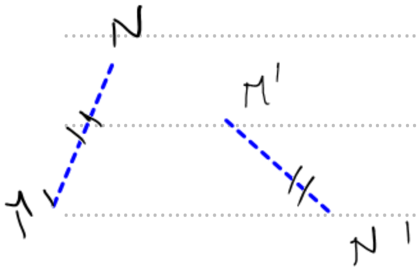
f est une isometrie $\Leftrightarrow$ $\boxed{f \text{ conserve les distances}}$



• M et N 2 pts $(M \neq N)$

• $f(M) = M'$

• $f(N) = N'$

$M'N' = MN$.

Théo • f, g 2 isometries

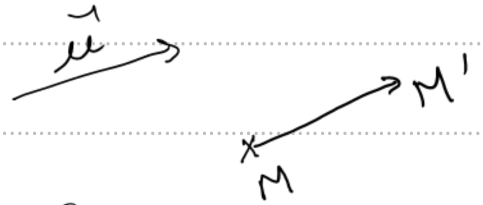
• $f \circ g$ une isometrie.

• f isometrie $\Leftrightarrow$ f est une bijection.

$$M \xrightarrow{f} N$$

① • tout pt M admet une unique image

② • tout pt N admet un unique antecedent

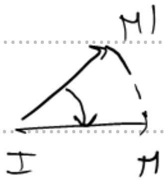


• f est une bij $\Leftrightarrow$ f admet une isometrie reciproque.

$$\begin{tikzcd} A \arrow[r, shift left, "f"] & A' \arrow[l, shift left, "f^{-1}"] \end{tikzcd}$$

- $\left(T_{\overrightarrow{AB}}\right)^{-1} = T_{\overrightarrow{BA}}$
- $\left(S_{\Delta}\right)^{-1} = S_{\Delta}$

- $R_{(I,\alpha)} = R_{(I,-\alpha)}$



Théo

f une isométrie .

① fixe 3 pts non alignés $\quad f(A)=A$, $f(B)=B$, $f(C)=C$ $\quad f = Id$

② fixe 2 pts $\quad f(A)=A$, $f(B)=B$ $\Rightarrow f = Id$ ou $f = S_{(AB)}$

③ f fixe uniquement 1 pt $\quad f(A)=A \Rightarrow f = R_{(A,\, \alpha \neq 0)}$

4) f n'a pas de pts fixe.

$f = T_{\vec{u}}$ $(\vec{u} \neq \vec{0})$ ou $f = $ sym. glissante

$= T_{\vec{v}} \circ S_{\Delta}$

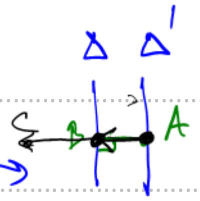
$\vec{v}$ dir de Δ

- $T_{\vec{0}} = Id$
- $R_{(A,0)} = Id$

5) $f(A) = A$

$f = R_{(A,\, \alpha \neq 0)}$ $\boxed{\text{ou}}$ S_{Δ} $(A \in \Delta)$ $\boxed{\text{ou}}$ Id.

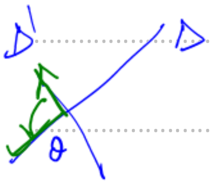
$$S_\Delta \circ S_\Delta = Id$$

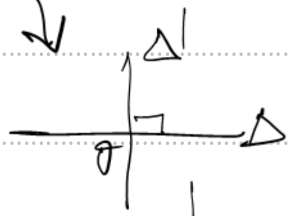


$f = T_{2\vec{AB}}$.

$f = \{ S_{\Delta_2} \circ S_{\Delta'} = ?$

$g = T_{\vec{AC}}$

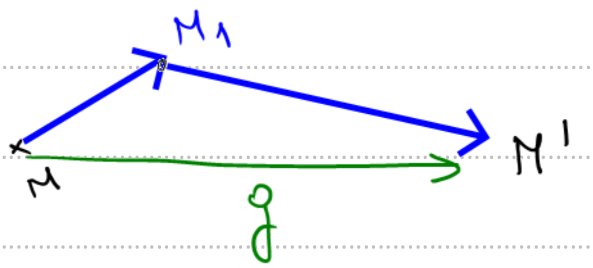




$f = R_{(O,\, 2(\vec{u_{\Delta'}},\, \vec{u_\Delta}))}$ $\quad$ $f = S_I = R_{(I,\pi)}$

