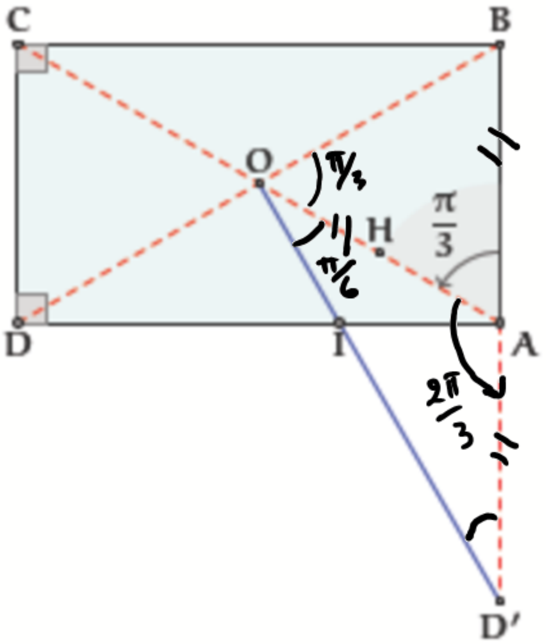
$(D'O)$ passe par $O = B * D$ reste $(D'O) \perp [BD]$??



ABCD est un rectangle de Centre O

$OA = OB$

Comme $\widehat{OAB} = \frac{\pi}{3}$

$\Rightarrow$ OAB equilateral.

$\Rightarrow \boxed{\widehat{AOB} = \frac{\pi}{3}}$

$AO = AB$ et $AB = AB'$

$AO = AD'$ d'où OAD' isocele en A

$\widehat{OAD'} = \frac{2\pi}{3} \Rightarrow \widehat{AOD'} = \frac{\pi - 2\pi/3}{2} = \pi/6$

d'où $\widehat{D'OB} = \widehat{D'OA} + \widehat{AOB} = \pi/6 + \pi/3 = \frac{\pi}{2}$

$\Rightarrow (D'O) \perp (BD)$ en $O = D * B$

$\Rightarrow (D'O) = \text{med } [BD]$.

2) $A \neq B$ on a $O = B * D$.

$\Rightarrow OD = OB$ or $OB = AB$.

a/ ① $A \neq B$ $\xRightarrow{\text{théo}}$ d'où l'existence et

② $AB = OD$ l'unicité du dép. f

déplacement

Id $\quad T_{\vec{u} \neq \vec{0}}$ $\quad R_{(I,\, \alpha \neq 0)}$

$\underbrace{Id \quad T_{\vec{u}\neq\vec{0}}}_{\text{angle} = 0\ (2\pi)}$ $\qquad R_{(I,\alpha\neq 0)} \to \alpha \neq 0\ (2\pi)$

b) $f(D') = B$.

$f(A) = O$

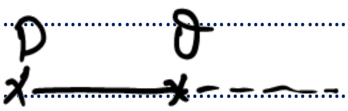
on a $A = B * D'$

$f(B) = D$

donc $f(A) = f(B) * f(D')$

$f(D') \overset{?}{=}$

$O = D * f(D')$



$f(D') = S_O(D) = B$.

c) f est un deplacement d'angle.

$$(\widehat{\overrightarrow{BD'}, \overrightarrow{DB}}) \equiv \pi + (\overrightarrow{BD'}, \overrightarrow{BB}) \ (2\pi)$$

$$\equiv \pi - \frac{\pi}{3} \ (2\pi)$$

$$= \frac{2\pi}{3} \ (2\pi)$$

f est un dep d'angle $\frac{2\pi}{3} \neq 0$ donc

f est une rotation d'angle $\frac{2\pi}{3}$

* Soit I' le centre de f

$f(B) = D \Rightarrow I' \in \text{med}[BD] = (OD')$

$f(D') = B \Rightarrow I' \in \text{med}[D'B] = (AD)$

$I' \in (AD) \cap (OD') = \{I\}$.

donc $\boxed{f = R_{(I, \frac{2\pi}{3})}}$

d) $f(D) \overset{?}{=} D'$

$\Leftrightarrow$ $ID = ID'$?

$(\overrightarrow{ID}, \overrightarrow{ID'}) \equiv \frac{2\pi}{3} \ (2\pi)$?

