able to see how we could prune the search even further.)

Nonetheless, the fact that problems may have such large search spaces is one of the reasons why Moore’s law, which you came across in Topic 1, will not necessarily make machines more intelligent. An increase of processing power by ten, a hundred, or even a million does not help much with ‘hard’ problems like this. There are some problems that are just too complex to succumb to brute-force searches and we must look for other approaches.

2.2 The structured search

Humans solve problems using their intelligence. We manage to understand massive search spaces in ways that enable us to find solutions to bewilderingly complex problems without resorting to ‘brute force’.

Consider the (nice) problem of choosing a holiday. There are thousands, if not millions, of possibilities: beach holidays, skiing holidays, sightseeing holidays, jungle treks and so on. Suppose you tried a brute-force search and looked at them all in the brochures taking, say, half a minute each. If there were 1 million holidays and you worked at it 20 hours a day, then it would take you over a year, so you’d be exhausted and miss your holiday anyway. Not a very good outcome!

Of course, people don’t search for holidays like this. They use a ‘structured’ search. For example, if you have young children then you might decide you want a beach holiday not too far away and restrict your attention to that part of the search space. Having made that decision, money might be a bit tight this year, so you would only look at inexpensive holidays.

These decisions can be represented as a tree (Figure 2.4). Every time one decision is made, large parts of the tree are ‘pruned’ from the search. In this case, by the time you are looking for an inexpensive beach holiday in Blackpool, the reduced search space might be small enough to search exhaustively.

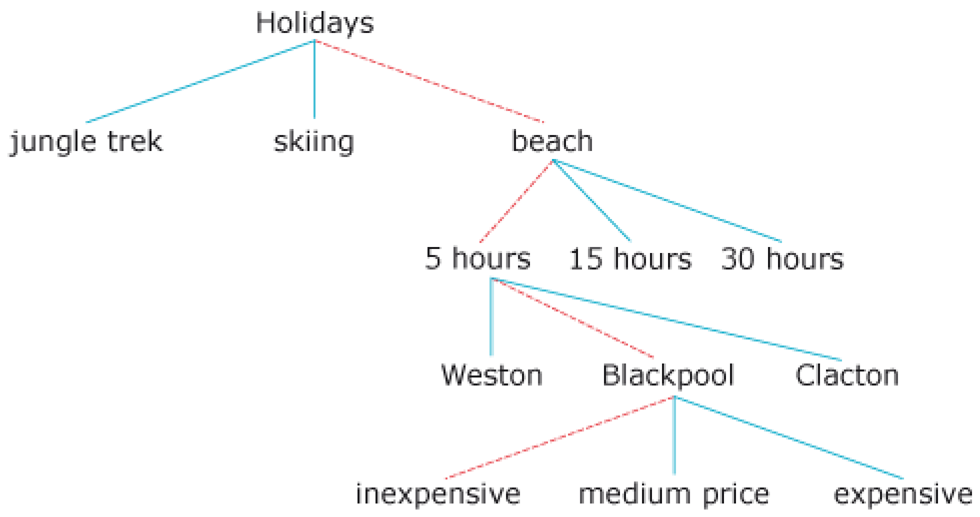

Figure 2.4 The search tree for choosing a holiday

Show description

Robots can also form structured searches by using their knowledge base. When they have a problem, they can rule out many potential solutions. For example, a robot might be searching for the best way to get its batteries charged (Figure 2.5). By structuring the search space, it can focus its attention on those possibilities that are relevant.

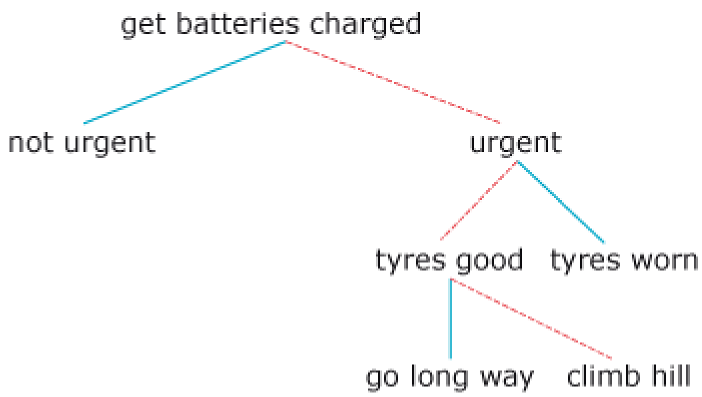

Figure 2.5 The search tree for a robot to charge its batteries

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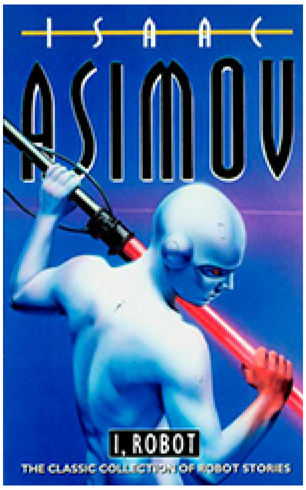
3 Trade-offs and equilibrium

In this section you will read what I think is one of Asimov's most interesting stories. It concerns the interplay between the Three Laws.

Activity 3.1 Read

Read Chapter 2, 'Runaround', of Asimov's book *I, Robot* and then answer the SAQ.



SAQ 3.1

Summarise this story and explain how the problem is resolved.

Reveal answer

3.1 Trade-offs

Suppose you are walking towards a bonfire, taking a temperature measurement at metre intervals. The closer you get to the bonfire, the hotter it becomes. This is illustrated in Figure 3.1. At 8 metres from the bonfire the temperature is 11 °C and at 7 metres it is 20 °C, and so on. You wisely stop 5 metres from the fire because 67 °C is getting dangerously hot. You don a protective suit so you can get closer then you continue to within 1 metre of the fire.