. Exercice 1

a) $g = T_{\vec{AB}} \circ T_{\vec{CI}} = T_{\vec{AB}+\vec{CI}} = T_{\boxed{\vec{CI}}+\vec{AB}}$



$\Rightarrow g = T_{\boxed{\vec{IA}}+\vec{AB}} = T_{\vec{IB}}$.

b) $g = T_{\vec{AB}} \circ S_{(AD)}$

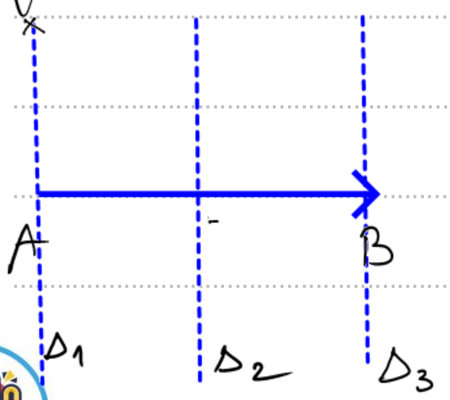
Remarque (rappel)

$$T_{\vec{u}} \circ S_{\Delta} = \begin{cases} \text{Symet. ortho } S_{\Delta'} \ (\text{fixe tous les pts de } \Delta') \\ \text{ou} \\ \text{Sym. glissante } (\text{pas de pt fixe}) \end{cases}$$

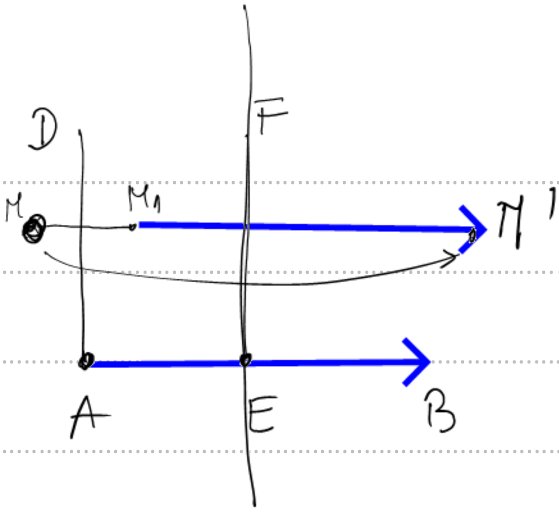
$$\text{Si } \begin{cases} t_{\vec{u}} \circ S_{\Delta}(A) = A \Rightarrow A \in \Delta' \\ t_{\vec{u}} \circ S_{\Delta}(B) = B \Rightarrow B \in \Delta' \end{cases}$$

$$T_{\vec{u}} \circ S_{\Delta} = S_{(AB)}$$

$$g = T_{\vec{AB}} \circ S_{(AD)}$$



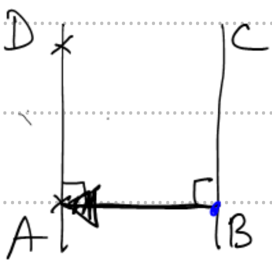
$$T_{\vec{AB}} = S_{\Delta_2} \circ S_{\Delta_1} = S_{\Delta_3} \circ S_{\Delta_2}$$



