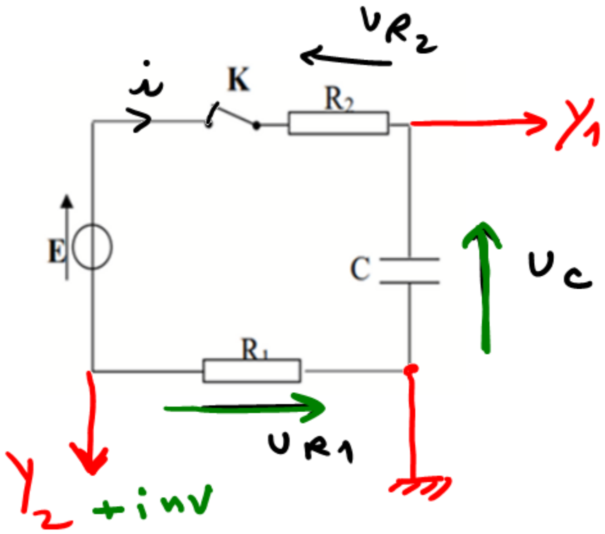
Exercice 3 :

1)



$$\begin{cases} u_C \rightarrow Y_1 \\ u_{R_1} \rightarrow Y_2 \end{cases}$$

2) a) à $t = 0$, le condensateur est déchargé $u_c(0) = 0$

$$\Rightarrow \begin{cases} (a) : u_c(t) \\ (b) : u_{R_1}(t) \end{cases}$$

b) Loi des mailles :

$$u_{R_2} + u_{R_1} + u_C - E = 0$$

à $t = 0 \Rightarrow u_c = 0$

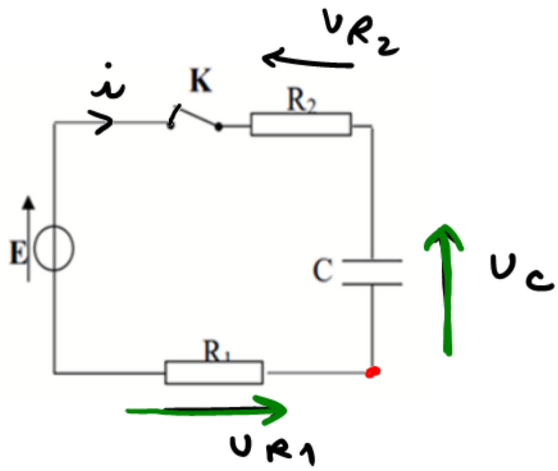
$$u_{R_2}(0) + u_{R_1}(0) = E$$

$$(R_2 + R_1)\, i(0) = E$$

$$i(0) = \frac{E}{(R_2+R_1)} = I_{max}$$

$$\boxed{U_{R_1}(0) = R_1 \frac{E}{R_2+R_1}}$$

3) a) loi des mailles :



$$i(R_2+R_1) + U_C - E = 0$$

$$(R_2+R_1)\,C\frac{dU_C}{dt} + U_C = E$$

$$\boxed{\frac{dU_C}{dt} + \frac{U_C}{(R_1+R_2)C} = \frac{E}{(R_1+R_2)C}}$$

b) En régime permanent :

U_C = constante

$$\underbrace{\frac{dU_C}{dt}}_{0} + \frac{U_C}{(R_1+R_2)C} = \frac{E}{(R_1+R_2)C}$$

$$\Rightarrow \boxed{U_C = E}$$

$* \; E = 7{,}5 \, V$

c) $u_C(t) = Ae^{\alpha t} + B$

à $t = 0$; $u_C(0) = A\overset{=1}{e^{0}} + B = 0$

$$\Rightarrow \boxed{A = -B}$$

$$u_C(t) = A e^{\alpha t} - A$$

$$* \; \frac{d}{dt} u_C(t) = \frac{d}{dt} Ae^{\alpha t} - \underbrace{\frac{d}{dt} A}_{0}$$

$$= \alpha \, Ae^{\alpha t}$$

$$* \; \alpha A e^{\alpha t} + \frac{Ae^{\alpha t}}{(R_1+R_2)C} - \frac{A}{(R_1+R_2)C} = \frac{E}{(R_2+R_1)C}$$

$$\alpha Ae^{\alpha t} - \frac{A}{(R_1+R_2)C} = \frac{E}{(R_2+R_1)C} - \frac{A e^{\alpha t}}{(R_1+R_2)C}$$