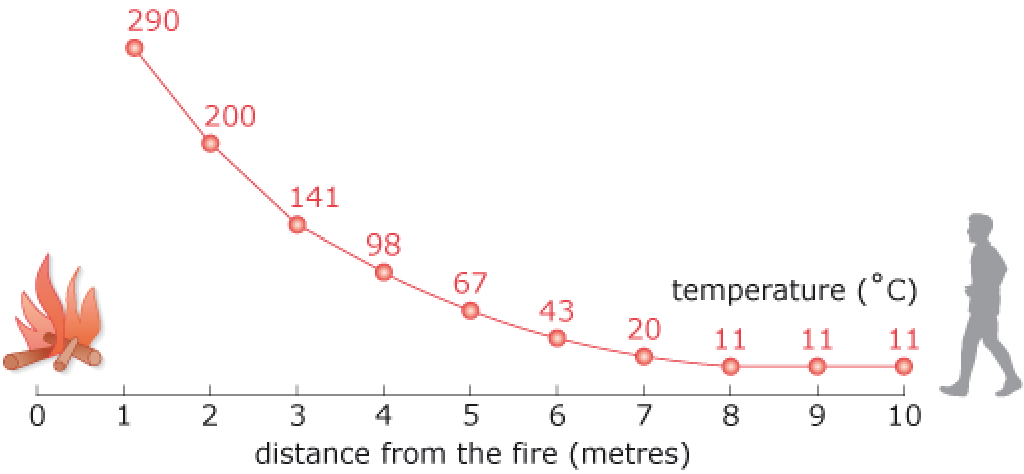


Figure 3.1 Temperature near a bonfire

Show description

Suppose you are working with a material that gets easier to work as its temperature increases, as illustrated in Figure 3.2. What would be the optimum distance from the fire to work the material? If you were trying to minimise cost, then according to this graph, the lowest cost would be 45 pence at 3 metres from the fire. If you were to go any closer it would get more expensive, and if you were to go further away it would get more expensive. In some sense, 3 metres would be the best, or 'optimum', distance from the fire.

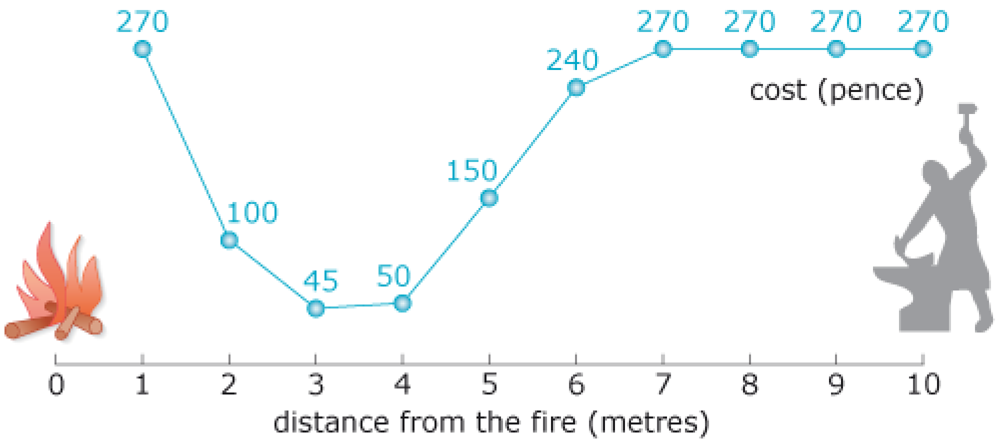


Figure 3.2 Cost of working metal near a bonfire

Show description

However, suppose you must be paid extra the more uncomfortable you get. For simplicity let's assume that you are paid a penny for each degree centigrade above zero. Then, to minimise this extra cost, the optimum place for you to work would be 8 metres or more from the fire (Figure 3.3).

However, at 8 metres from the fire the material is as hard to work as it can be, and the cost of that is very high. What is the optimum, taking into account both temperature-related wage costs and the cost of working the material?

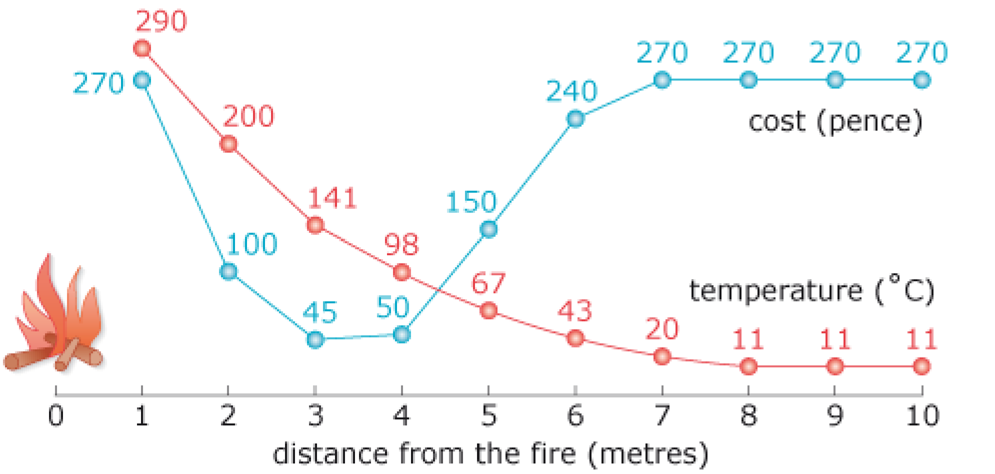


Figure 3.3 Temperature and cost near a bonfire

Show description

There is a 'trade-off' between the reduced cost of working the material at a higher temperature on the one hand, and the increased cost of being too hot on the other hand. For simplicity I have assumed that the cost of getting too hot is directly related to temperature, so that the total cost of working at 1 metre from the fire is 560 pence. Moving closer to the fire decreases the cost of working the material, but increases the amount you are paid.

