

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/316663146>

# Kinematic modelling of a robotic arm manipulator using MATLAB

Article in *Journal of Engineering and Applied Sciences* · April 2017

CITATIONS

48

READS

4,108

5 authors, including:



**Alejandro Leiva Toquica**

Universidad Antonio Nariño

1 PUBLICATION 48 CITATIONS

[SEE PROFILE](#)



**Luis Fernando Ortiz Martinez**

Universidad de La Sabana

1 PUBLICATION 48 CITATIONS

[SEE PROFILE](#)



**Ruthber Rodriguez**

Corporación Universitaria del Huila

85 PUBLICATIONS 602 CITATIONS

[SEE PROFILE](#)



**Adrian Fernando Chavarro Chavarro**

Servicio Nacional de Aprendizaje SENA

12 PUBLICATIONS 106 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Elaboración del diagnóstico de la situación actual del desempeño de la gestión estratégica, en los hoteles de la ciudad de Neiva, y la optimización mediante el diseño de sistemas de gestión de la calidad para ellos [View project](#)



Improvement of the adaptive grasp functionality of objects with an active underactuated anthropomorphic prosthesis. [View project](#)



# KINEMATIC MODELLING OF A ROBOTIC ARM MANIPULATOR USING MATLAB

Ruthber Rodríguez Serrezuela<sup>1</sup>, Adrián Fernando Chávarro Chavarro<sup>2</sup>, Miguel Ángel Tovar Cardozo<sup>1</sup>,  
Alejandro Leiva Toquica<sup>1</sup> and Luis Fernando Ortiz Martínez<sup>1</sup>

<sup>1</sup>Faculty of Biomedical, Electronic and Mechatronics Engineering, Universidad Antonio Nariño (UAN), Bogota, Colombia

<sup>2</sup>Department of Electronic and Telecommunications, Tecnoparque SENA, Bogota, Colombia

E-Mail: [ruthbrodriguez@uan.edu.co](mailto:ruthbrodriguez@uan.edu.co)

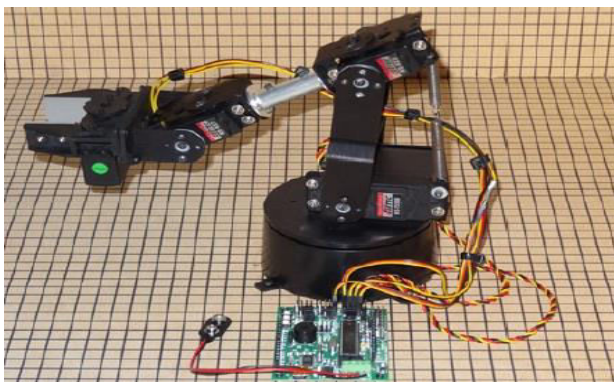
## ABSTRACT

In this paper the design and implementation of a kinematic model for a manipulator robot arm type with four degrees of freedom is developed, model robot performance can be checked mathematically using results from coordinate's frames, which set the proposed matrices by Denavit-Hartenberg method to determine the robot joints angle vector. This procedure describes the direct and inverse kinematics. The goal is to determine the final robot's position and orientation according to the joint angles related to a coordinate system, the final effector position, where joint angles are located. The results were implemented in a MATLAB application that performs fast calculations, it allows the verification of the theory and at the same time becomes as a tool to simplify the analysis and learning for its friendly interface which displays virtually the movements of the robotic arm AL5A.

**Keywords:** inverse and direct kinematics, robot arm.

## 1. INTRODUCTION

In recent years, virtual reality has been delivering great contributions in various fields of science and technology. Projects related to the design and implementation of a kinematic model for a robotic arm four or more degrees of freedom have been achieved successfully, implementing analysis and kinematic model of a robotic arm of 5 degrees of freedom giving as an objective modeling of direct and inverse kinematics of a robotic arm from a theoretical and practical experience in robotic systems, automation and control. [1], [2], [3]. Likewise, the development of software for the kinematic analysis of a robotic arm called Lynx 6, which suggests more effective methods to reduce multiple inverse kinematics solutions described. A visual software package called MSG is also developed to test the characteristics of arm movement [4], [5], [6]. In the same way, kinematic analysis for a robot arm based on a prototype with three degrees of freedom is presented. It uses an application that allows the program run on the card, receive data and operate allowing the clamp to be moved to a desired position [7], [8], [9].



**Figure-1.** Prototype robotic arm manipulator with four degrees of freedom.

## 2. MATERIALS AND METHODS

There are different mechanical aspects related to Robotic Arm, to understand the physics of this machine.

**Kinematics:** Kinematics is the part of physics that is responsible for studying the movement of bodies regardless of their cause, which essentially takes as a reference trajectory in function of time.

**Kinematic model:** It presents the algorithm of Denavit-Hartenberg as a method by which it is possible to represent the robotic arm using homogeneous transformation matrices.

