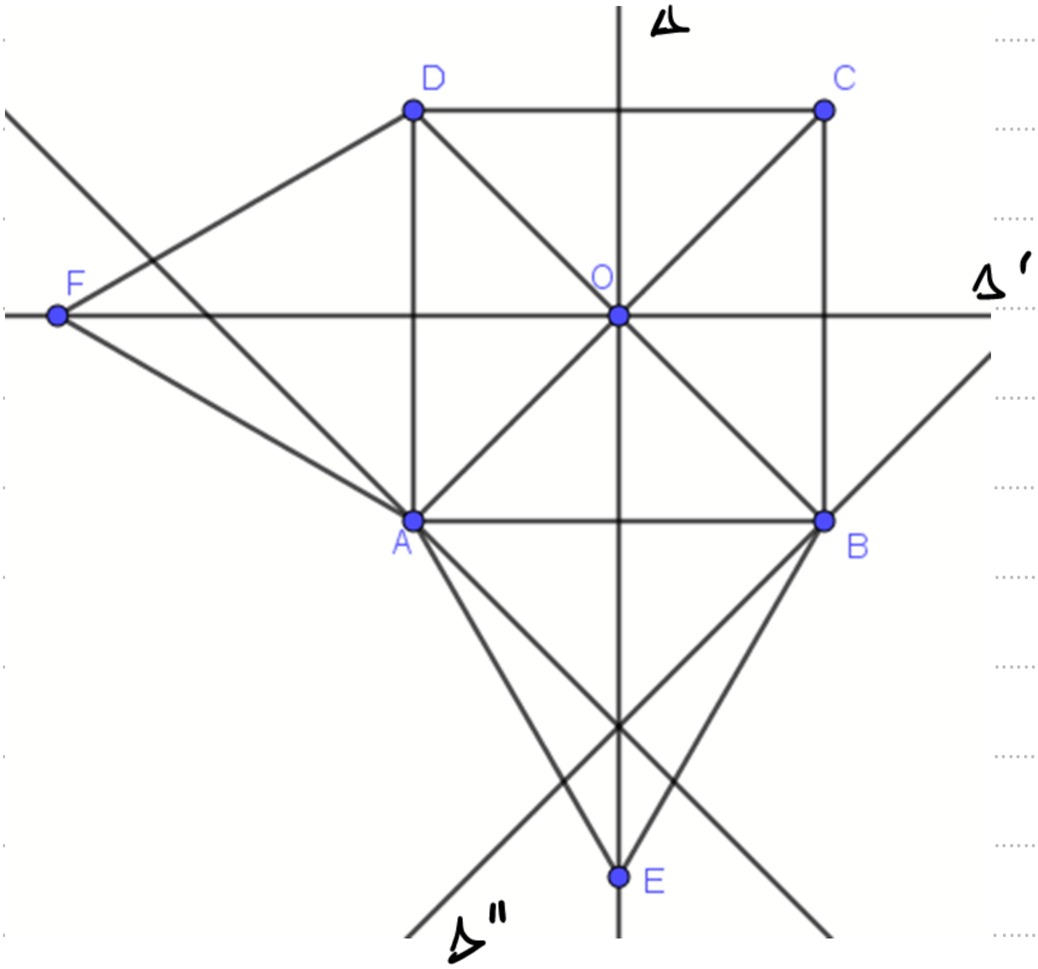
* Ex2 : (suite correction)

2)



a) R(A) = D
R(E) = C ?

on a : • A ≠ E

- $AE = AB = DC$

$\Rightarrow$ il existe un unique déplacement R tq $\begin{cases} R(A) = D \\ R(E) = C \end{cases}$

$$\theta_R \equiv (\widehat{\vec{AE}, \vec{DC}})\ [2\pi]$$
$$\equiv (\widehat{\vec{AE}, \vec{AB}})\ [2\pi]$$
$$\equiv \frac{\pi}{3}\ [2\pi] \not\equiv 0\ [2\pi]$$

$\Rightarrow$ R est une rotation.

b) R rotation d'angle $\frac{\pi}{3}$.
soit Ω le centre de R

on a : $R(A) = D \Leftrightarrow \begin{cases} \Omega A = \Omega D \\ (\widehat{\vec{\Omega A}, \vec{\Omega D}}) \equiv \frac{\pi}{3}\ [2\pi] \end{cases}$

or $\begin{cases} FA = FD \\ (\widehat{\vec{FA}, \vec{FD}}) \equiv \frac{\pi}{3}\ [2\pi] \end{cases} \Rightarrow \Omega = F$

$$R = R_{(F, \frac{\pi}{3})}$$

* on a : $R(E) = C$

donc $\begin{cases} FE = FC \\ (\overrightarrow{FE}, \overrightarrow{FC}) \equiv \pi/3 \; [2\pi] \end{cases}$

$\Rightarrow$ FEC équilatéral direct

3) $\varphi = S_{(BD)} \circ \underbrace{S_{(DA)} \circ S_{(AB)}}$

S_A car $(DA) \perp (AB)$ en A.

