Oscillations électriques forcées : série 3

$$tg(\varphi_{u_B} - \varphi_i) = \frac{L\omega}{r} > 0 \Rightarrow \varphi_{u_B} - \varphi_i > 0$$

$$0 < \varphi_{u_B} - \varphi_i < \frac{\pi}{2}$$

$\Rightarrow$ toujours $u_B(t)$ en avance par rapport à $i(t)$

c-à-d à $u_R(t)$

donc courbe de la voie B $\longrightarrow u_B(t)$

" " " A $\longrightarrow u_R(t)$

2°) ⓐ $N = \frac{1}{T} = \frac{1}{8 \times 2{,}5\,10^{-3}} \Rightarrow \boxed{N = 50\,Hz}$

ⓑ $\boxed{U_{Rm} = 16\,V}$ $\boxed{U_{Bm} = 4.\sqrt{2}\,V}$

ⓒ $Z = \frac{U_m}{I_m} = \frac{U_m}{\frac{U_{Rm}}{R}} \Rightarrow \boxed{Z = \frac{R\,U_m}{U_{Rm}}}$

$I_m = \frac{16}{160}$

$\boxed{I_m = 0{,}1\,A}$ $\boxed{Z = 282{,}84\,\Omega}$

ⓓ $|\varphi_{u_B} - \varphi_{u_R}| = |\varphi_{u_B} - \varphi_i|$ car $u_R = R\,i(t)$

