Exercice 3 :

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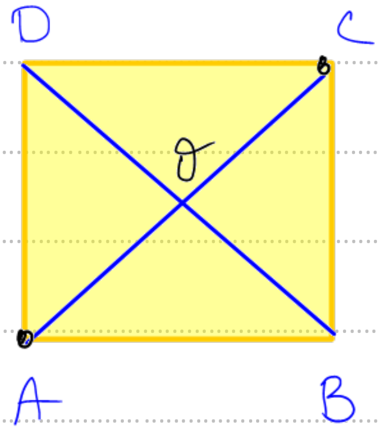


$$f = \boxed{T_{\overrightarrow{AC}}} \circ \boxed{R_{\left(A, \frac{\pi}{2}\right)}} = R_{\left(?, \frac{\pi}{2}\right)}$$

$$\begin{tikzcd} A \arrow[r, "R_{(A,\frac{\pi}{2})}"] \arrow[rr, bend right, "f"'] & A \arrow[r, "T_{\overrightarrow{AC}}"] & C \end{tikzcd}$$

on pose Ω centre de f $\quad \begin{cases} \Omega A = \Omega C & (1) \\ \left(\overrightarrow{\Omega A}, \overrightarrow{\Omega C}\right) \equiv \frac{\pi}{2} \, (2\pi) & (2) \end{cases}$

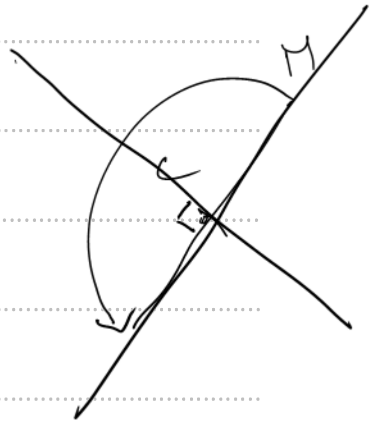
(1) $\Omega \in$ med $[AC]$

(2) $\Omega \in$ Cercle de diametre $[AC]$

① et ② D = Centre de f

$$f = T_{\vec{AC}} \circ R_{(A,\frac{\pi}{2})} = R_{(D,\frac{\pi}{2})}$$

$$g = S_C \circ \underbrace{t \circ R}$$



$$g = S_C \circ R_{(D,\frac{\pi}{2})}$$

$$= R_{(C,\pi)} \circ R_{(D;\frac{\pi}{2})}$$

Cours

theo

① $$\boxed{\varphi = R_{(\boxed{I},\alpha)} \circ R_{(\boxed{I},\beta)}}$$

$$\Longrightarrow \varphi = R_{(I,\alpha+\beta)}$$

② $$\varphi = R_{(I,\boxed{\alpha})} \circ R_{(J,\boxed{\beta})}$$

$\boxed{\alpha+\beta=0}$ $\alpha+\beta \neq 0$

$T_{\vec{u} \neq 0}$ $R(?\,,\alpha+\beta)$

$$J \xrightarrow{R_{(J,E)}} J \xrightarrow{R_I} J'$$

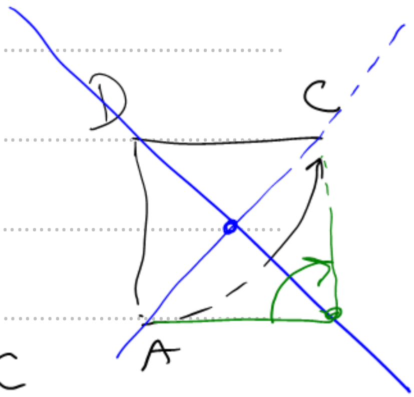
$$\vec{u} = \vec{JJ'}$$

$$g = S_C \circ R_{(D;\frac{\pi}{2})}$$

$$= R_{(C,\pi)} \circ R_{(D,\frac{\pi}{2})}$$

$$\pi + \frac{\pi}{2} = \frac{3\pi}{2} \equiv -\frac{\pi}{2}\ (2\pi) \not\equiv 0\ (2\pi)$$