$= S_{(BD)} \circ S_A$.

S_{Δ_1} $\quad\quad$ $S_{(AC)}$

$\begin{cases} \Delta_1 \; // \; (BD) \text{ passant par } A \\ (AC) \perp \Delta_1 \text{ en } A. \end{cases}$

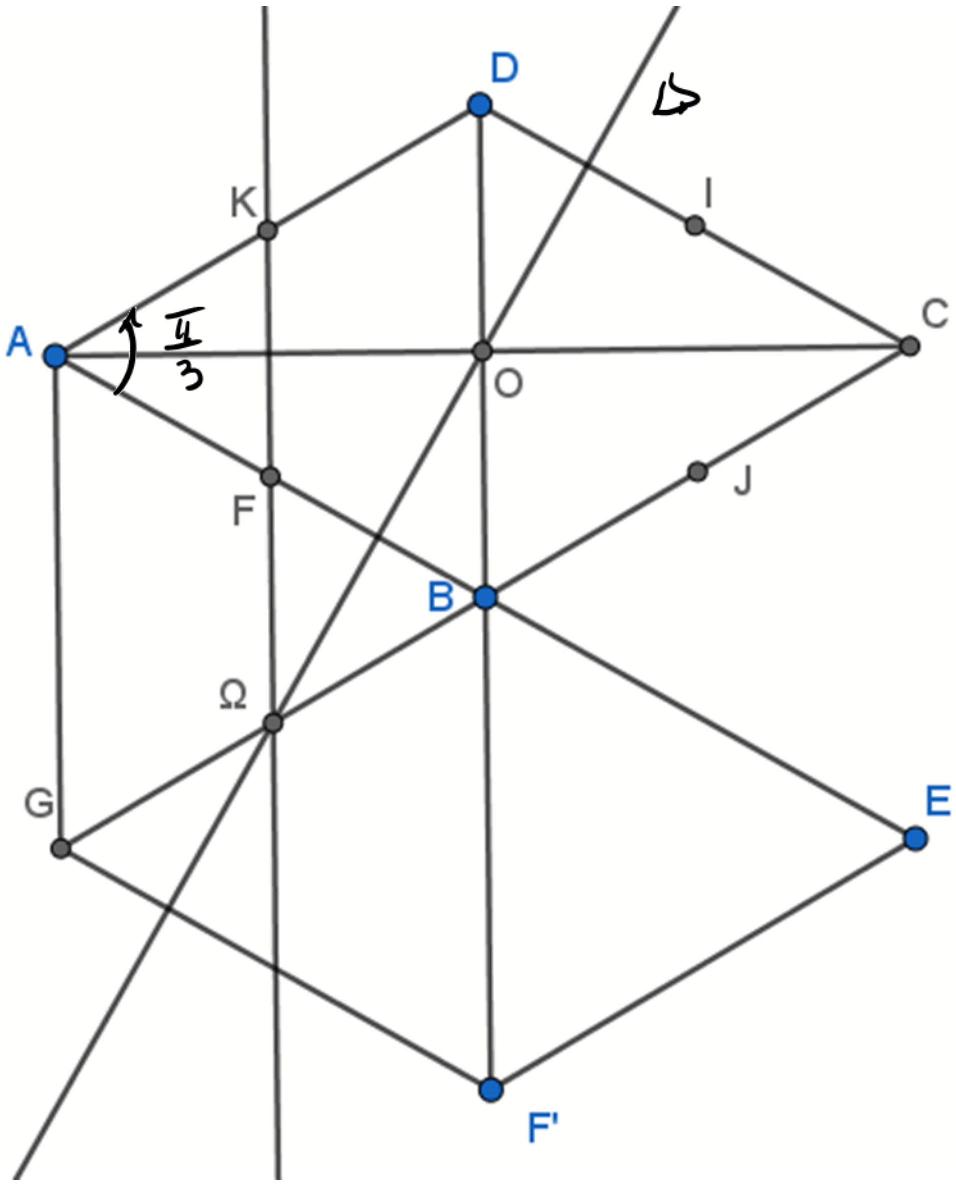
donc

$$u = \underbrace{S_{(BD)} \circ S_{\Delta}}_{t_{2\vec{AO}} = t_{\vec{AC}}} \circ S_{(AC)}$$

$= t_{\vec{AC}} \circ S_{(AC)}$, $\vec{AC}$ direction de (AC)

donc u symétrie glissante de vecteur $\vec{AC}$ et d'axe (AC)

* Ex 3 :