In this simple example, the optimum position is found by plotting a graph of the aggregated costs; this is shown in Figure 3.4. There is clearly a minimum cost at 4 metres from the fire, so this is the optimum in this example.

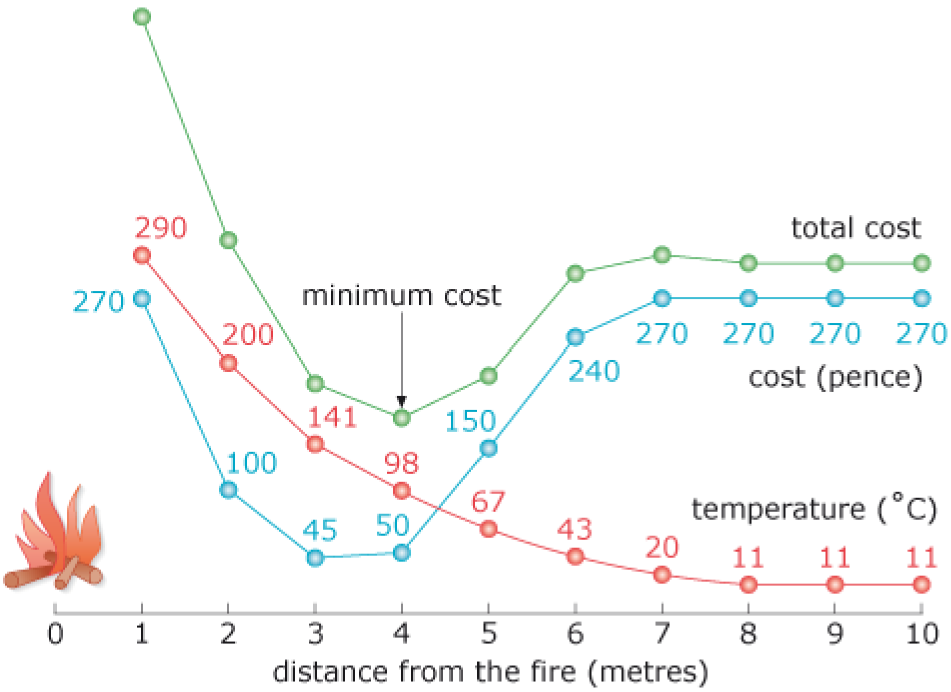


Figure 3.4 Trade-off between temperature and cost of working near a bonfire

Show description

In many situations it is necessary to trade off one thing against another. For example, if a robot moves very fast in a cluttered environment then it gets to its destination faster, but it is more likely to have an accident. In this case the robot might have to trade off speed against safety.

When the conflicting requirements balance each other, a system is said to be in equilibrium.

SAQ 3.2

What was being traded off by Speedy in Asimov's 'Runaround' story?

Reveal answer

3.2 Landscapes with global and local optima

In the previous example there was just one optimum, but in many situations there are many alternative optima. For example, Pam, a mountain climber, may get more satisfaction from climbing mountain A than mountain B (Figure 3.5). If there were no other considerations, she might choose A. However, suppose she starts from point D. The cost of getting to A might be prohibitive. In this case, B is a good second best, or a good 'sub-optimum' solution to the problem of choosing which mountain to climb.

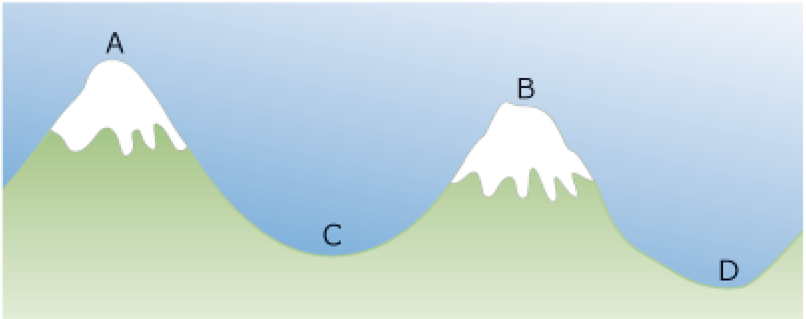


Figure 3.5 A mountain landscape

Show description

Consider the plight of Dave, who has a respiratory problem, trying to survive in this landscape. Suppose for him, high altitude causes discomfort, and the lower he is the more comfortable he feels. Where is the best place for him to be in this landscape?

D is lower than C, so D is the best, or 'global', optimum. However, suppose Dave were staying in a hotel at C. Would he be better off moving to D? If there were a mountain ridge between C and D, peaking at B, then he would have to climb over the ridge. No matter what he did he would feel more uncomfortable. So, to reach the global optimum, he would have to leave the relatively comfortable 'local' optimum at C, and become more uncomfortable before he could reach the most comfortable position at D. In a situation like this, it is often better to stay in a relatively good local optimum, than expend a lot of energy trying to find the global optimum.

The idea of a landscape is actually one that is widely used in optimisation. The height of the landscape is associated with some measure of 'goodness' or 'badness' about the quality of a solution to a problem. Your position in the landscape corresponds to one possible solution to the problem.

