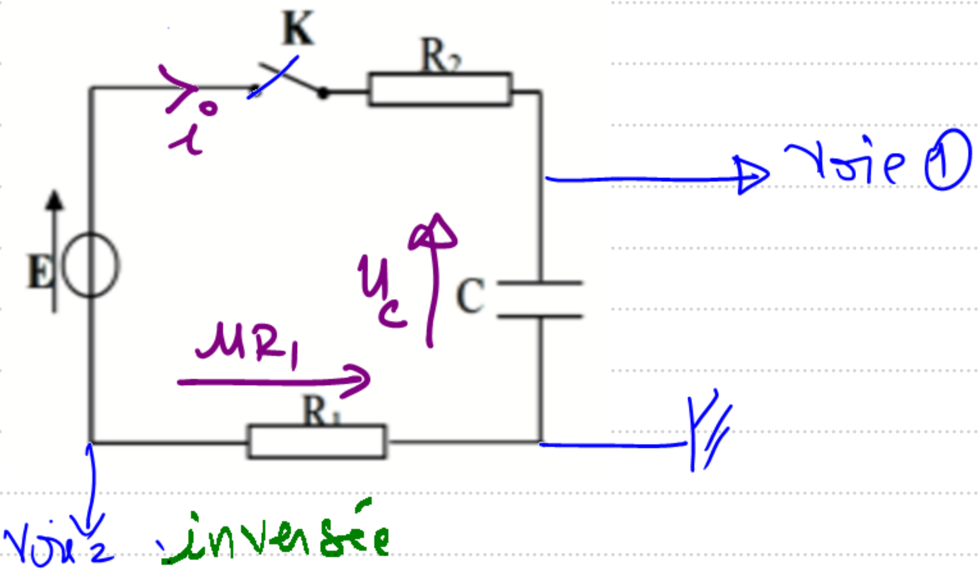
Dipôle RC : Série 3

Exercice N° 1

1°)

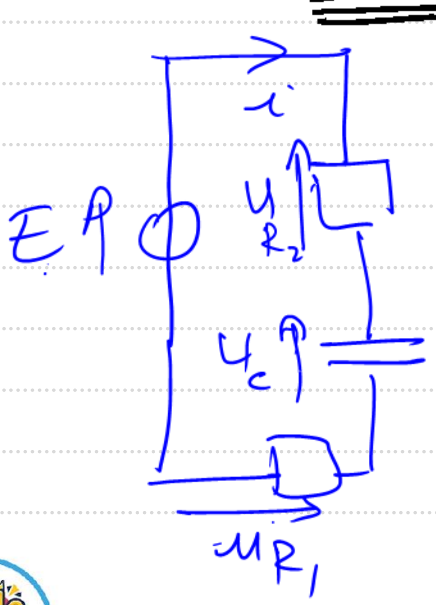


2°) En régime permanent, le condensateur se comporte comme un interrupteur ouvert

$\Rightarrow$ en R.P. : $i_p = 0 \Rightarrow u_{R_1} = R_1 i_p = 0$

ce qui correspond à la courbe (b)

(b) Mq : $u_{R_1}(0) = \frac{R_1}{R_1+R_2} E$?



Loi des mailles

$u_{R_1} + u_{R_2} + u_C - E = 0$. $\forall t$

à $t=0$: $u_C(0) = 0$

$(R_1 + R_2)\, i(0) = E$

or $i(0) = \frac{u_{R_1}(0)}{R_1}$

$$(R_1+R_2)\frac{u_{R_1}(0)}{R_1}=E \Rightarrow \boxed{u_{R_1}(0)=\frac{R_1 E}{R_1+R_2}}$$