$$g = T_{\overrightarrow{AB}} \circ S_{(AD)}.$$

$$= S_{(EF)} \circ \underbrace{S_{(AD)} \circ S_{(AD)}}_{Id.}$$

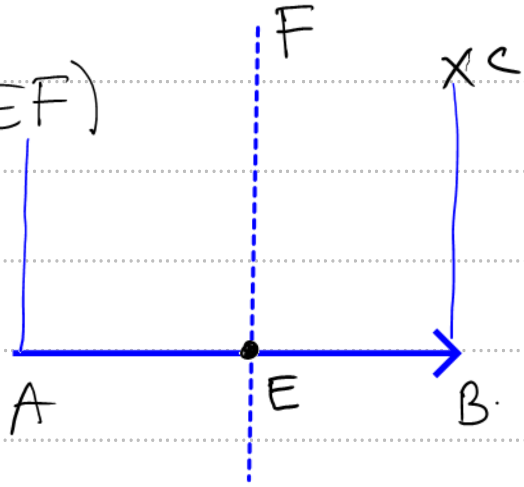
$$\boxed{g = S_{(EF)}.}$$



$$S_{\boxed{(AD)}} \circ S_{\boxed{(BC)}} = T_{2\overrightarrow{BA}}$$

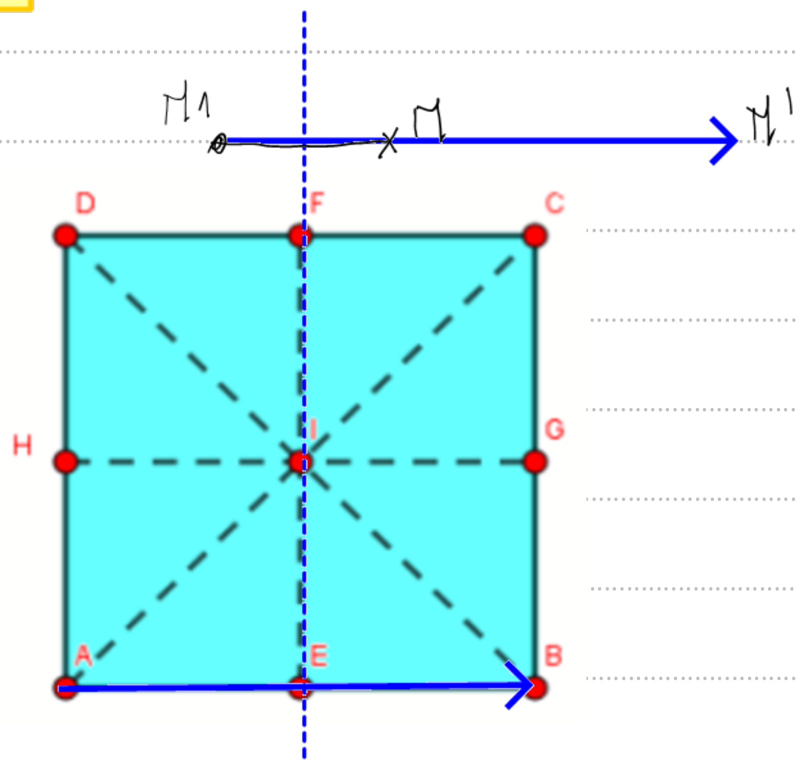
$$S_{(BC)} \circ S_{\boxed{(AD)}} = T_{2\overrightarrow{AB}}$$

c) $g = \boxed{T_{\overrightarrow{AB}}} \circ S_{(EF)}$



$g = S_{(BC)} \circ \underbrace{S_{(EF)} \circ S_{(EF)}}_{Id.}$

$$\boxed{g = S_{(BC)}}$$



2ème méthode : $(g = T_{\overrightarrow{AB}} \circ S_{(EF)})$

$$\begin{tikzcd} \boxed{B} \arrow[r, "S_{(EF)}"] \arrow[rr, bend right, "g"'] & A \arrow[r, "T_{\overrightarrow{AB}}"] & B. \end{tikzcd}$$

$$\boxed{C} \xrightarrow{S_{(EF)}} D \xrightarrow{T_{\overrightarrow{AB}}} \boxed{C}$$

g est une isométrie qui fixe B et C.

donc $\begin{cases} g = Id \ \times \ (\\ \text{ou} \\ g = S_{(BC)} \end{cases}$

(Absurde)

Si $g = Id$. $\quad (g(M) = M \quad \forall M)$

$g(I) = I$

$$\begin{tikzcd} M \arrow[r, "S_{(EF)}"] \arrow[rr, bend right] & M_1 \arrow[r, "T_{\overrightarrow{AB}}"] & M \end{tikzcd}$$

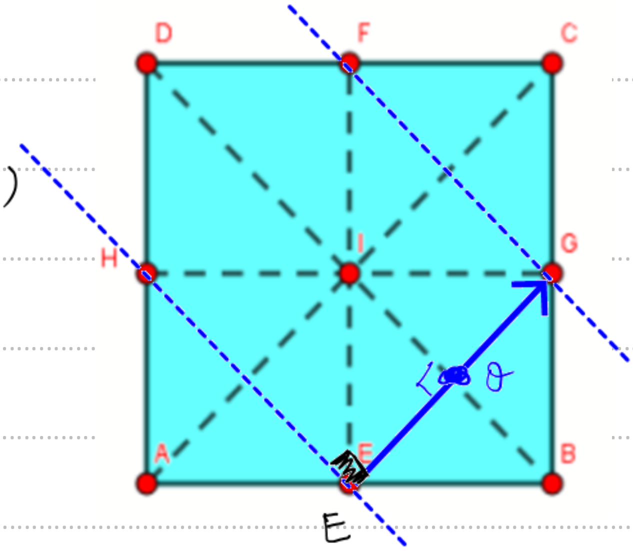
$$\begin{tikzcd} I \arrow[r, "S_{(EF)}"] \arrow[rr, bend right, "g"'] & \overset{g}{I} \arrow[r, "T_{\overrightarrow{AB}}"] & I \end{tikzcd}$$

