Suite Ex 3

- $5^{66} \equiv 1 \ (67)$
- $5^{p} \equiv 1 \ (67)$
- p divise 66

$$\begin{array}{r|l} 125 & 67 \\ \hline 58 & 1 \end{array}$$

c) • $5^3 = 125 \equiv \underline{58} \ (67)$

$58 \equiv -9 \ (67)$

• $5^6 = (5^3)^2 \equiv 58^2 \ (67)$

$\equiv (-9)^2 \ (67)$

$\equiv 81 \ (67)$

$\equiv 14 \ (67)$

• $5^{11} = 5^3 \times 5^6 \times 5^2$

$\equiv 58 \times 14 \times 25 \ (67)$

$\equiv 66 \ (67)$

$\begin{cases} 5^{11} \equiv 66 \ (67) \\ 66 \equiv -1 \ (67) \end{cases}$

• $5^{22} \equiv (5^{11})^2 \ (67)$

$\equiv (-1)^2 \ (67)$

$\equiv 1 \ (67)$

p est le p.p. entier naturel $5^p \equiv 1 \ (67)$

p divise 66 $\quad p \in D_{66}$

$D_{66} = \{1 \quad 2 \quad 3 \quad 6, \quad 11 \quad 22 \quad 33 \quad 66\}$

$5^1 \not\equiv 1 \ (67) \qquad 5^2 \not\equiv 1 \ (67)$

$5^3 \not\equiv 1 \ (67) \qquad 5^6 \not\equiv 1 \ (67)$

$5^{11} \not\equiv 1 \ (67) \qquad 5^{33} \equiv (5^{11})^3 \ (67)$

$\equiv (-1)^3$

$\equiv -1 \ (67)$

$\equiv 66 \ (67)$

$\not\equiv 1 \ (67)$

$p = 22$

3) $y \in \mathbb{N} \quad 5^y \equiv 1 \ (67)$

$y = 22q + r \qquad 0 \leq r < 22$

$5^y = 5^{22q} \times 5^r = (5^{22})^q \times 5^r$

$5^y \equiv 1 \ (67) \qquad 5^{22} \equiv 1 \ (67)$

$$\Rightarrow \left(5^{22}\right)^{q} \equiv 1 \ (67)$$

d'où $\underbrace{5^{y}} \equiv 1 \quad (67)$

$$\left(5^{22}\right)^{q} . 5^{r} \equiv 1 \quad (67).$$

$$\Rightarrow \quad 5^{r} \equiv 1 \quad (67)$$

$r < 22$ et vérifie $5^{r} \equiv 1 \ (67)$

$$\Rightarrow r = 0$$

$$\Rightarrow y = 22q$$

4) $\qquad 67^{x} + 5^{y} \equiv 1 \quad (335)$

$$\overline{A \text{ et } B} = \overline{A} \text{ ou } \overline{B}$$

Supposons que $x = 0$ ou $y = 0$

Si $x = 0$ $\qquad 67^{0} + 5^{y} \equiv 1 \ (335)$

$$\Rightarrow \quad 1 + \underbrace{5^{y}} \equiv 1 \quad (335)$$

$\Rightarrow \quad 5^y \equiv 0 \quad (335)$

$335 = 5 \times \underline{67}$ (67 diviseur de 67)

Si a divise b $\Rightarrow$ tout diviseur de a divise aussi b.

$\Rightarrow \quad 5^y \equiv 0 \quad (67)$ absurde.

premier

$5^y \left| \begin{array}{l} 5 \\ 5 \\ 5 \\ \vdots \\ 5 \end{array} \right\} y \text{ fois}$

d'où $\boxed{x \neq 0}$

$\boxed{\text{Si } y = 0}$ $\Rightarrow \quad 67^x + 5^0 \equiv 1 \; (335)$

$\Rightarrow \quad 67^x + 1 \equiv 1 \; (335)$

$\Rightarrow \quad 67^x \equiv 0 \; (335)$

D.F. premier

$\Rightarrow \quad 67^x \equiv 0 \; (5)$

$67^x \left| \begin{array}{l} 67 \\ 67 \\ 67 \\ \vdots \\ 67 \end{array} \right\} x \text{ fois}$

impossible.