3°) On a $u_C(t)+u_{R_1}(t)+u_{R_2}(t)=E$

$u_C(t)+(R_1+R_2)\,i(t)=E$; $i=C\frac{du_C}{dt}$

$$\boxed{u_C(t)+(R_1+R_2)C\frac{du_C}{dt}=E}$$

b) En régime permanent $u_C=C^{te} \Rightarrow \frac{du_C}{dt}=0$

$\Rightarrow \boxed{u_{C_p}=E}$ graphiquement $\boxed{E=7{,}5V}$

c) • $u_C(t)=Ae^{\alpha t}+B$ donc $\frac{du_C}{dt}=\alpha Ae^{\alpha t}$

eq diff $\Rightarrow Ae^{\alpha t}+B+(R_1+R_2)C\alpha Ae^{\alpha t}=E$

$\Rightarrow B+Ae^{\alpha t}\left[1+(R_1+R_2)C\alpha\right]=E$

équation vérifiée ssi $1+(R_1+R_2)C\alpha=0$

$$\Rightarrow \boxed{\alpha=\frac{-1}{(R_1+R_2)C}=-\frac{1}{\tau}}$$

c-à-d : $\boxed{B=E}$

• à $t=0$ $u_C(0)=0=A+B \Rightarrow \boxed{A=-B=-E}$

d'où $u_C(t)=-Ee^{-\frac{1}{\tau}t}+E$

$=\boxed{E\left(1-e^{-\frac{t}{\tau}}\right)}$. $\tau=(R_1+R_2)C$

4°) $u_{R_1}(0) = E \times \frac{R_1}{(R_1+R_2)} \Rightarrow R_1+R_2 = \frac{R_1 E}{u_{R_1}(0)}$.

$$\boxed{R_2 = \underset{=500}{R_1}\left(\frac{\overset{=7,5}{E}}{\underset{=5}{u_{R_1}(0)}} - 1\right)} \Rightarrow \boxed{R_2 = 250\,\Omega}$$

NB dans ce cas $u_{R_1}(0) + u_{R_2}(0) + \underset{=0}{u_C(0)} = 7,5\,V$

$u_{R_1}(0) = 5\,V \longrightarrow u_{R_2}(0) = 2,5\,V$

$$u_{R_1}(0) = 2\,u_{R_2}(0)$$

$$\Rightarrow R_1 \cancel{i(0)} = 2 R_2 \cancel{i(0)} \Rightarrow \boxed{R_2 = \frac{R_1}{2} = 250\,\Omega}$$

b) • à $t = \tau$ $u_C(\tau) = 0,63E = 4,72\,V$

$$\Rightarrow \boxed{\tau = 2.10^{-3}\,s}$$

• $\tau = (R_1+R_2)\,C \Rightarrow \boxed{C = \frac{\tau}{R_1+R_2}}$

$$\boxed{C = 2,66\,\mu F}$$

5°) a) $u_{R_1}(t) = R_1\, i(t) = R_1 C \frac{du_C(t)}{dt}$

$= R_1 C \left(E - E e^{-\frac{t}{\tau}}\right)'$

$= R_1 C \frac{E}{\tau} e^{-\frac{t}{\tau}}$.

$$\Rightarrow u_{R_1}(t) = R_1 \cancel{C}\,\frac{E}{(R_1+R_2)\cancel{C}}\, e^{-\frac{t}{\tau}}$$

$$\boxed{u_{R_1}(t) = \frac{R_1}{R_1+R_2}\, E\, e^{-\frac{t}{\tau}}} \quad \text{d'où} \quad \boxed{u_{R_2}(t) = \frac{R_2}{R_1+R_2}\, E\, e^{-\frac{t}{\tau}}}$$

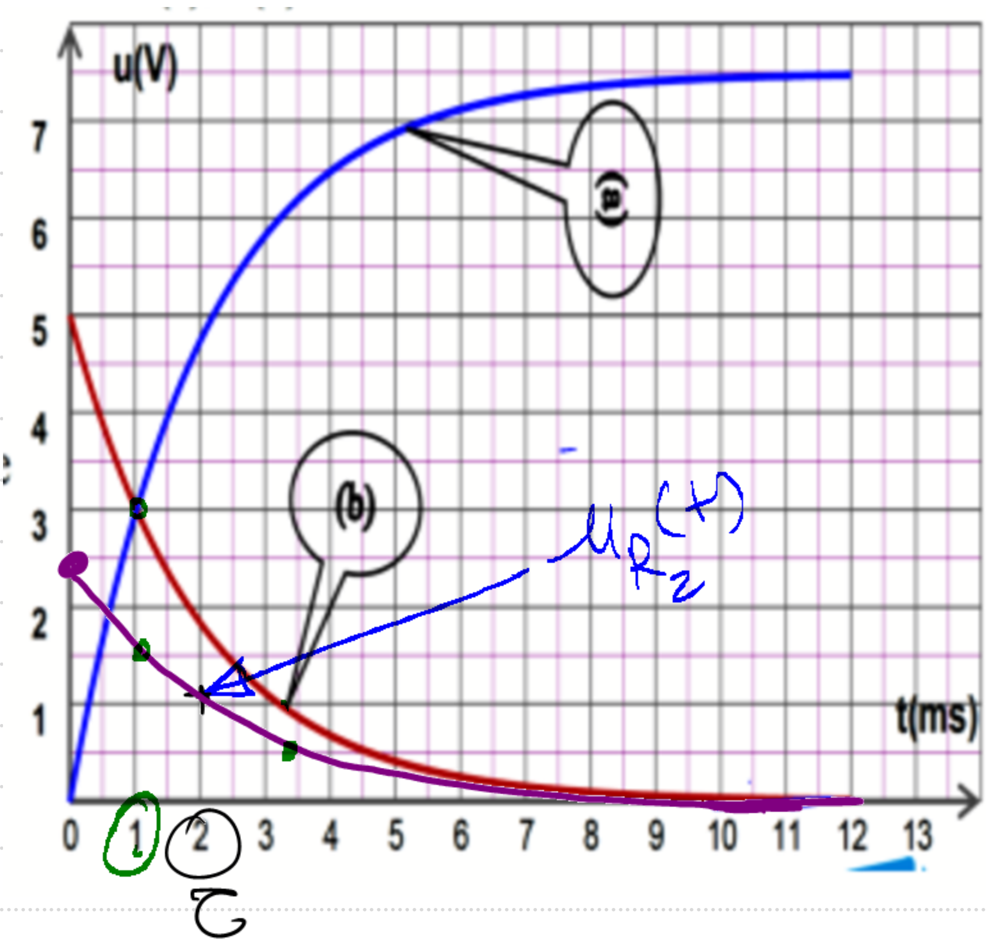
(b) • $u_{R_2}(0) = E - u_{R_1}(0) = 2{,}5\,V$