$$T_{\vec{AB}}(I) = I \Rightarrow \underbrace{\vec{AB} = \vec{II} = \vec{0}}_{\text{Imposs}}$$

d'où

$$g \neq Id \Rightarrow \boxed{g = S_{(BC)}}.$$

d) $g = \boxed{T_{\vec{EG}}} \circ S_{(EH)}$



$g = \boxed{S_{(BD)} \circ S_{(HE)}} \circ S_{(EH)}$

② ①

. EBGI est un Carré

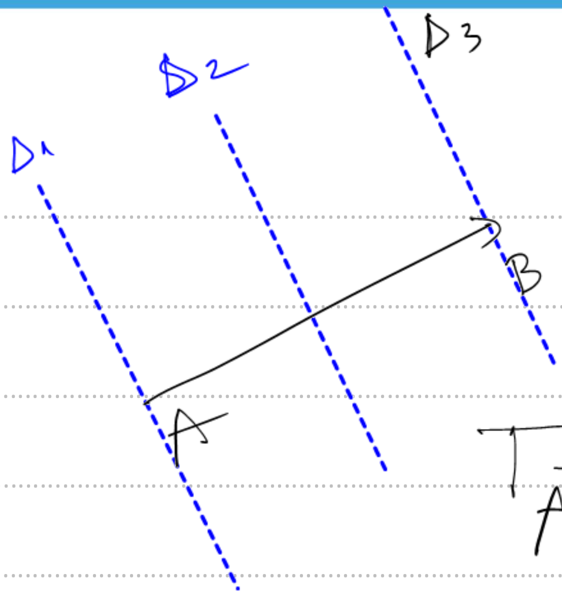
Soit O = Centre EBGI

O = proj ⊥ de E sur (BD)

$\circ S$

$S_{(GF)}$

$g = S_{(EH)}$.



$$T_{\vec{AB}} = S_{\Delta_2} \circ S_{\Delta_1}$$

$$= \underbrace{S_{\Delta_3} \circ S_{\Delta_2}}$$

e) $g = \boxed{S_{\Phi}} \circ S_{(BD)}$

$$g = S_{(AC)} \circ \underbrace{S_{(BD)} \circ S_{(BD)}}_{Id} = S_{(AC)}$$

con

$(AC) \perp (BD)$ en I $\Longrightarrow$ $S_{(AC)} \circ S_{(BD)}$

$$= S_{(BD)} \circ S_{(AC)}$$

$$= S_I \, .$$

f) $g = S_{(AD)} \circ S_{(IG)} = S_H = R_{(H,\pi)}$

car $(AD) \perp (IG)$ en H

g) $g = S_{(AC)} \circ S_{(EF)}$

ona $(AC) \cap (EF) = \{I\}$

donc $g = R_{\left(I\,,\,2\left(\widehat{\overrightarrow{IF}\,,\,\overrightarrow{IA}}\right)\right)}$

$= R_{\left(I\,,\,2\times\frac{3\pi}{4}\right)}$

$= R_{\left(I\,,\,\frac{3\pi}{2}\right)}$

$\frac{3\pi}{2} \equiv -\frac{\pi}{2}\ (2\pi)$ $\boxed{\text{ou}}$

$= R_{\left(I\,,\,2\left(\widehat{\overrightarrow{IF}\,,\,\overrightarrow{IC}}\right)\right)}$

$= R_{\left(I\,,\,2\times -\frac{\pi}{4}\right)}$

$= R_{\left(I\,,\,-\frac{\pi}{2}\right)}$

h) $g = S_{(CD)} \circ \underbrace{R_{(C, -\frac{\pi}{2})}}$

$g = \underbrace{S_{(CD)} \circ S_{(CD)}}_{Id} \circ S_{(CA)}$

$\boxed{g = S_{(CA)}}$. car :

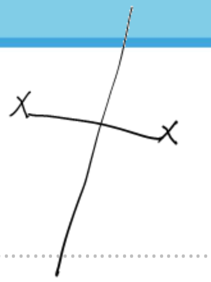
$(CD) \cap (CA) = \{C\}$.

donc $S_{(CD)} \circ S_{(CA)} = R_{(C, 2(\widehat{\vec{CA}, \vec{CD}}))}$

i) $\left.\begin{array}{l} g(A) = A \\ g(I) = I \end{array}\right\}$ g fixe deux pts.