**Homogeneous transformation matrix:** The homogeneous transformation matrix established as a 4x4 matrix allows to know the location, position and orientation of an axis system of coordinates  $O'UVW$  related to the fixed coordinates  $OXYZ$ .

**Direct Kinematics:** For using the direct kinematics the vector and matrix algebra is used based on the spatial location of an object in three dimensional space fixed reference. Because the robot is regarded as a kinematic chain consisting of links joined together by means of joints, can create a reference system on the base that describes the location of the links in reference to the system described.

**Inverse Kinematics:** The inverse kinematics seeks the values that should adopt the robot joint coordinates  $\mathbf{q} = [q_1, q_2, \dots, q_n]$  to position the robot end effector so that its end is oriented in a particular spatial location. To obtain the equations is required a procedure which is entirely dependent on the configuration of the robot. Thus, as you should obtain the values of the joint



variables so that the end effector has a certain position and orientation.

Multiple solutions should be considered, as minimize movements made by the arm to the current position, the concept of allows the nearest solution, to move the links of lower weight, and obstacles are considered important to avoid collisions.

Some generic procedures that can be programmed have been developed so that a computer can, from knowledge of the kinematics of the robot (with its parameters Denavit - Hartenberg) to obtain the n-tuple of joint values that position and orient its end. The disadvantage of these processes is that it is iterative numerical methods, whose convergence speed and even S\_ convergence is not always guaranteed. When solving the inverse kinematic problem is much more suitable to find the solution closed. This is to find a mathematical relationship explicit form.  $Q_k = F_k(x, y, z)$   $K = 1)(D.O.F.)$  [10].

To this solution, the kinematic problem reverse is solved in real time, so has to follow a certain path. To a solution in iterative time there is no guarantee that the solution develops properly. Otherwise what will happen with direct kinematic problem, quite often inverse kinematics is not unique but there are different joints that guide and position the end of the robot.

**Position representation:** The representation of the position is determined by various systems that allow locate a point in space. Thus systems Cartesian, cylindrical and spherical coordinates are part of this important method. This article will use the Cartesian coordinate system.

The reference system  $X, Y, Z$  allows the representation of a point in space from the position vector  $P$ , so that the equation describes a column vector of three elements;

$$P = (P_x, P_y, P_z)^T \quad (1)$$

Likewise, the location of point  $P$  can be represented by another coordinate system  $A, B, C$ . The new coordinate system, depends on the source location relative to system taken as the main reference  $X, Y, Z$  and orientation representing the coordinate axis. This figure shows the

Cartesian system  $A, B, C$  whose origin is at the point  $P$  system  $X, Y, Z$ .

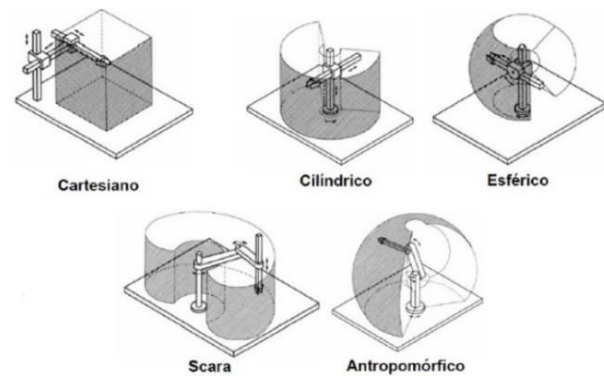


Figure-2. Robot arm achievable space [11].

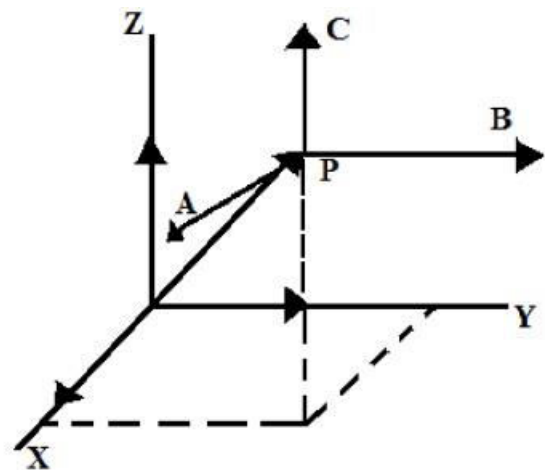


Figure-3. Representation of ABC system with P point in space.

**Representation orientation:** Point for a spatial location is described by a three-dimensional coordinates. For a solid body it is necessary to know the orientation of it related to a reference system.

It is important to unite the solid body coordinate system  $A, B, C$  that be described in space. Therefore, the orientation will take a direction of the unit vectors of the coordinate system attached to the body associated with the axes of the source coordinate  $X, Y, Z$ .

**Rotation matrix:** This method solves the problem of orientation, because it allows to use matrix algebra tools to establish relative rotation between main axes of two systems sharing the source.