Conclusion $x \neq 0$ et $y \neq 0$.

b) $67^x + 5^y \equiv 1 \ (335)$

$\Rightarrow 67^x + \underbrace{5^y} \equiv 1 \ (5)$

$$\boxed{5^y \equiv 0 \ (5) \qquad y \neq 0}$$

$\Rightarrow \quad 67^x \equiv 1 \ (5)$ $\qquad 67 \equiv 2 \ (5)$

$67^x \equiv 2^x \ (5)$

d'après 1) b) $\Rightarrow x = 4k \quad k \in \mathbb{N}^*$

* $\quad 67^x + 5^y \equiv 1 \ (335) \qquad (67 / 335)$

$\Rightarrow \underbrace{67^x} + 5^y \equiv 1 \ (67)$

$x \neq 0 \ \Rightarrow \ 67^x \equiv 0 \ (67)$

d'où $\quad 5^y \equiv 1 \ (67)$

d'après 3) $\quad y = 22q$

$\Rightarrow (x,y)$ sol de $(\mathcal{E})$

$\Rightarrow x = 4k$ et $y = 22q$

c) $67^{4k} + 5^{22q} \overset{?}{\equiv} 1 \ (5)$

$67^{4k} + 5^{22q} \equiv 1 \ (67)$

$\bullet \ 5^{22q} \equiv 0 \ (5) \qquad q \neq 0$

$67^{4k} + 5^{22q} \equiv 67^{4k} \ (5)$

or $67^4 \equiv 2^4 \ (5)$

$\equiv 1 \ (5)$

$\Rightarrow (67^4)^k \equiv 1 \ (5)$

$\bullet \ 67^{4k} \equiv 0 \ (67) \qquad k \neq 0$

$\underbrace{67^{4k}}_{0} + 5^{22q} \equiv 5^{22q} \ (67)$

or $5^{22} \equiv 1 \ (67)$

$\Rightarrow 5^{22q} \equiv 1 \ (67)$

<u>théo</u>

Si $\begin{cases} a \equiv 0 \ (n) \\ a \equiv 0 \ (m) \\ n \wedge m = 1 \end{cases} \Rightarrow a \equiv 0 \ (n \times m)$

d) $\begin{cases} 67^{4k} + 5^{22q} - 1 \equiv 0 \ (5) \\ 67^{4k} + 5^{22q} - 1 \equiv 0 \ (67) \\ 5 \wedge 67 = 1 \end{cases}$

$\Rightarrow 67^{4k} + 5^{22q} - 1 \equiv 0 \ (5 \times 67)$

d'où $67^{4k} + 5^{22q} \equiv 1 \ (335)$

<u>e</u>

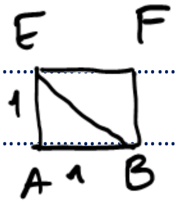
. si (x, y) sol de $(\mathcal{E}) \Rightarrow \begin{cases} x = 4k \\ y = 22q \end{cases}$

. Si $\begin{cases} x = 4k \\ y = 22q \end{cases} \Rightarrow (x, y)$ sol de $\mathcal{E}$.

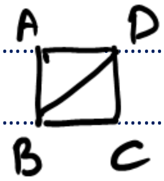
$S_{\mathbb{N} \times \mathbb{N}} = \left\{ (4k ; 22q) ; \begin{matrix} k \in \mathbb{N}^* \\ q \in \mathbb{N}^* \end{matrix} \right\}$.

