Designing Path Trajectories *of Multiple Cooperative Drones* for Crop Irrigation

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This paper considers the problem of assigning target locations to be visited by Unmanned Aerial Vehicles (UAVs). We formulate the problem as a multiple traveling salesman problem (MTSP), also we considered variations such as *Fixed Open - MTSP, multi objective - MTSP* and use *Genetic Algorithm* to solve the NP-Hard problems. The main goal is to find the effective assignment of UAVs to a set of locations into a crop so as to optimize the overall cost and maximize the performance of the UAVs. The target locations need to be efficient irrigated and be supplied with sufficient water to avoid crop water stress from the perspective of precision agriculture, motivated by the fact that agriculture is responsible for 70 percent of fresh water extractions at a global level. We define the position functions for irrigation UAVs based on an image of the Crop Water Stress Index (CWSI) of the crop. The methodology consists on dividing the crop into segments as target locations, and then finding the position function and euclidean distance between segment to estimate water and energy spent by the UAVs. Simulation results have shown that between more UAVs are implicated in crop irrigation, increase the overall cost of water, energy and distance but cost individually decrease, so we can use inexpensive UAVs.

1. INTRODUCTION

Improving agriculture is one of the biggest challenges today, due to the increase of the population that now exceeds 7.2 billion and that by 2050, it is expected that reach 9.6 billion. With the increase in food demand for the exponential growth of the population, it makes more pressure on ecosystems and their natural resources. Therefore, the agriculture must be developed for being sustainable and efficient implemented.[8] Water in particular has received special attention as can be seen in the yearly reports of the United Nations World Water Assessment Programme (WWAP). According to [9], agriculture is responsible for 70 percent of fresh water extractions at a global level, which motivates the efforts to reduce the water consumption in this area.

On the other hand, the world health organization determined that glyphosate used as a herbicide generates long-term cancer. In Colombia, this substance is used to for aerial fumigation to eradicate illegal drug plantations, which is inefficient, excessively polluting and dangerous for the health of the surrounding population, but this has diminished when it began to performs precise aerial spraying.[10]

Also, variations in topography for instance create humid and dry regions, and thus crops become heterogeneous with respect to the amount of water required to properly irrigate them. Crop monitoring and remote sensing are now being used at a wide scale to define management zones according to the space-time variations in the crop.[1] Indexes such as the Crop Water Stress Index (CWSI) [12] and the Normalized Difference Vegetation Index (NDVI) have been developed in order to monitor crop conditions.

2. PROBLEM FORMULATION

A. MTSP formulations

The heterogeneous multiple traveling salesman problem is a generalization of the MTSP and the TSP problem, in which more than one salesman is allowed, with the fact that each salesman has different matrix cost for the same objective function.

Here we present an assignment-based integer programming formulation for three-index MTSP, that each salesman had a matrix cost different of each others. Also, we consider that exist more that one depot (same arrival and departure) with the fact that each depot only have one salesman. Consider a graph $G_k = (V_k, A_k)$, where V_k is a set of n nodes, and A_k is the set of edges (i,j) Associated with a cost $c_k = c_{(ij)k}$, also k is the number of M salesman. We assume that the departure depot is node 1 and arrival depot is node N. We define a binary variable $x_{(ij)k}$ for each edge $(i,j) \in A_k$; x_{ij} takes the value 1 if edge $(i,j) \in A_k$ is included in the k-tour and $x_{(ij)k}$ takes the value 0 otherwise.

For the subtour elimination constraints (SECS) we use the Miller-Tucker-Zemlin SEC, the node potential u of each node indicates the order of the corresponding node in the tour, n denotes the the maximum number of nodes that can be visited by any salesman. The subtours are degenerate tours that are formed between intermediate nodes, and not connected to the depot.

$$Min \sum_{k=1}^{M} \sum_{(i,j)=1}^{n} c_{ijk} x_{ijk} \tag{1}$$

s.t

$$x_{ijk} \in [0,1], \qquad \forall (i,j) \in A_k \tag{2}$$

$$\sum_{k=1}^{M} \sum_{i=1}^{n} x_{ijk} = 1 \qquad j = 1, \dots, n. \tag{3}$$

$$\sum_{i=1}^{n} x_{ipk} - \sum_{j=1}^{n} x_{pjk} = 0 \qquad k = 1, \dots, M;\ p = 1, \dots, n. \tag{4}$$

$$\sum_{j=1}^{n} x_{1jk} = 1 \qquad k = 1, \dots, M \tag{5}$$

$$u_i - u_j + n \sum_{k=1}^{M} x_{ijk} \leq n - 1, i \neq j = 2, \dots, n \tag{6}$$