$$\begin{cases} \alpha \cancel{A} \cancel{e^{\alpha t}} = -\dfrac{\cancel{Ae^{\alpha t}}}{(R_1+R_2)C} \\ -\dfrac{A}{\cancel{(R_1+R_2)C}} = \dfrac{E}{\cancel{(R_2+R_1)C}} \end{cases}$$

$$\begin{cases} \alpha = -\dfrac{1}{(R_1+R_2)C} \\ E = -A \end{cases}$$

$$u_C(t) = A e^{\alpha t} - A$$

$$u_C(t) = -E\, e^{-\frac{t}{(R_1+R_2)C}} + E$$

$$u_C(t) = E\left(1 - e^{-\frac{t}{(R_1+R_2)C}}\right)$$

4) a) $$U_{R_1}(0) = \frac{E R_1}{R_1+R_2} = 5\,V$$

$$R_1 + R_2 = \frac{E R_1}{U_{R_1}(0)}$$

$$R_2 = \frac{E R_1}{U_{R_1}(0)} - R_1$$

$$= \left(\frac{E}{U_{R_1}(0)} - 1\right) R_1$$

$$R_2 = \left(\frac{7,5}{5} - 1\right) 500 = 250\,\Omega$$

b) à $t = \tau$; $u_C(\tau) = 0{,}63\,E$

$$u_C(\tau) = 0{,}63 \times 7{,}5$$

$$u_C(\tau) = 4{,}7\ V$$

$$\Rightarrow \boxed{\tau = 2.10^{-3}\ s.}$$

* $\tau = (R_1 + R_2)\,C$

$$C = \frac{\tau}{(R_1 + R_2)} = \frac{2.10^{-3}}{250 + 500} = 2{,}6.10^{-6} F$$

5) $u_C(t) \rightsquigarrow u_{R_1}(t)$

loi des mailles :

$$i(R_2 + R_1) + u_C - E = 0$$

$$i = \frac{E - u_C}{(R_2 + R_1)} = \frac{E - E(1 - e^{-\frac{t}{\tau}})}{(R_2 + R_1)}$$

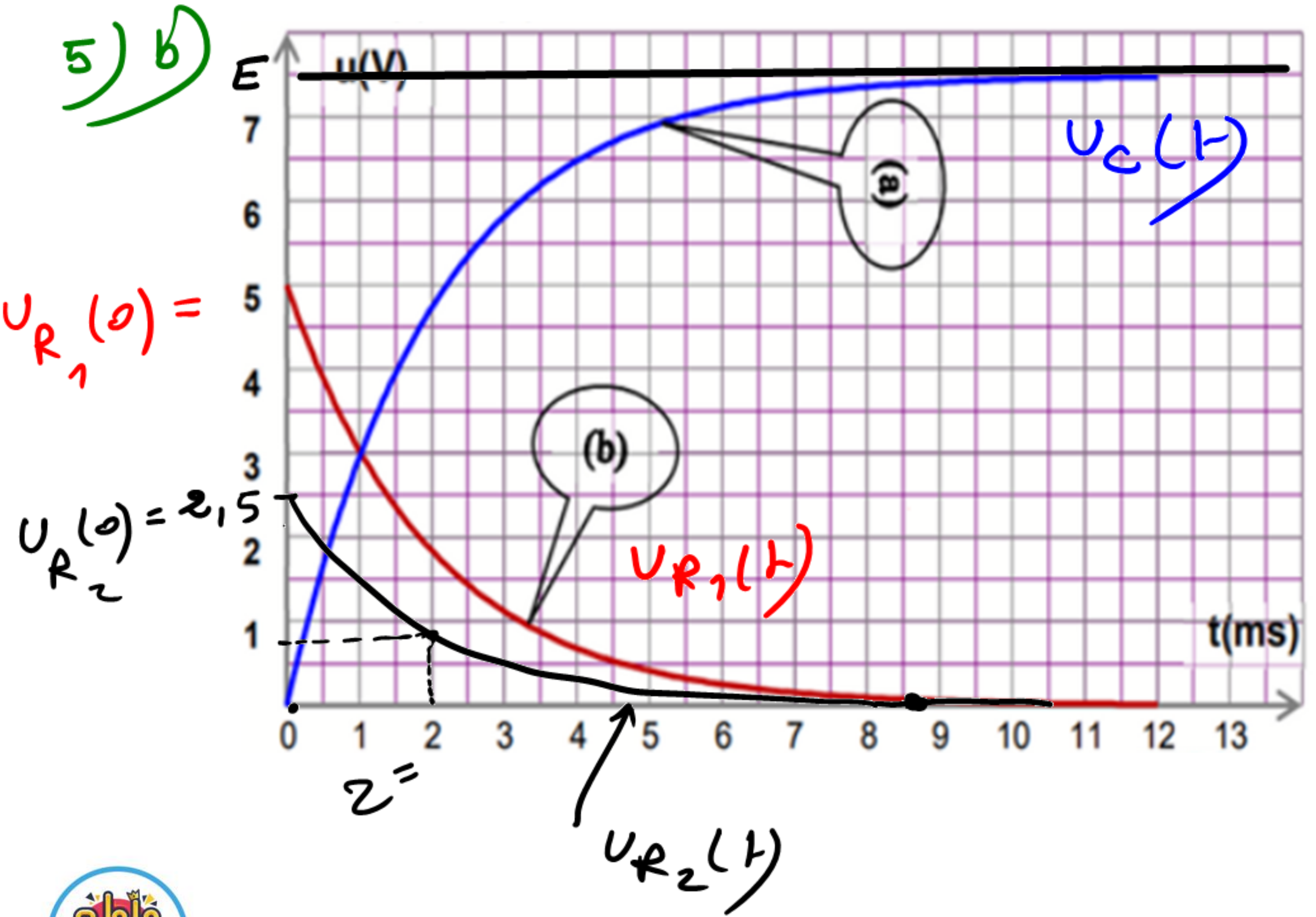
$$= \frac{\cancel{E} - \cancel{E} + Ee^{-\frac{t}{\tau}}}{R_2 + R_1}$$