$$g = R_{(?,\, -\frac{\pi}{2})}$$

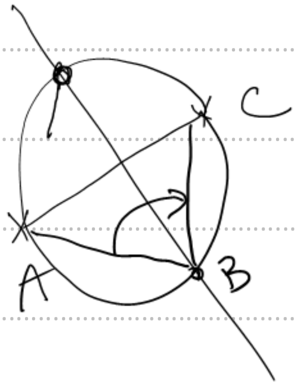


$$A \xrightarrow{R_{(D,\frac{\pi}{2})}} C \xrightarrow{S_C} C$$

$$R_{(\Omega ?,\, -\frac{\pi}{2})}(A) = C$$

$$\Omega A = \Omega C \Rightarrow \Omega \in \text{med}[AC]$$

$$(\widehat{\vec{\Omega A}, \vec{\Omega C}}) \equiv -\frac{\pi}{2} \ (2\pi)$$



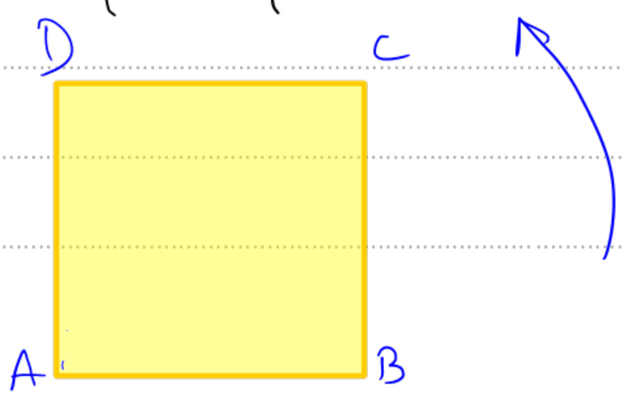
B est le centre de g

$$\Rightarrow g = R_{(B\,;\,-\frac{\pi}{2})}$$

* $f \circ g = R_{(D,\ \frac{\pi}{2})} \circ R_{(B,\ -\frac{\pi}{2})}$

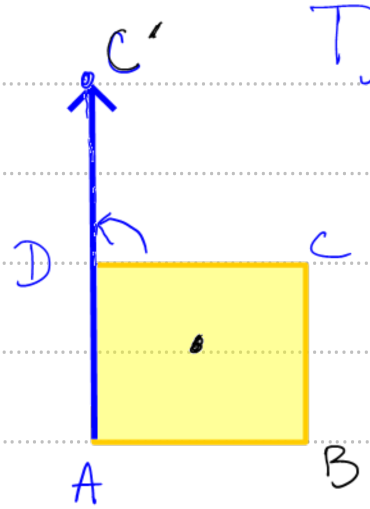
on a $\frac{\pi}{2} + \left(-\frac{\pi}{2}\right) \equiv 0 \ (2\pi)$

donc $f \circ g$ est une Translation de vect. non nul.



$$A \xrightarrow{R_{(B,-\frac{\pi}{2})}} C \xrightarrow{R_{(D,\frac{\pi}{2})}} C'$$

$$\underbrace{\qquad\qquad\qquad\qquad}_{T_{\vec{u}}}$$

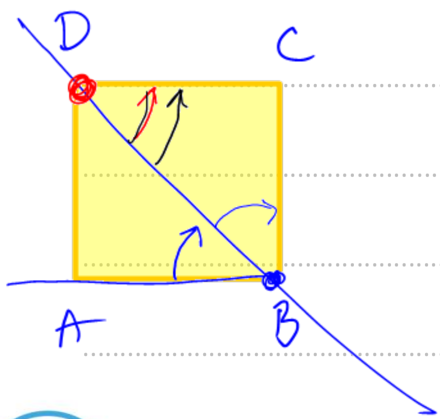


avec $C' = R_{(D,\frac{\pi}{2})}(C)$

2ème méthode

$$f \circ g = \boxed{R_{(D,\frac{\pi}{2})}} \circ \boxed{R_{(B,-\frac{\pi}{2})}}$$

$$= \underbrace{S_{(DC)} \circ S_{(DB)}} \circ \underbrace{S_{(BD)}}_{2} \circ \underbrace{S_{(BA)}}_{1}$$



$R_{\left(\boxed{D}, \frac{\pi}{2}\right)} = S_{\boxed{(DC)}} \circ S_{\boxed{(DB)}}$ car $\begin{cases} (DC) \cap (DB) = \boxed{\{D\}} \\ 2\left(\overrightarrow{DB}, \overrightarrow{DC}\right) = \frac{\pi}{2} \ (2\pi) \end{cases}$

$R_{\left(\boxed{B}, -\frac{\pi}{2}\right)} = S_{(BD)} \circ S_{(BA)}$

car $(BD) \cap (BA) = \{\boxed{B}\}$.