Where x is a binary variable (2), constraints (3) state that each customer should be visited exactly once and (4) are the flow conservation constraints which ensure that once a salesman visits a customer, then he must also depart from the same customer. Constraints (5) ensure that each vehicle is used exactly once and (6) include subtour elimination constraints (Miller-Tucker-Zemlin). [5]

B. Time Spent and Water Consumption

To find the time spent by a UAV for irrigate a segment into a crop, it is necessary understand the time function proposed by Catalina Albornoz [1]. The function determines how much time should an UAV remain on top of a region, depending on its Crop Water Stress Index (CWSI), assuming that the flow rate leaving the nozzles is constant, and thus the longer the UAV hovers over an area, the more water this area will receive. C. Albornoz conclude that with this function, the amount of wasted water is notably diminished, due to the fact that less water is used on regions that already have enough.

C. Albonoz divide the crop into 12 segments and calculate the median along y of the CWSI for all the segment, in order to eliminate the y-dimension and resolve the problem in one dimension, along x.

The time that a point stays under the area of irrigation i is the difference between the time at which it leaves the area of irrigation, and the time at which it enters the area of irrigation,

$$c_i(x + \frac{l_x}{2}) = \frac{MA}{Q}\phi(x,y) + c_i(x - \frac{l_x}{2}) \tag{7}$$

Where A is the area of irrigation, Q is the flow rate, l_x are the length of the area of irrigation for the nozzle. $\phi(x,y) \in [0,1]$ that correspond to the CWSI at position (x), as we can see in the left figure [1] low CWSI (dark regions) indicates good irrigation and a high CWSI (light regions) indicates lack of irrigation. M be the amount of litres of water per squared metre required by a point in the crop with maximum water stress (CWSI=1), On the other hand, the water consumed travelling along the segment i depend to the time function i and the flow rate Q which according to [1] is estimated as 0.057333 [L/s].

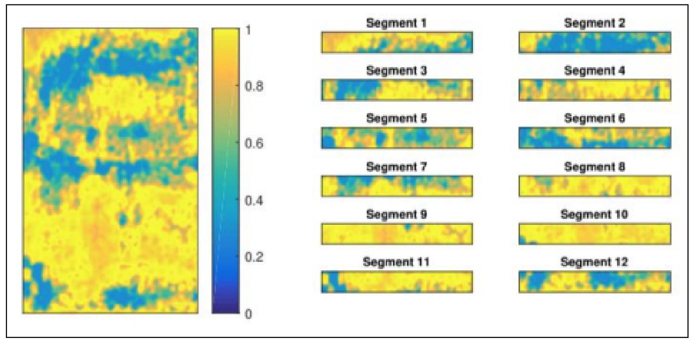

Fig. 1. Segments example used by C. Albornoz in [1].

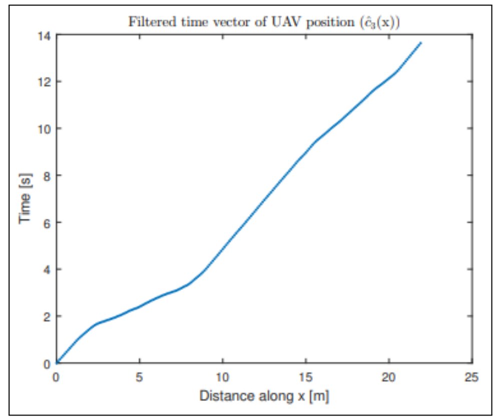

Fig. 2. Time vector of UAV for segment 3, see right figure [1]

$$w_i(x) = Q \cdot c_i(x) \tag{8}$$

C. Energy Consumtion

F. Yacef proposed an energetic model in order to have an idea about the energy consumed by the vehicle during the mission and provide a direct relation between energy and vehicle dynamic's [2]. The motor brushless direct current (BLDC) dynamics are modeled as a simple first order differential equation (9). Where the inertia J, includes the motor and the propeller, k_t is the drag coefficient. Also, $\dot{\omega}$ is driven by the motor torque and the load friction torque $Q_f(\omega(t))$.