$$\boxed{i(t) = \frac{E}{R_2 + R_1}\, e^{-\frac{t}{\tau}}}$$

$$U_{R_1} = R_1\, i \qquad\qquad U_{R_2} = R_2\, i$$

$$U_{R_1} = R_1 \frac{E}{R_2 + R_1}\, e^{-\frac{t}{\tau}} \qquad\qquad U_{R_2} = R_2 \frac{E}{R_2 + R_1}\, e^{-\frac{t}{\tau}}$$

5) b)



U_{R_2m}

* à $t=0$; $U_{R_2}(0) = \frac{R_2 E}{R_2+R_1} = \frac{250 \times 7,5}{250+500}$

$\Rightarrow \boxed{U_{R_2}(0) = 2,5\,V}$

* à $t=\tau$; $U_{R_2}(\tau) = 0,37 \frac{R_2 E}{R_2+R_1}$

$U_R(\tau) = 0,37 \times 2,5$

$U_R(\tau) = 0,9\,V$

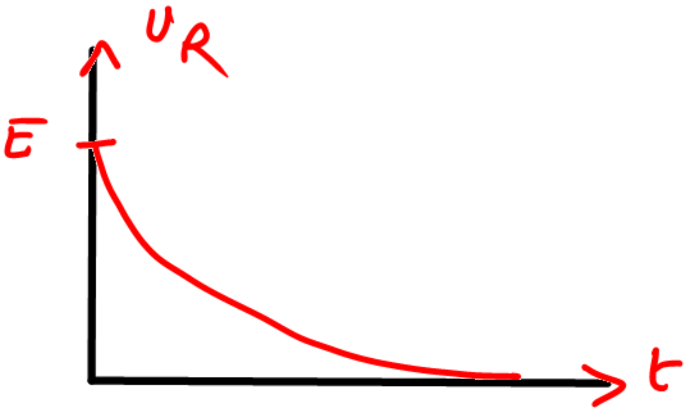
* En régime permanent $U_R = 0$

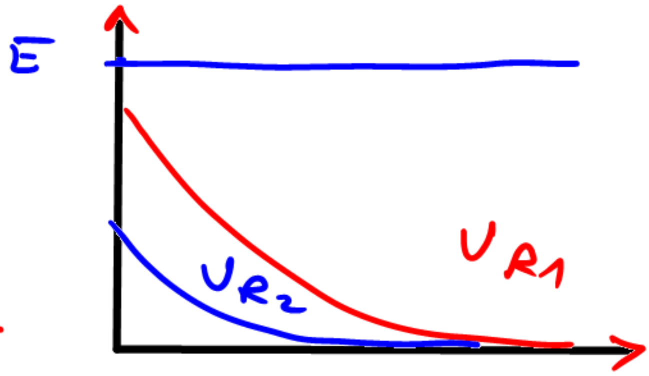
* Remarque :

