Cours

$$x \xrightarrow{f} \ln(u(x))$$

① $u(x) > 0$ sur I

② u d^{ble} sur I

$\Longrightarrow f = \ln(u)$

est d^{ble} sur I

et $f'(x) = \dfrac{u'(x)}{u(x)}$

$$\int_1^2 \frac{2x+1}{x^2+x}\,dx = \Big[\ln(x^2+x)\Big]_1^2$$

$u(x) = x^2 + x$

$u'(x) = 2x+1$

		-1		0	
x^2+x $x(x+1)$	$+$	0	$-$	0	$+$

* **Théo** $\quad x \xrightarrow{f} \ln|u(x)|$

① $u(x) \neq 0$ sur I

② u d^{ble} sur I

$\Longrightarrow f$ d^{ble}

$f'(x) = \dfrac{u'(x)}{u(x)}$

$$\int_{-2}^{-1} \frac{\overset{\to u'}{1}}{\underset{\to u}{x-1}}\,dx$$

	-2	-1	1	
$x-1$		$-$	0	$+$

$$= \Big[\ln|x-1|\Big]_{-2}^{-1}$$

f	I	F
$\frac{1}{x}$	$x>0$	$\ln(x)$
$\frac{1}{x}$	$x<0$	$\ln\lvert x\rvert$
$\frac{u'}{u}$	$u(x)>0$	$\ln(u)$
$\frac{u'}{u}$	$u(x) \neq 0$ sur I	$\ln\lvert u\rvert$

$$\int_{-1}^{-\frac{1}{e}} \frac{1}{x}\,dx = \Big[\ln|x|\Big]_{-1}^{-1/e} = (\quad) - (\quad)$$

$$\lim_{0^+} x^n \cdot \ln^m(x) = 0$$

Suite Exercice 2 :

$$\times \quad \lim_{x\to 0^+} \underset{\downarrow\, 0^+}{x} \sqrt{3+2\ln^2(x)}$$

$$0^+ \times \sqrt{3 \quad +\infty} \qquad \text{FI}$$

$$\lim_{0^+} \frac{x \cdot (3+2\ln^2 x)}{\sqrt{3+2\ln^2 x}}$$

$$= \lim_{x\to 0^+} \frac{3\overset{\to 0}{x} + 2\overset{\to 0}{(x\ln^2 x)}}{\underset{\to +\infty}{\sqrt{3+2\ln^2(x)}}} = 0$$

$\boxed{\text{ou}}$

$$x = |x| = \sqrt{x^2} \qquad x>0$$

$$\lim_{0^+} x\sqrt{3+2\ln^2(x)} = \lim_{0^+} \sqrt{x^2}\sqrt{3+2\ln^2 x}$$

$$= \lim_{0^+} \sqrt{3\underset{\to 0}{x^2} + 2\underset{\to 0}{(x^2\ln^2 x)}}$$

$$= 0$$

$$* \quad \lim_{x\to+\infty} \frac{\overset{\to +\infty}{(\ln x)^2}}{\underset{\to +\infty}{\sqrt[3]{x}}} \;;$$

F I

$$\boxed{\lim_{x\to+\infty} \sqrt[n]{x} = +\infty}$$

$$* \quad \lim_{x\to+\infty} \frac{\left(\frac{\ln^2(x)}{x}\right) \to 0}{\left(\frac{\sqrt[3]{x}}{x}\right) \to ?}$$

$$\boxed{\lim_{+\infty} \frac{\ln^n(x)}{x^m} = 0}$$

$x > 0$

$$\bullet \; x = \sqrt[2]{x^2}$$

$$\bullet \; x = \sqrt[3]{x^3}$$

$$= \lim_{x\to+\infty} \frac{\frac{\ln^2(x)}{x}}{\frac{\sqrt[3]{x}}{\sqrt[3]{x^3}}}$$

$$= \lim_{+\infty} \frac{\left(\frac{\ln^2(x)}{x}\right) \to 0}{\left(\sqrt[3]{\frac{\cancel{x}}{x^{2}}}\right) \to 0} \qquad X$$

$$\ln\sqrt{x} = \frac{1}{2}\ln x$$

$$\ln\left(\sqrt[n]{x}\right) = \frac{1}{m}\ln(x)$$

$$\boxed{m\ln\left(\sqrt[n]{x}\right) = \ln(x)}$$

$$2\ \ln\sqrt[2]{x} = \ln(x)$$

$$\boxed{3.\ \ln\left(\sqrt[3]{x}\right) = \ln(x)}$$

$$\times \lim_{x\to+\infty} \frac{\left(\boxed{\ln x}\right)^2}{\sqrt[3]{x}} = \lim_{+\infty} \frac{\left(\boxed{3\ln\sqrt[3]{x}}\right)^2}{\sqrt[3]{x}}$$

$$= \lim_{+\infty} 9 \cdot \frac{\left(\ln \boxed{\sqrt[3]{x}}\right)^2}{\boxed{\sqrt[3]{x}}}$$