$2\left(\overrightarrow{BA}, \overrightarrow{BD}\right) = -\frac{\pi}{2}$)

Rapple.

- $R_{(I, \alpha)} = S_{\Delta} \circ S_{\Delta'}$?

$\Delta' = R_{\left(I, -\frac{\alpha}{2}\right)}(\Delta)$

- $R_{(I, \alpha)} = S_{\Delta'}$? $\circ S_{\Delta}$

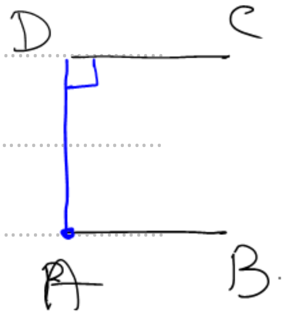
$\Delta = R_{\left(I, \frac{\alpha}{2}\right)}(\Delta)$

$$f \circ g = S_{(DC)} \circ \underbrace{S_{(BD)} \circ S_{(BD)}}\circ S_{(BA)}$$

$$f \circ g = S_{(DC)} \circ S_{\boxed{(BA)}}$$

$$\boxed{f \circ g = T_{2\vec{AD}}}$$

car $(DC) \parallel (BA)$



2) Montrer qu'il existe un unique point M du plan vérifiant

$f(M) = g(M) = M'$

$$f(M) \overset{?}{=} g(M) = M'$$

$$\begin{tikzcd} M \arrow[r, "g"] \arrow[rr, bend right] & M' \arrow[r, "f^{-1}"] & M \end{tikzcd}$$

$$f^{-1} \circ g(M) = M$$

$$\varphi = f^{-1} \circ g = \text{est un } \underline{\text{dep}}$$

f^{-1} : dep, g : dep

$$f = R_{(D,\frac{\pi}{2})} \longrightarrow f^{-1} = R_{(D,\boxed{-\frac{\pi}{2}})}$$

$$g = R_{(B,\boxed{-\frac{\pi}{2}})}$$

$$\varphi = f^{-1} \circ g = R_{(D,\boxed{-\frac{\pi}{2}})} \circ R_{(B,\boxed{-\frac{\pi}{2}})}$$

$$\boxed{-\frac{\pi}{2} - \frac{\pi}{2} =} -\pi \equiv \boxed{\pi}\ (2\pi)$$

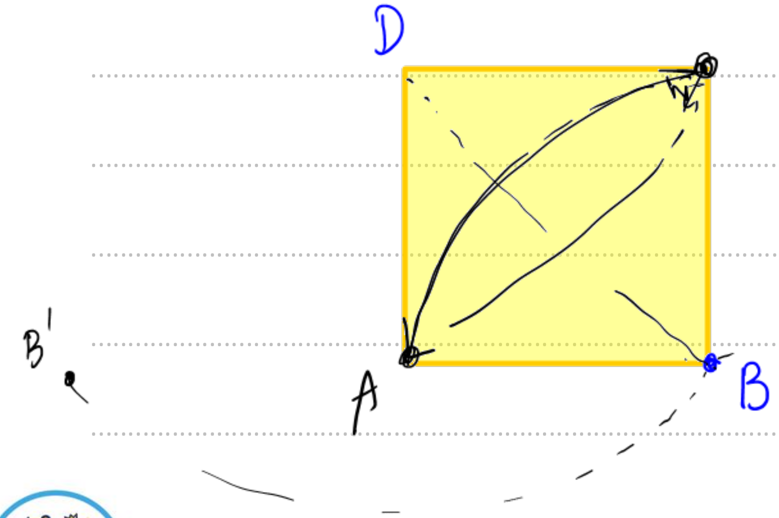
φ est une rotation d'angle π

$\Rightarrow \varphi$ = symétrie centrale $\quad \Omega = ?$

on a $A \xrightarrow{R_{(B,-\frac{\pi}{2})}} C \xrightarrow{R_{(D,-\frac{\pi}{2})}} A$

$\varphi(A) = A$

$\varphi = S_A$.



