Le dipôle RL (1)

Exercice 1

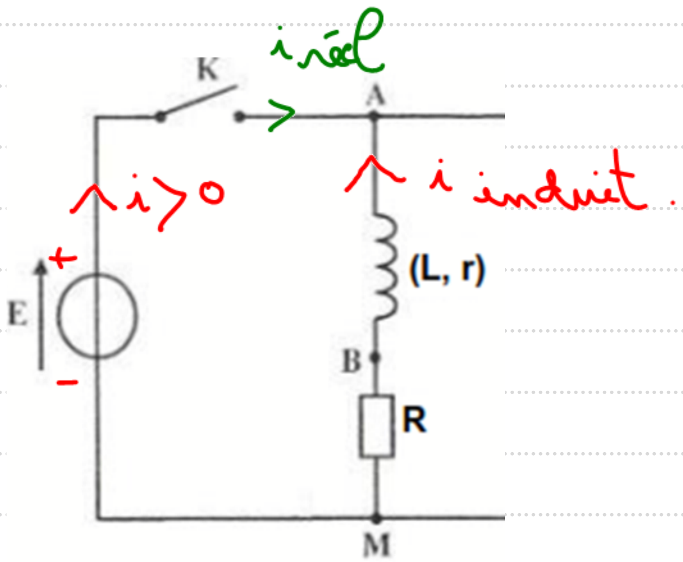
1) a) Auto-induction.

b) La bobine s'oppose à l'établissement du courant dans le circuit.

c)

$i > 0$ (courbe)

→ Le courant réel circule dans le sens positif



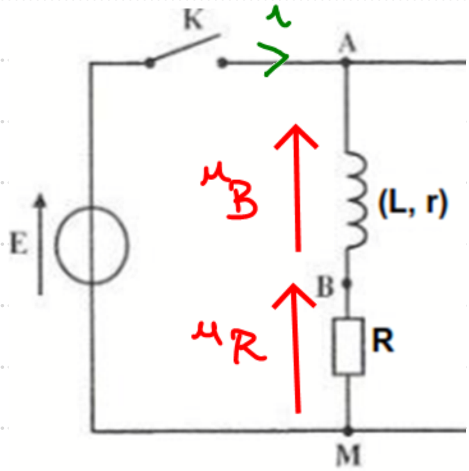
i_{induit} ?

* Cause : $i_G \nearrow$ (courbe) ; $\frac{di}{dt} > 0$

* Réponse : $e = -L\frac{di}{dt} < 0$

→ Le courant induit circule dans le sens négatif (pour s'opposer à l'augmentation du i_G)

2) Loi des mailles :



$$u_R + u_B - E = 0$$

$$u_R + u_B = E$$

$$Ri + ri + L\frac{di}{dt} = E$$

$$L\frac{di}{dt} + (R+r)i = E$$

$$\frac{di}{dt} + \frac{(R+r)}{L} \cdot i = \frac{E}{L}$$

3)

$$i(t) = I_p\left(1 - e^{-t/\tau}\right).$$

$t \rightarrow +\infty$: En régime permanent

$$i = I_p(1-0) = I_p.$$

→ I_p : l'intensité du courant en régime permanent.

on a :

$$\frac{di}{dt} + \frac{R+r}{L} \cdot i = \frac{E}{L}$$

En régime permanent $\begin{cases} i = cte = I_p \\ \frac{di}{dt} = 0 \end{cases}$

on remplace :

$$0 + \frac{R+r}{\cancel{L}} \cdot I_p = \frac{E}{\cancel{L}}$$

$$\boxed{I_p = \frac{E}{R+r}}$$

$\tau = ?$

$$\frac{di}{dt} + \frac{R+r}{L} \cdot i = \frac{E}{L}$$

$$i(t) = I_p \cdot (1 - e^{-t/\tau}) = I_p - I_p \cdot e^{-t/\tau}$$

$$\boxed{\frac{di}{dt} = \frac{I_p}{\tau} \cdot e^{-t/\tau}}$$

on remplace :