$t = \boxed{\sqrt[3]{x}}$ si $x \to +\infty$ $t \to +\infty$

$$= \lim_{t\to+\infty} 9 \ \underbrace{\frac{(\ln t)^2}{t}}_{\to 0} = 0 \ ✓$$

$$\lim_{x\to0} \frac{\ln\left(\boxed{\overset{\overset{1}{\uparrow}}{\cos x}}\right)}{x} \begin{matrix}\to 0 \\ \to 0\end{matrix} ;$$

FI

$$\lim_{X\to1} \frac{\ln\left(\boxed{X}\right)}{\boxed{X} - \boxed{1}} = 1$$

$$= \lim_{X\to0} \underbrace{\frac{\ln(\cos x)}{\cos x - 1}}_{\downarrow \ ?} \times \underbrace{\frac{\cos x - 1}{x}}_{\to 0}$$

$$\lim_{x\to 0}\frac{\ln(\boxed{\cos x})}{\boxed{\cos x}-1}$$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{x}=0}$$

$t=\cos x$

$x\to 0 \quad t\longrightarrow 1$

donc $\displaystyle\lim_{x\to 0}\frac{\ln(\boxed{\cos x})}{\boxed{\cos x}-1}=\lim_{t\to 1}\frac{\ln t}{t-1}=1$

d'où $\displaystyle\lim_{x\to 0}\underbrace{\frac{\ln(\cos x)}{\cos x-1}}_{\downarrow\ 1}\times\underbrace{\boxed{\frac{\cos x-1}{x}}}_{(-0)}$

$1 \times (-0)$

$$\boxed{=0.}$$

$$\lim_{x\to 0^+}\frac{\ln(\overset{1}{\boxed{1-\sqrt{x}}})\to 0}{x\ \to 0}.$$

$t=1-\sqrt{x}$

$t\longrightarrow 1$

$$=\lim_{x\to 0}\frac{\ln(\boxed{1-\sqrt{x}})}{(\boxed{1-\sqrt{x}})-1}\times\frac{-\sqrt{x}}{x}$$

$$\boxed{\lim_{x\to 1}\frac{\ln x}{x-1}=1}$$

$$=\lim_{x\to 0}\underbrace{\frac{\ln(\boxed{1-\sqrt{x}})}{(\boxed{1-\sqrt{x}})-1}}_{1}\times\underbrace{\boxed{\frac{-1}{\sqrt{x}}}}_{-\infty}=-\infty$$

$$* \quad \lim_{X\to 1} \frac{\ln X}{X-1} = \underline{1} \longrightarrow \lim_{X\to 0} \frac{\ln(X+1)}{x} = \underline{1}$$

$$\lim_{x\to 0^+} \frac{\overset{\to 0}{\ln(x+1)}}{\underset{\to 0}{x^2}} = \lim_{x\to 0^+} \underbrace{\frac{\ln(x+1)}{x}}_{\downarrow \atop 1} \cdot \underbrace{\frac{1}{x}}_{\downarrow \atop +\infty}$$

$$= +\infty$$

Exercice 3 25 min 5 pts

$$\lim_{x\to 0^+} \frac{\ln(\overbrace{1-\sqrt{x}}^{t})}{-\sqrt{x}} \rightarrow g(x) \qquad \frac{\ln t}{t-1}$$