Exercice 2

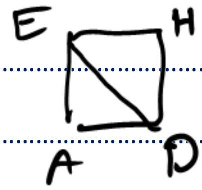
1)



$BE = \sqrt{2}$



$BD = \sqrt{2}$



$DE = \sqrt{2}$

$BE = BD = DE = \sqrt{2}$ (les diag. de 3 faces du Cube)

2) $B(1,0,0)$ $E(0,0,1)$ $D(0,1,0)$

$$\vec{BE}\begin{pmatrix}-1\\0\\1\end{pmatrix} \wedge \vec{BD}\begin{pmatrix}-1\\1\\0\end{pmatrix} = \begin{pmatrix}-1\\-1\\-1\end{pmatrix}$$

est un Vect normal à (BDE)

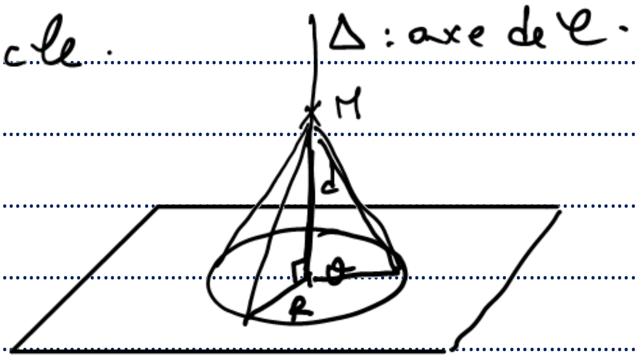
$(BED) : -x - y - z + d = 0$

$B(1,0,0) \in (BED) \Rightarrow d = 1$

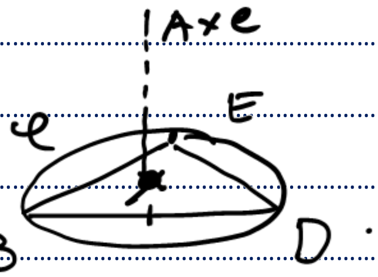
$(BED) : -x - y - z + 1 = 0$

$$(BED) : x + y + z - 1 = 0.$$

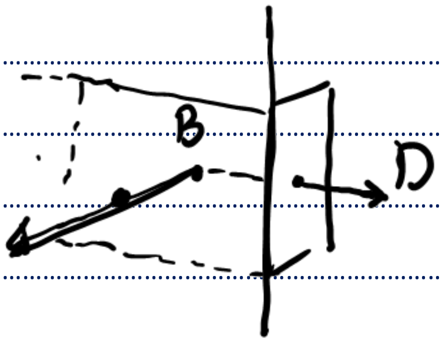
Axe d'un cercle.

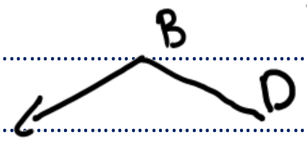


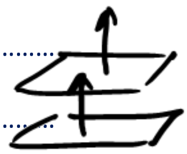
$\mathcal{C}$ = cercle circonscrit à un triangle EBD.