$$\frac{I_p}{\tau} \cdot e^{-t/\tau} + \frac{R+r}{L} \cdot I_p - \frac{R+r}{L} \cdot I_p \cdot e^{-t/\tau} = \frac{E}{L}$$

$$\underbrace{I_P \cdot e^{-t/\tau}}_{\neq 0} \cdot \underbrace{\left(\frac{1}{\tau} - \frac{R+r}{L}\right)}_{0} = \frac{E}{L} - \frac{\cancel{R+r}}{L} \cdot \frac{E}{\cancel{R+r}} = 0$$

$$\frac{1}{\tau} - \frac{R+r}{L} = 0$$

$$\frac{1}{\tau} = \frac{R+r}{L}$$

$$\boxed{\tau = \frac{L}{R+r}}$$

4)

$$\boxed{u_R(t) = R \cdot i(t)}$$

$$i(t) = I_p \cdot \left(1 - e^{-t/\tau}\right).$$

$$\boxed{u_R(t) = R \cdot I_p \cdot \left(1 - e^{-t/\tau}\right)}$$

5)

$$\boxed{u_B(t) = L\frac{di}{dt} + ri}$$

$$i(t) = I_p(1 - e^{-t/\tau}) = I_p - I_p e^{-\frac{t}{\tau}}$$

$$u_B(t) = \boxed{L\frac{I_p}{\tau}} e^{-t/\tau} + r I_p(1 - e^{-t/\tau})$$

$$\tau = \frac{L}{R+r} \quad ; \quad \boxed{\frac{L}{\tau}} = R + r$$

$$\boxed{I_p} = \frac{E}{R+r}$$

$$L\frac{I_p}{\tau} = \cancel{(R+r)} \cdot \frac{E}{\cancel{R+r}} = E$$

$$\boxed{u_B(t) = E \cdot e^{-t/\tau} + r I_p(1 - e^{-t/\tau})}$$

2ème méthode : Loi des mailles :

$$u_R + u_B - E = 0$$

$$u_B = E - u_R$$

or : $u_R = R I_p (1 - e^{-t/\tau})$

$$u_B(t) = E - R.I_p(1 - e^{-t/\tau}).$$

$$= E - R.I_p + R.I_p.e^{-t/\tau}$$

$$= E - \frac{R}{R+r}.E + R.I_p.e^{-t/\tau}.$$

$$= \frac{(R+r).E - RE}{R+r} + R.I_p.e^{-t/\tau}$$

$$u_B(t) = \frac{r.E}{R+r} + R.I_p.e^{-t/\tau}.$$

$$= \frac{r.E}{R+r} + R.I_p.e^{-t/\tau}$$

$$u_B(t) = \frac{r.E}{R+r} + R.I_p.e^{-t/\tau}$$

$$u_B(t) = r.I_p + R.I_p.e^{-t/\tau}$$

6)a) $U_{Rp} = ?$

1ère méthode : $u_R = Ri$

En régime permanent : $I_p = \frac{E}{R + r}$

$$U_{Rp} = R.I_p$$

$$\boxed{U_{Rp} = \frac{RE}{R+r}}$$

2ème méthode : $u_R(t) = R.I_p.(1 - e^{-t/\tau})$

En R.P : $t \longrightarrow +\infty$

$$U_{Rp} = R.I_p.(1 - 0)$$

$$= R.I_p = \frac{R.E}{R+r}.$$

Rq : Si :

$$\left(\frac{di}{dt} + \frac{R+r}{L}.i = \frac{E}{L}\right) \times R$$

$$\frac{du_R}{dt} + \frac{R+r}{L}.u_R = \frac{R.E}{L}$$

En RP $\begin{cases} u_R = cte = U_{Rp} \\ \frac{du_R}{dt} = 0 \end{cases}$

$$0 + \frac{R+r}{\cancel{L}} \cdot U_{Rp} = \frac{R.E}{\cancel{L}}$$

$$\boxed{U_{Rp} = \frac{R.E}{R+r}}$$

b)