$$\tau(t) = J\dot{\omega} + Q_f(\omega(t)) + D_v\omega(t) \tag{9}$$

Where $\tau(t)$ is the torque generated by the brushless motor, D_v is the viscous damping coefficient of the motor and $\omega(t)$ is the angular speed at time t. F. Yacef defined in his work [2] the total energy consumed by the vehicle during the mission as,

$$E_c = \int_{t_o}^{t_f} \sum_{j=1}^{4} (J\dot{\omega} + Q_f(\omega(t)) + D_v\omega(t))\omega_j(t) \mathrm{d}t \tag{10}$$

The equation of energy (10) are only function of $\omega(t)$ because J, k_t and D_v are constant of the quadcopter. In order to find $\omega(t)$, we could study the system motion dynamics, so two frames are used; an inertial frame attached to the earth defined by $E^a(e_{a1}, e_{a2}, e_{a3})$; a body fixed frame $E^b(e_{b1}, e_{b2}, e_{b3})$ fixed to the center of mass of the quadrotor, see fig.[3]. The absolute

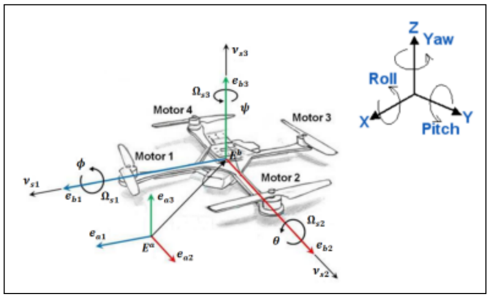

Fig. 3. Quadrotor scheme

position of the quadrotor is described by $p = [x, y, z]$ and its attitude by the Euler angles $\eta = [\phi, \theta, \psi]^T$. The attitude angles are respectively Yaw angle (ψ rotation around z-axis), Pitch angle (θ rotation around y-axis), and roll angle (ϕ rotation around x-axis). The dynamic model for quadrotor vehicle can be derived as the equation seen in fig [4]. Where l_x, l_y, l_z is the inertia of the UAV, J is the rotor inertia, l is the distance from the center of mass to the rotor shaft and g is the gravity.

$$
\begin{aligned}
m\ddot{x} &= (\cos\emptyset \sin\theta \cos\psi + \sin\phi \sin\psi)T \\
m\ddot{y} &= (\cos\emptyset \sin\theta \sin\psi - \sin\phi \sin\psi)T \\
m\ddot{z} &= (\cos\emptyset \cos\theta)T - mg \\
I_x\ddot{\phi} &= (I_y - I_z)\dot{\theta}\dot{\psi} + J\dot{\theta}\varpi + lu_1 \\
I_y\ddot{\theta} &= (I_z - I_x)\dot{\phi}\dot{\psi} - J\dot{\theta}\varpi + lu_2 \\
I_z\ddot{\psi} &= (I_x - I_y)\dot{\phi}\dot{\theta} + u_3
\end{aligned}
\qquad
\begin{aligned}
T &= k_b(\omega_1^2 + \omega_2^2 + \omega_3^2 + \omega_2^2) \\
u_1 &= k_b(\omega_2^2 - \omega_4^2) \\
u_1 &= k_b(\omega_3^2 - \omega_1^2) \\
u_3 &= k_\tau(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \\
\varpi &= \omega_1 - \omega_2 + \omega_3 - \omega_4 \\
-\pi/2 < \emptyset &< \pi/2, -\pi/2 < \theta < \pi/2
\end{aligned}
$$

Fig. 4. Dynamic model for a quadrotor vehicle.

To resolve all the equations seen in fig. [4] and obtain the angular velocity, a simulation tool developed by a group at Drexel University, as a Senior project, was used. The tool is called Quad-Sim, is free and Open Software [3]. It has a GUI that works on Simulink and can be seen in fig. [5], it requires the user to input a 3D path with a related time series vector, the initial conditions of the Quadcopter, the coordinate system fig. [3] and the UAV´s physical structure.