* BDD' est équilateral

. $A = B * D' \Longrightarrow [DA]$ mediane issue de D

$O = B * D \Longrightarrow [D'O]$ ——— D'

donc $\{I\} = (DA) \cap (OD')$

$\Longrightarrow I =$ Centre de gravité

$=$ " du cercle circonscrit

d'où $\begin{cases} ID = ID' \\ (\widehat{\overrightarrow{ID}, \overrightarrow{ID'}}) = 2 (\overrightarrow{BD}, \overrightarrow{BD'}) \ (2\pi) \\ \qquad = 2 \times \frac{\pi}{3} \end{cases}$

d'où $\boxed{f(D) = D'}$

3) $h = h_{(B, \frac{1}{2})}$

$g = h \circ f$

$g = h_{(B, \frac{1}{2})} \circ R_{(I, \frac{2\pi}{3})}$

Notation: $S^{+}_{(k, I, \alpha)} =$ Sim. dir. de rapport k Centre I angle α

$$g = \underset{\downarrow}{h} \circ \underset{\downarrow}{f}$$

$$= S^{+}_{\left(\underset{>0}{\frac{1}{2}} ; B, 0\right)} \circ S^{+}_{\left(1, I, \frac{2\pi}{3}\right)}$$

$$= S^{+}_{\left(\frac{1}{2} \times 1 , ? , 0 + \frac{2\pi}{3}\right)}$$

$$= S^{+}_{\left(\frac{1}{2} ; ? ; \frac{2\pi}{3}\right)}$$

Rappel.

- $$\varphi = \underset{\downarrow}{h_{(B, -2)}} \circ \underset{\downarrow}{R_{\left(J, \frac{\pi}{2}\right)}}$$

$$= S^{+}_{(2 ; B, \pi)} \circ S^{+}_{\left(1, J, \frac{\pi}{2}\right)}$$

$$= S^{+}_{\left(2 , ? , \pi + \frac{\pi}{2}\right)}$$

. Forme réduite

$$h_{(\Omega,3)} \circ R_{\left(\Omega,\frac{\pi}{6}\right)}$$

$$= S^{+}_{\left(3,\,\Omega,\,\frac{\pi}{6}\right)}.$$

b) $g = h \circ f$

$$\underbrace{B \xrightarrow{f} D \xrightarrow{h_{\left(B,\frac{1}{2}\right)}} \theta}_{\theta = B * D.}$$

$$\boxed{g(B) = \theta}$$

$* \; g(D) = ?$

$$g(D) = h\big(f(D)\big) = h_{\left(B,\frac{1}{2}\right)}(D') = A$$

car $A = B * D' \Rightarrow \overrightarrow{BA} = \frac{1}{2}\overrightarrow{BD'}$.

$* \quad \theta = B * D \Rightarrow g(\theta) = g(B) * g(D)$

$$= \theta * A$$

$$g(O) = H \cdot$$

c) $g(D') = h\left(f(D')\right) = h(B) = B \cdot$

$$D \xrightarrow{g} B \xrightarrow{g} O \xrightarrow{g} H$$

$\Longrightarrow$ $g \circ g \circ g\,(D') = H \cdot$

d)

$$g = S^{+}_{\left(\frac{1}{2},\, \Omega,\, \frac{2\pi}{3}\right)}$$

$$g \circ g \circ g = S^{+}_{\left(\frac{1}{8};\, \Omega;\, \frac{2\pi}{3}\times 3\right)}$$

$$= S^{+}_{\left(\frac{1}{8},\, \Omega,\, 0\right)}$$

$$= h_{\left(\Omega,\, \frac{1}{8}\right)}$$

or $g \circ g \circ g\,(D') = H \cdot$

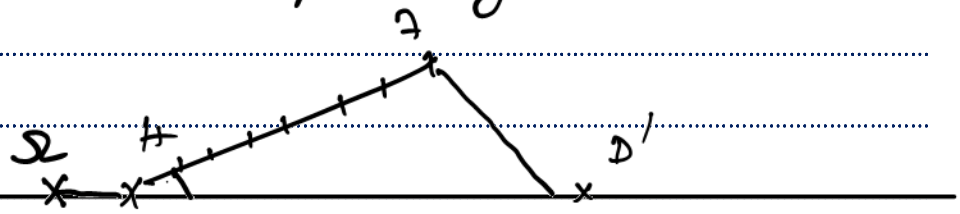
$\Longrightarrow$ $h_{\left(\Omega,\, \frac{1}{8}\right)}(D') = H$

$$\overrightarrow{\Omega H} = \frac{1}{8}\,\overrightarrow{\Omega D'}$$

$\Rightarrow 8\overrightarrow{\Omega H} - \overrightarrow{\Omega D'} = \vec{0}$

$\Rightarrow \Omega$ bary des points $(H,8), (D',-1)$

Construction barycentre.