$\rightarrow \varphi_{u_R} = \varphi_i$

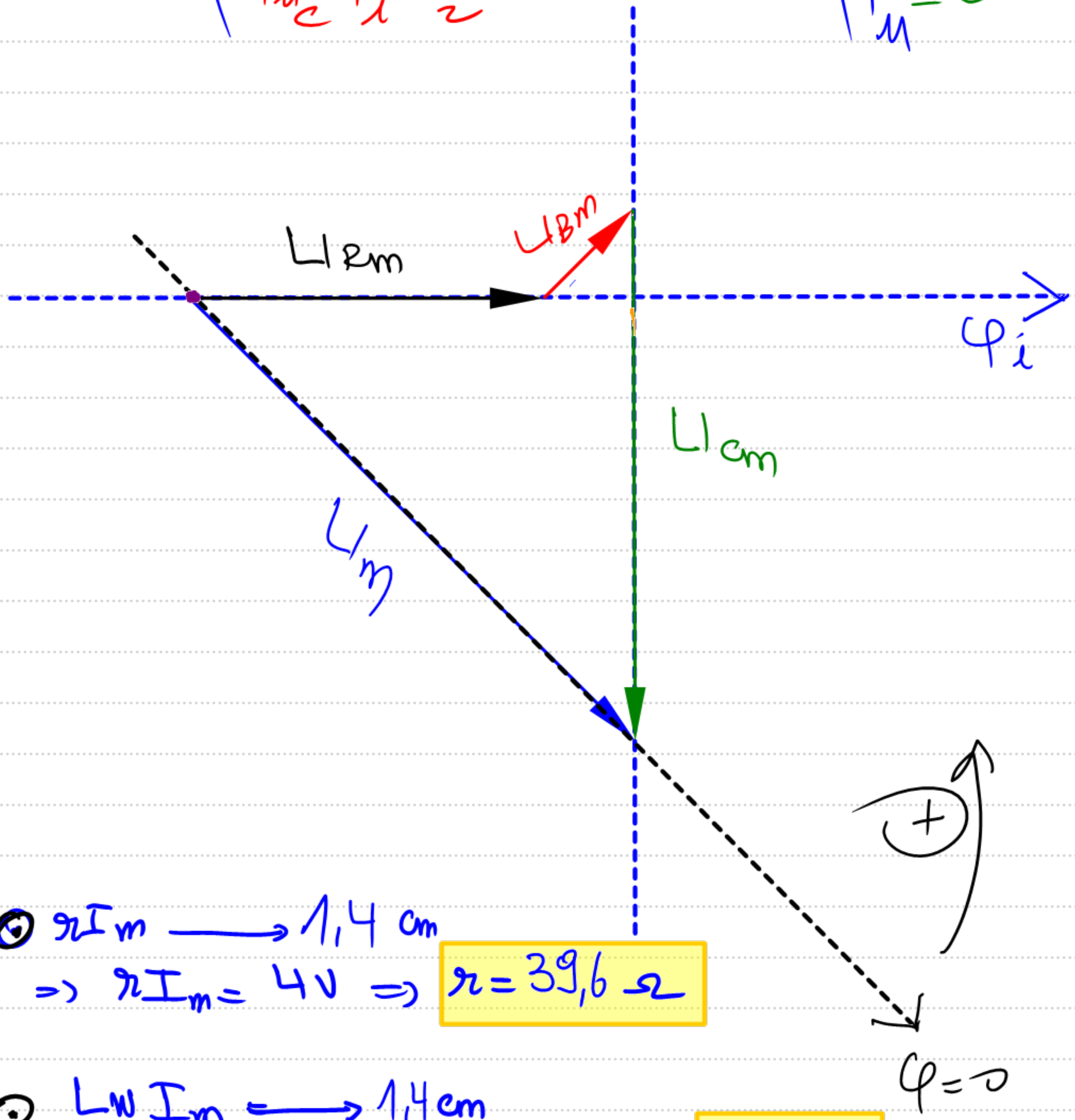
$= \frac{2\pi}{T} \times \frac{T}{8}$

$= \frac{\pi}{4}\,rad$

or $u_B(t)$ en avance sur $u_R(t) \Rightarrow \boxed{\varphi_{u_B} - \varphi_i = \frac{\pi}{4}\,rad}$

$u_R(t) \longrightarrow \begin{cases} U_{Rm} = 16V \longrightarrow 5,65\,cm \\ \varphi_i\ ? \end{cases}$ $\quad u_B(t) \longrightarrow \begin{cases} U_{Bm} = 4\sqrt{2} \longrightarrow 2\,cm \\ \varphi_{u_B} = \varphi_i + \frac{\pi}{4} \end{cases}$

$u_C(t) \longrightarrow \begin{cases} U_{cm}\ ? \\ \varphi_{u_C} = \varphi_i - \frac{\pi}{2} \end{cases}$ $\quad u(t) \longrightarrow \begin{cases} U_m = 20\sqrt{2} \longrightarrow 10\,cm \\ \varphi_u = 0 \end{cases}$



b)

- $rI_m \longrightarrow 1,4\ cm$

$\Rightarrow rI_m = 4V \Rightarrow \boxed{r = 39,6\ \Omega}$

- $L\omega I_m \longrightarrow 1,4\,cm$

$L\omega I_m = 4V \Rightarrow L = \dfrac{4}{2\pi N I_m} \Rightarrow \boxed{L = 0,126\ H}$

⊙ $\frac{I_m}{C\omega} \longrightarrow 8,4\,cm \Rightarrow \frac{I_m}{C\omega} = 16,8\sqrt{2}\,V$

$C = \frac{I_m}{16,8\sqrt{2} \cdot 2\pi N} \Rightarrow \boxed{C = 13,39.10^{-6}\,F}$

c) $\varphi_u - \varphi_i = -45° = -\frac{\pi}{4}\,rad$

⊙ $\varphi_u - \varphi_i < 0 \longrightarrow$ circuit capacitif

d) $\boxed{P_m = \frac{U_m I_m}{2} \underbrace{\cos(\varphi_u - \varphi_i)}}$ par définition

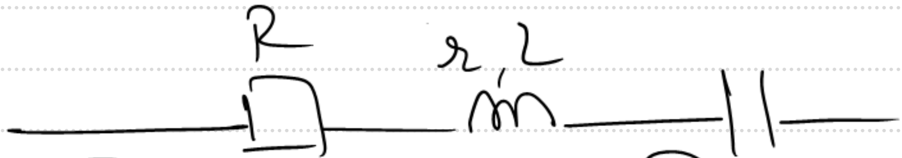
⊙ + jrs $\cos \varphi_u - \varphi_i = \frac{R+r}{Z}$ facteur de puissance.