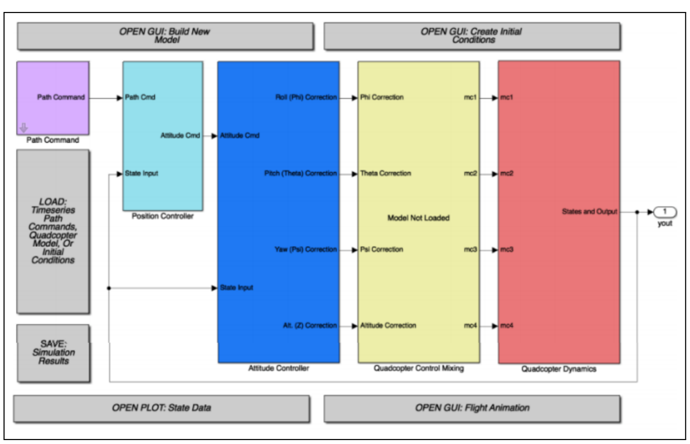

Fig. 5. GUI of the simulation tool in Simulink

J. Lopez in his work [4] used the Quad-Sim [3] to calculated the angular velocity vector between two points and then calculated the energy consumed for the UAV using equation [10].

3. PROPOSED SOLUTION

First, to analyze the images we divided the overall crop into a equally regions that are called nodes (vertices) or target locations, on such a way that we can make sure that the UAVs passes through all the crop, in the fig (6) the blue regions has the same length and width throughout the algorithm execution but are shown with different size to indicate to the reader the median CWSI of the area that are located.

As we can see in the right figure C. Albornoz [1] only use a UAV but now, the objective is use more that one UAV. So in this work, we treat the multiple traveling salesmen problem that is a NP-hard problem and can be solved using heuristic methods to find the effective assignment of UAVS to a set of locations so as to optimize the overall cost and maximize the performance for the UAVs. In the MTSP formulation of the present document, we define a matrix cost C_{ijk} in the equation (1). J. Lopez in [4] design a method that find the cost ij (c_{ij}) between the nodes i and the node j, in other words, the idea is to create segments from i to j, for every possible combinations of pair of the 132 nodes in the left of the fig. (6) so, we obtain a 132X132 matrix cost c_k for each k salesman. We use the algorithm 2 and in the step 7 is apply the algorithm 1 to minimize the distances, water and energy individually. Our main goal in the singular objective MTSP is compare our result with respect to those of C. Albornoz presented in [1]. So, we present three results; the first, is the variations Fixed Open - MTSP because the path designed by C. Albornoz the arrival depot and departure depot are not the same; the second, is the general MTSP where arrival and departure are the same; the third, is the same general MTSP as the second, but with the difference that only for the energy consumed, the salesman have different matrix cost.

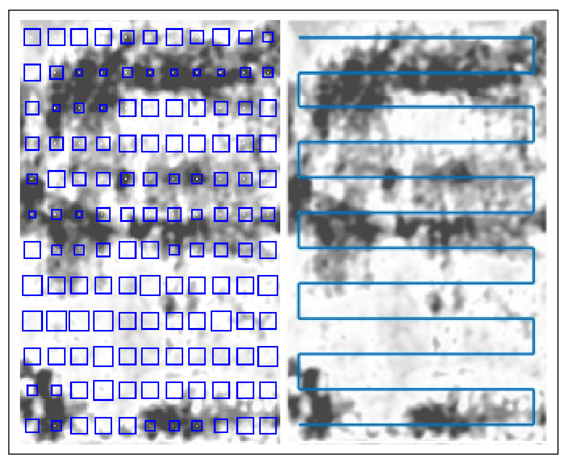

Fig. 6. Left: image divided into equally regions. Right: image divide into 12 segments and original path, proposed by C. Albornoz in [1]

4. RESULTS

As we can see in the figure (8) the total cost for distance traveled, water spent and energy consumed increases linearly as the number of quadcopter used to irrigate the crop, increases. So, the total tour cost is directly proportional to the number of quadcopter used.