Ω bary $(H,8)$ $(D',-1)$

$$\overrightarrow{H\Omega} = \frac{-1}{8+(-1)} \overrightarrow{HD'}$$

$$\overrightarrow{H\Omega} = -\frac{1}{7} \overrightarrow{HD'}$$

Rque

$S^{+}_{(3,\Omega,\frac{\pi}{2})} \Rightarrow S\circ S^{+} = S^{+}_{(9,\Omega,\pi)}$

$= h_{(\Omega,-9)}.$

$$S^{+}_{\left(\frac{1}{2};\,\Omega,\,\frac{\pi}{3}\right)} = S \circ S \circ S = S^{+}_{\left(\frac{1}{8},\,\Omega;\,\pi\right)}$$

$$= h_{\left(\Omega,\,-\frac{1}{8}\right)}$$

B)

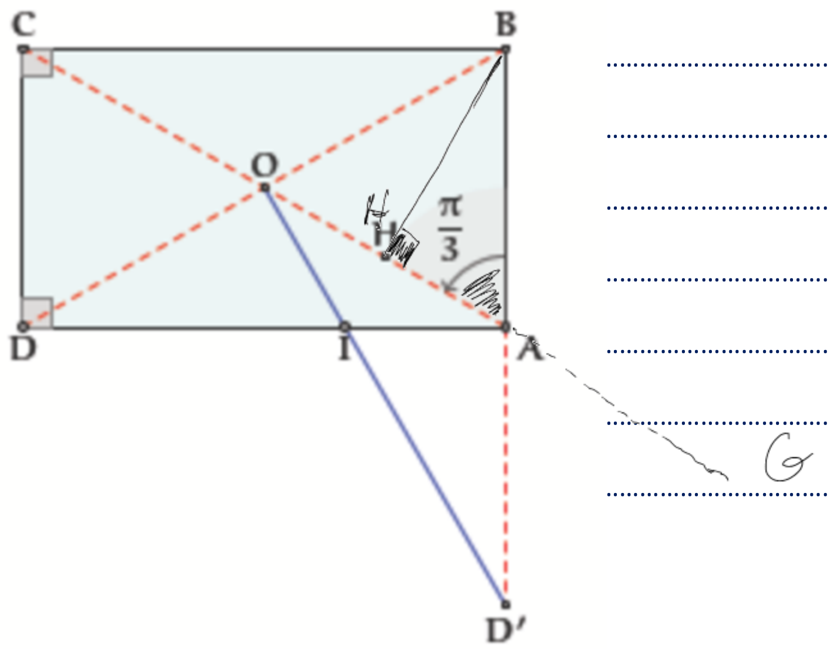
$$G = S_A(\theta)$$

$H \to H$

$A \to B$

$$S = S^{+}_{(k=?\,;\,H\,;\,?)}$$

$$k_S = \frac{HB}{HA} = \tan(\widehat{BAH}) = \sqrt{3}.$$



$\theta_S = (\widehat{\overrightarrow{HA}, \overrightarrow{HB}})\ (2\pi)$

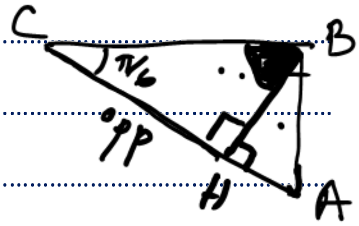
$= \frac{\pi}{2}\ (2\pi)$

$S = S^+_{(\sqrt{3}\ ;\ H\ ;\ \frac{\pi}{2})}$

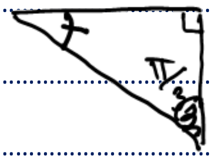
b) $S(B) = C$ $\qquad \frac{HC}{HB} \overset{?}{=} \sqrt{3}$