1) f isométrie fixe ABCD

a) $f([AC]) = [AC]$

$f(ABCD) = ABCD$

$\Rightarrow \boxed{f(\{A, B, C, D\}) = \{A, B, C, D\}}$

(rectangle circonscrit au losange ABCD est globalement invariant)

* si $f(A) = B$ $\quad \cancel{A}, B, C, D \rightarrow A, \cancel{B}, C, D$

$\Rightarrow f\{B, C, D\} = \{A, C, D\}$

$\Rightarrow f(BCD) = ACD$ $\boxed{\text{imp}}$

car BCD équilatéral

or ACD isocèle et non équilatéral

* si $f(A) = D$, $A, B, C, D \longrightarrow A, B, C, D$

$\Rightarrow f\{B, C, D\} = \{A, B, C\}$

$\Rightarrow f(BCD) = ABC$ imp

car de même ABC n'est pas équilatéral

donc $f(A) \in \{A, C\}$

De même $f(C) \in \{A, C\}$.

$\Rightarrow f\{A, C\} = \{A, C\}$.

$\Rightarrow \boxed{f([AC]) = [AC]}$

Rq

$\boxed{f[BD] = [BD]}$ car $f\{B, D\} = \{B, D\}$

car. $\cancel{A}, \underline{B}, \cancel{C}, \underline{D} \longrightarrow \cancel{A}, \underline{B}, \cancel{C}, \underline{D}$

$$\left.\begin{array}{l} * \ \rho\,[AC] = [AC] \\ O = A * C \end{array}\right\} \Rightarrow \boxed{\rho(O) = O}$$

b/ $\begin{cases} \rho(\{A, C\}) = \{A, C\}. \\ \rho\{B, D\} = \{B, D\} \\ \rho(O) = O \end{cases}$

<u>1 cas</u> : si $\rho(A) = A \Rightarrow \rho(C) = C$

$\Rightarrow \boxed{\rho = Id}$ ou $\boxed{\rho = S_{(AC)}}$

dans ce cas. $\rho(B) = B$
$\rho(D) = D$

dans ce cas
$\rho(B) = D$ et $\rho(D) = B$

<u>2 cas</u> : si $\rho(A) = C \Rightarrow \rho(C) = A$.

- si $\rho(B) = B \Rightarrow \rho(D) = D$

$\Rightarrow \boxed{\rho = S_{(BD)}}$

• si $\rho(B) = D$ et $\rho(D) = B$

$$\left.\begin{array}{l} \rho(A) = C = S_O(A) \\ \rho(B) = D = S_O(B) \\ \rho(C) = A = S_O(C) \\ A, B, C \text{ non alignés.} \end{array}\right\} \Rightarrow \rho = S_O$$

$\Gamma = \{ \rho \text{ isom} \,/\, \rho(ABCD) = ABCD \}$

on a: $\Gamma \subset \{ Id, S_{(AC)}, S_{(BD)}, S_O \}$

$\supset$? $Id(ABCD) = ABCD$

$S_{(AC)}(ABCD) = ABCD$

$S_{(BD)}(ABCD) = ABCD$

$S_O(ABCD) = ABCD$

donc $\Gamma = \{ Id, S_{(AC)}, S_{(BD)}, S_O \}$

2) a) $\rho_1 = S_{(AC)} \circ S_{(AB)}$

$(AC) \cap (AB) = \{A\}$.

$\rho_1 = R(A, \alpha)$,

$\alpha \equiv 2(\widehat{\overrightarrow{AB}, \overrightarrow{AC}})\ [2\pi]$

$\equiv 2\frac{\pi}{6}\ [2\pi] \equiv \frac{\pi}{3}\ [2\pi]$

$\boxed{\rho_1 = R(A, \frac{\pi}{3})}$

* $\rho_2 = S_{(CD)} \circ S_{(CA)}$

$(CD) \cap (CA) = \{C\}$.

$\Rightarrow \rho_2 = R(C, \alpha)$?

$\alpha \equiv 2(\widehat{\overrightarrow{CA}, \overrightarrow{CD}})\ [2\pi]$

$\equiv -\frac{\pi}{3}\ [2\pi] \Rightarrow \boxed{\rho_2 = R(C, -\frac{\pi}{3})}$

b) $g = R_{(C, -\frac{\pi}{3})} \circ R_{(A, -\frac{\pi}{3})}$

$= S_{(CD)} \circ \cancel{S_{(CA)}} \circ \cancel{S_{(AC)}} \circ S_{(AB)}$

$= S_{(CD)} \circ S_{(AB)}$

$(CD) \parallel (AB)$

$\Rightarrow \boxed{g = t_{2\overrightarrow{FD}}}$ car $F \in (AB)$

$D = p_{(DC)}(F)$

car $(FD) \perp (AB)$

ABD équil, $F = A * B$

et $(AB) \parallel (DC)$

$\Rightarrow (FD) \perp (DC)$