Nevertheless, when the number of quadcopter increases, the individually cost for each quadcopter used decrease, this result

Algorithm 1. MTSP using Genetic Algorthm

```
Inputs: Nodes, MatrixCost, TotalPopulation, NumberItertion
Initializations for route break point selection
Initialize the Population
Select the colors for the plotted Routes
for 1 : NumberIterations do
    for 1 : TotalPopulations do
        for Each salesman do
            Evaluate function with members of the Pop.
            Population satisfy the MTSP restriccions.
        End
        Save result for each members of the population
    End
    Find the Best Route in the population
    Plot and save the Best Route
    for PopZise : PopZise : TotalPopulations do
        for 1 : PopZise do
            GA Operators for general route and Break points
        End
        Generate New solutions
        New Solutions satisfy the MTSP restrictions
    End
End
Output: 1.) General Route that contain the Route for each dron. 2.) Break points for the General Route that establish the individual route for each dron
```

Algorithm 2. single objetive MTSP-GA

```
Inputs: 1.) Nodes to divide the crop, 2.) depots, 3.) CWSI image and 4.) Parameter for quadcopter, MTSP.
Get red channel of CWSI image
Determine the number and size of nodes to divide the crop.
Location, enumeration and value of nodes into the crop
Relocate the nodes acording to the Inicial path for begin the M-TSP
Matrix cost for distance, water and energy spend
Select the variable to minimize
MTSP using Genetic Algorthm
Outputs: 1.) Tour for each quadcopter, 2.) Cost Water, Distance and Energy
```

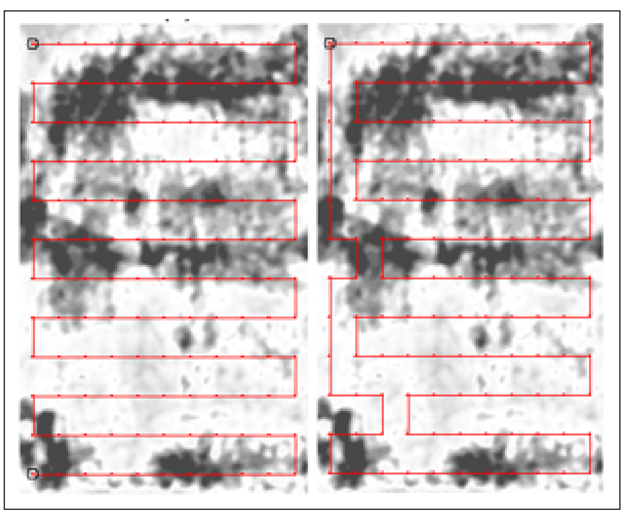

Fig. 7. Left: Different Arrival depot and departure depot, Right: Same Arrival and departure depot.

is seen in the figure (9). So, the individually tour cost is inversely proportional to the number of quadcopter used.

With this result, the drones require to be load with a lowest water volume that 9.81 L [1], so we can use more cheaper UAVs for the irrigation crop process than the DJI UAV proposed by C. Albornoz [14], because, if for example we use four drones, they only have to load with 3 L of water. The same analysis apply for the energy consumed and distance traveled, so we need a drone with a more low LiPo battery, taking into account that this type of batteries are expensive.

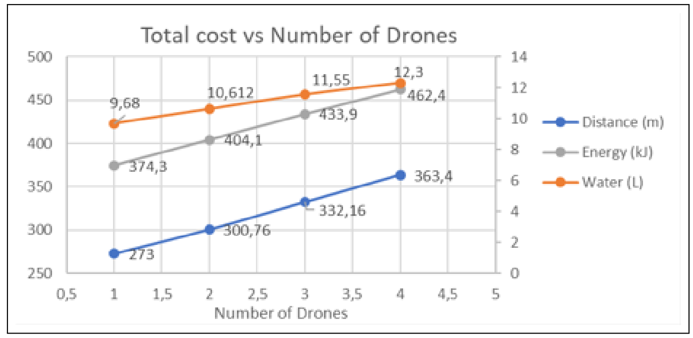

Fig. 8. Total cost when we have different Arrival depot and departure depot as seen in the left figure (7)