$$u_B = L.\frac{di}{dt} + ri$$

En R.P $\begin{cases} i = cte = I_p \\ \dfrac{di}{dt} = 0 \end{cases}$

$$\boxed{U_{Bp} = r.I_p = \frac{r.E}{R+r}}$$

2^e méthode :

$$u_B(t) = \underbrace{E.e^{-t/\tau}}_{\to 0} + r.I_p(1 - \underbrace{e^{-t/\tau}}_{\to 0})$$

En RP : $t \to +\infty$: $u_B = U_{Bp}$

$$U_{Bp} = r.I_p = \frac{r.E}{R+r}.$$

3^e méthode (Réponse exacte).

Loi des mailles : $u_R + u_B - E = 0$

En R.P : $U_{Bp} = E - U_{Rp}$

$$U_{Bp} = E - \frac{R.E}{R+r} = \frac{(R+r)E - RE}{R+r}$$

$$\boxed{U_{Bp} = \frac{r.E}{R+r}}$$

$$u_B(t) = E - R.I_p.(1 - e^{-t/\tau}) \quad e^{-t/\tau} \longrightarrow 0$$

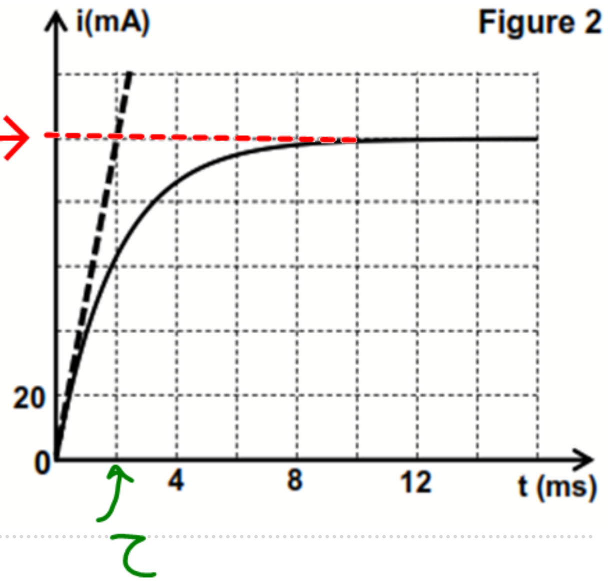
$t \longrightarrow +\infty$: $U_{Bp} = E - R.I_p$

$$\boxed{U_{Bp} = \frac{r.E}{R+r}}$$

7) a)

$I_p = 100\,mA = 0{,}1\,A$.

$\tau = 2\,ms$ (méthode de la tangente à $t=0$).



b)

$R = 50\,\Omega$; $E = 6V$

$$I_p = \frac{E}{R+r} = \frac{E}{R_T}$$

$$R_T = \frac{E}{I_p} = \frac{6}{0{,}1} = 60\,\Omega.$$

$$r = R_T - R = 60 - 50 = 10\,\Omega.$$

$$\tau = \frac{L}{R+r}\ ;\ L = \tau(R+r)$$

$$L = 2\times10^{-3}\times 60$$

$$\boxed{L = 0{,}12\,H}$$

8) a)

$$U_{Rp} = R.I_p = 50 \times 0{,}1 = 5V.$$

$$U_{Bp} = r.I_p = 10 \times 0{,}1 = 1V.$$

Remarque Bac : Si la bobine est purement inductive (idéale) : $r = 0$.