$\underbrace{\boxed{ME = MB = MD}} \Rightarrow M \in$ Axe de $\mathcal{C}$





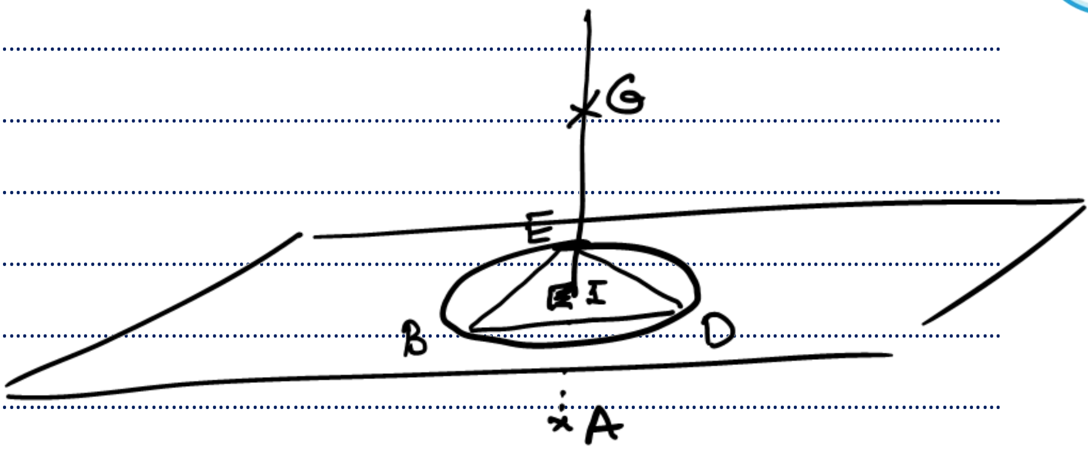


App

$AB = AD = AE = 1 \Rightarrow A \in$ Axe du $\mathcal{C}_{(EBD)}$

$GE = GD = GB = \sqrt{2} \Rightarrow G \in$ Axe du $\mathcal{C}_{(EBD)}$

$\Rightarrow (AG) =$ axe du cercle circonscrit à EBD.



- $AI = d(A\,;\,(BED)) = \dfrac{|0+0+0.-1|}{\sqrt{1^2+1^2+1^2}} = \dfrac{1}{\sqrt{3}}$

$A(0,0,0)$

$(EBD) : x + y + z - 1 = 0$

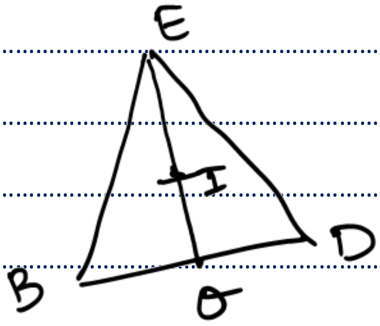
- $\overrightarrow{AG}\begin{pmatrix}1\\1\\1\end{pmatrix} \Rightarrow AG = \sqrt{1^2+1^2+1^2} = \sqrt{3}.$

$\dfrac{1}{3} AG = \dfrac{1}{3}.\sqrt{3} = \dfrac{1}{\sqrt{3}} = AI$

$$\text{donc } \begin{cases} AI = \dfrac{1}{3} AG \\ \text{Sens } \overrightarrow{AI} = + \text{ Sens } \overrightarrow{AG} \end{cases}$$

$\Rightarrow \overrightarrow{AI} = \dfrac{1}{3} \overrightarrow{AG}.$

Idée 2 : Coordonnées de I



$E(0,0,1)$

$\overrightarrow{EI} = \frac{2}{3}\overrightarrow{EO}$:

$\begin{pmatrix} x- \\ y \\ z-1 \end{pmatrix} = \frac{2}{3}\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{pmatrix}$ ...

$\overrightarrow{AI}\begin{pmatrix} \\ \\ \end{pmatrix}$ $\overrightarrow{AG}\begin{pmatrix} \\ \\ \end{pmatrix}$

Idée Vecteur.

II) $h = h_{(A,k)}$

- $h_{(A,k)}(G) = M_k \iff \overrightarrow{AM_k} = k\,\overrightarrow{AG}$

1) $k = \frac{1}{3}$ $\quad h = h_{(A,\frac{1}{3})}$

$$\overrightarrow{AM_{\frac{1}{3}}} = \frac{1}{3}\overrightarrow{AG} = \overrightarrow{AI}$$

$$M_{\frac{1}{3}} = I$$

- $P_{\frac{1}{3}} \parallel (BDE)$ et passe par I

$$P_{\frac{1}{3}} = (BDE)$$

- $N_{\frac{1}{3}} = (BDE) \cap (BC) = B.$

$M_{\frac{1}{3}} = I \quad , \quad P_{\frac{1}{3}} = (BDE) \quad N_{\frac{1}{3}} = B.$

2) a) $\overrightarrow{AM_k} = k\,\overrightarrow{AG}$

$A(0,0,0)$
$G(1,1,1)$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = k \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} k \\ k \\ k \end{pmatrix}$$