If there is a peak in the landscape that is surrounded by points that have a lower 'goodness' value, then we call it a 'local maximum'. If there is a valley in the landscape that is surrounded by points that have only higher 'goodness' values, then we call it a 'local minimum'. The single largest maximum in the landscape, if it exists, is called the 'global maximum'; the smallest minimum in the landscape, if it exists, is called the 'global minimum'.

In some problems, the height of the landscape may refer to how bad a solution is. For example, when a neural network is trained, the 'error' between the desired output and the actual output may be represented using the idea of a landscape. In this case, the optimisation process is looking for minima – points where the error is lowest.

Both maxima and minima can thus be seen as optima, depending on the problem, and optimisation may be seen as a way of trying to find the highest point in a landscape (maximisation) or the lowest point (minimisation).

SAQ 3.3

In the UK, the price of petrol and diesel fuel depends on many things, including where you live. Suppose the cost of fuel used in your car varies as shown in Figure 3.6.

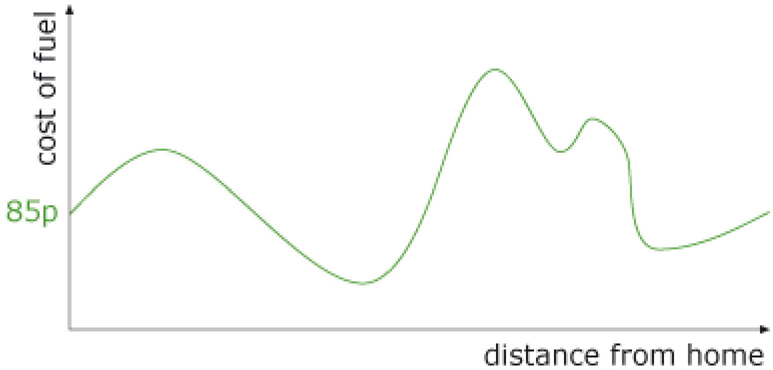


Figure 3.6 Cost of fuel as a function of distance

Show description

Where are the local maxima and minima in the price landscape shown in Figure 3.6? Are there any global optima in the landscape? If so, where?

Reveal answer

Hill climbing

In optimisation, the metaphor of a landscape is taken a further step by the term 'hill climbing'. Suppose you were blindfolded on the slope between C and B in the mountain range in Figure 3.5. Suppose now that the height of the mountain represents a benefit; ideally, you want the largest benefit. You can't see the whole mountain range because of the blindfold, but you can feel around with your feet. In this way you can find that going one way is up and the other way is down. So you take a step up, getting a better solution to the problem. If you do this repeatedly then you will climb up the mountain until you reach the peak at B. When you get to B you have 'hill-climbed' to a local maximum.

There are a number of optimisation techniques based on hill climbing. Instead of considering the full search space, these start from an approximate solution (or even just a guess) and look for a better solution close by. This is done repeatedly until no improvement is possible, which means a local optimum has been found. Although these techniques cannot guarantee that you will find the global optimum, they can produce a good solution much more efficiently than a brute-force search.

3.3 The travelling salesman problem

The so-called ‘travelling salesman’ is a classic problem in artificial intelligence. Consider a salesperson wanting to visit London (1), Plymouth (2), Canterbury (3), Stratford-upon-Avon (4), Brighton (5), Cambridge (6), Cardiff (7), Oxford (8) and Bath (9). These towns or cities are illustrated in Figure 3.8. Starting at London, what is the shortest way to visit each of them once, returning back to London? The problem is relatively easy for a small number of towns and cities like the nine shown here, but quickly gets much more difficult if you start adding more locations.

Figure 3.8 Destinations for the travelling salesman to visit

Show description

The route shown in Figure 3.9(a) takes the cities in the order I’ve numbered them. This is longer than the second route (b). The solutions to the problem can be represented by a string of numbers. For (a) it is 1-2-3-4-5-6-7-8-9-1 and for (b) it is 1-3-5-2-9-7-4-8-6-1.

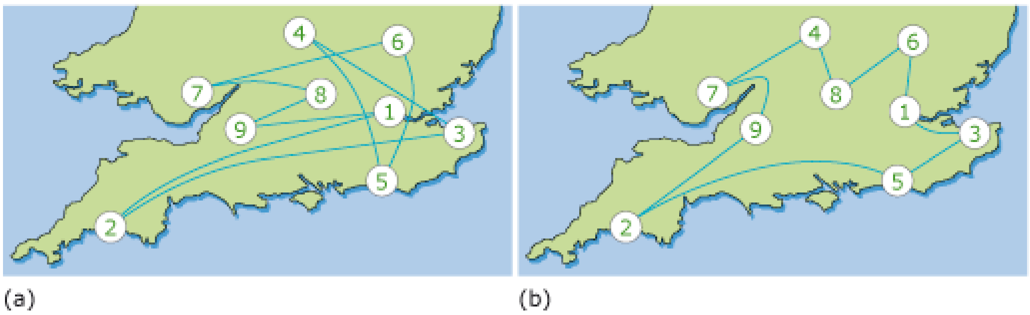


Figure 3.9 Solutions (a) and (b) to the travelling salesman problem

Show description

So the problem becomes one of finding a way of ordering the numbers so that the route they represent is the shortest. It is assumed that it does not matter which way round the circuit the salesperson travels; for example, the route 1-2-3-4-5-6-7-8-9-1 is considered to be the same as 1-9-8-7-6-5-4-3-2-1. Now that a simple ‘representation’ of the problem has been formulated, it is easy to see that the search space is the set of all sequences of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and 9. The solution space contains the sequence (or sequences) that involve the shortest distance between them.