$g = \begin{cases} \cancel{Id} & g(D) = B \quad g \neq I \\ \text{ou} & \\ g = S_{(AI)} & \end{cases}$

$g = S_{(AI)}$



j) $g(A) = A$

S_Δ ou $R_{(A,\alpha \neq 0)}$ ou $\cancel{Id}$.

$A \in \Delta$ $\qquad$ $g(D) = B$

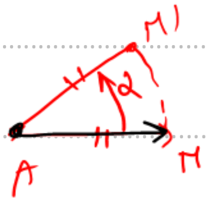
$f = R_{(A,\alpha)}$

* $\alpha = 0$ $\qquad f = Id.$

* $\alpha \neq 0$

→ $f(A) = A$

• $M \neq A$, $M' = f(M)$ $\Longleftrightarrow$ $\begin{cases} AM = AM' \\ (\overrightarrow{AM}, \overrightarrow{AM'}) \equiv \alpha \ (2\pi) \end{cases}$



* $R_{(A,\alpha)}\boxed{(M)} = \boxed{M'}$

$R_{(A,\alpha)}\boxed{(N)} = \boxed{N'}$

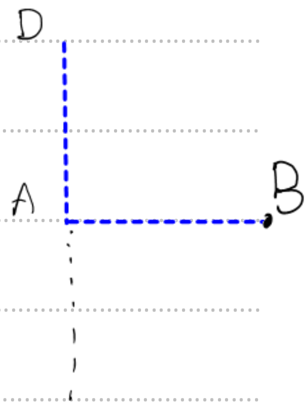
$\Rightarrow (\widehat{\boxed{\overrightarrow{MN}}, \boxed{\overrightarrow{M'N'}}}) \equiv \alpha \ (2\pi)$

Si $g = R_{(A,\, \alpha \neq 0)}$

$\boxed{g(B) = D}$
$\boxed{g(D) = B}$ $\Longrightarrow \alpha \equiv (\widehat{\overrightarrow{BD}, \overrightarrow{DB}})\ (2\pi)$

$\alpha \equiv \pi \quad (2\pi)$

$g = R_{(A,\pi)} = S_A$



Comme $g(B) = D \Longrightarrow \underbrace{A = B * D}_{\text{Absurde}}$.

d'où

$g \neq R_{(A,\, \alpha \neq 0)}$

d $\quad g = S_\Delta$ avec $A \in \Delta$

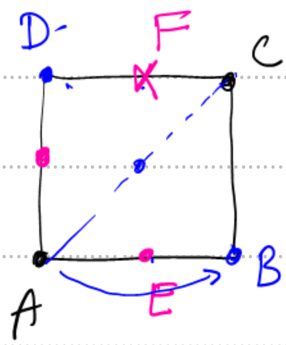
$g(B) = D \Longrightarrow \Delta = \text{med}\,[BD]$

d'où $\boxed{g = S_{(AC)}}$.

k) $g(\boxed{A}) = \boxed{C}$
$g(\boxed{B}) = \boxed{D}$

donc $g(\boxed{A * B}) = \boxed{C * D}$ (Conservation du milieu

$g(E) = F$

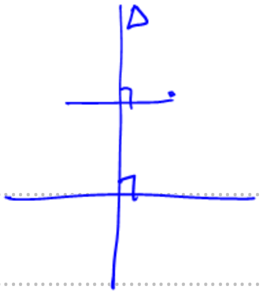


• $g \neq Id$ car $g(A) = C \neq A$

• $g \neq T_{\vec{u} \neq \vec{0}}$ $g(A) = C \Rightarrow \vec{u} = \overrightarrow{AC}$
$g(B) = D \Rightarrow \vec{u} = \overrightarrow{BD}$

donc $\overrightarrow{AC} = \overrightarrow{BD}$ (Impossible)

Si $g = S_\Delta$ $g(A) = C \Rightarrow \Delta = \text{med}[AC]$
$g(B) = D \Rightarrow \Delta = \text{med}[BD]$