$(\widehat{\overrightarrow{HB}, \overrightarrow{HC}}) \equiv \frac{\pi}{2}\ (2\pi)$

Dans le Triangle ABC rectangle en B



$$\begin{cases} \frac{HC}{HB} = \tan(\widehat{HBC}) = \tan\frac{\pi}{3} = \sqrt{3} \\ (\widehat{\overrightarrow{HB}, \overrightarrow{HC}}) \equiv \frac{\pi}{2}\ (2\pi) \end{cases}$$



$\Rightarrow$ $S(B) = C.$

2) Sim Indir de rapport 1 = Antidep $\to S_\Delta$; $\to$ gliss.

S^- de rapport $\neq 1 \Rightarrow S^-_{(k\neq 1,\ \Omega,\ Axe)}$

$A \xrightarrow{\sigma} B$

$B \xrightarrow{\sigma} C$

$k_\sigma = \dfrac{BC}{BA} = \tan\dfrac{\pi}{3} = \sqrt{3} \neq 1$

donc σ admet un Centre.

b) $A \xrightarrow{\sigma} B \xrightarrow{\sigma} C$.

$\sigma \circ \sigma (A) = C$.

Forme reduite Sim In

$S = S_{(k,\Omega,\Delta)} \quad (k>0)$

$= h_{(\Omega,k)} \circ S_\Delta = S_\Delta \circ h_{(\Omega,k)}$

$S \circ S = h_{(\Omega,k^2)}$. theo

soit Ω' = Centre de σ

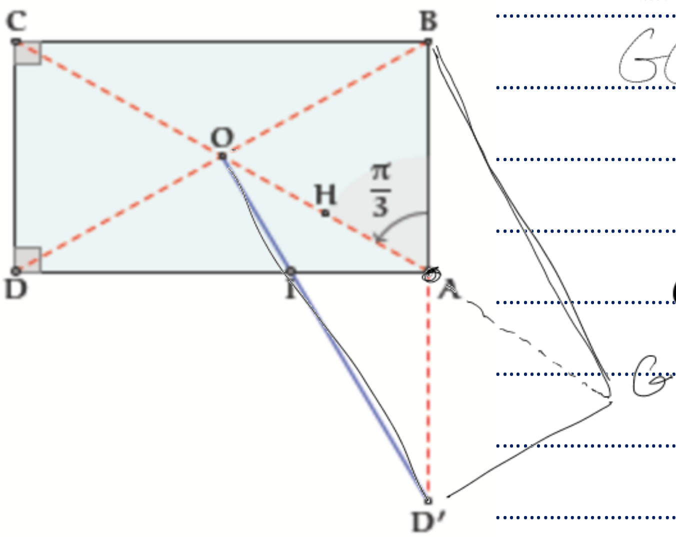
$\sigma \circ \sigma = h_{(\Omega',\sqrt{3}^2)} = h_{(\Omega',3)}$

or $\sigma \circ \sigma (A) = C$ $\quad \Omega' \overset{?}{=} G$

$$\Rightarrow h_{(\Omega', 3)}(A) = C$$

$$\Rightarrow \boxed{\overrightarrow{\Omega' C} = 3\,\overrightarrow{\Omega' A}}$$

$$\Omega' \overset{?}{=} G \Longleftarrow \overrightarrow{GC} = 3\,\overrightarrow{GA} \ ?$$



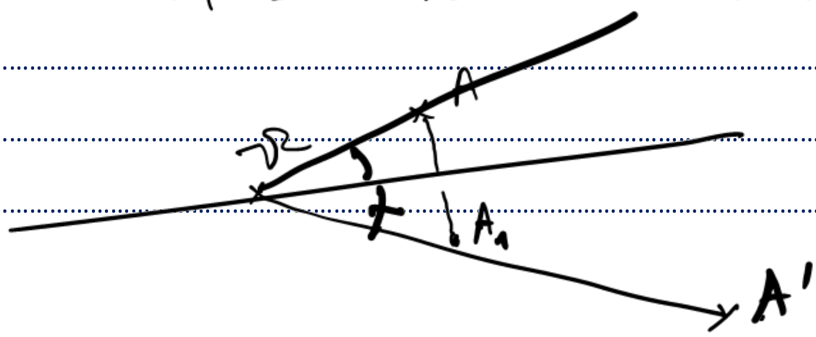
$$\overrightarrow{GC} = \overrightarrow{GO} + \underbrace{\overrightarrow{OC}}$$
$$= 2\,\overrightarrow{GA} + \overrightarrow{AO}$$
$$+ \overrightarrow{GA}$$
$$\overrightarrow{GC} = 3\,\overrightarrow{GA}$$