⊙ $P_m = UI \cos \Delta\varphi$

⊙ $P_m = U I \frac{(R+r)}{Z} \Rightarrow P_m = (R+r) I^2$ ∀N

⊙ A la résonance I_m est max, $\cos \Delta\varphi$ est max (car $\varphi_u - \varphi_i = 0 \Rightarrow \cos \Delta\varphi = 1$)

↳ P_m est max ($N = N_0$)

⊙ 

$P_{m(R)} = RI^2$; $P_{m(B)} = rI^2$

$P_{m(C)} = 0$

$$\boxed{P_m = \frac{U_m I_m}{2} \cos(\varphi_u - \varphi_i)}$$

$$\boxed{P_m = 1\,W}$$

4°) $\varphi_u - \varphi_{u_C} = \frac{\pi}{2}$ (car u_C est tjrs en retard \% $u(t)$) $\left\{ \begin{array}{l} P_m \ max \\ N = N_0 \\ \varphi_u - \varphi_i = 0 \\ Z = R_T \end{array} \right.$

$\varphi_i = \varphi_{u_C} + \frac{\pi}{2}$

$\Rightarrow \varphi_u - (\varphi_i - \frac{\pi}{2}) = \frac{\pi}{2} \Rightarrow \varphi_u - \varphi_i = 0$

Le circuit est en etat de resonance d'intensite

(b) A la resonance d'intensite $Z_T = R + r$

$\Rightarrow I_{m_0} = \frac{U_m}{R+r} = \frac{20\sqrt{2}}{200} \Rightarrow \boxed{I_{m_0} = 0{,}141\,A}$

(c) $P_{m_{res}} = \frac{U_m I_{m_0}}{2}$ $(\cos\Delta\varphi = 1)$

$$\boxed{P_{m_{res}} = 2\,W}$$

(d) Coefficient de surtension

$$Q = \frac{U_{Cm}}{U_m} = \frac{\frac{\cancel{I_{m_r}}}{C\omega_0}}{R_T \cdot \cancel{I_{m_r}}}$$

$$Q = \frac{1}{C\omega_0 R_T}$$

$$Q = \frac{1}{C \frac{1}{\sqrt{LC}} \times R_T} = \frac{1}{\sqrt{\frac{C}{L}}\, R_T}$$

$$Q = \frac{1}{R_T}\sqrt{\frac{L}{C}} = 0{,}48 < 1$$

pas de phenomène de surtension

e) $u(t) = U_m \sin(\omega t + \varphi_u)$

$(R+r)\,i(t) = (R+r)\,I_m \sin(\omega t + \varphi_i)$

or à la resonance d'intensité $\begin{cases} Z = R+r \\ \varphi_u = \varphi_i \end{cases}$

$\Rightarrow u(t) = (R+r)\,I_m \sin(\omega t + \varphi_u)$

$(R+r)\,i(t) = (R+r)\,I_m \sin(\omega t + \varphi_u)$

d'où $(R+r)\,i(t) = u(t)$

d) eq diff $(R+r)\,i(t) + L\frac{di}{dt} + \frac{1}{C}q = u(t)$

à la resonance d'intensité

$$L\frac{di}{dt} + \frac{1}{C}q = 0 \Rightarrow L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

