Exercice 3:

$f(x) = \sqrt{e^x - 1}$

$D_f = \{x \in \mathbb{R} \; ; \; e^x - 1 \geq 0\}$.

$x = ?$ $\quad e^x - 1 \geq 0 \iff e^x \geq 1$

$\iff x \geq 0$

$D_f = [0 ; +\infty[$.

1)

- $\lim\limits_{x \to +\infty} \sqrt{\underbrace{e^x}_{+\infty} - 1} = +\infty$

- $\lim\limits_{+\infty} \dfrac{\sqrt{e^x - 1}}{x} = \lim\limits_{x \to +\infty} \sqrt{\dfrac{e^x - 1}{x^2}}$

$= \lim\limits_{+\infty} \sqrt{\dfrac{e^x}{x^2} - \dfrac{1}{x^2}} = +\infty$

Int. graphi:

$\lim\limits_{+\infty} f = +\infty$ et $\lim\limits_{+\infty} \dfrac{f(x)}{x} = +\infty$

C_f admet une B.Parab de dir $(O\vec{j})$

au Vois $(+\infty)$.

2) $\lim\limits_{x \to 0^+} \dfrac{f(x)}{x} = \lim\limits_{0^+} \dfrac{\sqrt{e^x - 1}}{x}$

$= \lim\limits_{x \to 0^+} \underbrace{\dfrac{e^x - 1}{x}}_{\to 1} \times \dfrac{1}{\underbrace{\sqrt{e^x - 1}}_{\to 0^+}} = +\infty$

$$\lim_{x\to 0^+} \frac{f(x)}{x} = \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0} = +\infty$$

donc f admet une demi-tgte verticale dirigée vers le haut au pt $(0,0)$

2°) b) $f(x) = \sqrt{e^x - 1}$ $\quad x>0$

$$f'(x) = \frac{(e^x-1)'}{2\sqrt{e^x-1}} = \frac{e^x}{2\sqrt{e^x-1}}$$

c) $f'(x) = \dfrac{e^x}{2\sqrt{e^x-1}} > 0$

x	0	$+\infty$
$f'(x)$	|	$+$
$f(x)$	0 ↗	$+\infty$

$$0 \leq x \leq \ln 2 \iff e^x - 1 \overset{?}{\leq} \sqrt{e^x-1}$$

$$0 \leq a \leq 1 \iff a^2 \leq \boxed{a \leq \sqrt{a}}$$

$$0 \leq x \leq \ln 2 \iff e^0 \leq e^x \leq e^{\ln 2}$$

$$\iff 1 \leq e^x \leq 2$$

$$\iff 0 \leq e^x - 1 \leq 1$$

$$\iff e^x - 1 < \sqrt{e^x - 1}.$$

3) $$f'(x) = \frac{1}{2} \cdot \frac{e^x}{\sqrt{e^x - 1}}$$

$$\Rightarrow f''(x) = \frac{1}{2}\left(\frac{e^x\sqrt{e^x-1} - e^x \cdot \frac{e^x}{2\sqrt{e^x-1}}}{e^x - 1}\right)$$

$$= \frac{e^x}{2}\left(\frac{\frac{\sqrt{e^x-1}}{1} - \frac{e^x}{2\sqrt{e^x-1}}}{e^x - 1}\right)$$

$$= \frac{e^x}{2} \; \frac{\frac{2(e^x-1) - e^x}{2\sqrt{e^x-1}}}{e^x - 1}$$

$$= \underbrace{\frac{e^x}{2}}_{+} \; \frac{(e^x - 2)}{\underbrace{2\sqrt{e^x-1}}_{+}\;\underbrace{(e^x-1)}_{+}}$$