$$\boxed{G = \Omega'}$$

Cours

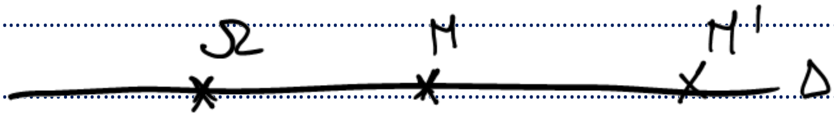
$$S = S_{(k \neq 1;\, \Omega,\, \Delta_{axe})}.$$

- $S(A) = A' \Rightarrow \Delta =$ biss. Interieur $A'\Omega A$

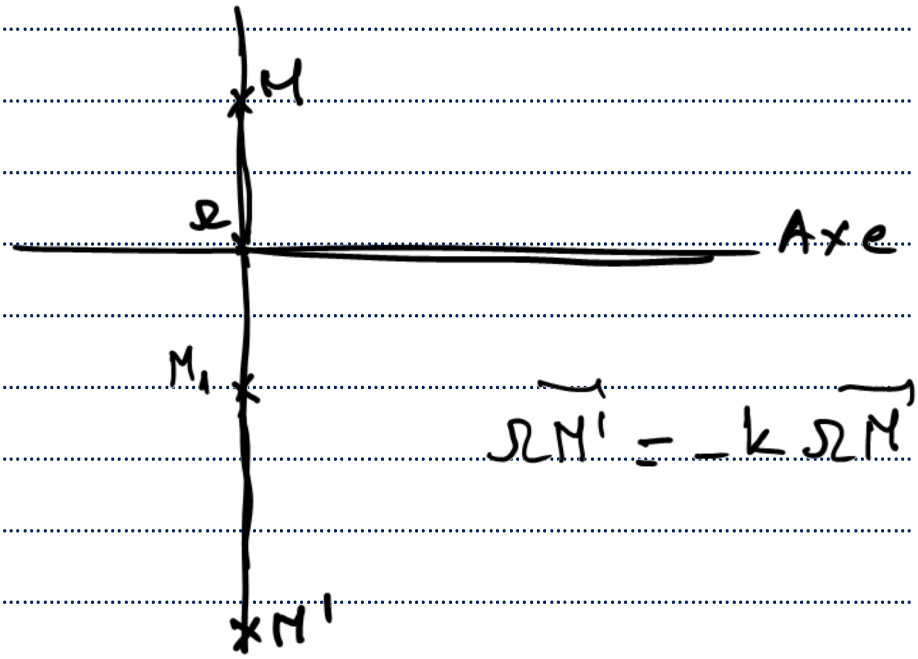


• $M \in Axe \Rightarrow S(M) = M' \in Axe$

$\overrightarrow{\Omega M'} = k\,\overrightarrow{\Omega M}$

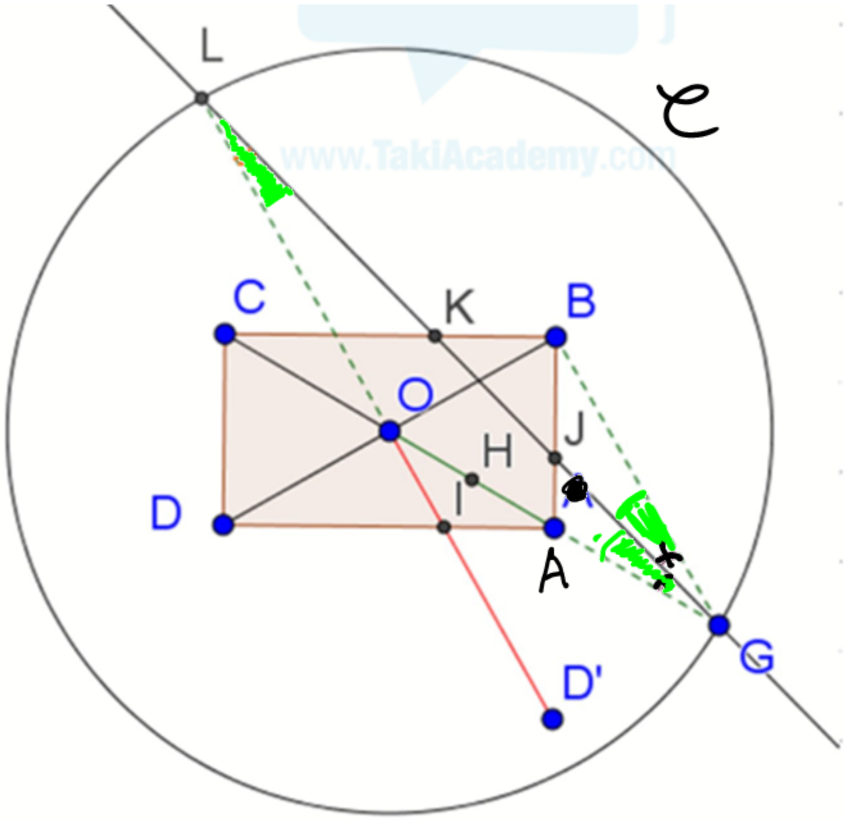


$\Delta = (MM')$



$\overrightarrow{\Omega M'} = -k\,\overrightarrow{\Omega M}$

$\Gamma = S_{(\sqrt{3},\, G,\, Axe?)}$



$A \longrightarrow B$

$B \longrightarrow C$

$G \longrightarrow G$

$L \in \mathcal{C}$ $G \in \mathcal{C}$

$\Rightarrow OL = OG$

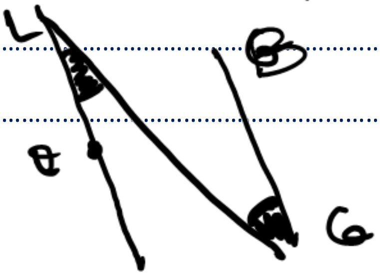
$\Rightarrow A\widehat{G}L = O\widehat{L}G$

dans le quadrilatère $OBGD'$

on a $O * G = D' * B = A$

$\Rightarrow OBGD'$ est un parallélogramme

d'où $O\widehat{L}G = L\widehat{G}B$