$$\Rightarrow \boxed{\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0}$$

$$E_{em} = \frac{1}{2} C u_C^2(t) + \frac{1}{2} L i^2(t)$$

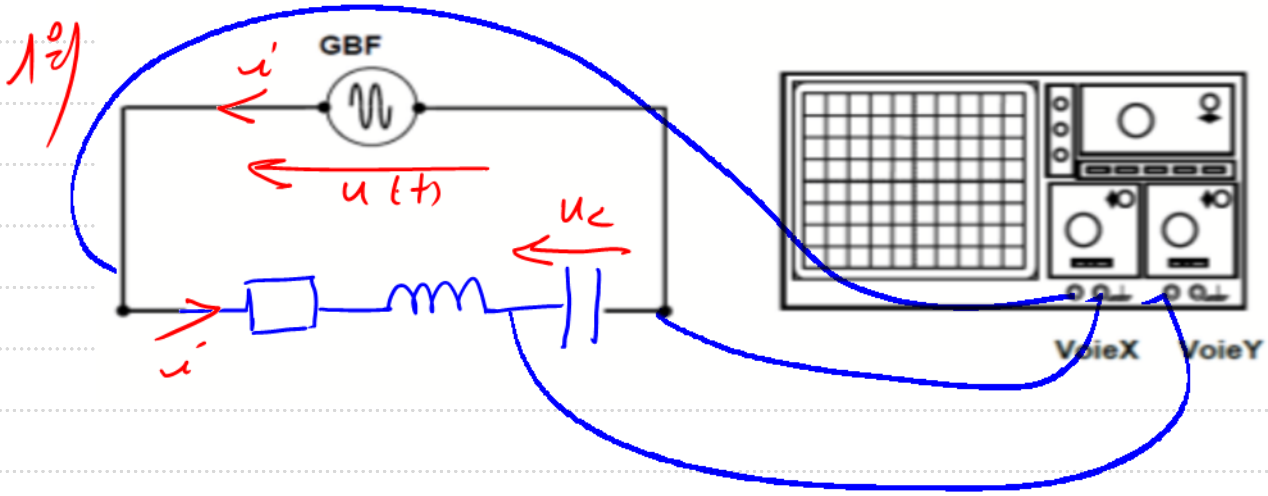
$$\frac{dE_{em}(t)}{dt} = \frac{1}{\cancel{2}} \cancel{2} C \frac{du_C}{dt} \cdot u_C + \frac{1}{\cancel{2}} \cancel{2} L \frac{di}{dt} \cdot i(t)$$

$$= i(t) \left[\underbrace{u_C(t) + L \frac{di}{dt}}_{0 \text{ (eq diff)}} \right]$$

c-à-d $\frac{dE_{em}}{dt} = 0 \Rightarrow$ L'energie est conservée

Exercice N°2

1°)



2°) $u_C(t)$ est toujours en retard de phase sur $u(t)$

$C_1 \longrightarrow u(t)$ $\qquad$ $C_2 \longrightarrow u_C(t)$

3°) a) $N_1 = \frac{1}{T_1} = \frac{1}{6\pi 10^{-3}} = \boxed{53\ Hz}$

b) $\boxed{U_m = 10\ V}$ $\boxed{U_{cm} = 15\ V}$

c) $|\varphi_u - \varphi_{u_c}| = \frac{2\pi}{T} \times \frac{T}{6} = \frac{\pi}{3}$ rad

$u(t)$ en avance sur $u_c(t) \Rightarrow \varphi_u - \varphi_{u_c} = \frac{\pi}{3}$ rad

d) $\varphi_u - \varphi_{u_c} = \frac{\pi}{3} \Rightarrow \varphi_u - (\varphi_i - \frac{\pi}{2}) = \frac{\pi}{3}$

$\varphi_u - \varphi_i = \frac{\pi}{3} - \frac{\pi}{2} = -\frac{\pi}{6}$ rad

$\Rightarrow \boxed{\varphi_i - \varphi_u = \frac{\pi}{6}\ \text{rad}}$

$\varphi_u - \varphi_i < 0 \longrightarrow$ Circuit capacitif

4°) $u(t) = U_m \sin(\omega t) \Rightarrow u(t) = 10 \sin(106\pi t)$

$u_c(t) = U_{cm} \sin(\omega t + \varphi_{u_c}) \Rightarrow u_c(t) = 15 \sin(106\pi t - \frac{\pi}{3})$