$\Longrightarrow$  $[AC] \parallel (BD)$ Impo

$g \neq S_\Delta.$

$*$ Si $g = rot_{(\Omega, \alpha ?)}$

$g(\boxed{A}) = \boxed{C}$

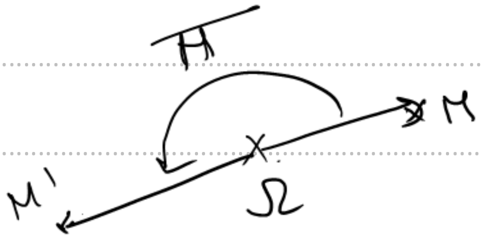
$g(\boxed{B}) = \boxed{D}$

$\Longrightarrow \alpha \equiv (\widehat{\boxed{\overrightarrow{AB}}, \boxed{\overrightarrow{CD}}}) (2\pi)$

$\equiv \pi \ (2\pi)$

donc $g = rot_{(?, \pi)} =$ Symetrie Centrale

$= S_\Omega.$



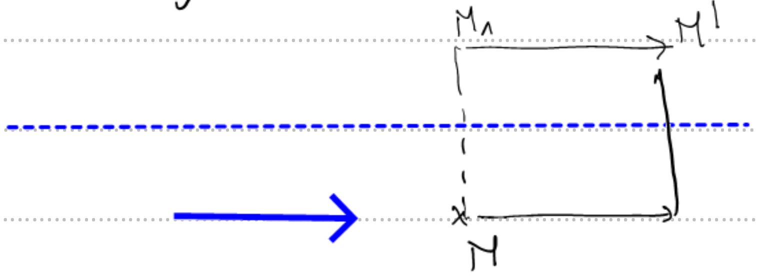
Comme $g(A) = C \Longrightarrow \Omega = A * C = I$

$g(B) = D \Longrightarrow \Omega = B * D = I$

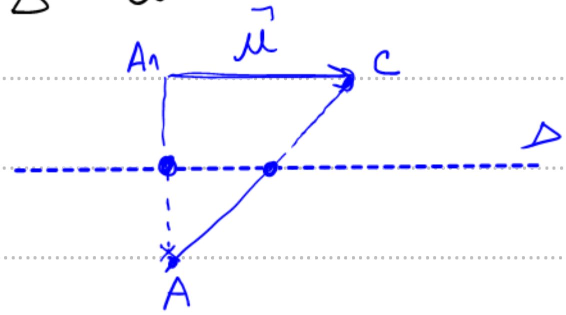
$g = R_{(I, \pi)}$

Si g = Symetrie glissante

$= T_{\vec{u}} \circ S_{\Delta}$ $\quad \vec{u}$ Vect Dir de Δ



$g = T_{\vec{u}} \circ S_{\Delta} = S_{\Delta} \circ T_{\vec{u}}$



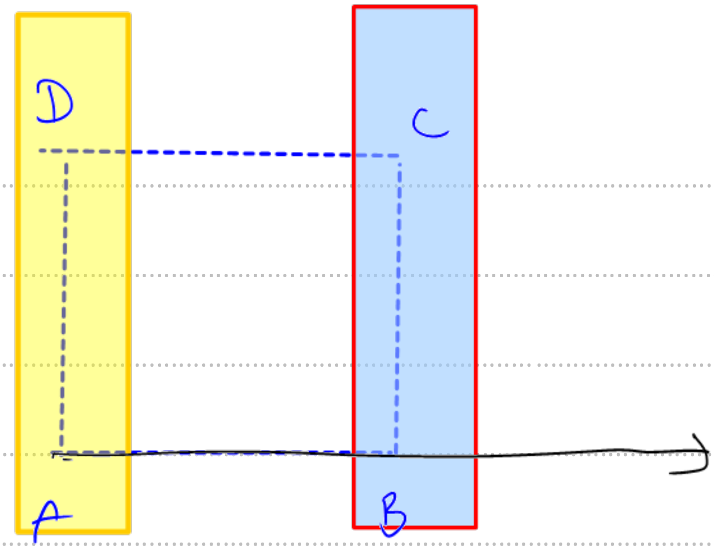
$g(A) = C$

$g(B) = D.$

$g(A) = C \Rightarrow I = A * C \in \text{Axe } \Delta$

$g(B) = D \Rightarrow I = B * D \in \text{Axe } \Delta$

Axe Δ passe I



$$S_{\boxed{(BC)}} \circ S_{\boxed{(AD)}} = t_{2\overrightarrow{AB}} = t_{2\overrightarrow{DC}}$$

$(AB) \parallel (CB)$