Soit $f(x) = \dfrac{\ln x}{x-1}$

donc $g(x) = f(1-\sqrt{x})$

$$\begin{cases} \lim_{x\to 0} 1-\sqrt{x} = 1 \\ \lim_{x\to 1} f(x) = 1 \end{cases} \quad \text{donc } \lim_{x\to 0} g(x) = \underline{1}$$

$$\lim_{x\to+\infty} \frac{\ln(1+\overset{\to+\infty}{x^2}-x)}{x} \quad \begin{matrix} \}\to+\infty \\ \}\to+\infty \end{matrix} \quad F \qquad \frac{\ln +}{+}$$

$$= \lim_{x\to+\infty} \frac{\ln(x^2-x+1)}{x^2-x+1} \times \frac{x^2-x+1}{x}$$

$$\downarrow 0 \quad \times \quad \downarrow +\infty$$

$$\lim_{x\to+\infty} \frac{\ln\left(x^2\cdot\left(1-\frac{1}{x}+\frac{1}{x^2}\right)\right)}{x} = \lim_{+\infty} \frac{\ln x^2 + \ln\left(1-\frac{1}{x}+\frac{1}{x^2}\right)}{x}$$

$$= \lim_{+\infty} \underbrace{\frac{2\ln x}{x}}_{\to 0} + \frac{\ln\left(1-\overset{\to 0}{\frac{1}{x}}+\overset{\to 0}{\frac{1}{x^2}}\right) \to 0}{x \to +\infty} =$$

$$0 + 0 = 0$$

Exercice 3 — 25 min — 5 pts

$$\begin{cases} U_n = e \\ (U_{n+1})^2 = \dfrac{U_n}{e} \end{cases}$$

$$v_n = \frac{1 + \ln(U_n)}{e}$$

$$\ln\left(x^n\right) = n\ln x$$

$$\boxed{\frac{1}{n}\ln\left(x^n\right) = \ln x}$$

$$v_{n+1} \overset{?}{=} \frac{1}{2} v_n$$

- $$v_{n+1} = \frac{1+\ln\left(U_{n+1}\right)}{e}$$

$$= \frac{1 + \frac{1}{2}\cdot\ln\boxed{\left(U_{n+1}^{2}\right)}}{e}$$

$$= \frac{1 + \frac{1}{2}\ln\boxed{\left(\frac{U_n}{e}\right)}}{e}$$

$$= \frac{1 + \frac{1}{2}\left(\ln\left(U_n\right) - \overset{1}{\ln e}\right)}{e}$$

$$= \frac{\boxed{1} + \boxed{\frac{1}{2}\ln\left(U_n\right)} - \boxed{\frac{1}{2}}}{e}$$

$$v_{n+1} = \frac{\boxed{\frac{1}{2}} + \boxed{\frac{1}{2}}\ln\left(U_n\right)}{e}$$

$$v_{n+1} = \boxed{\frac{1}{2}} \; \boxed{\frac{1+\ln\left(U_n\right)}{e}}$$

$$v_{n+1} = \frac{1}{2} \; v_n$$

donc (v_n) est une suite géo de raison $\frac{1}{2}$

2/ $v_n = v_0 \times q^n = v_1 \times q^{n-1}$

$q = \frac{1}{2}$ $n \geq 1$ $v_1 = ?$

$$v_1 = \frac{1+\ln(U_1)}{e} = \frac{1+\ln e}{e} = \underline{\underline{\frac{2}{e}}}$$

d'où $v_n = v_1 \times q^{n-1}$

$$= \frac{2}{e} \times \left(\frac{1}{2}\right)^{n-1} = \frac{2}{e} \times \frac{1}{2^{n-1}}$$

$$\frac{2^1}{2^{n-1}} = \frac{1}{2^{n-2}}$$

$$\boxed{v_n = \frac{1}{e} \times \frac{1}{2^{n-2}} \quad n \geq 1}$$

$$v_n = \frac{1+\ln(U_n)}{e}$$

$$e \cdot v_n = 1 + \ln(U_n)$$

$$(e \cdot v_n - 1) = \ln(U_n)$$

$$e^{(e \cdot v_n - 1)} = U_n$$

$$\left\{ \ln(x) = Y \Longrightarrow x = e^{y}. \right.$$

$$e^{e \times \frac{1}{e} \times \frac{1}{2^{n-2}}} = U_n$$

d'où $\boxed{U_n = e^{\frac{1}{2^{n-2}}}.}$ $n \geq 1$

3) $$v_n = \frac{1}{e} \frac{1}{2^{n-2}} = \underbrace{\frac{1}{e}}_{} \cdot \frac{1}{\underset{\to +\infty}{2^n}} \times \underbrace{\frac{1}{2^{-2}}}_{}$$

$$\left\{ \lim_{+\infty} v_n = 0 \right.$$

$\boxed{1 < q = 2}$

$$\lim_{n \to +\infty} e^n = +\infty$$

$1 < 2 < q = e$

× $$U_n = e^{\frac{1}{2^{n-2}}} = e^{\overset{\to 0}{\frac{1}{2^n}} \times \frac{1}{2^{-2}}}$$

$$= e^0 = 1$$

× $\ln(e^{x}) = n$ $\quad \left(f(f^{-1}(x)) = x \right.$

$e^{\ln(x)} = x \quad x > 0$

Exercice 5 · 30 min · 5 pts

. $\ln(1+x) \leq x \quad \forall\, x > 0$?