$\cdot\ M_k(k, k, k)$

b) $P_k \parallel (BDE) \Rightarrow \vec{n}_{(BDE)} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \vec{n}_{P_k}$

$P_k : x + y + z + d = 0$

$M_k(k, k, k) \in P_k \Rightarrow d = -3k$

$$\boxed{P_k : x + y + z - 3k = 0}$$

$*\ (P_k) \cap (BC) = \{N_k\}$

$(BC) = \mathcal{D}(B, \vec{BC})$

$N_k(x, y, z)$

$\vec{BC} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$$\begin{cases} x = 1 + 0\alpha \\ y = 0 + \alpha \\ z = 0 + 0\alpha \end{cases}$$

$\bullet\ x + y + z - 3k = 0$

$\Rightarrow$

$$\begin{cases} x = 1 \\ y = \alpha \\ z = 0 \end{cases}$$

$$x + y + z - 3k = 0$$

$$1 + \alpha + 0 - 3k = 0$$

$$\alpha = 3k - 1$$

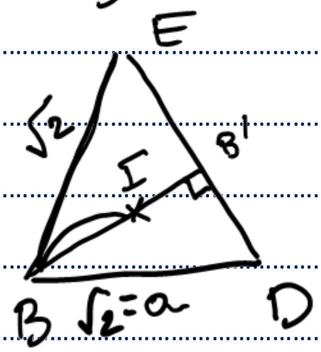
$$N\left(1, 3k-1 ; 0\right)$$

II

1) $M_{\frac{1}{3}} N_{\frac{1}{3}} = ?$

$M_{\frac{1}{3}} = I$

$N_{\frac{1}{3}} = B$



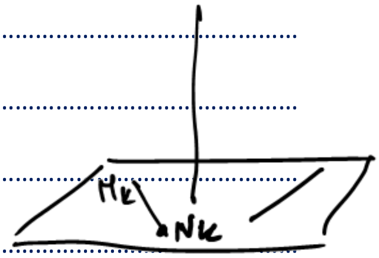
$$IB = \frac{2}{3} \underbrace{IB'}$$

$$= \frac{2}{3} \times \sqrt{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}.$$

3) b) $k = ?$

$$\begin{cases}(M_kN_k) \perp (AG)\\ (M_kN_k) \perp (BC)\end{cases}$$



or $(AG) \perp P_k$

$\Rightarrow (AG) \perp (M_kN_k) \quad \forall k$

$k = ?$ $\quad (M_kN_k) \perp (BC)$

$$\left.\begin{array}{l} M_k(k, k, k)\\ N_k(1, 3k-1, 0)\end{array}\right\} \quad \overrightarrow{N_kM_k}\begin{pmatrix}k-1\\1-2k\\k\end{pmatrix}$$

$$\overrightarrow{BC}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$K = ?$ $\quad \overrightarrow{N_kM_k} \cdot \overrightarrow{BC} = 0$

$\Rightarrow 0 + 1 - 2k + 0 = 0$

$$\boxed{k = \frac{1}{2}}$$

$$* \ \overrightarrow{N_k M_k} \begin{pmatrix} k-1 \\ 1-2k \\ k \end{pmatrix}$$

$$\Rightarrow N_k M_k = \sqrt{(k-1)^2 + (1-2k)^2 + k^2}$$

$$= \sqrt{6k^2 - 6k + 2} = f(k)$$

$$f'(k) = \frac{12k-6}{2\sqrt{\quad}} = 0$$

$$k = \frac{1}{2}$$

Exercice 4 :

$* \ 1]$. $\varphi_n(x) = \frac{1}{x} - \frac{1}{n}\ln(x)$

φ_n d^{ble} sur $]0,+\infty[$

$$\varphi_n'(x) = \frac{-1}{x^2} - \frac{1}{n} \times \frac{1}{x}.$$

$$= \frac{-n \quad -x}{n x^2} < 0$$

$$* \lim_{0^+} \frac{1}{x} - \frac{1}{n}\underbrace{\ln x}_{\to -\infty} = +\infty$$

$$\frac{1}{x} \to +\infty$$

$$= +\infty + \infty$$

$$* \lim_{+\infty} \frac{1}{x} - \frac{1}{n}\ln(x) = -\infty$$

$$\frac{1}{x} \to 0 \qquad -\infty$$

x	0	1	α_n	$+\infty$
$\varphi'_n(x)$	‖		$-$	
$\varphi_n(x)$	‖ $+\infty$	$\searrow$	0	$\searrow$ $-\infty$