$$\left\{\begin{array}{l}\widehat{BGL} = \widehat{LGA} \\ \sigma(A) = B\end{array}\right.$$

$\Rightarrow$ l'axe de $\sigma = (GL)$

on a $J \in (GL) \cap (AB)$

$\sigma(J) \in \sigma(GL) \cap \sigma(AB)$

d'où $\sigma(J) \in (GL) \cap (BC) = \{K\}$

donc $\boxed{\sigma(J) = K}$

3) $\sigma = S_{(\sqrt{3},\, G,\, (GL))}$

$\varphi = \{M \in P \,;\, \sigma(M) = S(M)\}$

Theo

f et g 2 dep. (Anti ou S^+ ou S^-)

$$f(A) = g(A) = A'$$

$$f(B) = g(B) = B'$$

$$\Longrightarrow \quad f = g.$$

f dep $\qquad g$ Anti dep.

$$f(A) = g(A) = A'$$

$$f(B) = g(B) = B'$$

$$\varphi = g^{-1} \circ f \quad (\text{Anti dep})$$

$$A \xrightarrow{f} A' \xrightarrow{g^{-1}} A$$

$$B \xrightarrow{f} B' \xrightarrow{g} B.$$

φ fixe 2 pts $\qquad$ Anti dep

$$\varphi = S_{(AB)}$$

* Si φ un deplacement qui fixe A et B

$$\varphi = \text{Identité}$$

* ψ et φ deux Similitude directes (ou indi) qui Coincident en A et B $\Longrightarrow \psi = \varphi$.