on pose $f(x) = \ln(1+x) - x \overset{?}{\leq} 0$

$$f'(x) = \frac{1}{1+x} - 1$$

$\ln u$

$$= \frac{1 - 1 - x}{1+x} = \frac{-x}{\boxed{1+x}_{+}} < 0$$

x	0		$+\infty$
$f'(x)$		$-$	
$f(x)$	0	$\searrow$	0

$f(0) = \ln(1+0) - 0 = 0$

$0 \leq x$ f est strict ↓

donc $\underbrace{f(0)}_{} \geq f(x)$

$0 \geq Ln(1+x) - x$

donc $\boxed{Ln(1+x) \leq x \qquad \forall x \geq 0}$

2) $Ln(1+x) \leq x \qquad \forall x > 0$

$x = \frac{1}{n} > 0$

donc $Ln\left(1 + \frac{1}{n}\right) \leq \frac{1}{n}$

$\Rightarrow Ln\left(\frac{n+1}{n}\right) \leq \frac{1}{n}$

$\Rightarrow \boxed{Ln(n+1) - Ln(n) \leq \frac{1}{n}.}$ *

3) $U_n = 1 + \frac{1}{2} + \dots + \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{k}$

on a $Ln(1+k) - Ln(k) \leq \frac{1}{k} \qquad k>0$

donc $$\sum_{k=1}^{m} \ln(1+k) - \ln(k) \leq \underbrace{\sum_{k=1}^{n} \frac{1}{k}}_{U_n}$$

$$\cancel{\ln(2)} - \underbrace{\ln 1}_{0} + \cancel{\ln 3} - \cancel{\ln 2} + \cancel{\ln 4} - \cancel{\ln 3} + \cdots + \ln(n+1) - \cancel{\ln(n)}$$

$$\boxed{\ln(m+1) \leq U_n}$$

$$\lim_{m \to +\infty} \ln(m+1) = +\infty \quad \text{donc} \quad \lim_{m \to +\infty} U_n = +\infty$$

(U_n) diverge. %

$$U_k \leq V_k \Longrightarrow \sum U_k \leq \sum V_k$$

- $\sum_{k=1}^{m} 2 = 2 + 2 + 2 + \cdots + 2 = 2 \times m$
- $\sum_{k=\textcircled{1}}^{\textcircled{n}} \frac{1}{n} = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} + \cdots + \frac{1}{n} = \frac{1}{n} \times n = 1$
- $\sum_{k=1}^{n} \frac{1}{k} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} = U_n$

Serie (Log et espace):

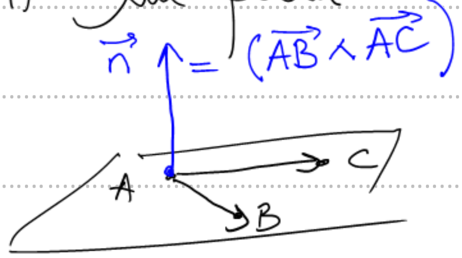
$$\left.\begin{array}{l} A(0,1,0)\\ B(1,0,0)\\ C(0,2,1)\end{array}\right\} \quad \overrightarrow{AB}\begin{pmatrix}\boxed{1}\\ \boxed{-1}\\ 0\end{pmatrix} \quad \overrightarrow{AC}\begin{pmatrix}\boxed{0}\\ \boxed{1}\\ 1\end{pmatrix}$$

$$\frac{0}{1} \neq \frac{1}{-1}$$

$\overrightarrow{AB}$ et $\overrightarrow{AC}$ ne sont colineaires

A, B et C ne sont pas alignés

donc ils forment un plan.



b) $M(x, y, 3) \in (ABC) \iff$

$$ax + by + cz + d = 0$$

R.O.N $\quad \vec{n}\begin{pmatrix}a\\ b\\ c\end{pmatrix}$ = Vect. normal à (ABC)

$$\overrightarrow{AB}\begin{pmatrix}1\\-1\\0\end{pmatrix}\wedge\overrightarrow{AC}\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}-1\\-1\\1\end{pmatrix}$$