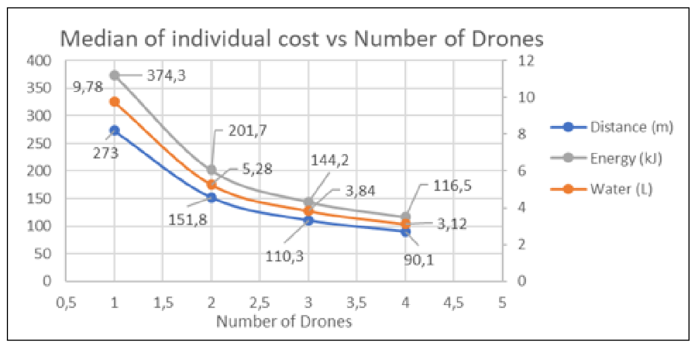

Fig. 9. Median of the individual tour cost, when we have different Arrival depot and departure depot as seen in the left figure (7)

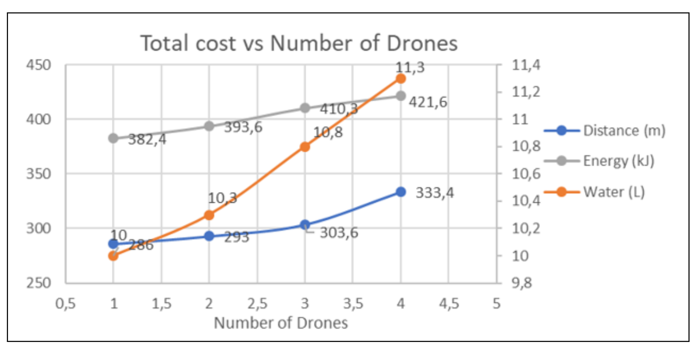

Fig. 10. Total cost when we have the Same Arrival and departure depot as seen in the right figure (7)

On the other hand, we do a similar analysis for the case when the UAVs have the same Arrival and departure depot. In the fig (10) we can observe than, like the previous analysis, the total

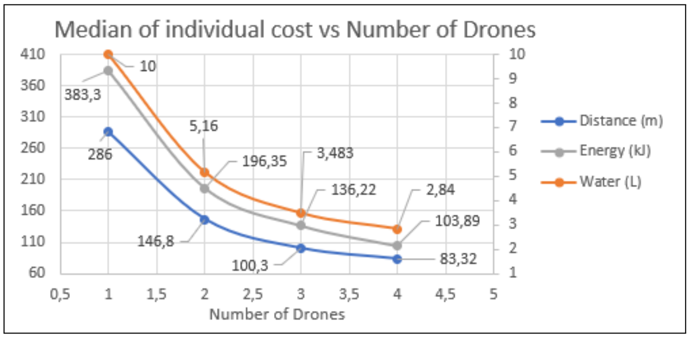

Fig. 11. Median of the individual tour cost when we have the Same Arrival and departure depot as seen in the right figure (7)

cost for distance traveled, water spent and energy consumed increases linearly as the number of quadcopter used to irrigate the crop, increases. So, the total tour cost is directly proportional to the number of quadcopter used.

Nevertheless, when the number of quadcopter increases, the individually cost for each quadcopter used decrease, this result is seen in the figure (9). So, the individually tour cost is inversely proportional to the number of quadcopter used, this result is the same as the previous one.

With this result, the drones require to be load with a lowest water volume that 9.81 L required in the results of C. Albornoz in [1], so we can use more cheaper UAVs than DJI drone [14] for the irrigation crop process, because, if for example we use four drones, they only have to load with 2.84 L of water, this analysis is similar apply for the energy consumed and distance traveled.

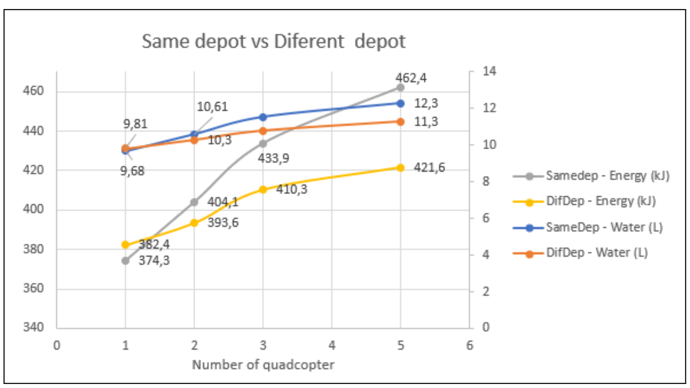

Fig. 12. Comparison for energy consumed and water spend when we use different and same arrival and departure depot.