5°) $I_m = \frac{U_{cm}}{Z_{c_1}} = \frac{U_{cm}}{\frac{1}{C_1\omega}}$

$I_m = C_1 \omega U_{cm} \Rightarrow I_m = 2,25\ 10^{-2}\ A$

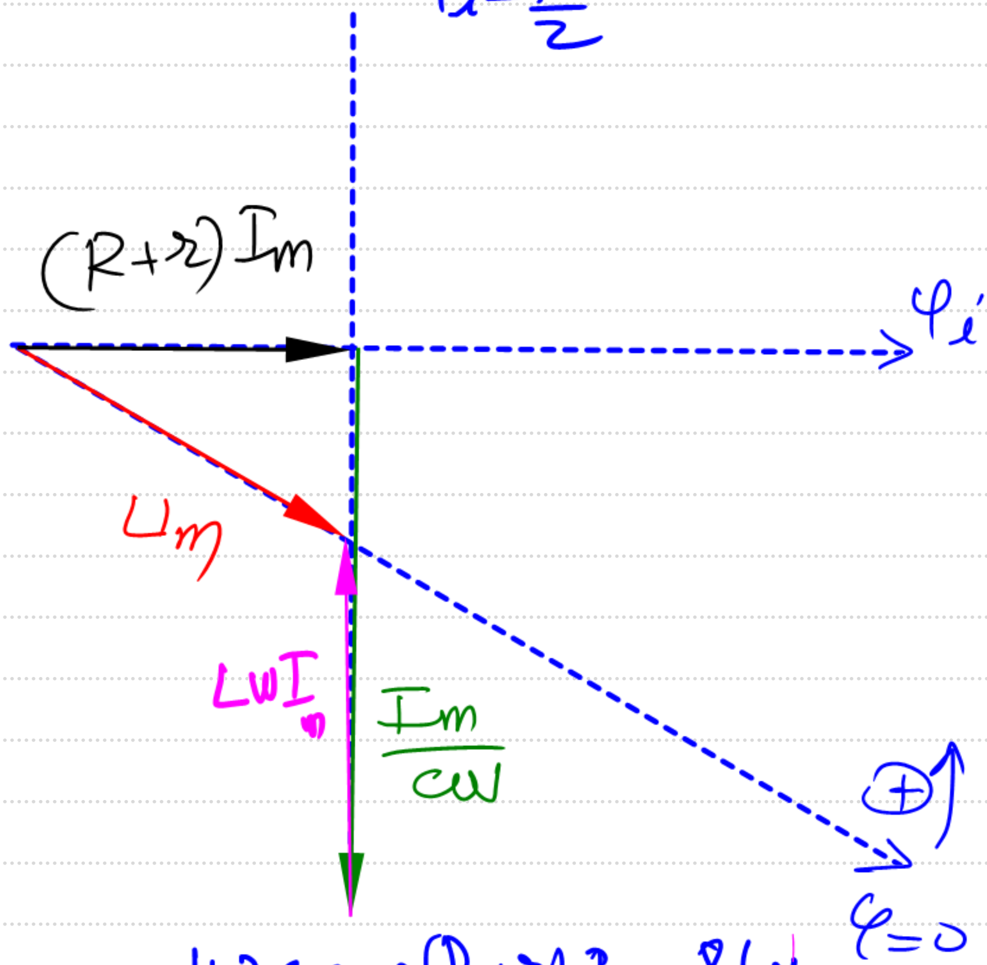
* $Z = \frac{U_m}{I} \Rightarrow Z = 444,4\ \Omega$

6) a) $u(t) \longrightarrow \begin{cases} U_m = 10V \longrightarrow 5\,cm \\ \varphi_u = 0 \end{cases}$

$(R+r)i(t) \longrightarrow \begin{cases} (R+r)I_m \;?? \\ \varphi_i = \frac{\pi}{6} \end{cases}$

$L\frac{di}{dt} \longrightarrow \begin{cases} L\omega I_m \;? \\ \varphi_i + \frac{\pi}{2} \end{cases}$

