B. Tech.-I (Semester-II) Branch-All

Subject: Mathematics-II (MA 114 S2)

Topic: P. I. Linear differential equation with variable coefficient, Cauchy Euler

Legendre's equation with variable coefficient, method of variation of parameters

1. Solve the following differential equations

   (i) $(D^6 - 1)y = 0$

Ans. $y = c_1 e^x + c_2 e^{-x} + e^{-\frac{x}{2}} [c_3 \cos(x\frac{\sqrt{3}}{2}) + c_4 \sin(x\frac{\sqrt{3}}{2})] + e^{\frac{x}{2}} [c_5 \cos(x\frac{\sqrt{3}}{2}) + c_6 \sin(x\frac{\sqrt{3}}{2})]$

   (ii) $(D^2 - 1)y = e^x(1 + x^2)$,   Ans. $y = c_1 e^x + c_2 e^{-x} + \frac{1}{12} e^x(2x^3 - 3x^2 + 9x)$

   (iii) $(x^2 D^2 - 3xD + 5)y = \sin(\log x)$

Ans. $y = x^2[c_1 \cos(\log x) + c_2 \sin(\log x)] + \frac{1}{8}[\cos(\log x) + \sin(\log x)]$

2. Find the P.I. of the following differential equations

   (i) $(x^2 D^2 + 3x D + 1)y = \frac{1}{(1-x)^2}$ Ans. $\frac{1}{x} \log[\frac{x}{1-x}]$

   (ii) $(xD + 1)y = \frac{1}{x}$ Ans. $\frac{\log x}{x}$

   (iii) $(x^2 D^2 + xD + 1)y = \log x \sin(\log x)$ Ans. $\frac{1}{4} \log x \sin(\log x) - \frac{1}{4}(\log x)^2 \cos(\log x)$

3. Apply the method of variation of parameters to solve

   (i) $y_2 + a^2 y = \csc ax$

Ans. $C_1 \cos ax + C_2 \sin ax - \frac{x}{a} \cos ax + \frac{1}{a^2} \sin ax \log(\sin ax)$

   (ii) $y_2 + a^2 y = \cot ax$

Ans. $C_1 \cos ax + C_2 \sin ax - \frac{1}{a^2} \cos ax \log(\sec ax + \tan ax)$

   (iii) $y_2 - y = \frac{2}{1+e^x}$

Ans. $y = c_1 e^x + c_2 e^{-x} - 1 - x e^x + (e^x - e^{-x}) \log(1 + e^x)$

4. Solve the following differential equation

   (i) $(x + 1)^2 y_2 - 3(x + 1)y_1 + 4y = x^2$

Ans. $y = (x + 1)^2[c_1 + c_2 \log(x + 1)] + \frac{1}{2}(x + 1)^2[\log(x + 1)]^2 - 2x - \frac{7}{4}$

   (ii) $(2x + 5)^2 y_2 - 6(2x + 5)y_1 + 8y = 0$

Ans. $y = (2x + 5)^2[c_1 (2x + 5)^{\sqrt{2}} + c_2(2x + 5)^{-\sqrt{2}}]$