As we can see in the figure (12) the result of the total energy consumed and water spend, when we have more that one drone, are the lowest when we have the same depot for arrival and departure that when we have a different arrival-departure depot. this also happens with the distance traveled but we not put it in the figure (12) for clarity of it.

If we do a similar graph as figure (12) but for the median of individually tour cost of each drone used, we conclude that the cost of the three variable worked are minimized when the arrival and departure depot are the same, when we use more than one drone.

finally, we consider the fact that the drones don´t have the same physical structure, so we have a different energy matrix

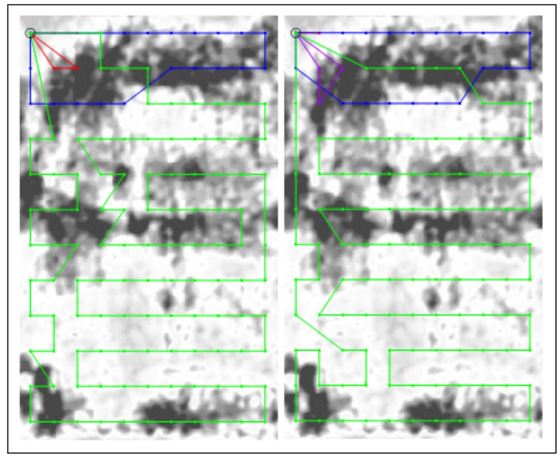

Fig. 13. Path or tour for the drones with different energy matrix cost.

cost for each drone, as seen in equation (10). In order to solve this problem, we modified an existing genetic algorithm TSP solver [7] for can receive more than one matrix cost and for satisfy the MTSP formulations presented thought the equation (1) to equation (6).

The principal result of the previously proposed solution, was that between less energy cost has a drone, the MTSP-GA algorithm assigns it more nodes and try to use less the drones that have the highest energy cost. In the left image of the figure (13) the drone´s tour in red color has the highest energy cost, so the algorithm treat to not assign it nodes and has the shortest path. The drone´s tour in green color has the lowest energy cost, so has the longest path and the drone´s tour in blue color has a energy cost between green and red drone.

Now we simulate four drones as shown in the right image of the figure (13), the green drone has the lowest energy cost, the violet drone has a higher cost than green a blue but the four red drone has a energy cost ten times the energy cost of violet drone, so the algorithm only assign it one node.

For that reason, if the drones have only one matrix cost we don´t know before what will the size of the tour designed by the algorithm 1, but if the drones have his own matrix cost we can estimate the tour length before we run the MTSP-GA algorithm.

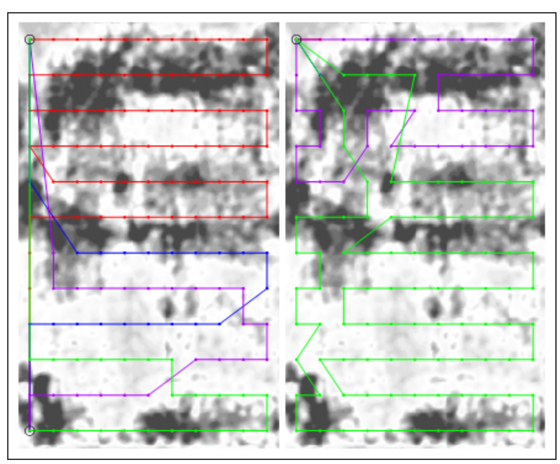

Fig. 14. Examples of path designs by the the MTSP-GA algorithm 1.

In the other hand, when we change the objetive fuction to minimize as the following

$$Min(Max(\sum_{k=1}^{M} c_{ijk}x_{ijk})) \qquad \textbf{(11)}$$

With the same restrictions given in the MTSP formulations. We can find that the standard deviation between the individual cost of each drone decays as the number of dron increase and is less when we minimize the objective function 11.