φ_n Cont et Strict $\downarrow$ Sur $]0;+\infty[$

$\varphi_n\left(]0,+\infty[\right) =]-\infty,+\infty[$

$0 \in \mathbb{R}$ donc $\exists!$ sol $\alpha_n \in]0,+\infty[$.

$\varphi_n(1) = \frac{1}{1} - \frac{1}{n}0 = 1 > 0$

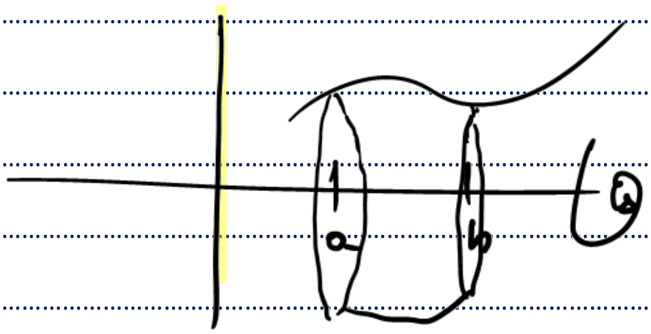
$$\varphi_n(1) = 1 > \varphi_n(\alpha_n) = 0$$

$\varphi_n^{-1} \downarrow$

$$\varphi_n^{-1}\left(\varphi_n(1)\right) < \underbrace{\varphi_n^{-1}\left(\varphi_n(\alpha_n)\right)}$$

$$\boxed{1 < \alpha_n}$$

Cours



$$V = \pi \int_a^b f^2(x)\, dx$$

$x \longrightarrow \varphi_1(x) = \frac{1}{x} - \ln x$ Cont sur $\left[\frac{1}{e}, 1\right]$.

$$V = \pi \int_{\frac{1}{e}}^{1} \varphi_1^2(x)\, dx$$

$$= \pi \int_{\frac{1}{e}}^{1} \left(\frac{1}{x} - \ln x\right)^2 dx.$$

$$= \pi \int_{\frac{1}{e}}^{1} \underbrace{\frac{1}{x^2}} + \ln^2(x) - \underbrace{2\,\frac{1}{x}\ln x}\, dx$$

$$= \pi \left(\int_{1/e}^{1} \frac{1}{x^2}\, dx - 2 \int_{\frac{1}{e}}^{1} \underbrace{\frac{1}{x}}_{u'} \underbrace{\ln(x)}_{u^1} dx + \int_{\frac{1}{e}}^{1} \ln^2 x \, dx \right)$$

$$I = \int_{\frac{1}{e}}^{1} \ln^2(x)\, dx = \int_{\frac{1}{e}}^{1} \ln(x) \cdot \ln x \, dx$$

$$U(x) = \ln x \longrightarrow u'(x) = \frac{1}{x}$$

$$v'(x) = \ln x \longrightarrow V(x) = x\ln x - x$$

$$I = \Big[\ln(x)\,(x\ln x - x) \Big]_{\frac{1}{e}}^{1} - \int_{\frac{1}{e}}^{1} \ln x - 1 \, dx$$

$$= \left(0 - \ln\left(\frac{1}{e}\right)\left(\frac{1}{e}\ln\left(\frac{1}{e}\right) - \frac{1}{e}\right)\right) - \Big[x\ln x - x - x\Big]_{\frac{1}{e}}^{1}$$

$$= 0 + \frac{2}{e} - \Big[x\ln x - 2x\Big]_{\frac{1}{e}}^{1}$$