The rotation matrix is defined as a  $3 \times 3$  matrix  $R$  which is used as an operator. The equation enables a transformation vector  $P$  on a position in three dimensional space. Thus, the coordinates become expressed in a system of coordinate axes rotated  $A, B, C$ .

$$P_{UVW} = (P_U, P_V, P_W)^T \quad (2)$$

The orientation is defined by rotating angles to the main axis of fixed reference.



$$P_{xyz} = RP_{uvw} = \begin{pmatrix} P_x \\ P_y \\ P_z \end{pmatrix} = R \begin{pmatrix} P_u \\ P_v \\ P_w \end{pmatrix} \quad (3)$$

**Rotation matrix operations:** Product: The coordinate system has different eventualities in a solid body due to experience more than one rotation about different axes of the source system. To obtain such rotations it is necessary multiply the rotation matrices representing individual rotations, achieving a composite matrix rotation.

The multiplication of matrices is not commutative, so these must be multiplied in the same order in which they spend the rotations of the system in space.

**Location in three-dimensional space:** The solids in three dimensional space can have movements of rotation and translation. These allow the coordinate system match a solid body translation vector to determine its position and a rotation matrix indicating the orientation linked to a fixed reference system. The relationship of this concept is defined in a 4x4 matrix array called homogeneous transformation (H.T.M.).

**Homogeneous transformation matrices:** H.T.M.4x4 represents the position and orientation of a moving three-dimensional system related to a fixed three-dimensional system.

A H.T.M. consists of four subarrays of different sizes:  
R3x3 submatrix corresponding to a rotation matrix.  
P3x1 submatrix corresponding to a translation vector.  
F1x3 represents submatrix perspective transformation.  
W1x1 submatrix corresponds to a global scale.

**Operation with homogeneous transformation matrices:** Product Multiplying several MTH have a sequence of positions, which results in a trajectory generation step. This result is called homogeneous transformation matrix composite, which depends on the order in which the products are made.

**Inverse transformation:** The homogeneous transformation matrix consists of four subarrays, of which both the perspective scaling should be constant, so cannot apply the transpose.

**Denavit-Hartenberg representation:** The Denavit-Hartenberg representation is a method that applies the properties of MTH to represent the relations of translation and rotation between adjacent elements of a Robot. In 1955 Denavit and Hartenberg proposed an algorithm to establish systematically a coordinate system if linked to each link  $i$  of an articulated chain, this allows moving from one link to the next by 4 basic transformations that rely exclusively on the geometrical characteristics of the link [11], [12].

**Denavit-Hartenberg algorithm:**  $Z_i$  axis coincides with the hinge axis towards the displacement for a prismatic joint and likewise the direction of the rotational axis for the joint. For the  $X_i$  axis perpendicular location takes a common reference  $Z_i$  and  $Z_i + 1$ .

**Determining the Denavit-Hartenberg Parameters:** The Denavit-Hartenberg parameters define the dimensional relationships between consecutive links and joint variables.

$\theta_i$  is the variable angle joint for a joint rotation.  
 $d_i$  is the distance of the variable joint for a prismatic joint.  
 $\alpha_{i-1}$  is the distance from the axis of joint  $i - 1$  to  $i$  joint axis measured along the perpendicular line common to these axes. When the axes  $\alpha_{i-1}$  are cut be zero.  
 $\alpha_{i-1}$  is the angle between the axis of joint  $i - 1$  and  $i$  joint axis; measured as a rotation axis  $i + 1$  around the common perpendicular until it coincides with the direction of the axis  $i$ .  
 $d_i$  indicates the time of articulation. It is assigned zero value for rotational joints and one for the prismatic joints [13], [14], [15].

**Meaning Denavit-Hartenberg representation:** D-H parameters provide information about the number and type of joints of a robot. It is thus that disclosed dimensions of the links, the link between consecutive angular joints and the separation between them. Also, determine the joint variables of the Robot.

**Transformation matrix D-H:** Being determined the representation D-H it results in the successive transformations corresponding to the  $i$ th system coordinate with the coordinate system  $(i - 1)$ th. It performs basic operations MTH obtaining a composite  $A_{i-1}$  transformation matrix called D-H. The matrix presented below represents the function of each of the D-H parameters in obtaining the matrix.

$${}^{i-1}A^i = T_{z,d} T_{z,\alpha} T_{x,\theta} T_{x,\alpha} \quad (4)$$

$$= \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Interpretation matrix transformation D-H:** The matrix  ${}^{i-1}A^i$  indicates that  $A$  represents a MTH corresponding to the coordinate axis system  $(i)$  linked to the joint  $(i)$  related to the coordinate axis system  $(i - 1)$  binding articulation  $(i - 1)$ . The matrix obtained is determined by a degree of freedom.

**Arm matrix:** For a robot with  $n$  degrees of freedom, the equation below shows the possible matrices D n-H multiplying sequentially for  ${}^0T_n$ . The end effector