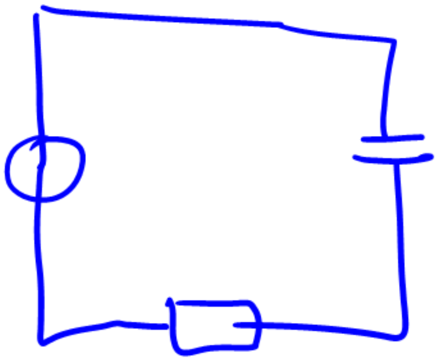
$U_{R_2} + U_{R_1} + U_C - E = 0$

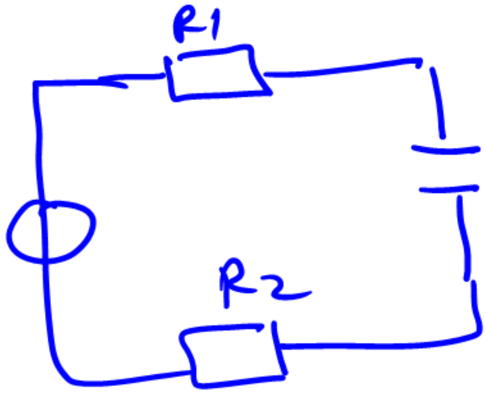
à $t=0$; $U_C = 0$

$\boxed{U_{R_2}^{(0)} + U_{R_1}^{(0)} = E}$









$$U_{R_1}(0) + U_{R_2}(0) = E$$

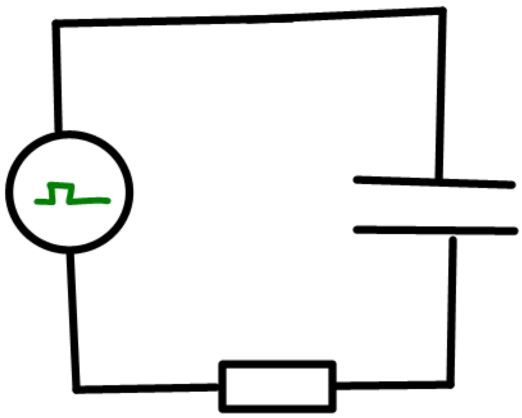
$$R_1 = 500 \qquad R_2 = 250$$

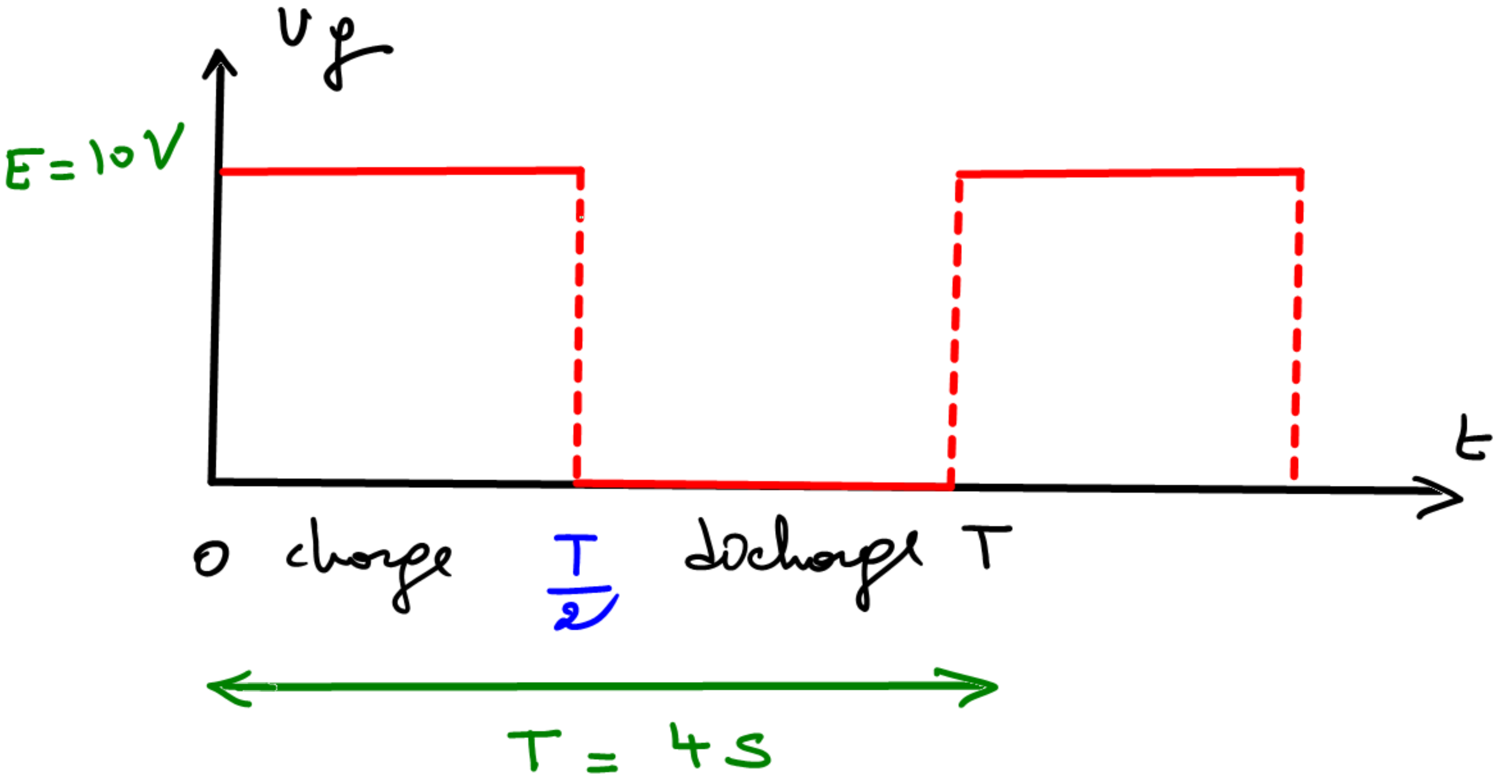
$$R_1 = 2R_2$$

$$R_1 i = 2R_2 i$$

$$U_{R_1} = 2\,U_{R_2}$$

* Remarque :