$\exists M$ tque $f(M) = g(M)$

$\Updownarrow$

$\exists A$ tq $f^{-1} \circ g(A) = A$

$\left(f^{-1}(g(A)) \right) = A$

$f\left(f^{-1}(g(A))\right) = f(A)$

$g(A) = f(A)$

Rque

- $f \circ g = S_\Delta$

$\Rightarrow \underbrace{f^{-1} \circ f} \circ g = f^{-1} \circ S_\Delta$

$g = f^{-1} \circ S_\Delta.$

- $T_{\vec{AB}} \circ R_{(C,\frac{\pi}{6})} = R_{(I,\frac{\pi}{6})}$

$\Rightarrow T_{\vec{AB}} \circ \underbrace{R_{(C,\frac{\pi}{6})} \circ R_{(C,-\frac{\pi}{6})}} = R_{(I,\frac{\pi}{6})} \circ R_{(C,-\frac{\pi}{6})}$

$T_{\vec{AB}} = R_{(I,\frac{\pi}{6})} \circ R_{(C,-\frac{\pi}{6})}$

$\times$ $S_\Delta \circ T_{\vec{AB}}(M) = N$

$\underbrace{S_\Delta \circ S_\Delta} \circ T_{\vec{AB}}(M) = S_\Delta(N)$

$T_{\vec{AB}}(M) = S_\Delta(N)$

3) Soient $I = A * B$ et $J = B * C$.

a) Montrer qu'il existe un unique déplacement φ qui envoie A en C et B en D.

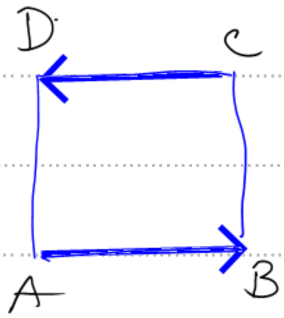
$\varphi(\boxed{A}) = \boxed{C}$

$\varphi(\boxed{B}) = \boxed{D}$

côtés d'un carré

on a $AB = CD \neq 0$ d'où l'existence et l'unicité de φ

$$\varphi \begin{cases} \to Id \\ \to T_{\vec{u}} \neq 0 \\ \to R(\ ;\alpha \neq 0) \end{cases}$$



L'angle de φ :

$$(\widehat{\vec{AB}, \vec{CD}}) \equiv \pi \ (2\pi)$$
$$\not\equiv 0 \ (2\pi)$$

$\varphi = rot(?, \pi) =$ Sym. centrale.

puisque $\varphi(A) = C$

centre de $\varphi = A * C = O$

$\varphi = S_O = R_{(O,\pi)}$.

b) Soit Ψ l'antidéplacement qui envoie A en C et B en D.

Déterminer $\psi o \varphi(C)$ et $\psi o \varphi(D)$.

on a

$\begin{cases} \varphi(A) = C \\ \varphi(B) = D \end{cases}$

$\boxed{\varphi = S_O}$

$\psi(A) = C$

$\psi(B) = D$

$\psi = ?$

S_Δ

Sym. gliss.

Si $\psi = S_\Delta$

$S_\Delta(A) = C \Rightarrow \Delta = \text{med}[AC]$

$S_\Delta(B) = D \Rightarrow \Delta = \text{med}[BD]$

or (AC) et (BD) secants O.