• En régime permanent ($t \to +\infty$ th)

$u_{R_2} \longrightarrow 0$

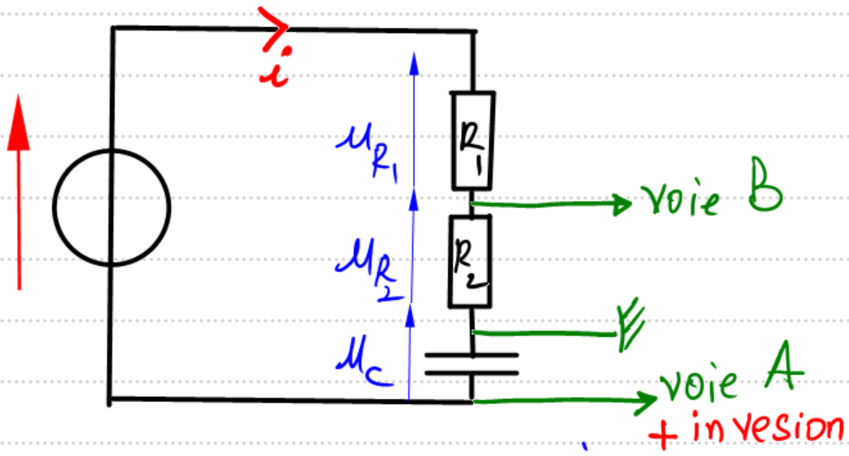
• $u_{R_2}(\tau) = E - u_{R_1}(\tau) - u_C(\tau) = 7{,}5 - 1{,}7 - 4{,}7$

$= 1{,}1\,V$



Exercice N°2

1°) a)

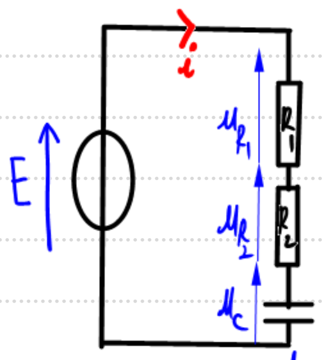


b) A $t=0$ $u_C(0)=0$: ce qui correspond à la courbe (a)

donc courbe (a) $\longrightarrow u_C(t)$

" (b) $\longrightarrow u_{R_2}(t)$

2°) a)



Loi des mailles : $u_C(t)+u_{R_1}(t)+u_{R_2}(t)-E=0$

$\Rightarrow u_C(t)+(R_1+R_2)\,i(t)=E$

Dérivons : $\frac{du_C(t)}{dt}+(R_1+R_2)\frac{di(t)}{dt}=0 \Rightarrow \boxed{i(t)+(R_1+R_2)C\frac{di(t)}{dt}=0}$

b) $i(t)=Be^{-\alpha t}$ solution

$\Rightarrow Be^{-\alpha t}-(R_1+R_2)C\alpha Be^{-\alpha t}=0 \Rightarrow Be^{-\alpha t}\left(1-(R_1+R_2)C\alpha\right)=0$ or $Be^{-\alpha t}\neq 0$

donc $1-(R_1+R_2)C\alpha=0 \Rightarrow \boxed{\alpha=\frac{1}{(R_1+R_2)C}}$

$i(t) = B e^{-\alpha t} \Rightarrow B = i(0)$ or L.M à $t=0$ $\underbrace{u_C(0)}_{=0} + (R_1+R_2)\,i(0) = E$