The travelling salesman problem is interesting because the size of the search space gets very big very quickly. Suppose you start in London. For your first destination, you have eight choices to go to. After you arrive at your first destination you have seven choices, at your second destination you have six choices and so on. This gives 8 × 7 × 6 × 5 × 4 × 3 × 2 × 1 choices in total. These include travelling each route in both directions, so the number of routes is divided by two. Hence, for nine destinations the number of routes is (8 × 7 × 6 × 5 × 4 × 3 × 2 × 1) ÷ 2.

For N destinations, there are $[(N-1) \times (N-2) \times \ldots \times 3 \times 2 \times 1] \div 2$ routes for the salesperson. This gets astronomically large when N exceeds 12.

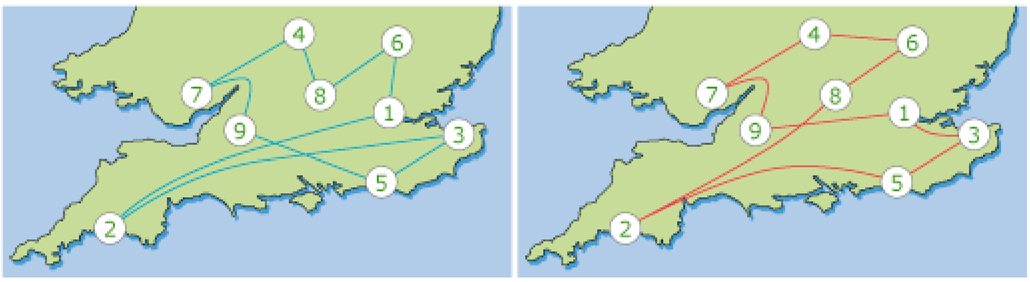
The travelling salesman problem clearly has practical applications. Imagine how many millions of round trips are made each year by lorries, delivering the food and goods that we all demand. If they could be sure of taking the minimum route then they would save a lot of time and money. Also, the travelling salesman problem has applications in microchip design, where large numbers of transistors have to be connected using the shortest connections. It has many other applications.

3.4 Genetic algorithms

A genetic algorithm is a computer program modelled on natural evolution. Solutions to problems are ‘evolved’ by creating new ‘child’ solutions from ‘parent’ solutions, with poor solutions being removed from the population. First you have to be able to represent the problem as a string of numbers, like 1-8-3-5-2-9-7-4-6-1. These are called ‘genomes’. Each genome describes a candidate solution to the problem. After evaluating each genome (for example, by calculating the length of the route) the better candidate solutions are preferentially selected and used to generate new candidate solutions that make up the next generation of genomes.

The ‘offspring’ candidate solutions are generated by combining elements from two parent genomes. Just as human children bear traits from both parents, the next generation of candidate solutions are made up from different aspects of their ‘parents’.

Consider the parent solutions (c) and (d) in Figure 3.10.



(c) (d)

Figure 3.10 Solutions (c) and (d) to the travelling salesman problem

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Each solution has some good and some bad parts. Two possible child solutions derived from these are (e) and (f) in Figure 3.11.

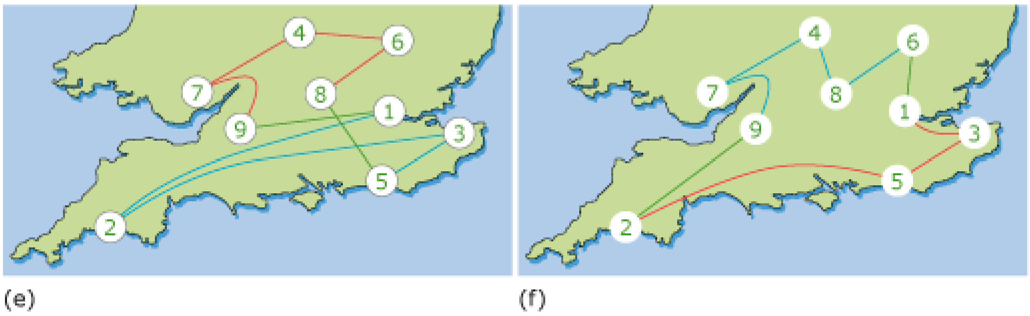


Figure 3.11 Solutions (e) and (f) to the travelling salesman problem

Show description

Solution (e) is rather poor, but solution (f) has inherited the best parts of both parents, and is a good new solution. By combining the parts of the routes in this way, new connections are made, and these are shown in green.

This kind of ‘breeding’ between two genomes (Figure 3.12) can be performed easily by computers.

1 - 2 - 3 - 5 - 9 - 7 - 4 - 8 - 6 - 1	mates with	1 - 3 - 5 - 2 - 8 - 6 - 4 - 7 - 9 - 1
1 - 2 - 3 - 5 | 9 - 7 - 4 - 8 - 6 - 1	genomes split	1 - 3 - 5 - 2 | 8 - 6 - 4 - 7 - 9 - 1
1 - 2 - 3 - 5 | 8 - 6 - 4 - 7 - 9 - 1	genomes swap	1 - 3 - 5 - 2 | 9 - 7 - 4 - 8 - 6 - 1
1 - 2 - 3 - 5 - 8 - 6 - 4 - 7 - 9 - 1	new generation	1 - 3 - 5 - 2 - 9 - 7 - 4 - 8 - 6 - 1

Figure 3.12 Stages in a genetic algorithm

Show description

Another mechanism used in genetic algorithms is mutation. Whereas breeding generates offspring that only rearrange the parents’ genomes, mutation randomly changes small parts of the offspring’s genome. For example, the route (g) (Figure 3.13) swaps the positions of destinations 7 and 8 (mutates) to give a new route shown as (h).