$1 \qquad 0$

$-1 \qquad 1$

$\overrightarrow{AB}\wedge\overrightarrow{AC}=\begin{pmatrix}-1\\-1\\1\end{pmatrix}$ est un Vect normal (ABC)

$(ABC) : -x - y + z + d = 0$

$A(0,1,0) \in (ABC) \Longrightarrow 0 - 1 + 0 + d = 0$

$d = 1$

d'où

$$(ABC) : \boxed{-x - y + z} + 1 = 0$$

Cours

$$T_{\overrightarrow{AB}} : \mathcal{E} \longrightarrow \mathcal{E}$$
$$M \longrightarrow M'$$

$$\overrightarrow{MM'} = \overrightarrow{AB}$$

$$\begin{pmatrix}x'-x\\y'-y\\z'-z\end{pmatrix}=\boxed{\begin{pmatrix}a\\b\\c\end{pmatrix}}$$

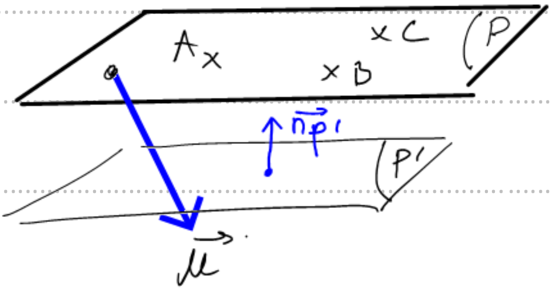
$$\begin{cases} x' = x + a \\ y' = y + b \\ z' = z + c \end{cases}$$

App $\overrightarrow{AB}\begin{pmatrix}1\\-1\\0\end{pmatrix}$ $\quad C(0,2,1)$

$$T_{\overrightarrow{AB}}(C) = C' \Rightarrow \begin{cases} x_{C'} = x_C + 1 \\ y_{C'} = y_C - 1 \\ z_{C'} = z_C + \end{cases}$$

d'où $$\begin{cases} x_{C'} = 0 + 1 = 1 \\ y_{C'} = 2 - 1 = 1 \\ z_{C'} = 1 \end{cases}$$

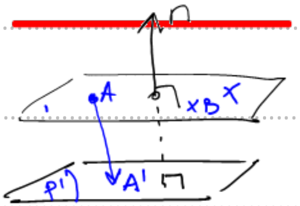
$$\boxed{C' = (1, 1, 1)}$$



on a $(ABC) : -x - y + z + 1 = 0$

$\vec{u}\begin{pmatrix}2\\-1\\3\end{pmatrix}$ $\quad T_{\vec{u}}(ABC) = P'$

Equation cartesienne de P'



$(P') \parallel (ABC)$

donc $\vec{AB} \wedge \vec{AC} = \begin{pmatrix}-1\\-1\\1\end{pmatrix}$ est aussi

un vect. normal au plan P'

d'où $(P') : \boxed{-x - y + z} + \overset{?}{d} = 0$

$A\begin{pmatrix}0,1,0\\ \downarrow\ \downarrow\ \downarrow\end{pmatrix} \in (ABC) \quad T_{\vec{u}}(A) = A' \in P'$ $\quad \vec{u}\begin{pmatrix}2\\-1\\3\end{pmatrix}$

$$\begin{cases} x_{A'} = x_A + 2 = 2 \\ y_{A'} = y_A - 1 = 0 \\ z_{A'} = z_A + 3 = 3 \end{cases}$$

$$A'(2,0,3) \in P' : -x - y + z + d = 0$$

$$-2 - 0 + 3 + d = 0$$

$$d = -1$$

$$\boxed{(P') : -x - y + z - 1 = 0}$$

$$h_{(I,k)} : \begin{array}{ccc} \mathcal{E} & \longrightarrow & \mathcal{E} \\ M & \longrightarrow & M' \end{array}$$

$$\overrightarrow{IM'} = k\,\overrightarrow{IM}$$

$$\begin{pmatrix} x' - x_I \\ y' - y_I \\ z' - z_I \end{pmatrix} = k \begin{pmatrix} x - x_I \\ y - y_I \\ z - z_I \end{pmatrix}$$

$$\begin{cases} x' = kx + (1-k)\,x_I \\ y' = ky + (1-k)\,y_I \\ z' = kz + (1-k)\,z_I \end{cases}$$