$\Rightarrow \quad \varphi \neq S_\Delta.$

d'où φ est une sym. glissante

$\psi = ?$ $\quad \psi(A) = C$

$\psi(B) = D.$

$A \xrightarrow{\psi} C \qquad\qquad A \xrightarrow{\varphi} C$

$B \xrightarrow{\psi} D \qquad\qquad B \xrightarrow{\varphi} D.$

$h = \psi \circ \varphi^{-1}$

$A \underset{\varphi^{-1}}{\overset{\varphi}{\rightleftarrows}} C$

$\boxed{C} \xrightarrow{\varphi^{-1}} A \xrightarrow{\psi} \boxed{C}.$

$B \underset{\varphi^{-1}}{\overset{\varphi}{\rightleftarrows}} D$

$\boxed{D} \xrightarrow{\varphi^{-1}} B \xrightarrow{\psi} \boxed{D}$

$h = \underset{\text{Anti}}{\psi} \circ \underset{\text{dep}}{\varphi^{-1}} = \text{antidep}$

$$h = S_{(CD)}$$

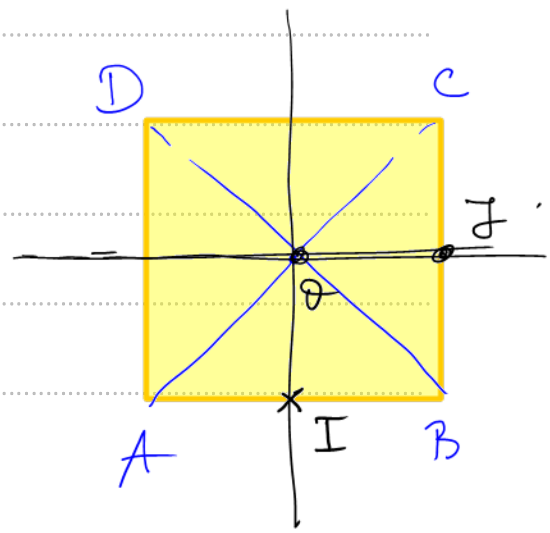
$$\psi \circ \varphi^{-1} = S_{(CD)}$$

$$\psi \circ \underbrace{\varphi^{-1} \circ \varphi} = S_{(CD)} \circ \varphi$$

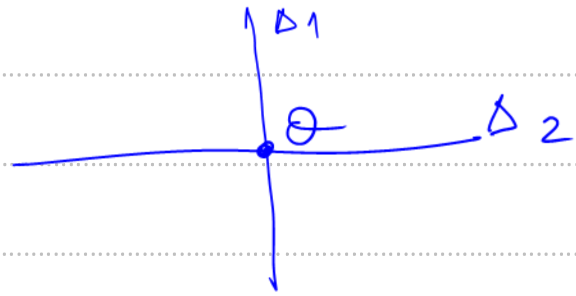
$$\psi = S_{(CD)} \circ \varphi.$$

$$\Rightarrow \psi = S_{(CD)} \circ S_{\theta}$$

obj $\psi = T_{\vec{u}} \circ S_{\Delta} \quad (\vec{u} \text{ dir } \Delta).$

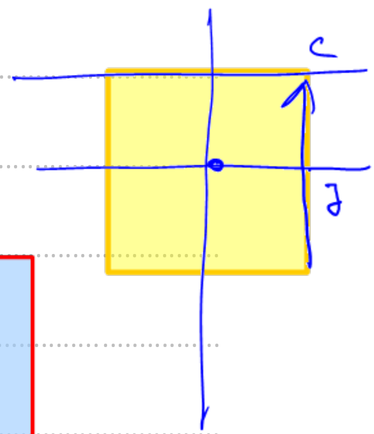


$$S_O = R_{(O,\pi)} = S_{\Delta_1} \circ S_{\Delta_2}$$



$$\psi = S_{(CD)} \circ S_O$$

$$= \underbrace{\boxed{S_{(CD)} \circ S_{(OJ)}}} \circ S_{(OI)}$$



$$(CD) \parallel (OJ)$$

$$\underset{②}{S_{(CD)}} \circ \underset{\uparrow\, 1}{S_{(OJ)}} = \boxed{T_{2\vec{JC}}}$$

$$\psi = T_{2\vec{JC}} \circ S_{(OI)}$$

$$\Psi = T_{\overrightarrow{BC}} \circ S_{(OI)}$$

$\overrightarrow{BC}$ est direct de (OI)

donc $\Psi = T_{\overrightarrow{BC}} \circ S_{(OI)}$ (Forme reduite)