$x > 0$

$e^x > e^0$

$\boxed{e^x > 1}$

Signe $f''(x)$ = Signe $(e^x - 2)$

	0		$\ln 2$		$+\infty$
$e^x - 2$	‖	$-$	0	$+$	

. $e^{\ln 2} - 2 = 2 - 2 = 0$

. $e^x - 2 \overset{\oplus}{>} 0 \iff e^x > 2$

$\iff \underbrace{x > \ln 2}$

f est 2 fois dérivable en ln2 et

f'' s'annule en changeant de signe

$\Rightarrow B(\ln(2); f(\ln(2)))$ est

un pt d'inflexion de C_f

$f(\ln 2) = \sqrt{e^{\ln 2} - 1} = 1$

$\boxed{\text{donc } B(\ln(2), 1)}$

$ae^x = -b \qquad e^x = -\frac{b}{a} > 0 \Rightarrow x = \ln(-b/a)$

	$-\infty$	$\ln(-\frac{b}{a})$	$+\infty$
$ae^x + b$	$-$ sig de a	0	sig de a

	0	$e^{-\frac{b}{a}}$	$+\infty$
$a\ln(x) + b$	$-$ sig de a	0	sig de a.

	$-\infty$		$+\infty$
$3e^x + 2$		+	

$2e^x - 4 = 0$

$e^x = 2$

$x = \ln 2$

		$\ln 2$	
$2e^x - 4$	$-$	0	+

4) $f(x) - (e^x - 1)$

Si $0 \leq x \leq \ln 2$ on a $e^x - 1 \leq f(x)$

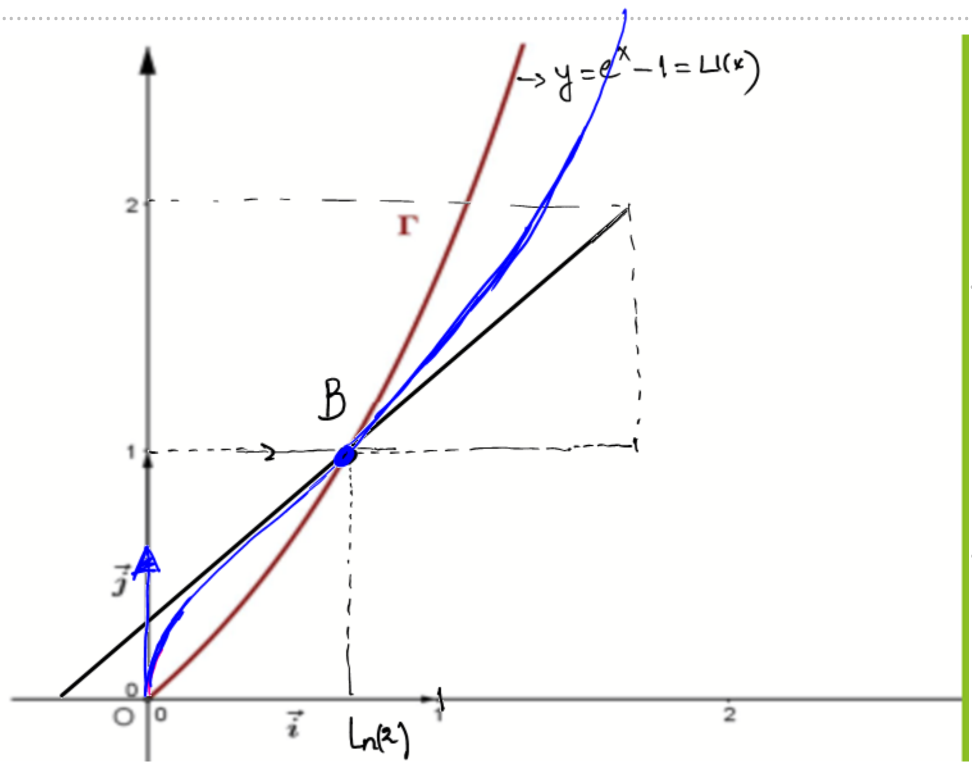
Si $\ln 2 \leq x \iff 2 \leq e^x$

$\iff 1 \leq e^x - 1$

$\iff \sqrt{e^x - 1} \leq e^x - 1$

$f(x) \leq e^x - 1$

x	0		$\ln 2$		$+\infty$
		$\mathcal{C}_f / \Gamma$		$\Gamma / \mathcal{C}_f$	



$4(\ln 2) = e^{\ln 2} - 1 = 2 - 1 = 1$

$T_B : y = f'(\ln 2)(x - \ln 2) + \underbrace{f(\ln 2)}$

$= 1 \; (x - \ln 2) + 1$

$\to$ pente $\longrightarrow$ Vect $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$y = x - \ln 2 + 1$

Ex1 (Complement Arith).

