## Department of Applied Mathematics and Humanities

## S. V. National Institute of Technology, Surat, Gujarat

## Tutorial-03

## B. Tech.-I (Semester-II) Branch-All

Subject: Mathematics-II (MA 114 S2)

**Topic**: P. I. Linear differential equation with variable coefficient, Cauchy Euler Legendre's equation with variable coefficient, method of variation of parameters

1. Solve the following differential equations

(i) 
$$(D^6 - 1)y = 0$$

Ans. 
$$y = c_1 e^x + c_2 e^{-x} + e^{-\frac{x}{2}} \left[ c_3 \cos(x \frac{\sqrt{3}}{2}) + c_4 \sin(x \frac{\sqrt{3}}{2}) \right] + e^{\frac{x}{2}} \left[ c_5 \cos(x \frac{\sqrt{3}}{2}) + c_6 \sin(x \frac{\sqrt{3}}{2}) \right]$$

(ii) 
$$(D^2 - 1)y = e^x(1 + x^2)$$
, Ans.  $y = c_1e^x + c_2e^{-x} + \frac{1}{12}e^x(2x^3 - 3x^2 + 9x)$ 

(iii) 
$$(x^2D^2 - 3xD + 5)y = sin(log x)$$

Ans.
$$y = x^2[c_1 cos(log x) + c_2 sin(log x)] + \frac{1}{8}[cos(log x) + sin(log x)]$$

2. Find the P.I. of the following differential equations

$$(i)(x^2 D^2 + 3x D + 1)y = \frac{1}{(1-x)^2} \text{ Ans. } \frac{1}{x} \log[\frac{x}{1-x}]$$

$$(ii)(xD+1)y = \frac{1}{x}$$
 Ans.  $\frac{\log x}{x}$ 

(iii) 
$$(x^2D^2 + xD + 1)y = logxsin(logx)$$
 Ans.  $\frac{1}{4}logxsin(logx) - \frac{1}{4}(logx)^2cos(logx)$ 

3. Apply the method of variation of parameters to solve

(i) 
$$y_2 + a^2y = \csc ax$$

Ans. 
$$C_1 \cos ax + C_2 \sin ax - \frac{x}{a} \cos ax + \frac{1}{a^2} \sin ax \log(\sin ax)$$

(ii) 
$$y_2 + a^2 y = \cot ax$$

Ans. 
$$C_1 \cos ax + C_2 \sin ax - \frac{1}{a^2} \cos ax \log(\sec ax + \tan ax)$$

(iii) 
$$y_2 - y = \frac{2}{1 + e^x}$$

Ans. 
$$y = c_1 e^x + c_2 e^{-x} - 1 - x e^x + (e^x - e^{-x}) \log(1 + e^x)$$

4. Solve the following differential equation

(i) 
$$(x+1)^2 y_2 - 3(x+1)y_1 + 4y = x^2$$

Ans. 
$$y = (x+1)^2 [c_1 + c_2 \log(x+1)] + \frac{1}{2} (x+1)^2 [\log(x+1)]^2 - 2x - \frac{7}{4}$$

(ii) 
$$(2x+5)^2 y_2 - 6(2x+5)y_1 + 8y = 0$$

Ans. 
$$y = (2x+5)^2 [c_1 (2x+5)^{\sqrt{2}} + c_2 (2x+5)^{-\sqrt{2}}]$$