1 - 2 - 3 - 4 - 5 - 6 - 7 - 8 - 9 - 1 mutates into 1 - 2 - 3 - 4 - 5 - 6 - 8 - 7 - 9 - 1

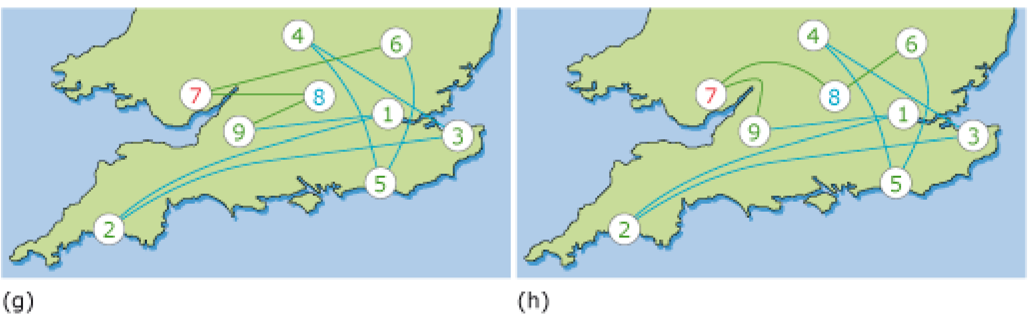


Figure 3.13 Solutions (g) and (h) to the travelling salesman problem, showing mutation

Show description

4 Game theory and the prisoner's dilemma

There are many situations in which people's actions depend on the actions of others. For example, if someone is nasty to you, then you are less likely to do something nice in return. So that other person might reason along the lines that 'if they are nice to you, then you are likely to be nice to them'.

One approach that is used widely in decision-making systems is 'game theory'. This is used, for example, when a decision needs to be made about the best course of action in the context of a range of possible decisions made by someone else.

I will introduce game theory using a simple game, known as the 'prisoner's dilemma'. Suppose that the police are holding you and your partner in prison on suspicion of a serious charge, of which you are both guilty. You are kept isolated from each other, so that neither of you knows how the other is answering the interrogator's questions. The dilemma that faces you is this:

- you and your partner know that if you both keep quiet (or in the parlance of game theory, 'cooperate' with each other but not with the police) then the serious charge against you will be dropped, but you will both eventually be found guilty on a lesser charge
- the police have told you both that if you both confess (i.e. if you both 'defect') then they will make a recommendation that you both go to prison for a reduced sentence on the most serious charge
- if one of you confesses (defects) and the other keeps quiet (cooperates), then the person who confessed will be released under a witness protection programme but the person who kept quiet will be sent to prison for the maximum term.

What should you do? Table 4.1 shows the different possible outcomes and associates each one with a pay-off; that is, a score that represents how good the outcome is for you and your partner.

Table 4.1 Pay-off matrix

	Partner cooperates	**Partner defects**
You cooperate	Reasonable outcome for you both – the lesser charge. You both score 2.	Bad outcome for you – your partner walks free but you go to jail. You score 0; your partner scores 4.
You defect	Excellent outcome for you – you go free while your partner goes to jail. You score 4; your partner scores 0.	Not that good an outcome for you – guilty as charged of the serious crime, though with a minimum sentence recommendation. You both score 1.

Looking at Table 4.1, the best outcome for you is if you defect against your partner (confess) and your partner cooperates with you (i.e. keeps silent). However, if your partner defects as well (the sneak!), then the outcome is the next-to-worst one.

One way of coping with this dilemma is to try to identify a strategy that is least worst for you whatever your partner does. In this example, the safest thing for you to do is defect (and score either one or four). By changing your strategy, you run the risk of doing worse (that is, scoring zero).

Many games, as well as real-world decisions, can be analysed using game theory. Indeed, a large amount of research was done by the US RAND Corporation during the Cold War in an attempt to identify optimal (or least bad) nuclear war strategies.

An interesting feature is that if the same game is played over and over again, then different sensible strategies emerge if the pay-offs are changed relative to one another. Playing the prisoner's dilemma over and over again is known as the 'iterated prisoner's dilemma' (IPD). In this case, you choose whether to cooperate or defect at each round of the game based on what has happened previously, and a cumulative score is kept. There are many well-known strategies for playing the IPD and more are continually being discovered.

SAQ 4.1

Consider the pay-off matrix shown in Table 4.2.

Table 4.2

	Partner cooperates	**Partner defects**
You cooperate	Both score 8	You score 1, partner scores 10
You defect	You score 10, partner scores 1	Both score 4

a. In a single play of this prisoner's dilemma, what strategy should you take if you assume that your partner will cooperate? What pay-off will you get?
b. In a single play of this prisoner's dilemma, what strategy should you take if you assume that your partner will defect? What pay-off will you get?
c. If you do not know what your partner will do, which strategy should you take and why? Do you think this is an optimal solution in terms of maximising total pay-off?

Reveal answer

5 Potential fields

One interesting approach to navigation in a robot, which has a lot to do with optimisation, is the use of so-called 'potential fields'. You are probably already familiar with the idea of a potential field, although you may not realise it. A potential field can be thought of as an attractive or repulsive force associated with a particular object. An example of an attractive force is gravity, while an example of a repulsive force is the force that exists between two similar magnet poles when they are in close proximity.

A potential field can be thought of as a force field. The field exists in the region around an object if a force is exerted on another object placed anywhere in that region.

Real potential fields, such as electrostatic or gravitational fields, exist everywhere around an object. A robot does not need to know, or calculate, the ‘forces’ acting on it at any point in time: it just needs to sense them. This ‘local’ information is used by the robot to decide where to move next. So, in a potential field approach, the robot uses information from its sensors to identify attractive and repulsive forces in its environment and then adds these forces together to work out where to go.