$\Rightarrow \boxed{B = i(0) = \dfrac{E}{R_1+R_2}}$

b) $\boxed{i(t) = \dfrac{E}{R_1+R_2}\, e^{-\frac{t}{(R_1+R_2)C}}}$

c) • $u_{R_2}(t) = R_2\, i(t) \Rightarrow u_{R_2}(t) = \dfrac{R_2 E}{R_1+R_2}\, e^{-\frac{t}{(R_1+R_2)C}}$

• $u_C(t) = E - (R_1+R_2)\, i(t) \Rightarrow u_C(t) = E - E e^{-\frac{t}{(R_1+R_2)C}}$

3°) a) En régime permanent $u_{C\,max} = \boxed{E = 6{,}5\,V}$

b) τ : abscisse de l'intersection de la tg à $t=0$ à la courbe $u_C(t)$ avec son asymptote $\boxed{\tau = 2.10^{-3}\,s}$

c) Loi des mailles à $t=0$

$(R_1+R_2)\, i(0) = E \Rightarrow (R_1+R_2)\dfrac{u_{R_2}(0)}{R_2}$

$\Rightarrow \dfrac{R_2}{R_1+R_2} = \dfrac{u_{R_2}(0)}{E} = \dfrac{6}{6{,}5} = \dfrac{12}{13}$

4°) a) $u_C(t_1) = u_{R_2}(t_1) = 3{,}1\,V$

b) $E_C(t_1) = \dfrac{1}{2} C u_C^2(t_1) \Rightarrow \boxed{C = \dfrac{2E_C(t_1)}{u_C^2(t_1)}} \Rightarrow \boxed{C = \dfrac{2 \times 96{,}1.10^{-6}}{(3{,}1)^2} = 2.10^{-5}\,F = 20\,\mu F}$

c) $\frac{R_2}{R_1+R_2}=\frac{12}{13} \Rightarrow 13R_2 = 12R_1+12R_2 \Rightarrow R_2=12R_1$

$\tau=(R_1+R_2)C \Rightarrow \tau = 13R_1C \Rightarrow \boxed{R_1=\frac{\tau}{13C}=7{,}7\,\Omega}$ $\boxed{R_2=92{,}4\,\Omega}$

d) graphiquement $\boxed{t_{1g}=1{,}6.10^{-3}s}$

e) $u_{R_2}(t_1)=u_C(t_1) \Rightarrow \frac{R_2}{R_1+R_2}\cancel{E}e^{-\frac{t_1}{\tau}}=\cancel{E}(1-e^{-\frac{t_1}{\tau}})$

$e^{-\frac{t_1}{\tau}}\left(1+\frac{R_2}{R_1+R_2}\right)=1$

$e^{-\frac{t_1}{\tau}}\left(\frac{2R_2+R_1}{R_1+R_2}\right)=1 \Rightarrow -\frac{t_1}{\tau}=\text{Ln}\left(\frac{R_1+R_2}{2R_2+R_1}\right)$

$t_{1th}=-\tau\,\text{Ln}\left(\frac{R_1+R_2}{2R_2+R_1}\right)=1{,}38\,10^{-3}s$

5) a) $u_{R'_2}(t)=R'_2\frac{E}{R'_1+R'_2}e^{-\frac{t}{\tau'}}$

$\text{Ln}(u_{R'_2})=\text{Ln}\left(\frac{R'_2E}{R'_1+R'_2}\right)+\text{Ln}\left(e^{-\frac{t}{\tau'}}\right)$

$$\boxed{\text{Ln}(u_{R'_2})=-\frac{1}{\tau'}.t+\text{Ln}\left(\frac{R'_2E}{R'_1+R'_2}\right)}$$

fonction affine decroissante ce qui justifie l'allure

b) $-\frac{1}{\tau'}$ = pente de la droite $= \frac{-1{,}188}{4{,}7.10^{-3}} = -250\ s^{-1}$

d'où $\boxed{\tau' = \frac{1}{250} = 4.10^{-3}s}$

c) $\tau' = (R'_1 + R'_2)C \Rightarrow R'_1 + R'_2 = \frac{\tau'}{C} = 200\,\Omega$

$Ln\left(\frac{R'_2 E}{R'_1 + R'_2}\right) = 1{,}188 \Rightarrow \frac{R'_2}{R'_1 + R'_2} E = 3{,}28$

$R'_1 + R'_2 = 200\,\Omega \Rightarrow R'_2 = \frac{(R'_1 + R'_2) \times 3{,}28}{E} = \frac{200 \times 3{,}28}{6{,}5}$

$\boxed{R'_1 = 100{,}92\,\Omega}$ $\boxed{R'_2 = 99{,}08\,\Omega}$