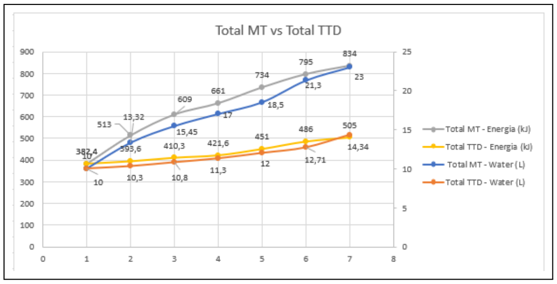

Fig. 15. standard deviation between 11 and 1

So we not only minimize the individual cost of each drone, also we make sure that all dron have the same tour length in the designed path in comparison with the right figure of 14.

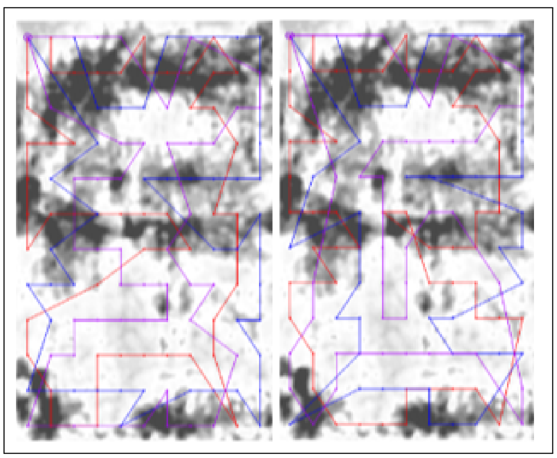

Fig. 16. Designed path with similar tour length for each dron.

With the previously result, we can use drones with have a low cost and more smalls than the proposed by C. Albornoz in [1] and we can see in the left figure of 17.

Fig. 17. Left, dron proposed by C. Albornoz. Right, new dron proposed.

In the next result, we can find that the size of the line indicate the time that the dron stay over an area for complete irrigation

Fig. 18. We need to use more than one drone to satisfy completely the irrigation crop.

(CWSI =0). So, the size is thin in the dark regions and the size is thick in the light regions. Is important mention that the size also depend the distance between two nodes.

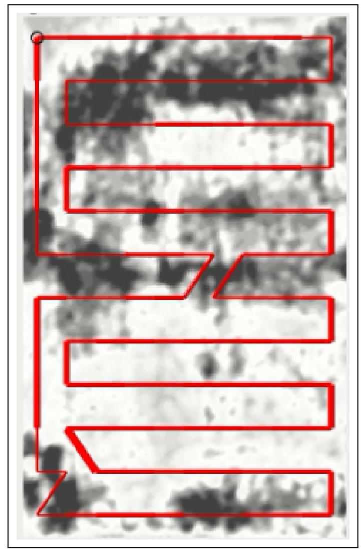

Fig. 19. Path irrigation with different line size.

5. CONCLUSIONS

We consider two cases: when the arrival-departure depot was the same and was different. For both, the total cost for distance traveled, water spent and energy consumed increases linearly as the number of quadcopter used to irrigate the crop, increases. So, the total tour cost is directly proportional to the number of quadcopter used. Nevertheless, when the number of quadcopter increases, the individually cost for each quadcopter used decrease. So, the individually tour cost is inversely proportional to the number of quadcopter used.

With this result, the drones require to be load with a lowest water volume that the proposed by C. Albornoz in [1], so we can use not expensive UAVs for the irrigation crop process. The same analysis apply for the energy consumed and distance traveled, so we need a drone with a more low LiPo battery, taking into account that this type of batteries are expensive.

The total energy consumed, water spend and distance traveled, when we have more that one drone, are the lowest when we have the same depot for arrival and departure that when we have a different arrival-departure depot. this happens equal with respect to the median of individually tour cost of each drone used.

we consider the fact that the drones don´t have the same physical structure, so we have a different energy matrix cost for each drone. The principal result was that between less energy cost has a drone, the MTSP-GA algorithm assigns it more nodes and try to use less the drones that have the highest energy cost. In the other hand, if the drones have only one matrix cost we don´t know before what will the size of the tour designed by the algorithm 1, but if the drones have his own matrix cost we can estimate the tour length before we run the MTSP-GA algorithm.

Finally, when we minimize the objective function 11 we find a less standard deviation between the individual cost of each dron in comparison with 1. So, we make sure that all dron have the same tour length in the designed path.

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