Figure 5.1 shows a potential field for an area with a goal and a solid object. The arrows show the direction of the hypothetical force. If a robot enters into this area at any point, for example at A, B or C, then it is attracted towards the goal, as indicated by the arrows. In the cases of A and C, the robot will automatically avoid the object.

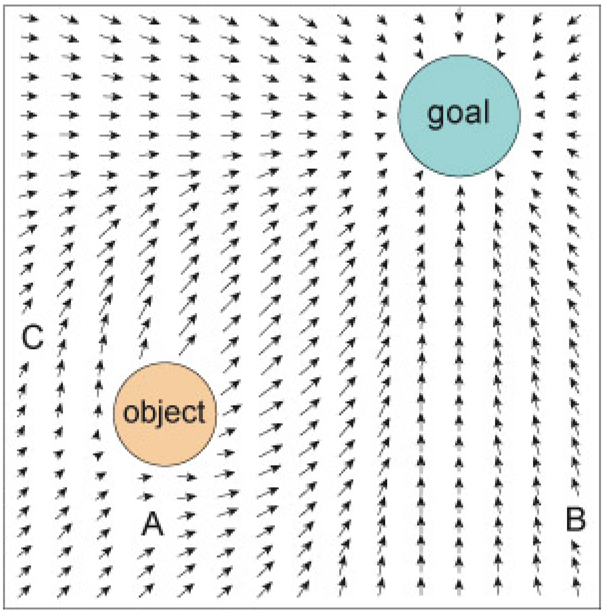


Figure 5.1 An attractive potential field showing a goal and an obstacle

Show description

However, the use of potential fields will not necessarily enable a robot to navigate to its intended destination. It is possible that the forces from several objects will lead the robot into a local minimum from where it cannot proceed further towards its target.

Typically, the strength of a potential field changes, depending on how far you are away from the source of the potential. By analogy, if your friends live a long way from you, then you probably feel less compelled to visit them than if they lived closer. For our purposes, the change may be gradual over distance, or it may change sharply at a particular distance from the source.

The potential field approach to navigation uses a robot's sensors to build up a 'picture' in terms of where it can and cannot go. For example, an obstacle placed in front of a mobile robot, which exerts a hypothetical repulsive force on it, would ensure that the robot does not collide with the obstacle. Figure 5.2 shows a hypothetical potential field in which the increasing potential is depicted by increasingly darker shades of grey.

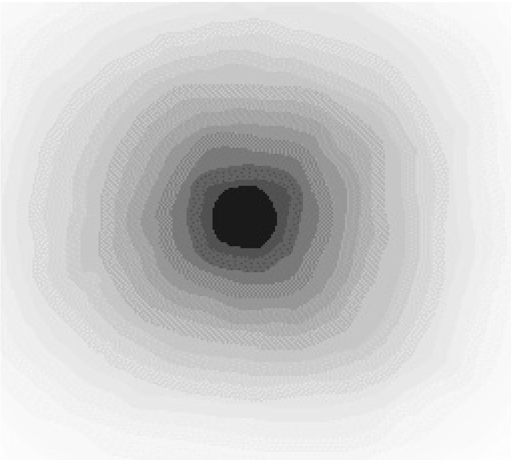


Figure 5.2 An attractive potential field

Show description

SAQ 5.1

a. A lamp can be thought of as the centre of a potential field for a moth. Does it exert an attractive or repulsive force on the moth?
b. Shopping centres create potential fields attracting consumers to buy things. Which of the following do you think create high or low potentials: Manchester, Birmingham, Edinburgh, Belfast, Netherwallop, Woburn, London, Cardiff and John o'Groats?

Reveal answer

It is possible to program a robot to use information from its distance sensors (such as ultrasound sensors) to control its motors just as if the object were exerting a repulsive force – analogous to the Braitenberg avoider. A target object, on the other hand, might be thought of as exerting an attractive force – as in the Braitenberg attractor. For example, a robot in need of a recharge might be able to recognise a recharging point in a room and establish this point as exerting an attractive force.

SAQ 5.2

If robots were at positions A, B and C in Figure 5.3, draw the paths they would follow in response to the potential field (red is repulsive potential, yellow is attractive potential).

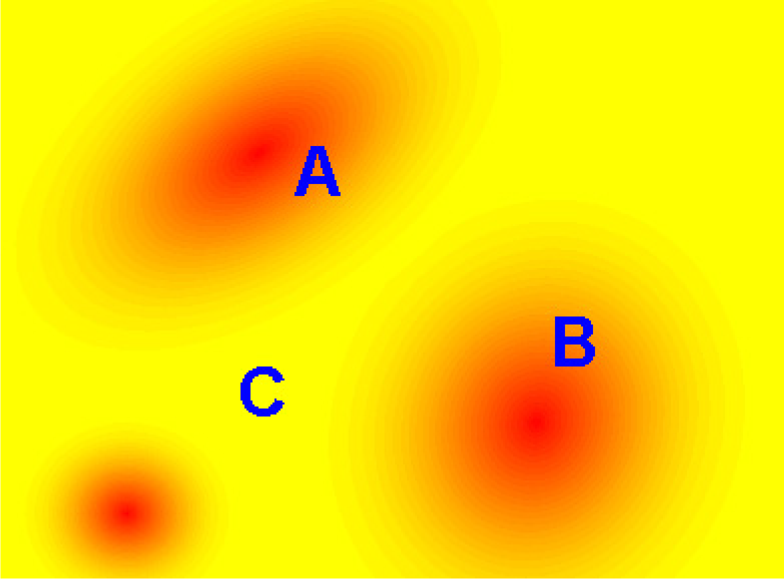


Figure 5.3 A complex potential field

Show description

Reveal answer

Figure 5.5 shows a practical application of the concept of a potential field.