Suite Ex

on pose

$$\psi = S^{-1} \circ \sigma = \text{Sim Indirecte}$$

$$A \xrightarrow{\sigma} B \xrightarrow{S^{-1}} A$$

$$B \xrightarrow{\sigma} C \xrightarrow{S^{-1}} B.$$

$$\text{rapport de } \psi = \frac{AB}{AB} = 1$$

$\Longrightarrow \psi$ est un anti deplacement

Sy. orth ~~Sym Gliss~~

qui fixe A et B

$$S^{-1} \circ \sigma = S_{(AB)}.$$

L'ensemble des pts $M \in P$

tel que $\sigma(M) = S(M)$

$$\Leftrightarrow \underbrace{S^{-1} \circ \sigma}(M) = M.$$

$$\Leftrightarrow S_{(AB)}(M) = M$$

$$\Rightarrow \Gamma = (AB)$$

$$M \xrightarrow{\sigma} M' \xrightarrow{S^{-1}} M$$

$$M \xrightarrow{S} M'$$

$$2^{6} \equiv 1 \; (\;)$$

Exercice 3

1) $2^0 = 1 \; (5)$ $\qquad 2^1 = 2 \; (5)$

$2^2 = 4 \; (5)$ $\qquad 2^3 \equiv 3 \; (5)$

$\underline{\underline{2^4}} \equiv 1 \; (5)$ $\qquad 2^{12} = (2^4)^3 \equiv 1$

$$2^{17} = \underbrace{(2^4)^4}_{1} \times 2 \equiv 2 \; (2)$$

$$2^{20} = \left(2^{4}\right)^{5} \equiv 1 \quad (5)$$

$$2^{21} = \underbrace{2^{20}}_{1} \times 2 \equiv 2 \, (5)$$

reste de n mod 4	0	1	2	3
reste 2^n mod 5	1	2	4	3.

$n = 4q$

- $2^n = \left(2^4\right)^q \equiv 1 \, (5)$

- $n = 4q + 1$

$$2^n = 2^{4q} \times 2 = \underbrace{\left(2^4\right)^q}_{} \times 2$$

$$\equiv 1 \times 2 \quad (5)$$

- $n = 4q + 2$ $\qquad q \in \mathbb{N}$

$$2^n = \underbrace{\left(2^4\right)^q}_{} \times 2^2$$

$$\equiv 1 \times 4 \quad (5)$$

- $n = 4q + 3 \Rightarrow 2^n = \left(2^4\right)^q \times 2^3$

$$\equiv 1 \times 3 \quad (5)$$

b) (E_1) $\quad 67^x \equiv 1 \ (5)$

$67 \equiv 2 \ (5)$

$\Rightarrow 67^x \equiv 2^x \ (5)$

x sol $(E_1) \Leftrightarrow 2^x \equiv 1 (5)$

d'après a) $\quad x = 4q$

$\Leftrightarrow x \equiv 0 \ (4)$

$S_{\mathbb{N}} = \{x = 4q \ ; q \in \mathbb{N}\}$.

2) $\quad 5^{66} \overset{?}{\equiv} 1 \ (\text{mod } \underline{67})$

$\begin{cases} \cdot \ 67 \text{ premier} \\ \cdot \ 67 \wedge 5 = 1 \end{cases}$ $\qquad \sqrt{67} \simeq 8,\ldots$

2
3
5
7

$\hookrightarrow$ P.T Fermat

$5^{67-1} \equiv 1 \ (67)$

d'où $\quad 5^{66} \equiv 1 \ (67)$

$$\begin{cases} 5 \text{ premier} \\ 5 \wedge 2 = 1 \end{cases} \Longrightarrow 2^4 \equiv 1 \ (5)$$

$4^n \equiv ? \ (7)$

$4^6 \equiv 1 \ (7)$

b) p est le plus petit entier naturel $\neq 0$

tq $5^p \equiv 1 \ (67)$

$\overset{?}{\Longrightarrow}$ p divise 66. $(p \leq 66)$

ona $5^p \equiv 1 \ (67)$

Supp $66 = p.q + r$, $0 < r < p$ $\quad (r \neq 0)$

$5^{66} = 5^{p.q} . 5^r = (\underset{\equiv}{5^p})^q \times 5^r$

or $5^{66} \equiv 1 \ (67)$

$$\underbrace{(5^p)^q}_{\equiv 1} \cdot 5^r \equiv 1\ (67)$$

d'où $5^r \equiv 1\ (67)$

Absurde.

$r < p$ et Verifie $5^r \equiv 1\ (67)$

or p est le p. petit entier $5^p \equiv 1\ (67)$

d' $66 = p.q \Rightarrow 66 \equiv 0\ (p)$