* pour que le condensateur se charge complètement :

$$\boxed{\frac{T}{2} \geq 5\tau}$$

* pour visualiser le régime permanent :

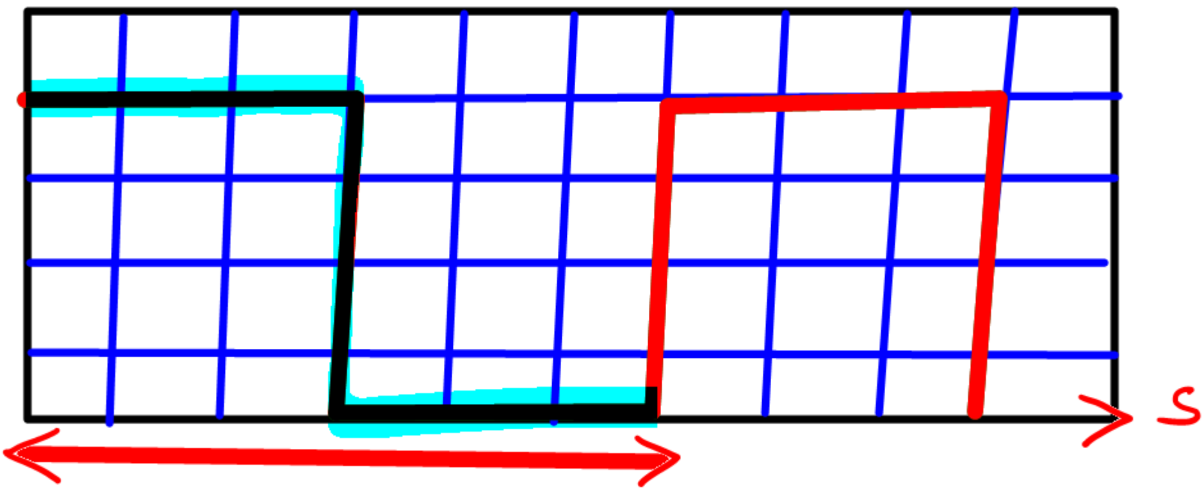
$$\boxed{\frac{T}{2} > 5\tau}$$

$$\Rightarrow \frac{1}{2N} > 5\tau$$

$$\Rightarrow 2N < \frac{1}{5\tau}$$

$$\Rightarrow N < \frac{1}{10\tau}$$

$$\Rightarrow N < \frac{1}{10RC}$$



* $T = 2.10^{-3}s \longrightarrow 6$ divisions

$1 \, div = \frac{T}{6} = \frac{2.10^{-3}}{6} = 0{,}33.10^{-3}s$