$\frac{1}{C_1}\int i\,dt \longrightarrow \begin{cases} \frac{I_m}{C_1\omega} = 15V \longrightarrow 7{,}5\,cm \\ \varphi_i - \frac{\pi}{2} \end{cases}$



b) • $(R+r)I_m \longrightarrow 4{,}3\,cm \Rightarrow (R+r)I_m = 8{,}6V$

$\Rightarrow R_T = R + r = \frac{8{,}6}{2{,}25\,10^{-2}} \Rightarrow \boxed{R_T = 382\,\Omega}$

• $L\omega I_m \longrightarrow 5\,cm \Rightarrow L\omega I_m = 10V$

$L = \frac{10}{\omega I_m} \Rightarrow \boxed{L = 1{,}33\,H}$

$I_{m_1} = I_{m_2} \Rightarrow Z_1^2 = Z_2^2 \Rightarrow \left(L\omega - \frac{1}{C_1\omega}\right)^2 = \left(L\omega - \frac{1}{C_2\omega}\right)^2$

$$L\omega - \frac{1}{C_1\omega} = \pm\left(L\omega - \frac{1}{C_2\omega}\right)$$

① si $L\omega - \frac{1}{C_1\omega} = L\omega - \frac{1}{C_2\omega} \Rightarrow C_1 = C_2$ impossible.

donc $$L\omega - \frac{1}{C_1\omega} = \frac{1}{C_2\omega} - L\omega$$

$$2L\omega = \frac{1}{C_1\omega} + \frac{1}{C_2\omega}$$

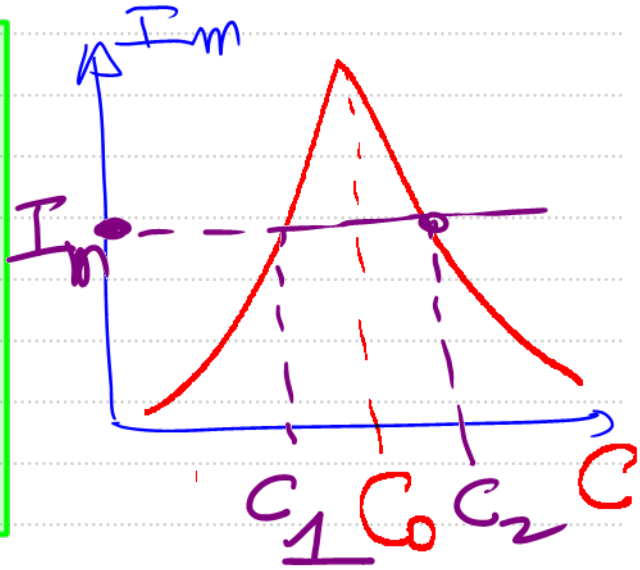
$$\boxed{\frac{1}{C_1} + \frac{1}{C_2} = 8\pi^2 N_1^2 L}$$

ⓑ $\frac{1}{C_2} = 8\pi^2 N_1^2 L - \frac{1}{C_1}$

$$\boxed{C_2 = \frac{1}{8\pi^2 N_1^2 L - \frac{1}{C_1}}} \Rightarrow \boxed{C_2 = 1{,}37\,10^{-5}\,F}$$

$$C_2 = \frac{1}{8\pi^2 \cdot 5(3 \times 1{,}3) - \frac{1}{4{,}5\,10^{-6}}}$$

~~Autrement~~



Rappel cours

$I_m(C_1) = I_m(C_2)$ et $C_1 \neq C_2$

pour C_1 : capacitif donc pour C_2 inductif

$Z(C_1) = Z(C_2)$

$$\underbrace{\frac{1}{C\omega} - L\omega}_{\text{cap}} = \underbrace{L\omega - \frac{1}{C_2\omega}}_{\text{ind}}$$

$$2L\omega = \frac{1}{C_1\omega} + \frac{1}{C_2\omega}$$

$$\boxed{\frac{1}{C_1} + \frac{1}{C_2} = 8\pi^2 N_1^2 L}$$