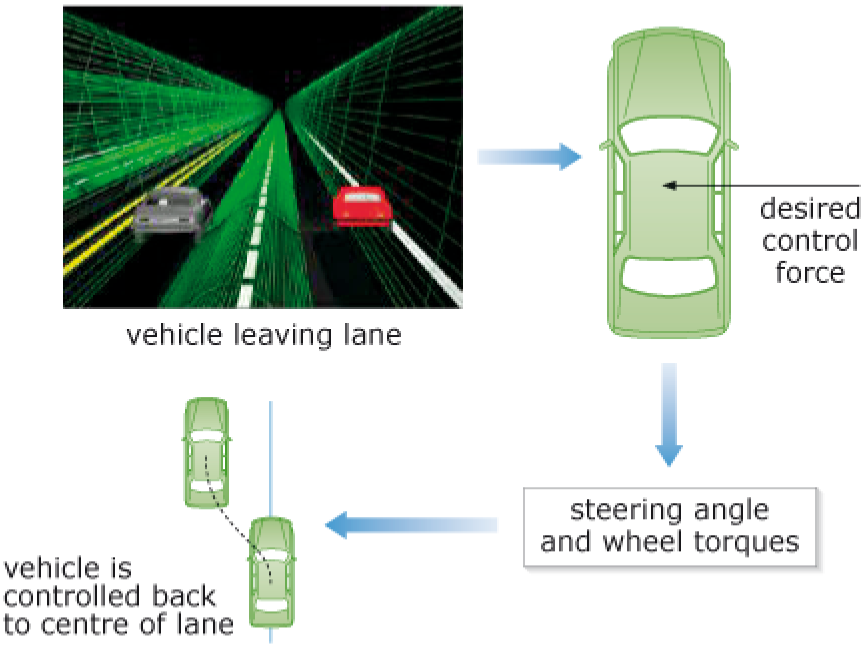


Figure 5.5 Potential fields used to keep cars in their lanes

Show description

Figure 5.5 shows how robotic cars could be kept in lane by use of potential fields. The figure shows two lanes on one side of a dual carriageway. The example is from the USA where cars drive on the right. The repulsive potential field is superimposed; it forms high banks at the edge of the carriageway and central reservation, with a low ridge between the two lanes. One car (red in the figure) has strayed to the edge of its lane, into the area of high repulsion where a corrective motion would be applied to bring it back to the area of zero repulsive potential at the centre of the lane.

The potential field approach has some benefits for a roboticist. Particularly if the robot can sense the potential in some way (for example, by using information from distance sensors), it only needs local information and is very efficient to calculate. The more traditional alternative of planning a course requires more 'global' information and uses optimisation techniques that you've seen can require considerable processing. On the other hand, you've also seen that using only local information can lead to a robot becoming 'stuck' in a local optimum and not able to reach the best solution.

Some of Asimov's *I, Robot* stories serve as illustrations of potentials.

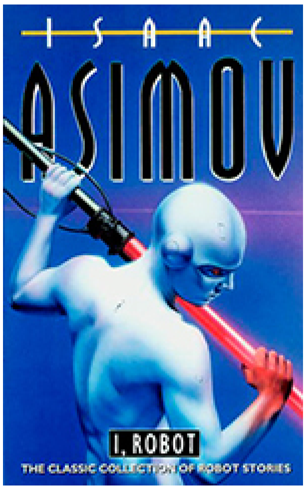
SAQ 5.3

Earlier you read Asimov's story 'Runaround'. Can you explain Speedy's behaviour in terms of potential fields?

Reveal answer

Activity 5.1 Read

Read Chapter 6, 'Little Lost Robot', of Asimov's book *I, Robot* and then answer the following SAQ.



SAQ 5.4

a. Summarise the story of 'Little Lost Robot'.
b. Explain how the potentials of the laws interact in this story.

Reveal answer

6 Practical activities

Activity 6.1 Robot practical activities

Follow the instructions on this week of the study planner and carry out the activities.

7 Summary

In this topic, you have considered optimisation in problem solving and how this can apply to the development of robots. In particular, you have considered:

- optimisation in terms of searching for good solutions to problems, and the ways in which the spaces of all possibilities can be searched; this includes structuring the search space, as in the holiday selection problem, or incremental hill-climbing towards better solutions in a landscape representing the goodness of solutions
- the travelling salesman problem, which is a classic problem in artificial intelligence because, even for small numbers of destinations, it has an astronomical number of solutions
- genetic algorithms, which can be used to search spaces in which the solutions can be represented as sequences of numbers – in the case of the travelling salesman problem, the numbering of the destinations along a route
- game theory, and how choices can be made in situations in which the players learn from previous experiences
- how potential fields can be used to create imaginary forces to navigate robots and explain the interactions between Asimov's laws.

Optimisation is a large field of great importance in robotics, and also in human planning and management. Although we have only been able to scratch the surface here, we hope this has given you an insight into a research area that will have a big impact on the development of robots in the future.

Where next

This is the end of Topic 5.

Topic 6 examines how the Internet of Things is changing the way in which humans and machines interact.

Acknowledgements

Grateful acknowledgement is made to the following sources:

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Activities 3.1 and 5.1: Taken from: https://www.bustle.com/articles/49003-11-fictional-female-scientists-wholl-stun-you-with-their-brilliance

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