- $a \wedge b = |a| \wedge |b|$
- $a \vee b = |a| \vee |b|$
- $(a \wedge b)(a \vee b) = |ab|$

- $ka \wedge kb = |k| (a \wedge b)$.

A) $13x \equiv 0 \; (3) \iff x \equiv 0 \; (3)$

reste de x mod 3	0	1	2
$13x$	0	1	2

$13x \equiv 0 \; (3) \iff x \equiv 0 \; (3)$.

ou

$13 \equiv 1 \ (3)$

$13x \equiv x \ (3)$

$13x \equiv 0 \ (3) \iff x \equiv 0 \ (3)$

idée

$x \equiv 0 \ (3) \iff x = 3q \quad q \in \mathbb{Z}$

$\iff 13x = 3 \times \underbrace{13q}$

$\iff 13x = 3\,Q$

$\iff 13x \equiv 0 \ (3)$.

2) u est un inverse de a mod n

$\iff au \equiv 1 \ (\text{mod } n)$

$5 \times 4 \equiv 1 \ (19)$

. $\exists$ un unique $u \in \{1, 2, \dots, n-1\}$.

$au \equiv 1 \ (n)$

2) a)

. $2 \times 111 \overset{?}{\equiv} 1 \ (221)$

$2 \times \boxed{111} = 222 \equiv 1 \ (221)$

$111 \in \{1, 2, \dots 220\}$.

d'où 111 est l'unique inverse de
2 mod (221).

$\rightarrow 19 \times \boxed{u} \equiv 1 \quad (42)$

$19 \times u = 1 + 42k$

$$\boxed{19 \times u - 42k = 1}$$

$42 = 19 \times 2 + \boxed{4}$

$19 = 4 \times 4 + \boxed{3}$

$4 = 3 \times 1 + 1$

$1 = 4 - \boxed{3} \times 1$

$1 = 4 - (19 - 4 \times 4)$

$1 = 4 - 19 + 4 \times 4$

$1 = 5 \times \boxed{4} - 19.$

$1 = 5 \boxed{(42 - 19 \times 2)} - 19$

$1 = 5 \times 42 - 19 \times 10 - 19$

$1 = 5 \times 42 - 19 \times 11$

$5 \times 42 - 19 \times 11 = 1$

$19 \times (-11) = 1 + 42 \times (-5)$

$19 \times \boxed{(-11)} \equiv 1 \quad (42)$

$-11 \equiv -11 \underset{+42}{} \quad (42)$

$-11 \equiv \boxed{31} \quad (42)$

$19 \times \underline{31} \equiv 1 \quad (42)$

31 inv de 19 mod 42.

- $42 \times \overset{?}{v} \equiv 1 \quad (19)$

$42 \cdot v = 1 + 19k$

$42 \times 5 - 19 \times 11 = 1$

$42 \times 5 = 1 + 19 \times 11$

$42 \times 5 \equiv 1 \quad (19).$

5 inv de 42 mod 19

$1 = 13u + 42v$

Suite Ex

2) b) $2x \equiv 1 \ (221) \Leftrightarrow x \equiv 111 \ (221)$

$2x \equiv 1 \ (221)$

$\Rightarrow \boxed{111 \times 2} x \equiv 111 \times 1 \ (221)$

$\Rightarrow x \equiv 111 \ (221)$

$\boxed{111 \times 2} \equiv \boxed{1} \ (221)$

$a \equiv b \ (n) \Rightarrow ac \equiv bc \ (n)$

* Si $x \equiv 111 \ (221)$

$\Rightarrow 2x \equiv 222 \ (221)$ or $222 \equiv 1 \ (221)$

$\Rightarrow 2x \equiv 1 \ (221)$

d) $2x \equiv 1 \ (221)$

$\Leftrightarrow x \equiv 111 \ (221)$

$4x \equiv 6 \ (13)$

$\Rightarrow 5 \times 4x \equiv 5 \times 6 \ (13)$

$\underline{20}\, x \equiv 30 \ (13)$ or $\underline{20} \equiv 1 \ (13)$

$\Rightarrow x \equiv \underline{30} \ (13)$

$\Rightarrow x \equiv 11 \ (13)$

$(\varepsilon) \ ax + by = c.$

<u>théo</u>.