$u_B = L.\frac{di}{dt}$; $\underline{EnRP}$: $i = cte$, $\frac{di}{dt} = 0$

$U_{Bp} = 0$

$$U_{Bp} = \frac{r.E}{R+r} \quad ; \quad r = 0$$

$$= 0.$$

$$U_{Rp} = \frac{R.E}{R+r} = E. \qquad r \rightarrow 0$$

b)

$U_{Rp} = R.I_p$; $U_{Bp} = r.I_p$.

$$\boxed{\frac{U_{Rp}}{U_{Bp}} = \frac{R}{r}} \quad ; \quad \frac{U_{Bp}}{U_{Rp}} = \frac{r}{R}$$

$E - U_{Rp}$

$$\frac{E - U_{RP}}{U_{RP}} = \frac{r}{R}$$

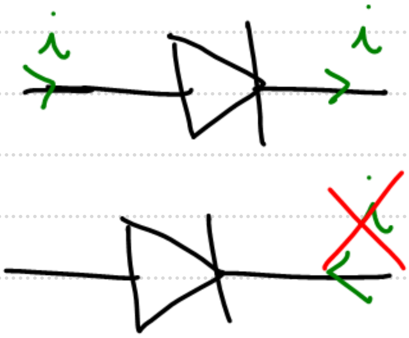
$$\boxed{\frac{E}{U_{RP}} - 1 = \frac{r}{R}}$$

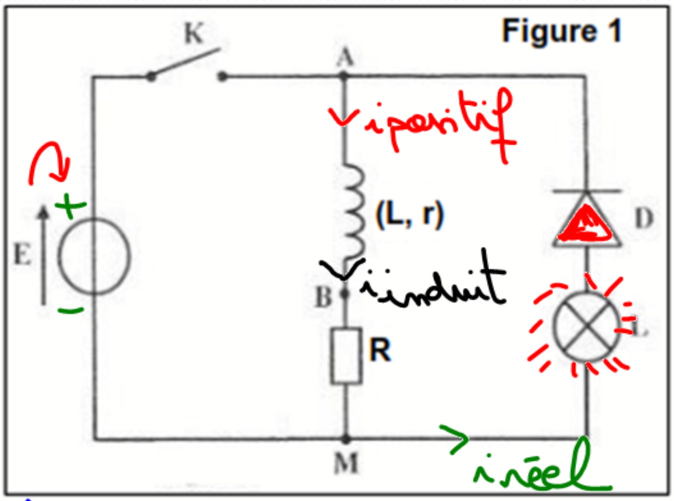
$$\frac{U_{RP}}{U_{BP}} = \frac{R}{r}$$

$$\frac{5}{1} = \frac{R}{r} \Rightarrow r = \frac{R}{5} = \frac{50}{5} = 10\,\Omega.$$

9) a) Auto-induction.

b)





Si : on ouvre K : $i \searrow$ mais $i \geq 0$.

→ Le courant réel circule dans le sens positif.

c) $e = -L \cdot \frac{di}{dt}$

Lorsqu'on ouvre K ; $i \searrow \Rightarrow \frac{di}{dt} < 0$

alors $\boxed{e > 0}$

→ Le courant induit circule dans le sens positif.

d) L'énergie magnétique emmagasinée dans la bobine.

En effet : $E_L = \frac{1}{2} L i^2$

En RP : $i = I_P = I_{max}$

$$\boxed{E_{Lp} = E_{L_{max}} = \frac{1}{2} L \cdot I_P^2}$$

Remarque Bac

$$\frac{du_R}{dt} + \frac{u_R}{\tau} = \frac{R.E}{L}$$

Loi des mailles : $u_R = E - u_B$

$$\frac{du_R}{dt} = -\frac{du_B}{dt}$$

$$-\frac{du_B}{dt} + \frac{E}{\tau} - \frac{u_B}{\tau} = \frac{R.E}{L}$$

$$-\frac{du_B}{dt} - \frac{u_B}{\tau} = \frac{RE}{L} - \frac{E}{\tau}$$

$$= \frac{R.E}{L} - \frac{E}{L}(R+r)$$

$$-\frac{du_B}{dt} - \frac{u_B}{\tau} = -\frac{r.E}{L}$$

$$\boxed{\frac{du_B}{dt} + \frac{u_B}{\tau} = \frac{r.E}{L}}$$