si $d = a \wedge b$ divise $c \Rightarrow$

<u>l'éq</u> (ε) admet des <u>sol</u>s.

dans $\mathbb{Z} \times \mathbb{Z}$

• $4x + 6y = 5$

$4 \wedge 6 = 2$ ne divise pas 5

$S_{\mathbb{Z} \times \mathbb{Z}} = \phi$

• $4x + 6y = 2$

$2x + 3y = 1$

B) 1) $17x - 13y = 3$.

$17 \wedge (-13) = 17 \wedge 13 = 1$

et 1 divise 3 $\Longrightarrow$ l'éq $(\mathcal{E})$

admet au moins 1 sol

b) $(-9, -12)$

$17 \times (-9) - 13 \times (-12) = \ldots = 3$

$\Rightarrow (-9, -12)$ est une sol de $(\mathcal{E})$.

c) $17x - 13y = 3$.

$17 \times (-9) - 13 \times (-12) = 3$.

$17x - 13y = 17 \times (-9) - 13 \times (-12)$

R $\otimes$ $17(\boxed{x} + 9) = 13 \cdot (12 + y)$

$\begin{cases} 13 \text{ divise } 17 \cdot (x + 9) \\ 13 \wedge 17 = 1 \end{cases}$

$\Rightarrow 13$ divise $x+9$

$\Rightarrow x+9 = 13k \qquad k \in \mathbb{Z}$

$$\boxed{x = 13k - 9}$$

$(*)\quad 17(x+9) = 13(y+12)$

$17(\cancel{13}k) = \cancel{13}(y+12)$

$17k = y+12$

$y = 17k - 12$

$$\Rightarrow \begin{cases} x = 13k - 9 \\ y = 17k - 12 \end{cases}$$

Verification:

$17(13k-9) - 13(17k-12)$

$= 17\times(-9) + 13\times 12 = 3$

Conclusion:

$$S_{\mathbb{Z}\times\mathbb{Z}} = \left\{ (13k-9\,;\,17k-12) \quad k \in \mathbb{Z} \right\}$$

$$17x - 13y = 3.$$

Sol particulière $(-9, -12)$

$x = 13k - 9$

$y = 17k - 12$

$$5x - 6y = 1$$

$$(-1, -1)$$

$\longrightarrow x = 6k - 1$

$y = 5k - 1$

2) $a = 13n - 9 \qquad n \in \mathbb{Z}$

$b = 17n - 12$

$d = a \wedge b.$

$d = (13n - 9) \wedge (17n - 12)$

(a, b) est Sol de $(\mathcal{E})$

$$17a - 13b = 3.$$

d / a et $d / b \Longrightarrow d / 17a - 13b = 3$

• $d/3$ $\left(\begin{array}{l} d=1 \\ d=3 \end{array}\right)$

$a = 13n - \boxed{9}$

$b = 17n - 12$

$n = ?$ $(13n-9) \wedge (17n-12) = 3.$

$\begin{cases} 3 \text{ divise } 13n-9 \Rightarrow \\ \quad \text{et} \\ 3 \text{ divise } 17n-12 \end{cases}$

$\begin{cases} 3/13n-9 \\ 3/9 \end{cases} \Rightarrow 3/(13n-9)+9$

$\Rightarrow 3/13n \xRightarrow{L.G} 3/n$

$3 \wedge 13 = 1$

donc $a \wedge b = 3 \Rightarrow n \equiv 0(3)$

c)

$(13 \times \boxed{2023^{2022}} - 9) \wedge (17 \times \boxed{2023^{2022}} - 12) \overset{?}{=}$

$(13 . \boxed{n} - 9) \wedge (17 \times \boxed{n} - 12) = d$

$d=1$ ou $\underline{d=3}$

$2023^{2022} \overset{?}{\equiv} (3)$

$2023 \equiv 1 \ (3) \Rightarrow 2023^{2022} \equiv 1 \ (3)$

$\not\equiv 0 \ (3)$ $\cancel{d=3}$

d'où $d=1$